2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Distance-based Formation Control Using Euclidean Distance Dynamics Matrix: Three-agent Case

Kwang-Kyo Oh[†], *Student Member, IEEE*, and Hyo-Sung Ahn[†], *Member, IEEE*

Abstract— **In this paper, we propose a triangular formation control law based on inter-agent distance information for a group of three single-integrator modeled agents on the plane. Although most of existing distance-based formation control laws have been designed by using the gradient of artificial potential functions, the proposed control law is derived from the time derivative of the Euclidean distance matrix associated with the realization of the agent group. Consequently, if the initial and desired formations are not collinear and the information graph of the group is complete, then the desired formation of the group is globally asymptotically stable with all squared inter-agent distance errors exponentially converging to zero. Furthermore, the proposed control law has a coordination property in the sense that the dynamics of all inter-agent distances are fully decoupled. Simulation results support the effectiveness of the proposed control law, demonstrating the coordination property.**

I. Introduction

Recently, a considerable amount of resources has been focused on stabilizing the formation of mobile agents based on local information [1]–[15]. In such works, depending on available local information for agents, researchers have primarily employed displacement- and distance-based approaches, with the key difference between such approaches in the availability of a common directional sense for agents [15]. Although several effective displacement-based control laws have been proposed [1]–[4], various issues, especially in stability analysis, remain unresolved within the realm of distance-based formation control, due to the complicacy arising from the absence of an available common directional sense. A noticeable work in distance-based formation control has been developed by Krick *et al.* In [5], [6], they have proven the local asymptotic stability of the desired formation of a general single-integrator modeled agent group with a gradient-based control law. Their approach, however, is difficult to be applied to global stability analysis of the desired formation since they have exploited center manifold theory.

As attempts for global stability analysis, researchers have addressed triangular formation control of three-agent group based on inter-agent distance information. For instance, Cao *et al.* have analyzed the exponential stability of triangular formation of a three-agent group by applying Lyapunov's direct method to the edge dynamics of the group [13], [14]. Dörfler and Francis have introduced the link dynamics of a three-agent group and proposed a differential geometric approach for stability analysis [7].

[†]School of Mechatronics, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro, Buk-gu, Gwangju, 500-712 Korea. E-mail: {kkoh, hyosung}@gist.ac.kr

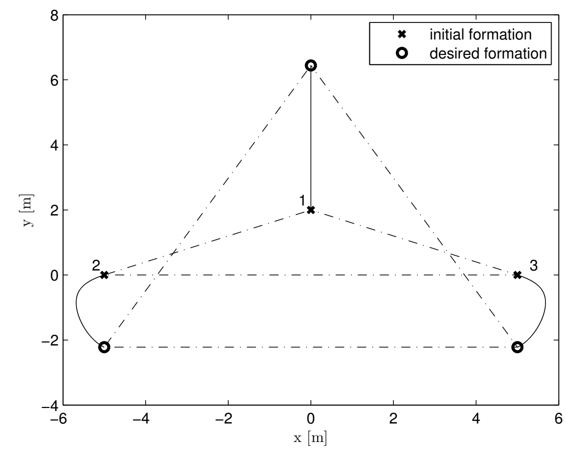


Fig. 1: The formation trajectory of three-agent group by an existing control law proposed in [5]–[8].

In the meanwhile, the majority of existing distance-based formation control laws are simply expressed as the sum of efforts of agents used to adjust distances to adjacent agents to given desired values. For instance, the control law u_i of agent i, proposed in [5]–[14], is given by

$$u_i = \sum_{j \in \mathcal{N}_i} u_{ij}, \tag{1}$$

where $\mathcal{N}_i$ and u_{ij} denote the set of agents adjacent to agent i and a function of the relative displacement to agent j from i, respectively. The function u_{ij} is the effort of agent i required to adjust the distance to agent j to a desired value. It should be noted here that conflicts may exist among control efforts, and that the control law of the form (1) is not capable of coordinating such conflicts. Subsequently, the trajectories of three agents with the control law in [5], [6] are depicted as Fig. 1; that is, the distance between agents 2 and 3, although already adjusted to the desired value, varied unnecessarily by the control law.

In this paper, motivated by such observations, we attempt to achieve the desired formation of a group of three single-integrator modeled agents in the plane, based on the direct control of inter-agent distances. We consider the control of the Euclidean distance matrix of the group and then derive a control law from the time-derivative of the matrix. Although most of existing distance-based control laws have been designed based on the gradient of artificial potential functions, we provide a novel way to design a control law by directly controlling inter-agent distances of the group. Consequently, the proposed control law has a coordination capability in the sense that all inter-agent distances are controlled in a fully decoupled way. Furthermore, since all squared inter-agent distance errors exponentially converge to zero, the stability analysis of the triangular desired formation with the proposed control law is straightforward. Accordingly, the triangular desired formation of the group is globally asymptotically stable if the connectivity among the agents are complete and the positions of the three agents are not collinear in the initial and desired formations.

The outline of the remainder of this paper is as follows. The mathematical background, problem formulation, and the control strategy are provided in Section II. In Section III, an Euclidean distance dynamics matrix of a three-agent group is designed and the proposed control law is derived from the matrix. The proof of global asymptotic stability of the desired formation is presented in Section IV, and simulation results are provided in Section V. Concluding remarks are presented in Section VI.

II. Preliminaries

In this section, mathematical background on Euclidean distance matrices is summarized and a formation problem is formulated. The control strategy in this paper is also presented. Details on Euclidean distance matrices are found in [16]–[18].

A. Euclidean Distance Matrices

A matrix $D = [d_{ij}] \in \mathbb{R}^{N\times N}$ is a Euclidean distance matrix (EDM) if there exist points $p_1, \ldots, p_N$ in $\mathbb{R}^n$ such that

$$d_{ij} \triangleq \|p_i - p_j\|^2,\ i, j = 1, \ldots, N, \tag{2}$$

where $\|\cdot\| : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the n-dimensional Euclidean norm. By the definition in (2), EDMs have following properties:

$$d_{ii} = 0, \tag{3}$$
$$d_{ij} \geq 0, \tag{4}$$
$$d_{ij} = d_{ji}, \tag{5}$$
$$\sqrt{d_{ij}} \leq \sqrt{d_{ik}} + \sqrt{d_{kj}} \tag{6}$$

for all distinct i, j and k in $\{1, \ldots, N\}$. Note that the properties in (3)-(6) are generally necessary conditions for EDMs, not sufficient conditions, though they are necessary and sufficient conditions for EDMs if the number of points is less than or equal to three. Conditions for EDMs are found in [16].

For an EDM D, the dimension n of the affine span of the set of points $p_1, \ldots, p_N$ that generates the matrix D is called the embedding dimension of D, and the matrix $p = [p_1^T \ldots p_N^T]^T \in \mathbb{R}^{nN}$ is called a realization of D. Although the embedding dimension of an EDM is unique, EDMs are invariant under linear rigid motions of its realizations. Hence, if $p \in \mathbb{R}^{nN}$ is a realization of D, then any matrix $[(Qp_1 + b)^T \ldots (Qp_N + b)^T]^T \in \mathbb{R}^{nN}$ is also a realization of D, where $Q \in \mathbb{R}^{n\times n}$ is an orthogonal matrix denoting a rotation or reflection operator and $b \in \mathbb{R}^n$ is a vector denoting translation.

B. Problem Formulation

In decentralized control schemes, though a group of agents has its overall goal to achieve, every agent of the group attempts to attain only its local goals or subtasks because of the lack of information and/or capability. Hence, subtasks should be properly assigned to agents to achieve the overall goal. In such sense, formation control of a group of agents consists of the assignment of subtasks to every agent and the design of control laws to stabilize agents as noted in [15]. In distance-based formation control problems, such subtasks are given by inter-agent distance constraints for each agent; attainment of all subtasks should be sufficient for the achievement of the overall goal.

We model an agent group by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ denote the set of agents of the group and connectivity among agents. We refer to the graph as the information graph of the group. Consider a group of three agents with an information graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each agent i of the group is modeled by a single-integrator of the form

$$\dot{p}_i = u_i,\ i \in \mathcal{V}, \tag{7}$$

where p_i and u_i denote the position and control input of agent i, respectively, in $\mathbb{R}^2$. Each agent i has a capability of sensing the relative displacements to its neighbors. That is, the measurements agent i are given by

$$d_{ji} = \|p_j - p_i\|^2,\ j \in \mathcal{N}_i,\ i \in \mathcal{V}, \tag{8}$$

where $\mathcal{N}_i$ denotes the set of neighbors of agent i. It should be noted that the measurement set contains only relative displacements because agents of the group have nonidentical coordinate systems due to the absence of an available common directional sense for agents.

Then the overall goal of the group is the stabilization of its realization $p = [p_1^T\ p_2^T\ p_3^T]^T \in \mathbb{R}^6$ to be congruent to a given realization of $p^* = [p_1^{*T}\ p_2^{*T}\ p_3^{*T}]^T \in \mathbb{R}^6$. Thus, for the given realization p^*, the desired formation of the group is defined by a set,

$$F^* \triangleq \{(\mathcal{G}, p) \mid \|p_i - p_j\| = \|p_i^* - p_j^*\|,\ \forall i, j \in \mathcal{V}\}. \tag{9}$$

Since there is no centralized control agent, the overall goal is divided into subtasks for agents. For instance, the subtask of agent i is the stabilization of its position p_i such that the following condition holds:

$$d_{ji} = \|p_j^* - p_i^*\|^2,\ \forall j \in \mathcal{N}_i,\ i \in \mathcal{V}. \tag{10}$$

The attainment of all such subtasks in (10) should be sufficient to achieve the overall goal of the group in (9). Since we address three-agent groups, the desired formation of the groups will be achievable if all inter-agent distances are adjusted to the desired values because a triangular is uniquely determined up to congruence by assigning lengths to its three sides. Accordingly, we assume that the information graph $\mathcal{G}$ is complete. For a general group of agents, such consistency between the overall goal of the group and the subtasks of agents can be guaranteed by the rigidity of information graphs.

Then, the formation control problem to be addressed in this paper can be formulated as follows:

Problem 2.1: For a group of three agents with the complete information graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, we assume that each agent is modeled by a single-integrator (7) and has measurements (8). Then, assuming that the subtask (10) is assigned to all agents for the desired formation (9), design a control law of each agent to achieve its assigned subtask.

C. Control Strategy

Consider a group of N single-integrator modeled agents on the plane. If every agent of the group smoothly moves in the plane, then its position is given by a differentiable function of time. Then, since the realization $p = [p_1^T \dots p_N^T]^T$ of the group is differentiable with respect to time, any inter-agent distance $d_{ij} = \|p_i - p_j\|^2$ for all $i, j = 1, \dots, N$ is also given by a differentiable function of time. For the EDM $D = [d_{ij}]$ associated with the realization p, the time derivative of D can be defined as element-wise time derivative of D; that is, $\dot{D} = [\dot{d}_{ij}]$. Subsequently, elements of $\dot{D}$ are given by

$$\begin{aligned}\dot{d}_{ij} =& \frac{\partial d_{ij}}{\partial t}\frac{\partial p_i}{\partial t} + \frac{\partial d_{ij}}{\partial t}\frac{\partial p_j}{\partial t} \\ =& 2\|p_i - p_j\| \frac{(p_i - p_j)^T}{\|p_i - p_j\|}(\dot{p}_i - \dot{p}_j) \\ =& 2(p_i - p_j)^T(\dot{p}_i - \dot{p}_j), \end{aligned} \tag{11}$$

for all $i, j = 1, \dots, N$.

Roughly speaking, our strategy is to control the formation of the group by the direct control of the time-derivative of D. That is, we aim to design a control law such that the inter-agent distance d_{ij} is stabilized in a desired fashion so that d_{ij} converges to d_{ij}^* for all $(i, j) \in \mathcal{E}$. To this aim, we need to specify a proper desired inter-agent distance dynamics, which is linked to the properties of the $\dot{D}$. Hence, we define Euclidean distance dynamics matrices as follows:

Definition 2.1 (Euclidean Distance Dynamics Matrix): For an $N \times N$ EDM D^0 embeddable in n-dimensional Euclidean space, a function $S : [0, \infty) \to \mathbb{R}^{N \times N}$ is a Euclidean distance dynamics matrix (EDDM) of D^0 if and only if $D : [0, \infty) \to \mathbb{R}^{N \times N}$ that is defined as

$$D(t) \triangleq D^0 + \int_0^t S(\tau) d\tau, \ \forall t \geq 0,$$

where integration of S is defined as element-wise integration, is an EDM embeddable in n-dimensional Euclidean space for all $t \geq 0$.

Thus, in the following section, we design an EDDM of a group of three agents and then derive a control law from the EDDM.

III. Formation Control via EDDM

A. EDDM Design

First, we consider how to design an EDDM for a group of three agents under the assumptions of *Problem 2.1*. According to [17], the convex combination of any two EDMs is also an EDM. Moreover, any realization of three points is definitely embeddable in $\mathbb{R}^2$.

Motivated by such facts, we design an EDDM of the group as follows. Let D^0 and D^* be EDMs associated with the initial and desired formations of the group. Then, an EDDM candidate of the group can be chosen as

$$S(t) = -k_s(D(t) - D^*), \tag{12}$$

where D denotes the EDM of the group[1] at any instant $t \geq 0$ and k_s is a design parameter. Since the elements of S in (12) are given as

$$\dot{d}_{ij} = k_s(d_{ij}^* - d_{ij}), \ \forall (i, j) \in \mathcal{E}, \tag{13}$$

each inter-agent distance can be expressed as

$$d_{ij} = d_{ij}^* - (d_{ij}^* - d_{ij}^0)e^{-k_s t}, \ \forall (i, j) \in \mathcal{E}, \tag{14}$$

which implies that there exists a constant α such that $D = \alpha D^* - (1 - \alpha)D^0$, where $0 \leq \alpha \leq 1$ for any instant $t \geq 0$. Since D is the convex combination of two EDMs and any realization of three points is embeddable in $\mathbb{R}^2$, D is also an EDM embeddable in $\mathbb{R}^2$ for all $t \geq 0$. Consequently, S in (12) is an EDDM of a three-agent group.

B. Control Law Design via EDDM

Next, we consider how to derive control laws from the EDDM in (12). By using (11) and (13), the constraint for the dynamics of agents incident to any edge (i, j) is given by

$$2(p_i - p_j)^T(\dot{p}_i - \dot{p}_j) = k_s(d_{ij}^* - d_{ij}), \ \forall (i, j) \in \mathcal{E}, \tag{15}$$

which associates inter-agent distance dynamics with agent dynamics. Since every agent is modeled by a single integrator, the constraint in (15) can be written as

$$2(p_i - p_j)^T(u_i - u_j) = k_s\tilde{d}_{ij}, \ \forall (i, j) \in \mathcal{E}, \tag{16}$$

where $\tilde{d}_{ij} = d_{ij} - d_{ij}^*$.

Then, among various possible choices, constraints for the control inputs of agents i and j can be chosen as

$$(p_j - p_i)^T u_i = \frac{k_s}{4}\tilde{d}_{ji}, \tag{17}$$

$$(p_i - p_j)^T u_j = \frac{k_s}{4}\tilde{d}_{ij},. \tag{18}$$

For any agent i of a three-agent group, the constraints for its control law are then given as,

$$(p_j - p_i)^T u_i = \frac{k_s}{4}\tilde{d}_{ji}, \ \forall j \in \mathcal{N}_i, \ \forall i \in \mathcal{V}. \tag{19}$$

If the dynamics of every agent of the group satisfies the constraints given in (19), then the dynamics of inter-agent

[1]For brevity, we refer the EDM associate with the realization of a group to the EDM of the group.

distances will be maintained as specified in (13) and eventually the desired formation will be achieved.

Since every agent has exactly two neighboring agents in the group, the constraints for the control law of agent i can be arranged as

$$\begin{bmatrix} (p_j - p_i)^T \\ (p_k - p_i)^T \end{bmatrix} u_i = \frac{k_s}{4} \begin{bmatrix} \tilde{d}_{ji} \\ \tilde{d}_{ki} \end{bmatrix}, \ \forall i \in \mathcal{V}, \tag{20}$$

where $j = i + 1(\text{mod } 3)$ and $k = i + 2(\text{mod } 3)$. Here, mod denotes the modulo operator. The constraints for the control input of agent i can be interpreted as a system of two linear equations with two free variables. Thus, the control law can be uniquely determined if the system matrix of the equation is non-singular. Since the system matrix consists of relative displacements to two neighbors of agent i, the system matrix is non-singular if and only if the two relative displacements are linearly independent, which means $p_i,\ p_j$ and p_k are not collinear. Hence, if $p_i,\ p_j$ and p_k are not collinear, then the control law of agent i can be uniquely determined from (20) as

$$u_i = \frac{k_s}{4} \begin{bmatrix} (p_j - p_i)^T \\ (p_k - p_i)^T \end{bmatrix}^{-1} \begin{bmatrix} \tilde{d}_{ji} \\ \tilde{d}_{ki} \end{bmatrix}, \ \forall i \in \mathcal{V}, \tag{21}$$

where $j = i+1(\text{mod } 3)$ and $k = i+2(\text{mod } 3)$. Note that the proposed control law (21) cannot be expressed as the linear sum of control efforts, unlike control laws such as (1).

Remark 3.1: Although the control law is straightforwardly derived from an EDDM of a three-agent group, such a strategy is not directly applicable to general agent groups. First, the convex combination of two EDMs embeddable in $\mathbb{R}^n$ is an EDM embeddable in at most $\mathbb{R}^{2n}$ [17]. Hence, for general agent groups, another way to design an EDDM is required. Second, even when an EDDM of a general agent group is designed, derivation of a control law from the EDDM is not straightforward. That is, if an agent has more than two neighboring agents, then the constraints for its control law is given as an over-determined system of linear equations. Results on such problems can be found in [19].

Remark 3.2: Though we represent the agent position dynamics in the global coordinate system for the purpose of stability analysis, it should be noted that the proposed control law (21) is implementable in the local coordinate system of agent i, $i \in \mathcal{V}$. That is, the proposed control law (21) depends only on inter-agent relative-displacements and therefore is not dependent on the specific coordinate system.

IV. Stability Analysis

In this section, we prove that the desired formation of a group of three agents under the assumptions in *Problem 2.1* is globally asymptotically stable by the proposed formation control law in (21). Moreover, we show that the squared inter-agent distances exponentially converge to the desired values.

For a group of three agents under the assumptions in *Problem 2.1*, since the control law (21) is well defined if the formation of the group is not collinear[2], we need to investigate if the control law keeps the formation from being collinear. The following *Lemma 4.1* reveals that if the initial and desired formations are not collinear, then the control law (21) preserves the non-collinearity of the formation of the group.

Lemma 4.1: For a group of three agents under the assumptions of *Problem 2.1*, if the initial and desired formations of the group are not collinear, then the control law (21) preserves the non-collinearity of the formation of the group.

Proof: Since the initial and desired formations of the group are not collinear, triangular inequality holds for the elements of EDMs of the initial and desired formation for all distinct i, j and k in $\mathcal{V}$:

$$\sqrt{d_{ik}^0} < \sqrt{d_{ij}^0} + \sqrt{d_{jk}^0}, \tag{22}$$

$$\sqrt{d_{ik}^*} < \sqrt{d_{ij}^*} + \sqrt{d_{jk}^*}. \tag{23}$$

Furthermore, from (14), d_{ij} can be expressed as a convex combination of d_{ij}^0 and d_{ij}^* by the control law (21):

$$d_{ij} = \alpha d_{ij}^0 + (1-\alpha) d_{ij}^*,\ (i,j) \in \mathcal{E}, \tag{24}$$

where $0 \le \alpha \le 1$. By squaring both sides and adding terms on each side of (22) and (23), we then have

$$d_{ik} < \ d_{ij} + d_{jk} + 2(1-\alpha)\sqrt{d_{ij}^0 d_{jk}^0} + 2\alpha\sqrt{d_{ij}^* d_{jk}^*}, \tag{25}$$

based on (24). Furthermore, the inequality (25) and the inequality,

$$(1-\alpha)\sqrt{d_{ij}^0 d_{jk}^0} + \alpha\sqrt{d_{ij}^* d_{jk}^*} \le \sqrt{d_{ij} d_{jk}},$$

which is evident from

$$2\sqrt{d_{ij}^0 d_{jk}^*}\sqrt{d_{ij}^* d_{jk}^0} \le d_{ij}^0 d_{jk}^* + d_{ij}^* d_{jk}^0,$$

lead to the following inequality,

$$d_{ik} < d_{ij} + d_{jk} + 2\sqrt{d_{ij} d_{jk}}. \tag{26}$$

Hence, an EDM generated by the convex combination of the two EDMs are also not collinear by (26):

$$\sqrt{d_{ik}} < \sqrt{d_{ij}} + \sqrt{d_{jk}}. \tag{27}$$

Since the initial formation of the group is not collinear, the control law (21) is initially well defined. Thus, the control law (21) preserves the non-collinearity of the formation the group by (27) if the initial and desired formations of the group are not collinear. ■

Remark 4.1: Non-collinearity of the initial and desired formations is a general assumption in formation control based on inter-agent distance information. Since agents do not share a global coordinate system, if an agent and its neighbors are collinear, then the motion of the agent will be restricted to the collinear state. This is why non-collinearity

[2]For brevity here, if points of the three agents a group lie on a single straight line, then we call the formation of the three-agent group collinear.

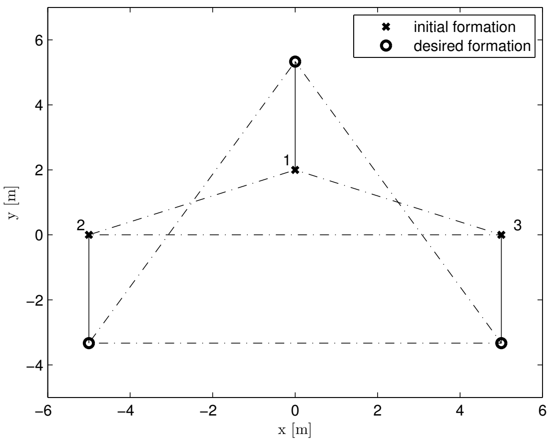
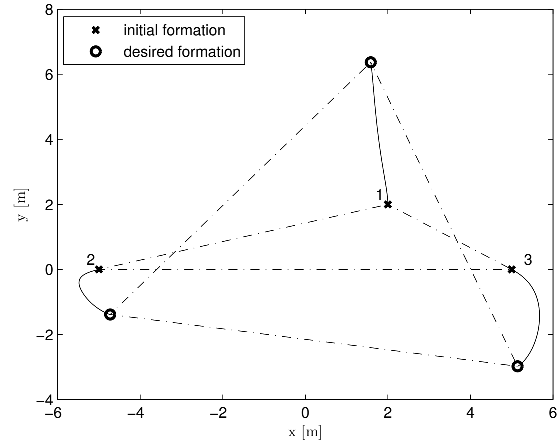
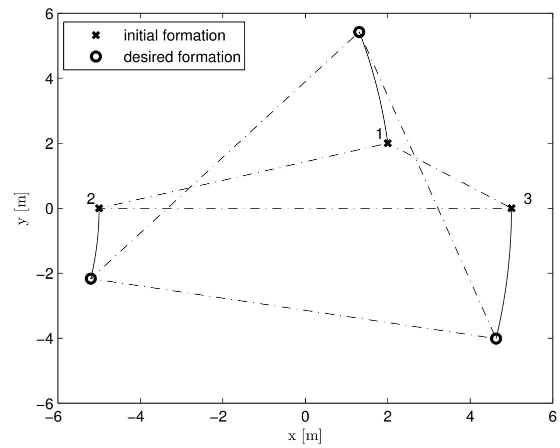


Fig. 2: The formation trajectory of three-agent group by the proposed formation law.



(a) By an existing control law proposed in [5]–[8].



(b) By the proposed control law.

Fig. 3: The formation trajectories of three-agent groups.

for general agent groups has been assumed in most past researches such as [5]–[14].

Then, the following *Theorem 4.1* confirms the global asymptotic stability of the desired formation of the three-agent group by the proposed control law (21).

Theorem 4.1 (Main Result): For a group of three agents under the assumptions of *Problem 2.1*, if the initial and desired formations of the group are not collinear, then the desired formation of the group is globally asymptotically stable by the control law (21). Furthermore, all squared inter-agent distances exponentially converge to the desired values.

Proof: Since the initial formation is not collinear, the control law (21) is well defined by *Lemma 4.1*. Then, from (14), the overall distance error dynamics of the group is given by $\dot{\tilde{d}}_{ij} = -k_s \tilde{d}_{ij}$ for all $(i, j) \in \mathcal{E}$, which means that $\tilde{d}_{ij}$ exponentially converges to zero. That is, the error dynamics is globally exponentially stable; therefore, the desired formation of the group is globally asymptotically stable since the formation of the group is uniquely determined by inter-agent distances. ■

Although the control law (21) has a singularity and the control input approaches infinity whenever the initial formation is arbitrarily close to the set of collinear formations, the drawback can be overcome by multiplying the control law (21) by $|\det(A_i)|$ [20].

V. Simulation Results

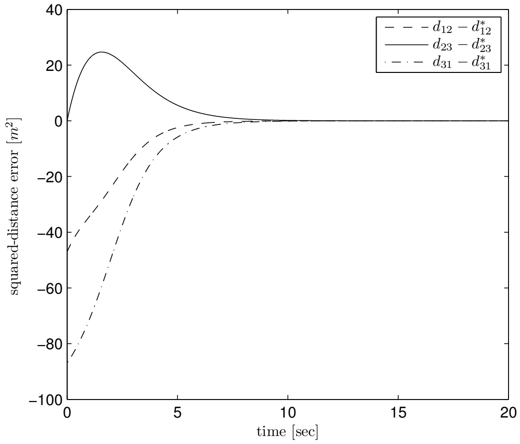
In this section, we present the simulation results of formation control of a three-agent group, comparing the proposed control law with an existing control law proposed in proposed in [5]–[8].

In the first simulation, for the three-agent group under the assumptions of *Problem 2.1*, the initial and desired realizations were assumed to be $[(0, \sqrt{75})^T \, (-5, 0)^T \, (5, 0)^T]^T$ and $[(0, 2)^T \, (-5, 0)^T \, (5, 0)^T]^T$, respectively. Fig. 1 and 2 depict trajectories of the agents by the existing control law [5], [6] and the proposed control law, respectively. Although the relative distance between agents 2 and 3 varied unnecessarily by the existing control law, such a phenomenon did not occur when the proposed control law was applied.

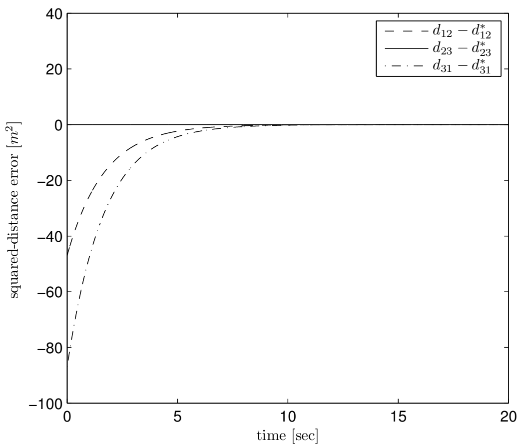
In the second simulation, the initial and desired realization were assumed to be $[(0, \sqrt{75})^T \, (-5, 0)^T \, (5, 0)^T]^T$ and $[(2, 2)^T \, (-5, 0)^T \, (5, 0)^T]^T$, respectively. Fig. 3 and 4 depict trajectories and squared inter-agent distance errors, respectively. Squared inter-agent distance errors exponentially converged to zero by the proposed control law as depicted in Fig. 4(b).

VI. Conclusion

In this paper, we proposed a triangular formation control law based on inter-agent distances for a group of three single-integrator modeled agents in the plane. Accordingly, the group desired formation is globally asymptotically stable if the initial and desired formations are not collinear and the information graph is complete. Moreover, squared inter-agent distance errors exponentially converge to zero.



(a) By an existing control law proposed in [5]–[8].



(b) By the proposed control law.

Fig. 4: The squared inter-agent distance errors of three-agent groups.

Conflicts among control efforts in most of existing control laws are inevitable. The proposed control law, designed by an EDDM of the group, has a coordination property in the sense that inter-agent distances are fully decoupled by the law. Simulation results not only validated the effectiveness of the proposed control law but demonstrated its coordination capability for a group of three agents.

Although we addressed three-agent group in the plane, the results can be extended to general cases. A control law for a group of four single-integrator modeled agents in three-dimensional space can be designed. Furthermore, the extension of the results in this paper to a general group can be found in [19].

VII. Acknowledgements

This research was supported by the National Research Foundation of Korea (NRF) (No. 2010−0015778).

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