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Determining battery parameters by simple algebraic method

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***Abstract*— This paper derives simple and explicit formulas for computing the parameters of the Thevenin's equivalent circuit model for a discharging battery. The general Thevenin's equivalent circuit model has n pairs of parallel resistors and capacitors. The main idea behind the new method is to transform the problem of solving a system of high order polynomial equations into one of solving several linear equations and a single variable nth order polynomial equation, via some change of variables. Experimental and computational results are obtained for 3 types of batteries.**

Keywords: battery model, Thevenin's equivalent circuit, polynomials, linear algebraic equations

I. INTRODUCTION

Dynamic models for batteries are important for analysis, design, and simulation of battery powered electronic systems (e.g., see [1]–[7]). They are also important for characterization of battery performance, life-time estimation, power management, and efficient use of batteries [8]–[12].

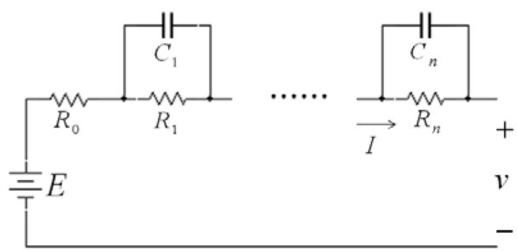

Fig. 1. Thevenin's equivalent model for a discharging battery

Depicted in Fig. 1 is a Thevenin's equivalent circuit model, which has been widely used to model the discharging dynamics of various types of batteries such as lead-acid, lithium-ion (Li-ion), Li-polymer, nickel metal hydride (NiMH), and fuel cells, e.g., see [1]- [12]. Although the parameters in the model depend on many factors [1], [7], [11], [13], such as the state of charge, the load, the temperature and the history of charge/discharge, under certain working condition and over a relatively short period of time, they are assumed to be constants.

The most commonly used model is the circuit with only one pair of parallel resistor and capacitor (RC). For this 1st-order model, the parameters can be easily estimated from the experiment. However, the voltage response by the 1st-order model can be quite different from the response of the real battery, as will be demonstrated with examples. For models with two or more pairs of parallel RCs, there seems to be a lack of systematic methods to identify the parameters, except for using brute force numerical methods, such as minimizing a cost function of the sum (or integral) of squares of the difference between the experimental data and analytical expression [13]. The drawback with such kind of numerical optimization method is that the cost function is highly nonlinear with respect to circuit parameters and there is no guarantee to find the global minimum. Plus, it is a very tricky issue to find proper initial parameters for optimization.

The purpose of this paper is to derive a simple analytical method to identify the parameters for the general case with two or more pairs of parallel RCs.

II. EXPLICIT FORMULAS FOR COMPUTING THE PARAMETERS

We consider the battery model with n pairs of parallel resistors and capacitors, $(R_1, C_1), (R_2, C_2), \cdots, (R_n, C_n)$, as depicted in Fig. 1. For simplicity, this will be called an nth-order model. A common set up to obtain the parameters is to connect the battery to an electronic load which absorbs a constant current I from the battery, see Fig. 2.

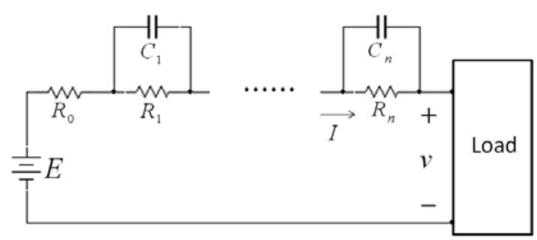

Fig. 2. A battery connected to a load with constant current

Assume that the load is connected at $t = 0$ and all the initial capacitor voltages are 0. Then the first two parameters to be obtained are

$$E = v(0^-), R_0 = (v(0^-) - v(0^+))/I.$$

The other parameters $(R_1, C_1), \cdots, (R_n, C_n)$ have to be evaluated via the time response $v(t)$ over a period of time. The advantage of using a constant current load (over a resistor) is that $v(t)$ can be simply expressed as

$$v(t) = E - R_0 I - \sum_{k=1}^{n} R_k I (1 - e^{-\frac{t}{R_k C_k}})$$

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If only one pair of parallel resistor and capacitor (R_1, C_1) is considered in the model, they can be estimated from the approximate time constant of the experimental response. However, the voltage response of this simple model can be very different from that of the actual battery.

In this paper, we present explicit formulas to compute the parameters for the general nth-order model.

A. Detailed results for the 3rd-order model

For the 3rd-order model,

$$v(t)=E-R_0I-R_1I(1-e^{-\frac{t}{R_1C_1}})-R_2I(1-e^{-\frac{t}{R_2C_2}}) -R_3I(1-e^{-\frac{t}{R_3C_3}}) \tag{1}$$

To find the 6 parameters $(R_k, C_k), k = 1, 2, 3$, we pick 6 equally spaced time instants, $t_k = kT, k = 1, \cdots, 6$. Let

$$d_1 = e^{-\frac{T}{R_1C_1}}, \quad d_2 = e^{-\frac{T}{R_2C_2}}, \quad d_3 = e^{-\frac{T}{R_3C_3}}$$

Then for $k = 1, 2, \cdots, 6$,

$$v(kT)=E-R_0I-R_1I(1-d_1^k)-R_2I(1-d_2^k)-R_3I(1-d_3^k), \quad k = 1, 2, \cdots, 6. \tag{2}$$

Theoretically, the 6 parameters $(R_1, d_1), (R_2, d_2), (R_3, d_3)$ can be determined from the above 6 equations with $v(kT)$ obtained from experiment. With these 6 parameters, C_1, C_2, C_3 are easy to compute.

At first sight, it may seem that the 6 nonlinear equations in (2) are impossible to solve by analytical method, since the last equation has three 7th order terms $R_1d_1^6, R_2d_2^6, R_3d_3^6$. However, after further examination on the structure of the equations, all the solutions can be explicitly obtained with some change of variables.

The main idea of the new method is to define the following variables

$$u_1=d_1+d_2+d_3, \;\; u_2=d_1d_2+d_2d_3+d_3d_1, \;\; u_3=d_1d_2d_3. \tag{3}$$

It turns out that these variables can be solved via a system of linear equations. Then d_1, d_2, d_3 can be obtained by solving a 3rd order single variable polynomial equation with coefficients formed with u_1, u_2, u_3. After that, the computation of $R_k, C_k, k = 1, 2, 3$ is straightforward. The main result is summarized as follows.

Main Result: Given $E, R_0, v(kT), k = 1, 2, \cdots, 6$ from experiment. The parameters $(R_k, C_k), k = 1, 2, 3$ can be computed from the following steps:

1. Compute $b_1 = E - R_0I - v(T)$. For $k = 2, \cdots, 6$, $b_k = v((k-1)T) - v(kT)$.
2. Compute $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} b_3 & -b_2 & b_1 \\ b_4 & -b_3 & b_2 \\ b_5 & -b_4 & b_3 \end{bmatrix}^{-1} \begin{bmatrix} b_4 \\ b_5 \\ b_6 \end{bmatrix}$.
3. Let the roots to $q^3 - 2u_1q^2 + (u_1^2 + u_2)q + (u_3 - u_1u_2) = 0$ be q_1, q_2, q_3. Then
$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$
4. Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ d_1 & d_2 & d_3 \\ d_1^2 & d_2^2 & d_3^2 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Then $R_k = \frac{x_k}{I(1-d_k)}, C_k = -\frac{T}{R_k \ln(d_k)}, k = 1, 2, 3.$

Proof of the main result:

Define

$$x_1 = R_1I(1-d_1), \;\; x_2 = R_2I(1-d_2), \;\; x_3 = R_3I(1-d_3)$$

The 6 equations in (2) can be rewritten as follows:

$$x_1 + x_2 + x_3 = E - R_0I - v(T) \tag{4}$$

$$(1 + d_1)x_1 + (1 + d_2)x_2 + (1 + d_3)x_3 = E - R_0I - v(2T) \tag{5}$$

$$(1 + d_1 + d_1^2)x_1 + (1 + d_2 + d_2^2)x_2 + (1 + d_3 + d_3^2)x_3 = E - R_0I - v(3T) \tag{6}$$

$$\sum_{k=0}^{N} d_1^k x_1 + \sum_{k=0}^{N} d_2^k x_2 + \sum_{k=0}^{N} d_3^k x_3 = E-R_0I-v((N+1)T) \quad N = 3, 4, 5 \tag{7}$$

By keeping (4), subtracting (4) from (5), subtracting (5) from (6), and so on, we obtain

$$x_1 + x_2 + x_3 = E - R_0I - v(T) =: b_1 \tag{8}$$
$$d_1x_1 + d_2x_2 + d_3x_3 = v(T) - v(2T) =: b_2 \tag{9}$$
$$d_1^2x_1 + d_2^2x_2 + d_3^2x_3 = v(2T) - v(3T) =: b_3 \tag{10}$$
$$d_1^3x_1 + d_2^3x_2 + d_3^3x_3 = v(3T) - v(4T) =: b_4 \tag{11}$$
$$d_1^4x_1 + d_2^4x_2 + d_3^4x_3 = v(4T) - v(5T) =: b_5 \tag{12}$$
$$d_1^5x_1 + d_2^5x_2 + d_3^5x_3 = v(5T) - v(6T) =: b_6 \tag{13}$$

The above 6 equations still have high order polynomials (in the variables $d_1, d_2, d_3, x_1, x_2, x_3$) but the structure is very clear. With further change of variables, the complexity will be reduced. Let u_1, u_2, u_3 be defined as in (3).

In what follows, we provide the key step to derive 3 linear equations for u_1, u_2, u_3. Then it is straightforward to find d_1, d_2, d_3, x_1, x_2, x_3 and the parameters $(R_k, C_k), k = 1, 2, 3$. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ d_1 & d_2 & d_3 \\ d_1^2 & d_2^2 & d_3^2 \end{bmatrix}$$

From (8)-(10), we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{14}$$

Applying this to (11), we have

$$d_1^3x_1+d_2^3x_2+d_3^3x_3 = \begin{bmatrix} d_1^3 & d_2^3 & d_3^3 \end{bmatrix} A^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = b_4 \tag{15}$$

It can be verified via straightforward computation that

$$\begin{bmatrix} d_1^3 & d_2^3 & d_3^3 \end{bmatrix} = \begin{bmatrix} u_3 & -u_2 & u_1 \end{bmatrix} A, \tag{16}$$

where u_1, u_2, u_3 are defined in (3). Thus

$$\begin{bmatrix} d_1^3 & d_2^3 & d_3^3 \end{bmatrix} A^{-1} = \begin{bmatrix} u_3 & -u_2 & u_1 \end{bmatrix} \tag{17}$$

It follows from (15) that

$$b_1 u_3 - b_2 u_2 + b_3 u_1 = b_4 \tag{18}$$

Similarly, from (9)-(11), we have

$$\begin{bmatrix} d_1 x_1 \\ d_2 x_2 \\ d_3 x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Applying this to (12), and using (17), we obtain

$$b_2 u_3 - b_3 u_2 + b_4 u_1 = b_5 \tag{19}$$

From (10)-(12), we have

$$\begin{bmatrix} d_1^2 x_1 \\ d_2^2 x_2 \\ d_3^3 x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

Applying this to (13), and using (17), we obtain

$$b_3 u_3 - b_4 u_2 + b_5 u_1 = b_6 \tag{20}$$

Combine (18), (19), (20), we have

$$\begin{bmatrix} b_3 & -b_2 & b_1 \\ b_4 & -b_3 & b_2 \\ b_5 & -b_4 & b_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Thus u_1, u_2, u_3 can be solved as

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} b_3 & -b_2 & b_1 \\ b_4 & -b_3 & b_2 \\ b_5 & -b_4 & b_3 \end{bmatrix}^{-1} \begin{bmatrix} b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

With $u_1 = d_1 + d_2 + d_3, u_2 = d_1 d_2 + d_2 d_3 + d_3 d_1, u_3 = d_1 d_2 d_3$ computed, we can find d_1, d_2, d_3 by the following procedure. Let $p = d_1 d_2, q = d_1 + d_2$, then $d_3 = u_1 - q$ and $u_3 = p(u_1 - q)$, $u_2 = p + q(u_1 - q)$. By substituting $p = u_2 - u_1 q + q^2$ into $u_3 = p(u_1 - q)$, we obtain a 3rd order equation for q:

$$q^3 - 2u_1 q^2 + (u_1^2 + u_2)q + (u_3 - u_1 u_2) = 0 \tag{21}$$

Let the three roots be q_1, q_2, q_3. Then $q = d_1 + d_2$ is one of the roots. Here we notice that the relationship between (d_1, d_2, d_3) and (u_1, u_2, u_3) is symmetric. This means that the values of u_1, u_2, u_3 are the same if d_1, d_2, d_3 are exchanged in any manner. If we let $p = d_2 d_3, q = d_2 + d_3$, or let $p = d_1 d_3, q = d_1 + d_3$, we obtain the same equation (21). This implies that both $q = d_2 + d_3$ and $q = d_1 + d_3$ satisfy (21). Therefore, (21) must have 3 positive roots which are $d_1 + d_2, d_2 + d_3$ and $d_1 + d_3$, respectively, i.e.,

$$d_2 + d_3 = q_1, \quad d_1 + d_3 = q_2, \quad d_1 + d_2 = q_3$$

From the three roots q_1, q_2, q_3, we can solve for

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

With d_1, d_2, d_3 solved, we obtain x_1, x_2, x_3 from (14). And finally,

$$R_k = x_k/(I(1 - d_k)), \ C_k = -T/(R_k \ln(d_k)), \ k = 1, 2, 3.$$

B. Algorithm for the 2nd-order model

For the case with two capacitors, we need to obtain the voltage response $v(t)$ at 4 equally spaced time instant: $v(kT), k = 1, 2, 3, 4$. The new variables are defined as

$$u_1 = d_1 + d_2, \quad u_2 = d_1 d_2.$$

The parameters $(R_k, C_k), k = 1, 2$ can be computed from the following steps:

1. Compute $b_1 = E - R_0 I - v(T)$. For $k = 2, 3, 4$, $b_k = v((k-1)T) - v(kT)$.
2. Compute $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} b_2 & -b_1 \\ b_3 & -b_2 \end{bmatrix}^{-1} \begin{bmatrix} b_3 \\ b_4 \end{bmatrix}$.
3. The roots to the second order equation $d^2 - u_1 d + u_2 = 0$ will be d_1, d_2.
4. Let $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ d_1 & d_2 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Then

$$R_k = \frac{x_k}{I(1 - d_k)}, C_k = -\frac{T}{R_k \ln(d_k)}, \quad k = 1, 2.$$

C. Algorithm for higher-order models

For the case with n pairs of parallel RCs, we need to obtain the voltage response $v(t)$ at $2n$ equally spaced time instants: $v(kT), k = 1, \cdots, 2n$. Define the new variables:

$$u_1 = d_1 + d_2 + \cdots + d_n \tag{22}$$

$$u_2 = \sum d_{k_1} d_{k_2}, \quad k_1 \neq k_2, \quad k_1, k_2 \leq n \tag{23}$$

$$\vdots$$

$$u_{n-1} = \sum d_{k_1} d_{k_2} \cdots d_{k_{n-1}}, \quad k_1, \cdots, k_{n-1} \text{ are distinct}, \quad k_i \leq n \tag{24}$$

$$u_n = d_1 d_2 \cdots d_n \tag{25}$$

The parameters $(R_k, C_k), k = 1, 2, \cdots, n$ can be computed from the following steps:

1. Compute $b_1 = E - R_0 I - v(T)$. For $k = 2, \cdots, 2n$, $b_k = v((k-1)T) - v(kT)$.
2. Compute

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} b_n & -b_{n-1} & \cdots & (-1)^{n+1} b_1 \\ b_{n+1} & -b_n & \cdots & (-1)^{n+1} b_2 \\ \vdots & & & \\ b_{2n-1} & -b_{2n-2} & \cdots & (-1)^{n+1} b_n \end{bmatrix}^{-1} \begin{bmatrix} b_{n+1} \\ b_{n+2} \\ \vdots \\ b_{2n} \end{bmatrix}$$

3. Solve $d_1, \cdots, d_n$ from $u_1, \cdots, u_n$ by forming an nth order polynomial from (22)-(25):

$$f(q) = q^n + c_1 q^{n-1} + \cdots + c_n = 0. \tag{26}$$

   The roots are $q_j = (\Sigma_{k=1}^n d_k) - d_j, j = 1, 2, \cdots, n$. Then d_j's can be obtained from q_j's by solving a system of linear equations. The procedure in this step can be implemented with the following Matlab code (where $u(i)$ stands for u_i):

```
c=[ 1 -u(1) u(2)]
for i=3:n
c=conv(c,[1 -u(1)])+[zeros(1,i) u(i)];
end
```

```
q=roots(c);
d=inv(ones(n,n)-eye(n))*q;
```

The first 4 lines form the coefficients of the polynomial $f(q)$ with a recursive procedure, where "conv" computes the coefficients of the product of two polynomials.

4. Let

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ d_1 & d_2 & \cdots & d_n \\ \vdots & & & \\ d_1^{n-1} & d_2^{n-1} & \cdots & d_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Then for $k = 1, 2, \cdots, n$,

$$R_k = \frac{x_k}{I(1-d_k)}, C_k = -\frac{T}{R_k \ln(d_k)}.$$

III. Experimental examples and computational results

We obtained experimental data for 3 types of batteries: lead acid (6V), NiMH (7.2V) and Li-polymer (11.1V). Terminal voltage responses under constant current load were recorded via a 16-digit data acquisition (DAQ) device. All the experiment is conducted under room temperature.

A. Data collection

The sampling period of the DAQ is $T_d = 0.01$ second. The input voltage range is -10V to 10V. Thus the resolution is $20/2^{16} = 3.0518^{-4}V$. We also measured the current via a 0.1Ω resistor and an operational amplifier to see its transience after the load is turned on and also to see the ripples at steady state so that we can make proper adjustment to the terminal voltage. No digital or analog filter is used to process the data/signals.

We use the data obtained from a 6V lead-acid battery rated 13Ah to demonstrate the procedure. Fig. 3 shows the

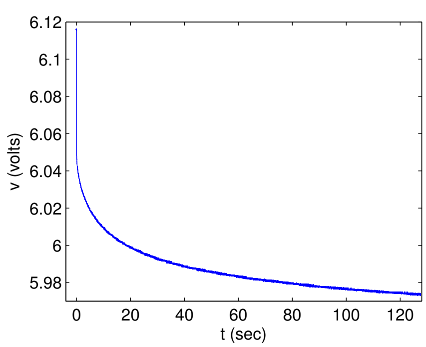

Fig. 3. Voltage response of a lead-acid battery to 1A load, 60% SOC

voltage response of the battery to a 1A load. The initial voltage is 6.117V (corresponding to about 60% residual capacity or state of charge (SOC)). Fig. 4 is plotted to show the voltage and the current around $t = 0$ and at steady state. The plot at lower-left shows that the electronic load generates a current that jumps from 0A to 1A within one sampling period. Since the current load can be turned on

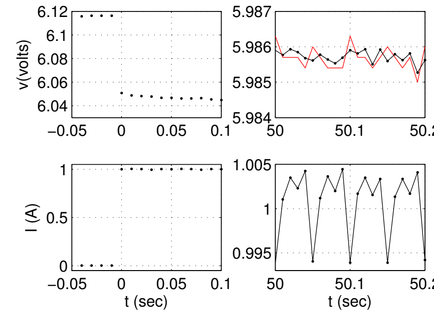

Fig. 4. Voltage and current around $t = 0s$ and $t = 50s$.

any instant within a sampling period of the DAQ, the value of the current at the first sampling instant (after the load is turned on) varies between 0 and the set value. Thus it may also take two sampling steps for the current to reach the set value, which has been observed in some of our tests. But mostly the set value of the current is reached in one step, as depicted in Fig. 4. The time 0 is chosen as the sampling instant when the current first reaches the set value.

The upper-left plot in Fig. 4 shows the initial voltage drop. From these initial values, we can obtain $E = v(-T_d)$ ($= 6.117V$ in this case), and $R_0 = (v(-T_d) - v(0))/I$ ($= 0.0656\Omega$ in this case). If the set value of the current is reached in two steps, we take $E = v(-2T_d)$, and $R_0 = (v(-2T_d) - v(0))/I$.

The two plots to the right show the ripples of the current at steady state and the spikes of the voltage (light-colored). We see that the current oscillates around 1A with a period of 5 sampling periods. Due to the relatively large time constants R_kC_k for the parallel resistors and capacitors, the small current ripples mostly affect the voltage across the inner resistance R_0. Thus we can make slight corrections to the battery voltage by replacing $v(jT_d)$ with $v(jT_d) - R_0 * (1 - I(jT_d))$ for all $j > 0$. The darker curve in the upper-right plot shows the corrected voltage. The spikes are reduced. The corrected voltage will be used for estimating the parameters.

B. Models for a lead-acid battery

A lead-acid battery rated 6V, 13Ah is used for the tests.

The voltage response is collected under a load of 1A. The open circuit voltage is 6.117V and $R_0 = 0.0656\Omega$. The voltage response is plotted in Fig. 3.

In this section, we use the explicit formula in Section II to estimate the parameters for the model of different order. For the $nth-$order model, we need the voltage $v(kT), k = 1, \cdots, 2n$, where $T = MT_d$ for a certain integer M. Due to the noises and other non-idealities, the resulting parameters (R_k, C_k) will be dependant on M. We may let M vary in a proper range and choose the one which yields the least RMSE. To further reduce the effect of noises, we may use the average around $v(kT)$, e.g., the average of $v(kT - mT_d)$, $\cdots, v(kT), \cdots, v(kT + mT_d)$ for a certain integer m. The best m may depend on the noise pattern of the experimental setup. For the data we have collected, the best m is between 0 and 5.

For the 1st-order model, the minimal RMSE obtained is 4.96×10^{-3}. The resulting parameters are $R_1 = 0.071\Omega, C_1 = 205.1F$. The response by the model and that from experiment are compared in Fig. 5, where the fuzzy light-colored curve is from experiment and the smooth darker curve is by the 1st-order model.

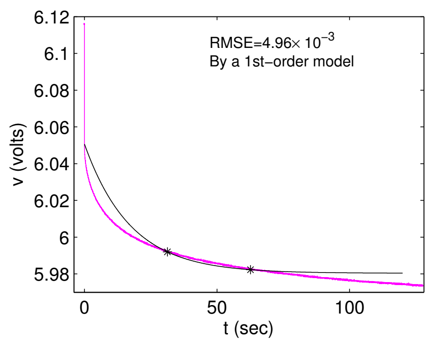

Fig. 5. Matching the response with a 1st-order model, a lead-acid battery, 1A load, 60% SOC

For the 2nd-order model, the minimal RMSE obtained is 9.14×10^{-4}. The parameters are

$$R_1 = 0.0334\Omega, R_2 = 0.044\Omega, C_1 = 84.4F, C_2 = 834F.$$

The response by the 2nd-order model (darker, smooth) and that from experiment (light-colored, fuzzy) are compared in Fig. 6. The stars "*" mark the points $(t_i, v(t_i)), t_i = T, 2T, 3T, 4T$ that are used to solve for the parameters. In subsequent figures, we use the same symbols and notations to compare voltage responses.

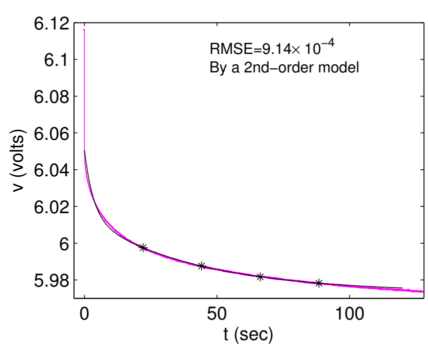

Fig. 6. Matching the response with a 2nd-order model, a lead-acid battery, 1A load, 60% SOC

For the 3rd-order model, the minimal RMSE obtained is 4.62×10^{-4}. The parameters are

$$R_1 = 0.0187\Omega,\ R_2 = 0.0260\Omega,\ R_3 = 0.0365\Omega,$$
$$C_1 = 79.7F,\ C_2 = 358.9F,\ C_3 = 1684.2F.$$

The response by the 3rd-order model and that from experiment are plotted in Fig. 7 for comparison.

From these three figures and the RMSE values, we see that the 1st-order model matches the experimental response

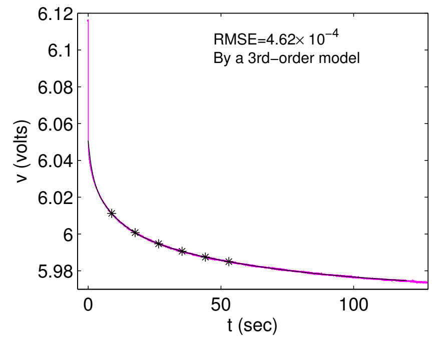

Fig. 7. Matching the response with a 3rd-order model, a lead-acid battery, 1A load, 60% SOC

poorly. The 2nd-order model shows significant improvement. However, there are still visible difference in the first interval (between $t = 0$ and the first "*"). The 3rd-order model matches the experimental response almost perfectly. Recall that the resolution of the DAQ is $3.0518^{-4}V$. The RMSE by the 3rd-order model is not much greater than this.

C. Models for an NiMH battery

The battery under test is a 7.2V (6-cell) NiMH battery rated 5000mAh. The voltage response is measured under a load of 1A. The open circuit voltage is $E = 7.87V$ and $R_0 = 0.0588\Omega$. We use the same procedure as that for the lead-acid battery to obtain the 1st,2nd and the 3rd-order models. For the 1st-order model, the minimal RMSE obtained is 5.1×10^{-3}, with $R_1 = 0.083\Omega, C_1 = 503F$. The response by the model also differs significantly from the experimental response, similarly to that for the lead-acid battery.

For the 2nd-order model, the minimal RMSE obtained is 9.57×10^{-4}. The parameters are

$$R_1 = 0.022\Omega, R_2 = 0.104\Omega, C_1 = 95.7F, C_2 = 1290F$$

The response by the 2nd-order model and that from experiment are plotted in Fig. 8.

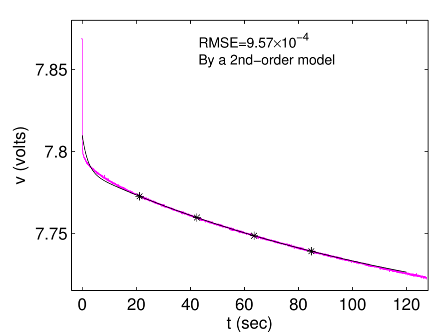

Fig. 8. Matching the response with a 2nd-order model, an NiMH battery, 1A load, 55% SOC

For the 3rd-order model, the minimal RMSE obtained is

2.88×10^{-4}. The parameters are

$$R_1 = 0.0129\Omega, R_2 = 0.0099\Omega, R_3 = 0.1117\Omega,$$
$$C_1 = 8.6F, C_2 = 663F, C_3 = 1350.7F$$

The response by the 3rd-order model and that from experiment are plotted in Fig. 9.

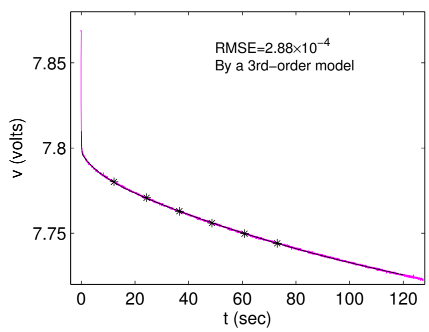

Fig. 9. Matching the response with a 3rd-order model, an NiMH battery, 1A load 55% SOC

As with the lead-acid battery, the 1st-order model does a very poor job matching the terminal voltage, the 2nd-order model shows significant improvement, while the 3rd-order model matches the experimental response almost perfectly.

D. Models for a Li-Polymer battery

The battery under test is a 11.1V (3-cell) Li-polymer battery rated 5000mAh. The voltage response was obtained under a load of 2A. The open circuit voltage was 11.51V and $R_0 = 0.0483\Omega$. For the 2nd-order model, the minimal RMSE obtained is 5.04×10^{-4}. The parameters are

$$R_1 = 0.0097\Omega, R_2 = 0.0220\Omega, C_1 = 571.6F, C_2 = 4051.1F$$

The response by the 2nd-order model and that from experiment are plotted in Fig. 10.

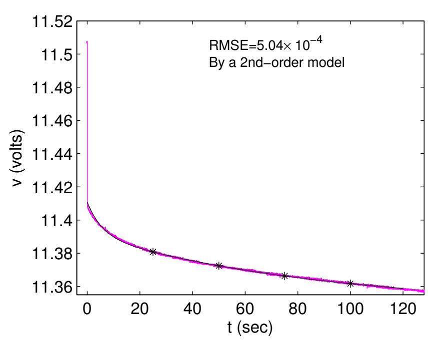

Fig. 10. Matching the response with a 2nd-order model, 2A load, 55% SOC, a Lipo battery

For the 3rd-order model, the minimal RMSE obtained is 4.11×10^{-4}. The parameters are

$$R_1 = 0.0056\Omega, R_2 = 0.0083\Omega, R_3 = 0.0332\Omega,$$
$$C_1 = 638.3F, C_2 = 2124.8F, C_3 = 7866.5F.$$

The response by the 3rd-order model and that from experiment are plotted in Fig. 11.

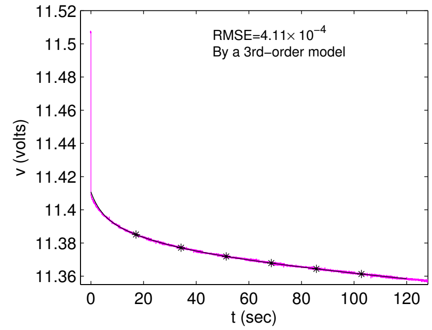

Fig. 11. Matching the response with a 3rd-order model

IV. CONCLUSIONS

We derived simple analytical algorithms to compute the parameters for batteries. Experiment and computation are implemented on 3 different types of batteries. For all these batteries, the computation shows that the 1st-order model does a very poor job matching the experimental responses, the 2nd-order model shows significant improvement and the 3rd-order model can produce a response that matches the experimental data almost perfectly.

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