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Frequency Domain Based Friction Compensation - Industrial Application to Transmission Electron Microscopes -

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Abstract— **Friction is a performance limiting factor in many industrial motion systems. Correct compensation or control of friction and other nonlinearities is generally difficult. Apart from the complex nature of friction, compensation of even the most basic type of friction, Coulomb friction, is non trivial. Most available tuning methods rely on time domain data and are often unable to distinguish between nonlinear effects of friction and that of for example linear viscous damping. Furthermore, the sensitivity of time domain data to the influence of friction is too low for correct tuning in many of the high precision motion applications currently used in industry. In this paper a frequency domain method is introduced that allows fast and high accuracy tuning of controller parameters when the closed loop system is subject to nonlinear influences. This methodology is applied to optimally compensate friction in a high precision motion stage of a transmission electron microscope. Theoretical and experimental results are presented and related to time domain performance to illustrate the advantage of frequency domain tuning over time domain tuning.**

I. INTRODUCTION

Many high end industrial motion systems suffer from performance limitations due to the effects of friction. As the presence of friction is sometimes unavoidable, or even necessary, the application of friction compensating techniques is required. A common way to deal with friction is the application of (Coulomb) friction feed forward which is often tuned based on the tracking error when the closed loop system is subject to a symmetric setpoint, i.e. moving back and forth. This method has three distinct disadvantages that this paper aims to solve:

1) the effects of Coulomb friction and viscous damping are not independent in the time domain,
2) tuning Coulomb friction feed forward to optimally approximate the more complex, true nonlinear dynamics, is nontrivial in the time domain,
3) the detection sensitivity of time domain analysis to the effects of friction is limited.

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To cope with these disadvantages, a frequency domain based tuning method is introduced in this paper. This method possesses improved sensitivity compared to time domain analysis and allows to distinguish between different types of (non)linear effects. The methodology introduced in this paper allows for tuning of both feedback and feed forward controllers and is not restricted to a specific type of nonlinearity. However, the discussion is limited to tuning Coulomb friction feed forward to illustrate the ability of the algorithms to the compensation of strong nonlinearities in a setting that is applicable in industry.

This paper applies frequency domain analysis of nonlinear systems to optimally tune controller systems that possess nonlinear behavior. Various approaches to the frequency domain analysis of nonlinear systems exist [1], [5], [6], [11], [12]. The results presented in the sequel rely on the quantification of nonlinear effects by measuring energy in the output spectrum at harmonics of the input frequency, when the system is subject to a sinusoidal input signal. This representation of nonlinear effects in the frequency domain is captured by the higher order sinusoidal input describing functions [2], [3], [4], [8], [10], which describe the systems response (gain and phase) at harmonics of the base frequency of a sinusoidal input signal. In [7], [9] recent results concerning frequency domain tuning of Coulomb friction feed forward are presented. This paper extends these results and shows their application in industry. Recent results and related software downloads are available at the website of the author[1].

The paper is structured as follows. Section II introduces the required preliminaries. In Section III the theoretical framework required to tune Coulomb friction feed forward in the frequency domain is presented and adapted for application in practice. Section IV presents the application of this methodology to an industrial high precision motion stage of a transmission electron microscope and relates performance in the time domain to the frequency domain performance measure. Finally, Section V presents conclusions and future research.

II. NOMENCLATURE AND PRELIMINARIES

This section briefly discusses the type of nonlinear systems for which the results presented in this paper are valid. Moreover, the Higher Order Sinusoidal Input Describing Functions (HOSIDF) are introduced, which are used in the sequel to quantify nonlinear effects in the frequency domain.

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The analysis in this paper is valid for uniformly convergent, time invariant nonlinear systems [5]. Convergent systems have a unique limit solution corresponding to a certain continuous input. If this input is periodic with period time T_0, the output is periodic with the same period time. Note that when considering dynamical systems in closed loop in the sequel, the closed loop system is required to be convergent rather than the plant itself.

The response of a uniformly convergent, time invariant nonlinear system to a sinusoidal input is described using the higher order sinusoidal input describing functions. It is composed of harmonics of the input frequency and given by:

$$y(t) = \sum_{k=0}^{K} |\mathfrak{H}_k(\omega_0, \gamma)| \gamma^k \cos\left(k(\omega_0 t + \varphi_0) + \angle \mathfrak{H}_k(\omega_0, \gamma)\right) \quad (1)$$

where $\mathfrak{H}_k(\omega_0, \gamma) \in \mathbb{C}$ is the k$^{\text{th}}$ order HOSIDF which describes the response (gain and phase) at the k$^{\text{th}}$ harmonic of the base frequency ω_0 of a sinusoidal input signal. This definition of the HOSIDF is slightly different from the one used in [3] and is formalized below.

Definition 1 ($\mathfrak{H}_k(\omega, \gamma)$: HOSIDF):
Consider a uniformly convergent, time invariant nonlinear system and let the input be a one-tone input signal $u(t) = \gamma \cos(\omega_0 t + \varphi_0)$. Define the systems output $y(t)$ and corresponding Fourier transforms of the input and output $\mathscr{U}(\omega)$, $\mathscr{Y}(\omega) \in \mathbb{C}$. Then, the k$^{\text{th}}$ higher order sinusoidal input describing function $\mathfrak{H}_k(\omega_0, \gamma) \in \mathbb{C}$, $k = 0, 1, 2, \ldots$ is defined as:

$$\mathfrak{H}_k(\omega_0, \gamma) = \frac{\mathscr{Y}(k\omega_0)}{\mathscr{U}^k(\omega_0)}. \quad (2)$$

Next, the theoretical framework required to tune Coulomb friction feed forward in the frequency domain is presented and extended for application to noisy measurement data in practice.

III. Frequency Domain Based Friction Compensation

Consider an exponentially, uniformly convergent, time invariant closed loop system as depicted in Figure 1. The plant is a dynamical system subject to Coulomb friction in closed loop with a stabilizing controller C. The input and output of the system are $u(t)$ and $y(t)$ and the system is subject to a feed forward that aims to compensate the Coulomb friction in the plant. In this section a frequency domain based methodology is introduced to optimally tune this feed forward.

A. I/O Linearization in the Frequency Domain

In order to motivate the methodology introduced in this paper, first consider a linear time invariant system. A key property of LTI systems is that when such system is subject to an input signal with spectral content at frequency lines $f_k \in \mathbb{F}_{in}$, the output spectrum will contain the same spectral lines, i.e. $\mathbb{F}_{out} = \mathbb{F}_{in}$. However, for nonlinear systems this property fails. To quantify this difference, consider an exponentially, uniformly convergent, time invariant system, subject to the following sinusoidal input signal:

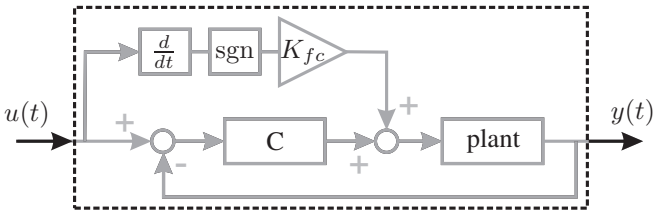

Fig. 1. Exponentially, uniformly convergent, time invariant, closed loop system subject to Coulomb friction feed forward.

$$u(t) = \gamma \cos(\omega_0 t + \varphi_0) \quad (3)$$

with input frequency $\omega_0 \in \mathbb{R}_{>0}$ and amplitude and phase $\gamma, \varphi_0 \in \mathbb{R}$.

The corresponding steady state output signal is described using the higher order sinusoidal input describing functions by (1). Hence, the steady state output spectrum $\mathscr{Y}(\omega)$ of an exponentially, uniformly convergent, time invariant system will only contain harmonics of the input frequency ω_0 and a possible DC value.

As opposed to nonlinear systems, LTI systems do not change the spectral content (lines) of their input and no harmonics of the input frequency are present in the output spectrum when an LTI system is subject to (3). Hence, tuning K_{fc} in Figure 1 to optimally compensate for Coulomb friction, is equivalent to linearizing the input-output (i/o) behavior of the closed loop system or more formally stated:

Definition 2 (Optimal i/o linearization): The system depicted in Figure 1 is called optimally linearized from input to output if it is subject to (3) and K_{fc} is selected such that:

$$\boxed{K_{fc}^{\star} = \underset{K_{fc} \in \mathbb{R}_{\geq 0}}{\operatorname{argmin}} \frac{1}{K-1} \frac{\sum_{k=2}^{K} |\mathscr{Y}(k\omega_0)|}{|\mathscr{Y}(\omega_0)|}} \quad (4)$$

where the sum of absolute values is used rather than the sum of squares as the cost function represents an average measure of nonlinear effects and no additional sensitivity to the size of the magnitudes is required. The cost function is normalized with respect to the number of harmonic lines $K - 1$.

B. Application in Practice

When applying Definition 2 in practice, the optimality condition (4) suffers from the presence of stochastic disturbances. This requires Definition 2 to be adapted for application to noisy experimental data.

Consider an exponentially, uniformly convergent, time invariant system subject to the following experiment: the system is excited for N periods of the periodic signal (3) after transient effects have vanished. This yields N output spectra $\mathscr{Y}_n(\omega)$, with frequency resolution ω_0. Next, consider only spectral lines that correspond to harmonics of the input frequency $\mathscr{Y}_n(k\omega_0)$, $k = 0, 1, \ldots, K$. For this series of

Description	Symbol	Definition
Expected value	$\bar{\mathscr{Y}}(k\omega_0)$	$\frac{1}{N}\sum_{n=1}^{N}\mathscr{Y}_n(k\omega_0)$
Variance on the average	$\sigma^2_{\bar{\mathscr{Y}}}(k\omega_0)$	$\frac{1}{N^2-N}\sum_{n=1}^{N}\left(\mathscr{Y}_n(k\omega_0)-\bar{\mathscr{Y}}(k\omega_0)\right)^2$
Average of the variance	$\bar{\sigma}^2_{\mathscr{Y}}$	$\frac{1}{K-1}\sum_{k=1}^{K}\sigma^2_{\mathscr{Y}}(k\omega_0)$
Variance on the variance	ς^2_σ	$\frac{1}{K-1}\sum_{k=1}^{K}\left(\sigma_{\mathscr{Y}}(k\omega_0)-\bar{\sigma}_{\mathscr{Y}}\right)^2$

TABLE I

STATISTICAL PROPERTIES OF THE EXPERIMENTAL RESULTS.

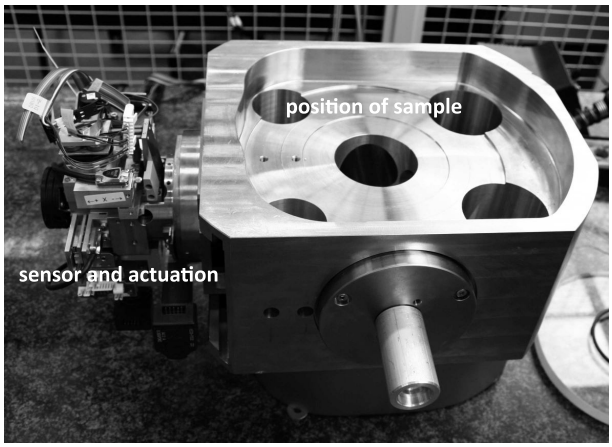

Fig. 2. High precision transmission electron microscope motion stage.

experiments, the statistical properties summarized in Table I are computed. These properties yield information about the level of harmonics and distortions in the output spectrum, which is required to select the relevant harmonic lines for the optimization procedure.

Next, consider the following extension of Definition 2, which is adapted for practical application to noisy measurement data.

Definition 3 (Practical optimal i/o linearization): The system depicted in Figure 1 is called practically, optimally linearized from input to output if it is subject to (3) and K_{fc} is selected such that:

$$K^\star_{fc} = \underset{K_{fc}\in\mathbb{R}_{\geq 0}}{\operatorname{argmin}} \frac{1}{N_{\mathbb{K}}} \frac{\sum_{k\in\mathbb{K}} \left|\mathtt{E}\{\mathscr{Y}(k\omega_0)\}\right|}{\left|\mathtt{E}\{\mathscr{Y}(\omega_0)\}\right|}$$
$$\mathbb{K} = \left\{k\in\mathbb{N}_{\geq 2} \;\middle|\; \left|\mathtt{E}\{\mathscr{Y}(k\omega_0)\}\right| > \varepsilon_k\right\} \tag{5}$$

where the expected value $\mathtt{E}\{\cdot\}$ is computed according to Table I. The cost function is normalized with respect to the number of relevant harmonic lines $N_{\mathbb{K}}$ and $\varepsilon_k \in \mathbb{R}_{\geq 0}$ is selected such that the harmonic lines included in the sum are sufficiently far above the noise level.

To obtain the bound ε_k in (5), the variance at each individual harmonic line is evaluated and combined with the overall variance on the noise level. This allows to evaluate the validity of the measured harmonic components, using the following frequency dependent bound in Definition 3:

$$\varepsilon_k = \sigma_{\bar{\mathscr{Y}}}(k\omega_0) + 2\varsigma_\sigma \tag{6}$$

Next, the framework introduced in this section will be used to optimally apply friction compensation in an industrial high precision motion stage of a transmission electron microscope.

IV. FRICTION COMPENSATION IN INDUSTRIAL TEM MOTION STAGES

The results presented in this section demonstrate the application of the preceding theory to an industrial high precision motion stage. The high precision motion stage of a Transmission Electron Microscope (TEM) is used as it provides a realistic industrial systems which performance is limited by friction. The TEM motion stage is introduced in more detail in the first part of this section. Next, the main problem is stated and finally, the frequency domain tuning method introduced in Section III-B is applied and the results are related to the time domain performance of the system.

A. Set-Up

The methodology presented in this paper is applied to the motion stage of a Transmission Electron Microscope (Figure 2). Performance requirements on such motion stages are high as they position the sample in the electron microscope. The accuracy and speed of the TEM motion stage therefore determine how accurate and fast the area of interest can be moved into view. Apart from speed and accuracy many applications require smooth motion as well. Combining these requirements with a system that operates in high vacuum and that requires the system to be at complete standstill during long term image acquisition, yields a control loop requiring optimal friction compensation to achieve the required performance.

Figure 2 depicts a TEM motion stage in a laboratory setting. The stage is used as a SISO system driven by a Maxon DC motor, with the motor voltage as input and the position of the stage as an output. It is controlled by a Bosch Rexroth NYCe4000 controller which enables automation of the experiments and is used for data acquisition as well.

B. Problem Statement

In the TEM motion stage, friction becomes a dominant performance limiting factor during high accuracy point to point motion and slow movement of the stage. Many industrial applications use a high gain (proportional) feedback to cope with the performance limiting effects of friction. The main downside of this approach is that the high gain feedback is only really required where friction is dominant and might not be required when the system is in slip mode. The accuracy required in the TEM stage would, however, need a loop gain that cannot result in a stable loop, using P or PD control only.

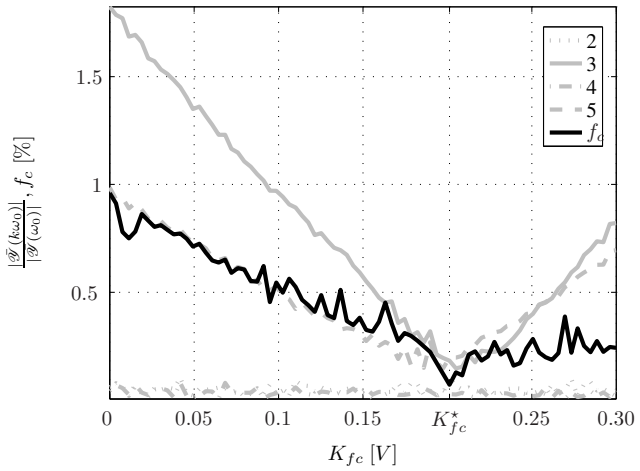

Fig. 3. OPTIMIZATION Average amplitude at the first four harmonic lines $\mathscr{Y}(k\omega_0)$ in the output spectrum, relative to the amplitude at the excitation frequency and cost function $f_d(K_{fc})$ as in Definition 3 (low gain feedback $K_p = 5 \cdot 10^6$).

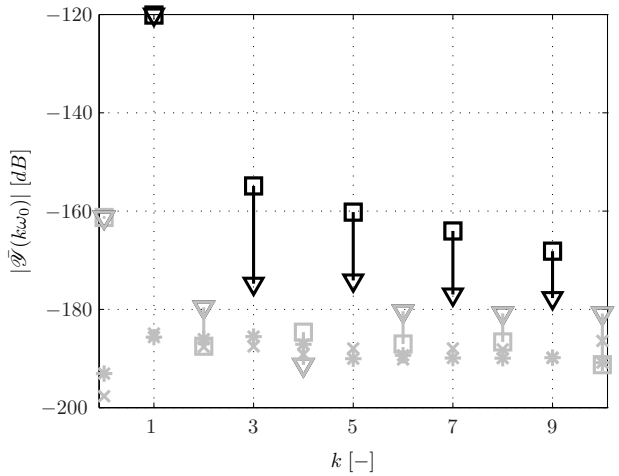

Fig. 4. OUTPUT SPECTRUM Average amplitude at harmonic lines in the output spectrum (low gain feedback $K_p = 5 \cdot 10^6$).
No feed forward: □ mean, ∗ standard deviation.
Optimal i/o linearization ($K_{fc} = K^\star_{fc}$): △ mean, × standard deviation.

Primarily, the issue discussed above is related to the fact that this type of linear feedback control is not properly equipped to cope with (strong) nonlinearities. Although the application of more advanced (nonlinear) feedback controllers is possible, the application of feed forward is to be preferred for two reasons. First, as opposed to feedback, feed forward does not compromise the stability of the closed loop system. Second, many industrial controllers have Coulomb friction feed forward available, whereas advanced nonlinear feedback control is often not available.

Next, the theory presented in Section III is applied to optimally tune the Coulomb friction feed forward in a TEM motion stage.

C. Experiment and Feed Forward Tuning

Consider the TEM motion stage in a closed loop setting with a stabilizing proportional controller $C = K_p$ and subject to a feed forward as depicted in Figure 1. To investigate the influence of the feed forward parameter K_{fc} the method introduced in Section III-B is applied by incrementally increasing K_{fc} from 0 to K_{fc}^{max} in M steps. The following experimental scheme is applied:

1) $m = 1$: the experiment series starts with no feed forward, i.e. $K_{fc}^{[1]} = 0$,
2) the system is excited for N periods of the periodic signal (3) after transient effects have vanished, yielding N output spectra $\mathscr{Y}_n^{[m]}(\omega)$,
3) $m = m + 1$: if the maximum feed forward parameter is not reached, increase the feed forward gain: $K_{fc}^{[m+1]} = K_{fc}^{[m]} + \Delta_{K_{fc}}$, with $\Delta_{K_{fc}} = \frac{K_{fc}^{max}}{M}$ and return to step 2.

This procedure yields $M \cdot N$ output spectra $\mathscr{Y}_n^{[m]}(\omega)$, which are analyzed according to Definition 3.

To relate the frequency domain results to time domain performance, a measure of performance in the time domain is required. Since the overall performance is of interest, the maximum rms value of the error is taken over the different periods of excitation for each value of $K_{fc}^{[m]}$:

$$\epsilon^{[m]} = \max_{n \in \mathbb{N}_{\geq 1}} \sqrt{\frac{1}{L} \sum_{L=1}^{L} \left(u_n^{[m]}(t_\ell) - y_n^{[m]}(t_\ell) \right)^2} \qquad (7)$$

with $t_\ell, = \{1, 2, \ldots, L\}$ the sample instances within the n^{th} period of the m^{th} experiment series. The experiment is performed by evaluating K_{fc} in the interval $[0\ 0.3]$ at $M = 80$ equally spaced values, i.e. $\Delta_{K_{fc}} = 0.00375$. Furthermore, the influence of the feedback controller is investigated by performing two series of experiments: one with a low feedback gain ($K_p = 5 \cdot 10^6$) and one with a high feedback gain ($K_p = 2 \cdot 10^7$). In all experiments the reference signal $u(t)$ is a sinusoidal input signal (3) with amplitude $\gamma = 6\ [\mu m]$ and frequency $f_0 = 0.5\ [Hz]$.

D. Results

Figure 3 - 8 show the experimental results for both high and low feedback gain. In Figure 3 the cost function $f_c(K_{fc})$ from Definition 3 is depicted as well as the energy at harmonics, relative to the energy at the excitation frequency. Since friction is an odd nonlinearity the dominant behavior of the odd harmonics is to be expected. As the feed forward gain is increased, a decrease in the energy observed at the relevant harmonics, relative to the energy at the excitation frequency, appears until a minimum is reached. The minimum of the cost function combines the behavior of all harmonic lines and is a measure for the overall linearity of the i/o behavior. This analysis indicates that $K^\star_{fc} = 0.2013$ yields an optimal i/o linearization of the plant using low gain feedback.

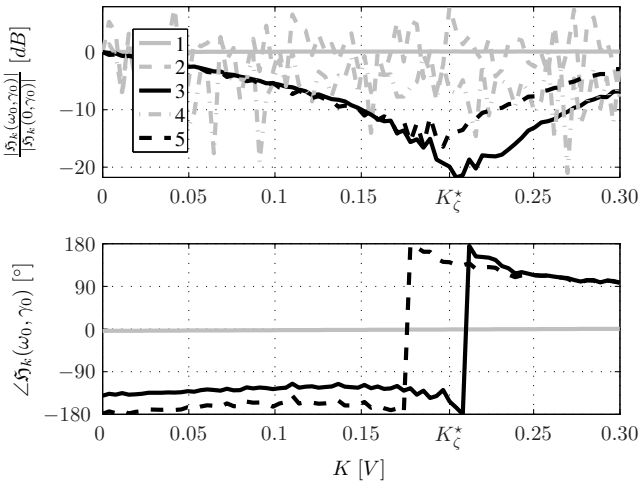

Fig. 5. HOSDIF First five higher order sinusoidal input describing functions at $f_0 = 0.5$ $[Hz], \gamma_0 = 1$ $[\mu m]$ for varying values of K_{fc} (low gain feedback $K_p = 5 \cdot 10^6$).

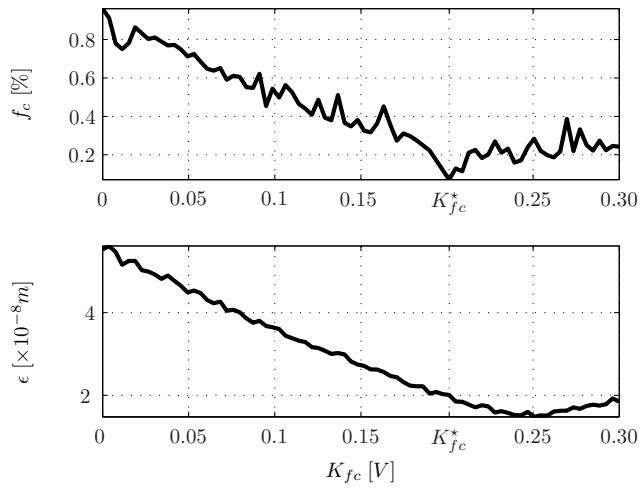

Fig. 6. PERFORMANCE Performance in time and frequency domain. Linearity is measured by the cost function $f_c(K_{fc})$ in the frequency domain and time domain performance as defined in (7) (low gain feedback $K_p = 5 \cdot 10^6$).

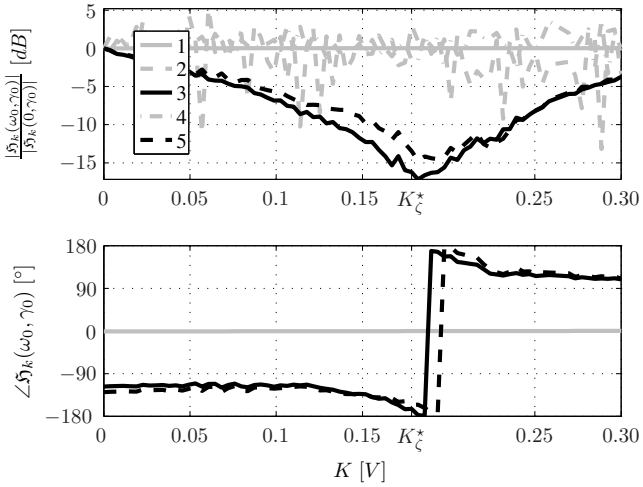

Fig. 7. HOSIDF First five higher order sinusoidal input describing functions at $f_0 = 0.5$ $[Hz], \gamma_0 = 1$ $[\mu m]$ for varying values of K_{fc} (high gain feedback $K_p = 2 \cdot 10^7$).

Fig. 8. PERFORMANCE Performance in time and frequency domain. Linearity is measured by the cost function $f_c(K_{fc})$ in the frequency domain and time domain performance as defined in (7) (high gain feedback $K_p = 2 \cdot 10^7$).

Harmonics

Figure 4 shows the average energy at harmonics of the input frequency both with and without optimally i/o linearizing feed forward. Using an optimal feed forward, for example, decreases the energy at the third harmonic by a factor of 10. The cost function in Figure 3 even drops by a factor of 13, showing less than 7.5% of the energy at harmonics in the situation where correct feed forward is applied. Figure 5 shows the normalized Higher Order Sinusoidal Input Describing Functions (HOSIDF) for varying feed forward gain. Corresponding to the decreasing energy at harmonic lines, the HOSIDFs show a minimum close to $K^\star_{fc}$. Moreover, the HOSIDFs show a strong phase shift around the optimum that can be used to efficiently detect the optimum.

Apples and Oranges

In Figure 6 performance measures in the time and frequency domain are compared. The top figure shows the frequency domain cost function $f_c(K_{fc})$ indicating the optimally i/o linearizing value of the feed forward parameter. The bottom plot, however, depicts a time domain measure of performance as defined in (7). It becomes clear that both measures indicate a different optimal value of the feed forward parameter. This paradoxal behavior is caused by the presence of viscous damping which is not compensated for during the experiment. As the Coulomb friction feed forward gain is the only tunable parameter, the minimal time domain error occurs at a value of K_{fc} where this feed forward compensates for part of the viscous damping as well. At first glance, this appears to yield a better tracking

performance. Note however, that for $K_{fc} > K^{\star}_{fc}$ one is compensating apples with oranges (or in this case damping with friction); the apparent improvement in performance is local and dependent on the excitation signal. It is therefore not an overall performance increase and the true optimal value of the feed forward gain is located at $K^{\star}_{fc}$ after all. To further improve tracking performance, additional feed forward or feedback is required.

Low and High Gain Feedback

Figures 7 - 8 depict the HOSIDFs and performance measures for the same experiment with high gain feedback. As becomes clear, the initial amount of energy at harmonics when using high gain feedback is less than in the low gain experiment. Hence, increasing the feedback gain linearizes the closed loop behavior. Furthermore, the optimal value $K^{\star}_{fc}$ in the high feedback gain experiments, differs from the optimum observed in the low gain experiments.

	$K_p = 5 \cdot 10^6$	$K_p = 2 \cdot 10^7$
$K^{\star}_{fc}$	0.2013	0.1899
$\underset{K_{fc} \in [0\ 0.3]}{\operatorname{argmin}}\ \epsilon$	0.2506	0.2359

TABLE II

EXPERIMENTAL RESULTS

Table II summarizes the numerical results from both experiments. It shows that the optimally i/o linearizing feed forward gains $K^{\star}_{fc}$ for the low and high gain experiment differ by approximately 5%. Comparing Figure 6 and 8 shows that the decrease in energy at harmonic lines in the low gain feedback ($13\times$) is reduced when using high gain feedback ($4\times$). However, the relative difference between the optimally i/o linearizing value of K_{fc} and the apparent optimum observed from the time domain error is 24% in both experiments.

V. CONCLUSIONS AND FUTURE RESEARCH

A frequency domain based method for controller tuning in the presence of nonlinearities is presented. This methodology is generally applicable for linearization of the input-output behavior of a system containing nonlinearities. However, it is initially applied to feed forward tuning for Coulomb friction compensation, to illustrate the practical applicability and ability to cope with strong nonlinearities. The application of the feed forward tuning procedure is successfully demonstrated in practice on an industrial high precision stage of a transmission electron microscope.

The experimental results emphasize the improved sensitivity and accuracy when using frequency rather than time domain data to tune controller parameters in the presence of nonlinearities. Furthermore, comparison between time domain and frequency domain performance shows that, as opposed to time domain analysis, frequency domain analysis allows to distinguish between performance degradation due to friction and damping. Finally, frequency domain analysis yields a clear optimum for the optimal approximation of the true nonlinear dynamics by Coulomb friction feed forward.

Summarizing, the method introduced in this paper is shown to be effective in optimally tuning friction compensation in high end industrial motion systems and possess significant advantages compared to traditional time domain tuning. Future research will focus on automation of tuning procedure and application of the methods to more advanced nonlinear (feed forward) models.

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