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Event-Triggered Real-Time Scheduling For Stabilization Of Passive and Output Feedback Passive Systems

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Abstract— **Event-triggered control has been recently proposed as an alternative approach to the traditional periodic implementation of control tasks. The possibility of reducing the number of executions while guaranteeing stability makes event-triggered control very appealing in the context of sensor/actuator networked control systems. In this paper, we revisit the event-triggered control from an input-to-output perspective and we propose a simple event-triggered control strategy for stabilization of passive and output feedback passive systems. The triggering condition is derived based on the output information of the control system and an estimate of the lower bound on inter-sampling time is also provided.**

I. I

The majority of feedback control laws nowadays are implemented on digital platforms since microprocessors offer many advantages of running real-time operating systems. This creates the possibility of sharing the computational resources among control and other kinds of applications thus reducing the deployment costs of complex control systems [12]. Since we are dealing with resource-limited microprocessors, it becomes important to assess to what extent we can increase the functionality of these embedded devices through novel real-time scheduling algorithms.

Traditionally, the control task is executed periodically, since this allows the closed-loop system to be analyzed and the controller to be designed using the well-developed theory on sampled-data systems, see [1]-[3]. However, the control strategy obtained based on this approach is conservative in the sense that resource usage(i.e., sampling rate, CPU time) is more frequent than necessary to assure a specified performance level, since stability is usually guaranteed under sufficiently fast periodic execution of control action. To overcome the drawback of the periodic paradigm, several researchers suggested the idea of event-based control. The terminology refers to the triggering mechanism as event-based-sampling[14], to event-driven sampling[15], Lebesgue sampling[7], deadband control[16], level-crossing sampling[17], state-triggered sampling[8] and self-triggered sampling[11] with slight different meaning. However in all cases control signals are kept constant until the violation of a condition on certain signals of the plant triggers the re-computation of the control actions. In event-triggered real-time scheduling algorithms, the control tasks are executed whenever a novelty error becomes large compared with the state of the plant[8] (so the triggering mechanism usually needs full-state information of the plant). The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-triggered control very appealing in networked control systems(NCSs). One should be aware that the event-triggered technique reduces resource usage while providing a high degree of robustness, since embedded hardware is used to monitor the state of the plant continuously.

Most of the results on event-triggered control are obtained under the assumption that the feedback control law provides input-to-state stability(ISS) in the sense of [18] with respect to the measurement errors, see [8]-[12]. Note that some results on designing ISS stabilizing control laws can be found in [19]-[22]. The ISS framework provides insight into the triggering condition by exploring the relation between stabilization and the current full-state information. However, in many control applications the full state information is not available for measurement, so it is important to study stability and performance of event-triggered control systems with dynamic and static output feedback controllers.

In this paper, a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive(OFP) NCSs. For passive system, the triggering condition is derived such that the size of the output novelty error should be less than or equal to the size of the current output, and any static positive definite output feedback gain could stabilize the system if the system is also detectable; for OFP system, the triggering condition and the stabilization output feedback gain are derived based on the output feedback passivity index of the system. Analyses on the inter-sampling time without and with network induced delays are both provided followed by discussions and examples. The rest of this paper is organized as follows: we introduce some background on passive and OFP systems in section II; the problem is formulated in section III; the main results are stated in section IV; concluding remarks are given in section V.

II. B M

We first introduce some basic concepts on passive and OFP systems.

Consider the following control system, which could be linear or nonlinear:

$$H : \begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \tag{1}$$

where $x \in X \subset \mathbb{R}^n$, $u \in U \subset \mathbb{R}^m$ and $y \in Y \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X, U and

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Y are the state, input and output spaces, respectively. The representation $\phi(t,t_0,x_0,u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Definition 1(Supply Rate)[4]: The supply rate $\omega(t) = \omega(u(t),y(t))$ is a real valued function defined on $U \times Y$, such that for any $u(t) \in U$ and $x_0 \in X$ and $y(t) = h(\phi(t,t_0,x_0,u))$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty. \tag{2}$$

Definition 2(Dissipative System)[4]: System H with supply rate $\omega(t)$ is said to be **dissipative** if there exists a nonnegative real function $V(x) : X \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in X$ and $u \in U$,

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau, \tag{3}$$

where $x_1 = \phi(t_1,t_0,x_0,u)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 3(Passive System)[4]: System H is said to be **passive** if there exists a storage function $V(x)$ such that

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau, \tag{4}$$

if $V(x)$ is C^1, then we have

$$\dot{V}(x) \leq u(t)^T y(t), \ \forall t \geq 0. \tag{5}$$

One can see that passive system is a special case of dissipative system with supply rate $\omega(t) = u(t)^T y(t)$.

Definition 4(Output Feedback Passive System)[5]: System H is said to be **Output Feedback Passive**(OFP) if it is dissipative with respect to the supply rate

$$\omega(u,y) = u^T y - \rho y^T y, \tag{6}$$

for some $\rho \in \mathbb{R}$.

Remark 1. Note that if $\rho > 0$, then H is strictly output passive, and H is said to have excessive output feedback passivity of ρ; if $\rho < 0$, H is said to lack output feedback passivity. We call ρ the *output feedback passivity index* of the system, and denote a dissipative system with supply rate given in (6) by OFP(ρ). Also note that if a system is OFP(ρ), then it is also OFP($\rho - \varepsilon$), $\forall \varepsilon > 0$.

Definition 5[5]: Consider the system H with zero input, that is $\dot{x} = f(x,0)$, $y = h(x,0)$, and let $Z \subset \mathbb{R}^n$ be its largest positively invariant set contained in $\{x \in \mathbb{R}^n | y = h(x,0) = 0\}$. We say H is **Zero-State Detectable**(ZSD) if $x = 0$ is asymptotically stable conditionally to Z. If $Z = \{0\}$, we say that H is **Zero-State Observable** (ZSO).

III. P S

We consider a control system as given in (1). We first assume H is a passive system, and there exists a nonnegative storage function $V(x)$ such that (5) is satisfied. We know that if H is ZSD, then under the feedback control law

$$u(t) = -Ky(t), \tag{7}$$

where K could be any positive scalar or any $m \times m$ positive definite matrix, the origin of H is asymptotically stable. For the rest of this paper, we assume for simplicity that $K > 0$ is scalar.

In real time, the implementation of the feedback control law is typically done by sampling the output $y(t)$ at time instants t_0, t_1, ..., computing the control action and updating the input to the plant at time instants $t_0 + \Delta_0$, $t_1 + \Delta_1$, ..., where $\Delta_k \geq 0$, for $k = 0,1,2,\ldots$ represents the network induced delay from the sampler to the remote controller at each sampling time(here, we assume the delay from the controller to the actuator is negligible). In event-triggered NCSs, a new sampled output information is sent to the remote controller only when the size of the output novelty error $\tilde{e}(t) = y(t) - y(t_k)$ for $t \in [t_k, t_{k+1})$ at the event-detector (an embedded hardware in the sampler) exceeds a certain threshold, where $y(t_k)$ denotes the last sampled output information of the plant, see the event-triggered network control scheme as shown in Fig. 1.

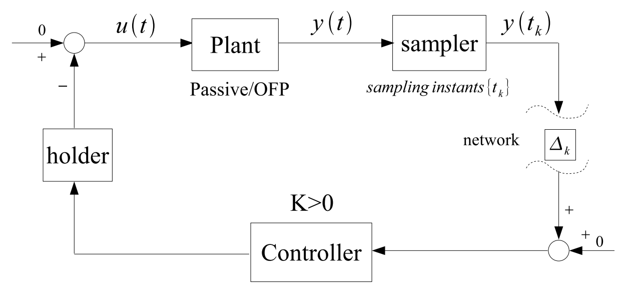

Fig. 1: Event-triggered control for NCSs (we assume that the actuator is collocated with the plant)

We summarize the problems we try to solve in this paper as follows: if we restrict the plant to be passive or OFP, when should a new sampled output information be sent to the remote controller for stabilization? What is the triggering condition and what is the stabilization controller? How about the inter-sampling time?

IV. M R

In this section, we first derive the triggering condition for stabilization of passive and OFP systems without considering network induced delay Δ_k.

Theorem 1. Consider the networked control system as shown in Fig.1, where the plant is passive and ZSD as given in (1). Assume that the network induced delay $\Delta_k \equiv 0$, $\forall k$. If the sampling time t_k is determined by the following triggering condition

$$\|\tilde{e}(t)\|_2 = \delta \|y(t)\|_2, \ \forall t \geq 0, \tag{8}$$

where $\delta \in (0,1]$, then with any $K > 0$ being the output feedback gain, the control system is asymptotically stable.

Proof: Since the plant is passive, with $u(t) = -Ky(t_k)$ and $\tilde{e}(t) = y(t) - y(t_k)$ for $t \in [t_k, t_{k+1})$, we can obtain

$$\begin{aligned} \dot{V}(x) &\leq u^T y = -K(y - \tilde{e})^T y = -Ky^T y + K\tilde{e}^T y \\ &\leq K\|\tilde{e}\|_2 \|y\|_2 - K\|y\|_2^2, \ t \in [t_k, t_{k+1}), \end{aligned} \tag{9}$$

where $V(x)$ is the storage function of the plant. So if a new sampled output information is sent to the remote controller whenever the triggering condition (8) is satisfied, then we can guarantee that $\|\tilde{e}\|_2 \leq \|y\|_2$, thus $\dot{V}(x) \leq 0$ for $t \geq 0$. Moreover, since the plant is ZSD, one can further conclude that the control system is asymptotically stable. ■

Remark 2. One can verify that the triggering condition (8) actually assures that the sampled output information $y(t_k)$ to have the same sign with the output $y(t)$ at any sampling instant t_k ($y(t_k)^T y(t) \geq 0$, $\forall t_k$). Since

$$\dot{V}(x) = -Ky(t_k)^T y, \text{ for } t \in [t_k, t_{k+1}) \ ,$$

this further assures that $\dot{V}(x) \leq 0$, for $t \geq 0$.

Theorem 2. Consider the networked control system as shown in Fig.1, where the plant is OFP(ρ) with $\rho < 0$ and ZSD as given in (1). Assume that the network induced delay $\Delta_k \equiv 0$, $\forall k$. If the sampling time t_k is determined by the following triggering condition

$$\|\tilde{e}(t)\|_2 = \delta\sigma_o \|y(t)\|_2, \ \forall t \geq 0, \tag{10}$$

where $\delta \in (0, 1]$, and $\sigma_o = \frac{\rho+K}{K}$, then with any $K > -\rho$ being the output feedback gain, the control system is asymptotically stable.

The proof is similar to the proof shown in the Theorem 1, thus it is omitted here.

Remark 3. Intuitively, if the triggering threshold $\delta\sigma_o$ has a larger value, then the triggering condition will be satisfied less frequently. The value of σ_o depends on the ration of $\frac{\rho}{K}$, so if K is larger, then we have a larger triggering threshold; if $K \gg -\rho$, the $\delta\sigma_o \to \delta$.

The triggering conditions (8) in Theorem 1 and (10) in Theorem 2 explicitly determine when a new sampled output information of the plant should be sent to the remote controller for control action update to assure stability of the system in the absence of network induced delay Δ_k. Another problem that needs to be addressed is how often should we sample the output of the plant? This problem is not easy in general, especially when the dynamics of the plant are highly nonlinear and only output information can be used to generate control action. The following proposition gives a way to estimate the lower bound on the inter-sampling time when we restrict the output of the plant to be a memoryless function belonging to a bounded sector of the state.

Proposition 1. Consider the networked control system shown in Fig.1, where the plant is passive with storage function $V(x)$. Assume that the network induced delay from the sampler to the controller $\Delta_k \equiv 0$, $\forall k$. Let the following assumptions be satisfied:

1) $f(x, u) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is locally Lipschitz continuous in x on a compact set $S_x \subset \mathbb{R}^n$ with Lipschitz constant L_x;
2) $\|f(x, u) - f(x, 0)\|_2 \leq L_u \|u\|_2$ for all $x \in S_x$ with some nonnegative constant L_u;
3) $h(x) : \mathbb{R}^n \to \mathbb{R}^n$ belongs to a sector (K_1, K_2), with $K_1 x^T x \leq x^T h(x) \leq K_2 x^T x$, where $K_1 \in \mathbb{R}$, $K_2 \in \mathbb{R}$ and $0 < K_1 K_2 < \infty$;
4) $\left\|\frac{\partial h}{\partial x}\right\|_2 \leq \gamma$, where $0 < \gamma < \infty$;

then for any initial condition $x(0)$ in a compact set $S_0 \subset S_x$, with the control action given by $u(t) = -Ky(t_k), K > 0$, the inter-sampling time $\{t_{k+1} - t_k\}$ implicitly determined by the triggering condition (8) is lower bounded by

$$\tau = \frac{1}{\gamma\zeta L_x} \ln\Big(1 + \frac{L_x\zeta\delta}{L_x\zeta + L_u K + \delta L_u K}\Big) \tag{11}$$

where $\zeta = \max\left\{\frac{1}{|K_1|}, \frac{1}{|K_2|}\right\}$.

Proof: Since $\tilde{e}(t) = y(t) - y(t_k)$ for $t \in [t_k, t_{k+1})$, we can get for $t \in [t_k, t_{k+1})$

$$\begin{aligned} \frac{d}{dt}\frac{\|\tilde{e}\|_2}{\|y\|_2} &= \frac{d}{dt}\frac{(\tilde{e}^T\tilde{e})^{\frac{1}{2}}}{(y^T y)^{\frac{1}{2}}} = \frac{(\tilde{e}^T\tilde{e})^{-\frac{1}{2}}\tilde{e}^T\dot{\tilde{e}}(y^T y)^{\frac{1}{2}} - (y^T y)^{-\frac{1}{2}} y^T \dot{y}(\tilde{e}^T\tilde{e})^{\frac{1}{2}}}{y^T y} \\ &= \frac{\tilde{e}^T\dot{\tilde{e}}}{\|\tilde{e}\|_2\|y\|_2} - \frac{y^T\dot{y}}{\|y\|_2\|y\|_2}\frac{\|\tilde{e}\|_2}{\|y\|_2}, \end{aligned} \tag{12}$$

since $y(t_k)$ is kept constant for $t \in [t_k, t_{k+1})$, $\forall k$, we have $\dot{\tilde{e}}(t) = \dot{y}(t)$, $\forall t \geq 0$, and we can further get

$$\frac{d}{dt}\frac{\|\tilde{e}\|_2}{\|y\|_2} \leq \frac{\|\tilde{e}\|_2\|\dot{y}\|_2}{\|\tilde{e}\|_2\|y\|_2} + \frac{\|y\|_2\|\dot{y}\|_2\|\tilde{e}\|_2}{\|y\|_2\|y\|_2\|y\|_2} = \Big(1 + \frac{\|\tilde{e}\|_2}{\|y\|_2}\Big)\frac{\|\dot{y}\|_2}{\|y\|_2}. \tag{13}$$

Based on assumptions 1) and 2), we have

$$\begin{aligned} \|f(x, u)\|_2 &= \|f(x, 0) + f(x, u) - f(x, 0)\|_2 \\ &\leq \|f(x, 0)\|_2 + \|f(x, u) - f(x, 0)\|_2 \\ &\leq L_x\|x\|_2 + L_u\|u\|_2 \\ &\leq L_x\|x\|_2 + L_u K\|y\|_2 + L_u K\|\tilde{e}\|_2, \end{aligned} \tag{14}$$

thus

$$\begin{aligned} \big\|\dot{y}\big\|_2 &= \Big\|\frac{\partial h(x)}{x}\dot{x}\Big\|_2 \leq \Big\|\frac{\partial h(x)}{x}\Big\|_2 \|\dot{x}\|_2 \\ &\leq \gamma(L_x\|x\|_2 + L_u K\|y\|_2 + L_u K\|\tilde{e}\|_2). \end{aligned} \tag{15}$$

Moreover, since $y = h(x)$ belongs to sector $[K_1, K_2]$ such that $K_1 x^T x \leq x^T h(x) \leq K_2 x^T x$, where $0 < K_1 K_2 < \infty$, one can verify that

$$\|x\|_2 \leq \max\Big\{\frac{1}{|K_1|}, \frac{1}{|K_2|}\Big\}\|y\|_2 = \zeta\|y\|_2, \tag{16}$$

so we can obtain

$$\begin{aligned} \frac{d}{dt}\frac{\|\tilde{e}\|_2}{\|y\|_2} &\leq (1 + \frac{\|\tilde{e}\|_2}{\|y\|_2})\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\leq \gamma(1 + \frac{\|\tilde{e}\|_2}{\|y\|_2})(L_x\frac{\|x\|_2}{\|y\|_2} + L_u K + L_u K\frac{\|\tilde{e}\|_2}{\|y\|_2}) \\ &\leq \gamma(1 + \frac{\|\tilde{e}\|_2}{\|y\|_2})(L_x\zeta + L_u K + L_u K\frac{\|\tilde{e}\|_2}{\|y\|_2}). \end{aligned} \tag{17}$$

So the evolution of $\frac{\|\tilde{e}\|_2}{\|y\|_2}$ during the time-interval $[t_k, t_{k+1})$ is upper bounded by the solution to

$$\dot{p} = \gamma(1 + p)(L_x\zeta + L_u K + L_u K p), \tag{18}$$

for $t \in [t_k, t_{k+1})$ with $p(t_k) = 0$. Thus the time for $\frac{\|\tilde{e}\|_2}{\|y\|_2}$ to evolve from 0 to δ is lower bounded by the solution to $p(t_k + \tau) = \delta$. One can verify that

$$\tau = \frac{1}{\gamma\zeta L_x}\ln\Big(1 + \frac{L_x\zeta\delta}{L_x\zeta + L_u K + \delta L_u K}\Big) \ .$$

The proof is completed. ■

Remark 4. When $y(t) = x(t)$, then we can directly measure the state of the plant and apply state feedback control. This corresponds to the case when $\zeta = 1$.

Remark 5. For OFP(ρ)($\rho < 0$) system, the analysis is the same, but the triggering condition is (10) and the output feedback gain should be chosen based on the index ρ of the plant. In this case, one can verify that

$$\tau = \frac{1}{\gamma\zeta L_x}\ln\Big(1 + \frac{L_x\zeta\delta\sigma_o}{L_x\zeta + L_u K + \delta\sigma_o L_u K}\Big),$$

with $\sigma_o = \frac{\rho+K}{K}$ and $K > -\rho$.

Remark 6. The analysis shown in Proposition 1 requires that the output of the plant belongs to a bounded sector of the state. This is actually a conservative condition and it requires that the input and output have the same dimension as the state. Note that in many control systems, we can only use partial state information to generate the control action. In this case, if the plant is passive or OFP with respect to those states which can be measured, then we can still use the method in Proposition 1 to get an estimate of inter-sampling time. This situation is illustrated by the following example.

Example 1. Consider the output feedback passive system given by

$$\begin{aligned} \dot{x}_1(t) &= -3x_1^3(t) + x_1(t)x_2(t) \\ \dot{x}_2(t) &= 3x_2(t) + 2u(t) \\ y(t) &= x_2(t), \end{aligned} \tag{19}$$

we can see that the system is ZSD but unstable, and we can only measure x_2. If we choose the storage function $V(x) = \frac{1}{4}x_2^2(t)$, we can get

$$\dot{V}(x) = u(t)y(t) + 1.5y^2(t), \tag{20}$$

and in this case $\rho = -1.5$, the plant is OFP(-1.5) with respect to x_2. Moreover, we have

$$\begin{aligned} \frac{\|\dot{y}\|_2}{\|y\|_2} &= \frac{\|\dot{x}_2\|_2}{\|y\|_2} = \frac{\|3x_2 - 2K(y-\tilde{e})\|_2}{\|y\|_2} \\ &\leq 3\frac{\|x_2\|_2}{\|y\|_2} + 2K + 2K\frac{\|\tilde{e}\|_2}{\|y\|_2} = 3 + 2K + 2K\frac{\|\tilde{e}\|_2}{\|y\|_2}, \end{aligned} \tag{21}$$

we can further get

$$\begin{aligned} \frac{d}{dt}\frac{\|\tilde{e}\|_2}{\|y\|_2} &\leq (1 + \frac{\|\tilde{e}\|_2}{\|y\|_2})\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\leq (1 + \frac{\|\tilde{e}\|_2}{\|y\|_2})(3 + 2K + 2K\frac{\|\tilde{e}\|_2}{\|y\|_2}). \end{aligned} \tag{22}$$

So in this case, with no network induced delay, the evolution of $\frac{\|\tilde{e}\|_2}{\|y\|_2}$ during each inter-sampling time is upper bounded by the solution to

$$\dot{p} = (1+p)(3 + 2K + 2Kp)$$

with $p(t_k) = 0$, and we can get an estimate on the lower bound of the inter-sampling time τ based on the solution to $p(t_k + \tau) = \delta\sigma_o$. According to Theorem 2, we need to choose $K > -\rho > 0$ as the stabilization output feedback gain. If we choose $K = 3$ and $\delta = 1$, then the triggering condition in Theorem 2 becomes $\|e(t)\|_2 = \frac{K+\rho}{K}\|y(t)\|_2 = 0.5\|y(t)\|_2$. We add external disturbance at the input to the plant which is an uniformly distributed random signal on the interval $[0, 0.5]$, the simulation result is shown in Fig.2, where $\sigma(t)$ shows the evolution of $\frac{\|\tilde{e}\|_2}{\|y\|_2}$, $\{t_{k+1} - t_k\}$ shows the evolution of the inter-sampling time, x_{p1} and x_{p2} show the evolution of the state of the plant.

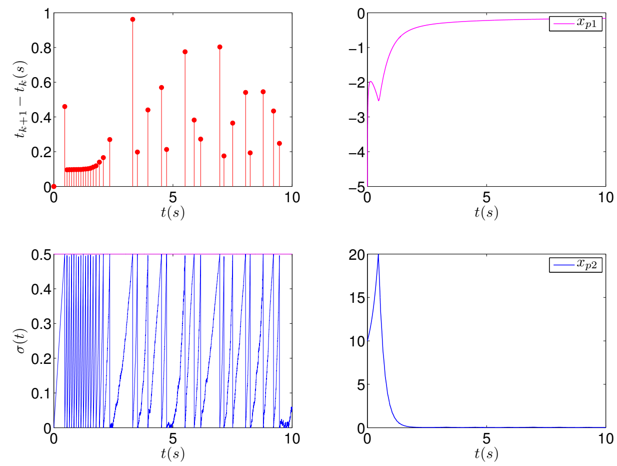

Fig. 2: simulation result of example 1

When there is non-trivial network induced delay from the sampler to the controller ($\Delta_k \neq 0$), we need to be careful about the triggering condition, and the admissible network induced delay is related to the triggering condition: usually a larger triggering threshold implies a longer inter-sampling time, thus more tolerance to the network induced delay. Since $\|\tilde{e}\|_2 = \|y - y(t_k)\|_2$, we have $\|\tilde{e}\|_2 \geq \|y(t_k)\|_2 - \|y\|_2$, thus $\|y\|_2 \geq \|y(t_k)\|_2 - \|\tilde{e}\|_2$, for $t \in [t_k, t_{k+1})$. One can verify that a sufficient condition for $\|\tilde{e}\|_2 \leq \|y\|_2$ to be satisfied for all $t \geq 0$ is given by $\|\tilde{e}\|_2 \leq 0.5\|y(t_k)\|_2$, for $t \in [t_k, t_{k+1})$, $\forall k$. So alternative triggering conditions to (8) and (10) are given by

$$\|\tilde{e}\|_2 = 0.5\delta\|y(t_k)\|_2, \text{ if the plant is passive} \tag{23}$$

$$\|\tilde{e}\|_2 = \frac{\delta\sigma_o}{1+\sigma_o}\|y(t_k)\|_2, \text{ if the plant is OFP}(\rho), \rho < 0 \tag{24}$$

with $\delta \in (0, 1]$, and for $t \in [t_k, t_{k+1})$, $\forall k$. Note that they are tighter triggering conditions compared with (8) and (10). We use them for analysis on the upper bound of the admissible network induced delay as provided in the following proposition.

Proposition 2. Consider the networked control system shown in Fig.1, where the plant is passive with storage function $V(x)$. Let assumptions 1)-4) in Proposition 1 be satisfied. Then for any initial condition $x(0) \in S_0 \subset S_x$, there exists $\eta_k > 0$, such that for $\Delta_k \in [0, \eta_k]$ and with control action $u(t) = -Ky(t_k), K > 0$, $t \in [t_k + \Delta_k, t_{k+1} + \Delta_{k+1})$, the inter-sampling time $\{t_{k+1} - t_k\}$ implicitly determined by the triggering condition (23) is strictly positive.

Proof: Let $e(t)$ denote the output novelty error at the controller, and we have

$$e(t) = \begin{cases} y(t) - y(t_{k-1}), & \text{for } t \in [t_k, t_k + \Delta_k) \\ y(t) - y(t_k), & \text{for } t \in [t_k + \Delta_k, t_{k+1}), \end{cases} \tag{25}$$

while

$$\tilde{e}(t) = y(t) - y(t_k), \text{ for } t \in [t_k, t_{k+1}) \ . \tag{26}$$

For $t \in [t_k, t_k + \Delta_k)$, we have

$$\begin{aligned} \frac{d}{dt}\|\tilde{e}\|_2 &\le \|\dot{\tilde{e}}\|_2 = \|\dot{y}\|_2 \le \gamma \|\dot{x}\|_2 \\ &\le \gamma[L_x\|x\|_2 + L_u K \|y(t_{k-1})\|_2] \\ &\le \gamma[L_x\zeta\|y\|_2 + L_u K\|y - e\|_2] \\ &\le \gamma[(L_x\zeta + L_u K)\|y\|_2 + L_u K\|e\|_2] \\ &= \gamma[(L_x\zeta + L_u K)\|\tilde{e} + y(t_k)\|_2 + L_u K\|\tilde{e} + y(t_k) - y(t_{k-1})\|_2] \\ &\le \gamma(L_x\zeta + 2L_u K)\|\tilde{e}\|_2 + \gamma(L_x\zeta + L_u K)\|y(t_k)\|_2 \\ &+ \gamma L_u K\|y(t_k) - y(t_{k-1})\|_2, \end{aligned} \tag{27}$$

so the evolution of $\|\tilde{e}\|_2$ during $[t_k, t_k + \Delta_k)$ is bounded by the solution to

$$\begin{aligned} \dot{p} &= \gamma(L_x\zeta + 2L_u K)p + \gamma(L_x\zeta + L_u K)\|y(t_k)\|_2 \\ &+ \gamma L_u K\|y(t_k) - y(t_{k-1})\|_2 \end{aligned} \tag{28}$$

with $p(t_k) = 0$. Assume $\|\tilde{e}(t_k + \Delta_k)\|_2 = 0.5\delta_1\|y(t_k)\|_2$, with $\delta_1 \in (0, \delta)$, then Δ_k is lower bounded by the solution to $p(t_k + \varepsilon_k^-) = 0.5\delta_1\|y(t_k)\|_2$, we can obtain

$$\varepsilon_k^- = \frac{1}{\gamma(L_x\zeta + 2L_u K)} \ln\left(1 + \frac{0.5\delta_1(L_x\zeta + 2L_u K)}{L_x\zeta + (1 + 0.5\delta_1)L_u K}\right) . \tag{29}$$

Assume $\Delta_k \le \varepsilon_k^-$, then for $t \in [t_k + \varepsilon_k^-, t_{k+1})$, we have

$$\begin{aligned} \frac{d}{dt}\|\tilde{e}\|_2 &\le \gamma(L_x\|x\|_2 + L_u K\|y(t_k)\|_2) \\ &\le \gamma(L_x\zeta\|y\|_2 + L_u K\|y(t_k)\|_2) \\ &\le \gamma L_x\zeta\|\tilde{e}\|_2 + \gamma(L_x\zeta + L_u K)\|y(t_k)\|_2, \end{aligned} \tag{30}$$

so the evolution of $\|\tilde{e}\|_2$ during $[t_k + \varepsilon_k^-, t_{k+1})$ is bounded by the solution to

$$\dot{p} = \gamma L_x\zeta p + \gamma(L_x\zeta + L_u K)\|y(t_k)\|_2, \tag{31}$$

with $p(t_k + \varepsilon_k^-) = 0.5\delta_1\|y(t_k)\|_2$. Assume $\|\tilde{e}(t_{k+1})\|_2 = 0.5\delta_2\|y(t_k)\|_2$, where $\delta_2 \in (\delta_1, 1)$, then $[t_{k+1} - (t_k + \varepsilon_k^-)]$ is lower bounded by the solution to $p(t_k + \varepsilon_k^- + \varepsilon_k) = 0.5\delta_2\|y(t_k)\|_2$, we can obtain

$$\varepsilon_k = \frac{1}{\gamma L_x\zeta} \ln\left(\frac{L_u K + (1 + 0.5\delta_2)L_x\zeta}{L_u K + (1 + 0.5\delta_1)L_x\zeta}\right). \tag{32}$$

Note that for $t \in [t_k + \varepsilon_k^-, t_{k+1})$, $e = \tilde{e}$, and at $t = t_{k+1}$, $\tilde{e}$ is reset to zero while $\|e\|_2$ is continuing growing due to the delay from the sampler to the controller. To assure stability, we need $\|e\|_2 \le 0.5\|y(t_k)\|_2$ for $t \in [t_{k+1}, t_{k+1} + \Delta_{k+1})$. The evolution of $\|e\|_2$ during $[t_{k+1}, t_{k+1} + \Delta_{k+1})$ is bounded by

$$\begin{aligned} \frac{d}{dt}\|e\|_2 &\le \|\dot{e}\|_2 \le \gamma\Big(L_x\|x\|_2 + L_u K\|y(t_k)\|_2\Big) \\ &\le \gamma\Big(L_x\zeta\|y\|_2 + L_u K\|y(t_k)\|_2\Big) \\ &\le \gamma L_x\zeta\|e\|_2 + \gamma(L_x\zeta + L_u K)\|y(t_k)\|_2. \end{aligned} \tag{33}$$

Assume $\|e(t_{k+1} + \Delta_{k+1})\|_2 = 0.5\delta\|y(t_k)\|_2$, where $\delta \in (\delta_2, 1)$, one can verify that Δ_{k+1} is lower bounded by

$$\varepsilon_k^+ = \frac{1}{\gamma L_x\zeta} \ln\left(\frac{L_u K + (1 + 0.5\delta)L_x\zeta}{L_u K + (1 + 0.5\delta_2)L_x\zeta}\right). \tag{34}$$

Let $\eta_k = \min\{\varepsilon_k^-, \varepsilon_k^+\}$, then one can verify that for any $\Delta_k \in [0, \eta_k]$, the inter-sampling time implicitly determined by the triggering condition (23) is lower bounded by

$$t_{k+1} - t_k \ge \tau_k = \Delta_k + \varepsilon_k > 0 \ ,$$

and the proof is completed. ■

Remark 7. For OFP(ρ) system with $\rho < 0$, the analysis on the admissible network induced delay should be the same with the analysis shown in Proposition 2. One can verify that in this case

$$\varepsilon_k^- = \frac{1}{\gamma(L_x\zeta + 2L_u K)} \ln\left(1 + \frac{\frac{\delta_1\sigma_o}{\sigma_o+1}(L_x\zeta + 2L_u K)}{L_x\zeta + (1 + \frac{\delta_1\sigma_o}{\sigma_o+1})L_u K}\right), \tag{35}$$

$$\varepsilon_k^+ = \frac{1}{\gamma L_x\zeta} \ln\left(\frac{L_u K + (1 + \frac{\delta\sigma_o}{\sigma_o+1})L_x\zeta}{L_u K + (1 + \frac{\delta_2\sigma_o}{\sigma_o+1})L_x\zeta}\right), \tag{36}$$

$$\varepsilon_k = \frac{1}{\gamma L_x\zeta} \ln\left(\frac{L_u K + (1 + \frac{\delta_2\sigma_o}{\sigma_o+1})L_x\zeta}{L_u K + (1 + \frac{\delta_1\sigma_o}{\sigma_o+1})L_x\zeta}\right). \tag{37}$$

Example 2. Consider again the OFP system studied in Example 1. We can get $L_x = 3$, $L_u = 2$, $\zeta = 1$, $\gamma = 1$. Choose $K = 3$, $\delta_1 = 0.2$, $\delta_2 = 0.8$ and $\delta = 1$, one can verify that $\varepsilon_k^- = \varepsilon_k^+ = 0.0067s$ in this case. We add randomly generated network induced delay from the sampler to the controller into the system studied in Example 1, and impose Δ_k to be uniformly distributed on the interval $[0, 0.0067s]$, the simulation result is shown in Fig.3.

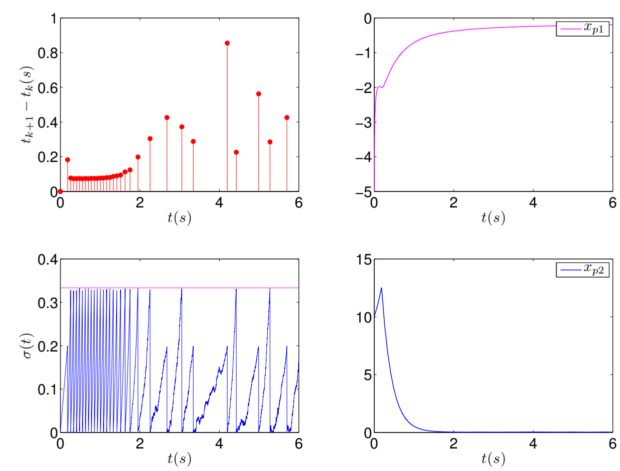

Fig. 3: simulation result of example 2

V. C

In this paper, we propose an output feedback based event-triggered control strategy for stabilization of passive and

OFP NCSs. The triggering condition and the stabilization output feedback controller are obtained based on the output feedback passivity index of the plant. Analyses on the inter-sampling and the admissible network induced delay from the sampler to the controller are derived by restricting the output to belong to a bounded sector of the state.

VI. A

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[1] E. I. Jury, *Sample-Data Control Systems*, Wiley, New York, 1958.

[2] J. R. Ragazzini and G. F. Franklin, *Sampled-Data Control Systems*, McGraw-Hill, New York, 1958.

[3] K. J. Aström and B. Wittenmark,*Computer Controlled Systems: theory and design*, Prentice-Hall, Englewood Cliffs, NJ, 1990.

[4] J. C. Willems, "Dissipative dynamical systems part I: General theory", *Archive for Rational Mechanics and Analysis*, Springer Berlin, Vol.45, No.5, pp.321-351, January, 1972.

[5] R. Sepulchre, M. Jankovic and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, 1997.

[6] K. E. Årzén, "A simple event based PID controller", *Proceedings of 14th IFAC World Congress,* pp.423-428, vol.18, 1999.

[7] K. J. Aström and B. M. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems (I)", *Proceedings of the 41st IEEE Conference on Decision and Control*, Vol.2, pp.2011-2016, 2002.

[8] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transaction on Automatic Control*, Vol.52, No.9, pp.1680-1685, September 2007.

[9] M. Mazo and P. Tabuada, "On event-triggered and self-triggered control over sensor/actuator networks" , in Proceedings of the 47th Conference on Decision and Control, 2008.

[10] X. Wang and M. Lemmon, "Event-triggering in distributed networked systems with data dropouts and delays" , in Hybrid Systems: Computation and Control, 2009.

[11] X. Wang and M. D. Lemmon, "Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability", IEEE Transactions on Automatic Control, Vol.54, No.3, pp.452-467, March 2009.

[12] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems", Vol.55, No.9, pp.2030-2042, Sept. 2010.

[13] J. P. LaSalle, "Some extensions of Liapunovs second method", *IRE Transactions on Circuit Theory*, CT-7, pp.520-527, 1960.

[14] K. J. Aström, "Event Based Control", *Analysis and Design of Nonlinear Control Systems*, Part 3, pp.127-147, 2008.

[15] W. P. M. H. Heemelsa, J. H. Sandeeb, P. P. J. Van Den Boscha, "Analysis of event-driven controllers for linear systems", International Journal of Control, Vol. 81, No. 4, pp.571-590, April 2008.

[16] P. G. Otanez, J. R. Moyne, D. M. Tilbury, "Using deadbands to reduce communication in networked control systems", in Proceedings of the American Control Conference, pp.3015-3020, 2002.

[17] E. Kofman, J. H. Braslavsky, "Level Crossing Sampling in Feedback Stabilization under Data-Rate Constraints", in Proceedings of the IEEE Conference on Decision and Control, pp.4423-4428, 2006.

[18] E. D. Sontag, "Smooth stabilization implies coprime factorization,"IEEE Transactions on Automatic Control, Vol.34, No.4, pp.435-443, Apr. 1989.

[19] N. C. S. Fah, "Input-to-state stability with respect to measurement disturbances for one-dimensional systems", ESAIM J. Control, Optim. Calculus Variations, Vol.4, pp.99-122, 1999.

[20] R. A. Freeman, "Global internal stabilizability does not imply global external stabilizability for small sensor disturbances,"IEEE Transactions on Automatic Control, Vol.40, No.12, pp.2119-2122, Dec. 1995.

[21] R. A. Freeman, P. V. Kokotovic "Robust Nonlinear Control Design: State-Space and Lyapunov Techniques", Boston, MA: Birkhäuser, 1996.

[22] Z. P. Jiang, I. M. Y. Mareels, and D. Hill, "Robust control of uncertain nonlinear systems via measurement feedback,"IEEE Transactions on Automatic Control, Vol.44, No.4, pp.807-812, Apr. 1999.