2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Frequency-Tunable LPV Controller for Narrowband Active Noise and Vibration Control

Pablo Ballesteros, Christian Bohn, *Member, IEEE*

***Abstract*—A design method for a discrete-time gain-scheduling controller for the rejection of a harmonic disturbance with known but time-varying frequency for a single-input single-output linear plant is presented. The controller is obtained through the gain scheduling design method for linear parameter-varying (LPV) systems. This results in a controller where the frequency of the disturbance is the gain-scheduling variable of the controller and closed-loop stability is guaranteed for the whole range of frequencies specified in the design. The work is motivated by active noise and vibration control (ANC/AVC), and experimental real-time results obtained with an ANC headset and an AVC test bed are presented. The design method is very straightforward and the experimental results show that the controller is well suited for narrowband ANC and AVC.**

I. Introduction

Active noise and vibration control (ANC/AVC) has attracted considerable interest from the control and signal processing community over the last decades (see, e.g., [1]). One specific application that motivates the work presented in this paper is the rejection of harmonic disturbances of time-varying, known frequencies. This control objective is applicable for environments where rotating machinery operates and the angular velocities of the machines vary, e.g., in automotive applications [2, 3] or aircrafts.

Good cancellation of harmonic (narrowband) or broadband disturbances can be achieved with adaptive feedforward control methods such as the Filtered-x LMS (FxLMS) algorithm [1]. The FxLMS algorithm often works well in practice, but may have critical issues such as convergence speed and tracking performance. Even for constant frequencies, it is difficult to predict the performance offline for such an adaptive feedforward controller, because the characteristics of the resulting controller are not known beforehand. Only approximate stability results for the FxLMS algorithm seem available to date [1, 4].

Another alternative is feedback control. For good disturbance rejection, the feedback controller has to include a model of the disturbance (this is the internal model principle [5]). This can be achieved through an observer-based controller, where the observer not only estimates the plant states but also the states of a disturbance model. The estimated states of the disturbance model can then be used to cancel the disturbance [2, 3, 6]. For harmonic disturbances, this is equivalent to using an adaptive (or rather, scheduled) version of the "spectral observer" [7]. Since the disturbance model is time-varying, also a time-varying observer is required. The time-varying observer gain can either be calculated on-line (e.g. using a Kalman filter) or a set of observer gains can be calculated offline for a set of fixed frequencies [6]. In the latter case, the current observer gain is calculated through interpolation [6] or by switching between different gains [2, 3]. In the first case, stability can be guaranteed but the computational effort is increased (due to the on-line calculation of the covariance matrix). In the second case the computational effort is lower (table lookup and interpolation operations) but stability is not guaranteed. Another alternative is to use observer-based feedback control but to schedule the state feedback gain instead of the observer [8-10].

The work presented here is motivated by [11, 12], where an LPV gain-scheduling approach is proposed for the rejection of a harmonic disturbance. In [11, 12], the controller design is carried out in continuous time and a polytopic LPV description is used. For practical real-time implementation, the controller has to be implemented in discrete time. Also, in active noise and vibration control, the plant model is often obtained through system identification. This usually gives a discrete-time plant model. It is therefore most natural to carry out the whole design in discrete time. If a continuous-time controller is computed, this controller would have to be discretized. Since the controller is time-varying, this discretization would have to be carried out at each sampling instant. Particularly in LPV gain scheduling control, an approximate discretization is proposed [13]. However, this leads to a distortion of the frequency scale that shifts the controller poles to other frequencies. For the rejection of harmonic disturbances, it is required that the frequencies of the controller poles match the disturbance frequencies exactly. Therefore, a frequency distortion cannot be tolerated for controllers that are designed to suppress harmonic disturbances. Discretization methods that maintain the frequencies of the poles such as step invariance (zero-order-hold) or the matched pole-zero method [14] are computationally too expensive (calculation of a matrix exponential, calculations of poles at each sampling instant). It is therefore not surprising that the continuous-time design methods [6, 9, 11, 12] are only tested in simulations (with a very simple two-mass system as a plant).

P. Ballesteros and C. Bohn are with the Institute of Electrical Information Technology, Clausthal University of Technology, Leibnizstr. 28, 38678 Clausthal–Zellerfeld, Germany (e-mail: ballesteros@iei.tu-clausthal.de, bohn@iei.tu-clausthal.de).

978-1-4577-0079-8/11/$26.00 ©2011 AACC

In this paper, the LPV gain-scheduling controller is calculated directly in discrete time and the parameter variations are described in linear fractional transformation (LFT) form. The resulting controller is evaluated experimentally on an ANC headset and an AVC test bed.

Advantages of the LPV approach are that the resulting controller structure is very simple, the computational load is not too high and that the stability of the closed-loop is guaranteed even for arbitrarily fast changes of the scheduling variable.

The remainder of this paper is organized as follows. In Sec. II, LPV gain scheduling control is briefly reviewed. The design procedure to achieve harmonic disturbance rejection is described in Sec. III. In Sec. IV, the experimental set-ups are described and experimental results are presented. Conclusions are given in Sec. V.

II. LPV CONTROL DESIGN

An LPV system in LFT form is shown in Fig. 1. It consists of a generalized plant $G(z)$ that includes input and output weighting functions and a parametric uncertainty block θ. For this general system, a gain-scheduling controller can be calculated following the method presented in [15]. In this method, two sets of linear matrix inequalities (LMIs) are solved. The first set of LMIs determines the feasibility of the problem which means that a bound on the control system performance in the sense of the H_∞ norm can be satisfied. With the second set of LMIs, the controller matrices are calculated from the solution of the first set of LMIs.

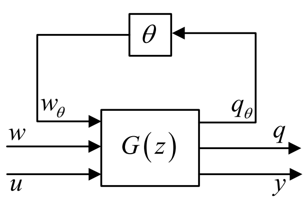

Fig. 1. General LPV system.

As a result of applying this control design method, the gain-scheduling control structure of Fig. 2 is obtained. The time-varying plant parameter is directly used as the gain-scheduling parameter of the controller. This approach is used in this paper for the rejection of a time-varying frequency. The gain-scheduling parameter θ is calculated from the disturbance frequency. This is described in the following section. An alternative approach is to use a polytopic description of the LPV model. This is done in [6, 8-12].

III. DESIGN PROCEDURE

In this section, it is described how the design method outlined in Sec. II can be applied for the rejection of harmonic disturbances. For this, it is necessary to combine the plant and a disturbance model and transform this to the LFT-LPV form shown in Fig. 1.

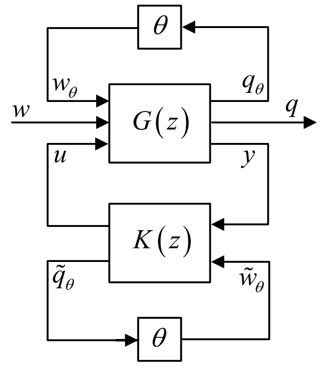

Fig. 2. LPV gain-scheduling control structure.

A harmonic disturbance with frequency f can be modeled as the output y_{d} of the state space model

$$\boldsymbol{x}_{\text{d},\,k+1} = \boldsymbol{A}_{\text{d}}(f)\,\boldsymbol{x}_{\text{d},\,k} + \boldsymbol{B}_{\text{d},\,w} w_{\text{d},\,k}\,, \tag{1}$$

$$y_{\text{d},\,k} = \boldsymbol{C}_{\text{d},\,y}\boldsymbol{x}_{\text{d},\,k}\,, \tag{2}$$

with

$$\boldsymbol{A}_{\text{d}}(f) = \begin{bmatrix} 0 & 1 \\ -1 & a(f) \end{bmatrix}, \boldsymbol{B}_{\text{d},\,w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{C}_{\text{d},\,y} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{3}$$

and

$$a(f) = 2\cos(2\pi f\, T)\,. \tag{4}$$

The frequency varies between f_{min} and f_{max}, therefore, a will vary between a_{min} and a_{max} and can be written as

$$a = a_0 + a_1\theta\,, \tag{5}$$

where a_0 and a_1 are real constants and $\theta \in [-1,\, 1]$. This model can be expressed in LFT-LPV form as

$$\boldsymbol{x}_{\text{d},\,k+1} = \boldsymbol{A}_{\text{d},\,0}\boldsymbol{x}_{\text{d},\,k} + \boldsymbol{B}_{\text{d},\,\theta} w_{\theta,\,k} + \boldsymbol{B}_{\text{d},\,w} w_{\text{d},\,k}\,, \tag{6}$$

$$q_{\theta,\,k} = \boldsymbol{C}_{\text{d},\,\theta}\boldsymbol{x}_{\text{d},\,k}\,, \tag{7}$$

$$y_{\text{d},\,k} = \boldsymbol{C}_{\text{d},\,y}\boldsymbol{x}_{\text{d},\,k}\,, \tag{8}$$

$$w_{\theta,\,k} = \theta q_{\theta,\,k}\,, \tag{9}$$

with

$$\boldsymbol{A}_{\text{d},\,0} = \begin{bmatrix} 0 & 1 \\ -1 & a_0 \end{bmatrix}, \boldsymbol{B}_{\text{d},\,\theta} = \begin{bmatrix} 0 \\ a_1 \end{bmatrix}, \boldsymbol{C}_{\text{d},\,\theta} = \begin{bmatrix} 0 & 1 \end{bmatrix}. \tag{10}$$

In the examples considered below, the frequency varies between 80 Hz and 130 Hz and the sampling time is 1 ms. For this case, the matrices for the disturbance model are

$$\boldsymbol{A}_{\text{d},\,0} = \begin{bmatrix} 0 & 1 \\ -1 & 1.5607 \end{bmatrix}, \boldsymbol{B}_{\text{d},\,\theta} = \begin{bmatrix} 0 \\ 0.2130 \end{bmatrix}, \tag{11}$$

$$\boldsymbol{B}_{\text{d},\,w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{C}_{\text{d},\,\theta} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \boldsymbol{C}_{\text{d},\,y} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{12}$$

It is assumed that the disturbance enters at the plant input. The state-space model of the plant is

$$\boldsymbol{x}_{\text{p},\,k+1} = \boldsymbol{A}_{\text{p}}\boldsymbol{x}_{\text{p},\,k} + \boldsymbol{B}_{\text{p}} u_{\text{p},\,k} + \boldsymbol{B}_{\text{p}} y_{\text{d},\,k}\,, \tag{13}$$

$$y_{p,\,k+1} = \boldsymbol{C}_p \boldsymbol{x}_{p,\,k} \,. \tag{14}$$

The plant and the disturbance model can then be combined as shown in Fig. 3, where $G_p(z)$ and $G_d(z)$ are the transfer functions of the plant and the time-invariant part of the disturbance model, respectively. The state space model of this combined system is

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}_u u_{p,\,k} + \boldsymbol{B}_\theta w_{\theta,\,k} + \boldsymbol{B}_w w_{d,\,k} \,, \tag{15}$$

$$y_k = \boldsymbol{C}\boldsymbol{x}_k \,, \tag{16}$$

with

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_p & \boldsymbol{B}_p \boldsymbol{C}_{d,\,y} \\ \boldsymbol{0} & \boldsymbol{A}_{d,\,0} \end{bmatrix}, \boldsymbol{B}_u = \begin{bmatrix} \boldsymbol{B}_p \\ \boldsymbol{0} \end{bmatrix}, \boldsymbol{B}_\theta = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{d,\,\theta} \end{bmatrix}, \tag{17}$$

$$\boldsymbol{B}_w = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{d,\,w} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_p & \boldsymbol{0} \end{bmatrix}. \tag{18}$$

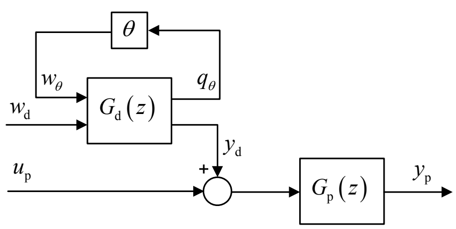

Fig. 3. Augmented model including the disturbance.

To obtain the generalized plant $G(z)$, additional weighting functions are included. A first-order low pass filter is used to weight the output signal, and constant gains are used as weights for the disturbance input w and the control signal u_p. The overall structure of the LPV control system is shown in Fig. 4 and corresponds to the structure of Fig. 1.

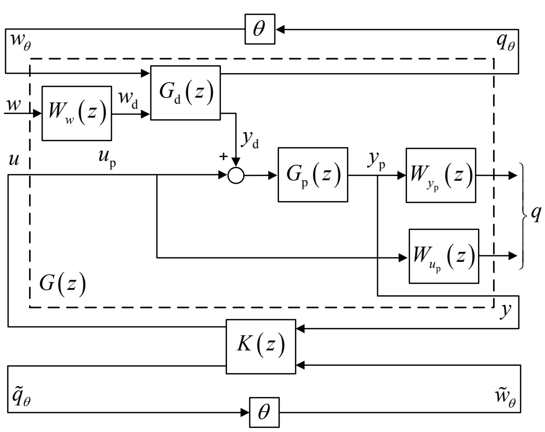

Fig. 4. LPV gain-scheduling structure with the model of the harmonic disturbance and the weighting functions.

IV. Experimental results

The controller obtained from the design procedure outlined in Sec. II and Sec. III is experimentally validated on a Sennheiser PXC 300 headset and on an AVC test bed.

A. Active Noise Control

The ANC headset has two microphones placed in the ear cups of the headset (Fig. 5). The aim of the ANC system is to cancel a harmonic disturbance generated by an external loudspeaker. An anti-aliasing filter is used for the output signal and a reconstruction filter for the control input. The control algorithm is implemented on a rapid prototyping unit (dSpace MicroAutoBox).

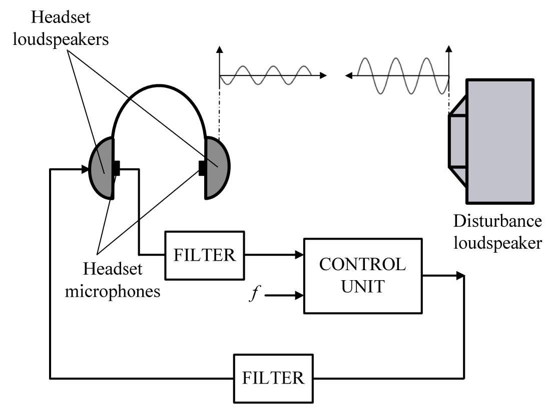

Fig. 5. ANC system.

The transfer function between the output of the control unit and the input to the control unit is the plant $G_p(z)$ (usually called secondary path in the AVC/ANC literature [1]). To design the control algorithm, this transfer function is required. To obtain $G_p(z)$, the system is excited with a multisine test signal and the output is recorded. The transfer function can be estimated using a standard black-box technique (oe). All usual methods (arx, oe, n4sid) resulted in models that were suitable for control design. If a transfer function model is identified, this is converted to a state-space description for the controller design. The identified transfer functions for the left and the right sides of the headset were almost identical, therefore, the same control algorithm was implemented on both sides. Both control algorithms work independently from each other. The experimental results shown are for the right side.

The control algorithm is designed for a harmonic disturbance in the frequency range of 80 Hz to 130 Hz. A sampling frequency of 1 kHz was chosen such that the Nyquist frequency of 500 Hz is well above the highest disturbance frequency. The identified plant model is of 12th order and the controller is of 15th order.

Design and experimental results are shown in Figs. 6-10 for the ANC system. Figs. 6 and 7 show the amplitude frequency responses of the open-loop and the closed-loop disturbance transfer functions for fixed frequencies of 80 Hz and 120 Hz for the ANC system. The resulting closed loop transfer functions represent notch filters with a zero at the disturbance frequency. Due to Bode's sensitivity integral, the disturbance attenuation at the disturbance frequency leads to some disturbance amplification for frequencies below and above the disturbance frequency (the "waterbed" or "spillover" effect [16]). Whether this is tolerable in a practical application depends on the spectral content of the background noise. Real-time results for the rejection of constant disturbances of 80 Hz and 120 Hz for the ANC system are shown in Figs. 8 and 9. As expected from the frequency responses, excellent disturbance rejection is achieved.

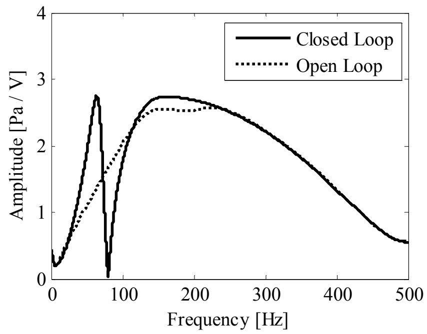

Fig. 6. Open-loop and closed-loop amplitude frequency responses for the ANC system; the closed-loop transfer function is calculated for a fixed disturbance frequencie of 80 Hz.

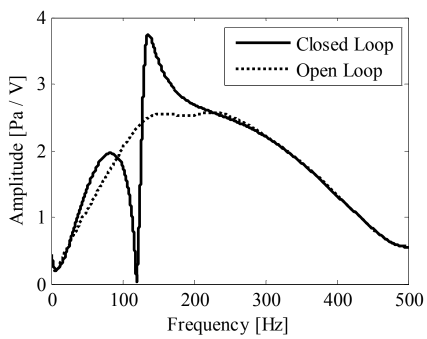

Fig. 7. Open-loop and closed-loop amplitude frequency responses for the ANC system; the closed-loop transfer function is calculated for a fixed disturbance frequencie of 120 Hz.

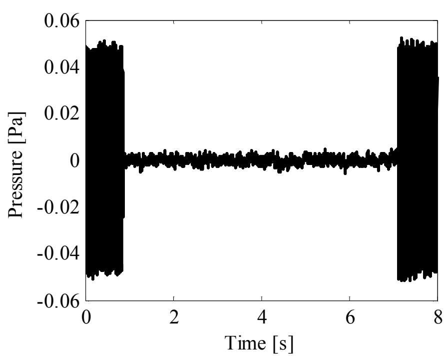

Fig. 8. Pressure measured for a fixed disturbance frequency of 80 Hz; the control sequence is off/on/off.

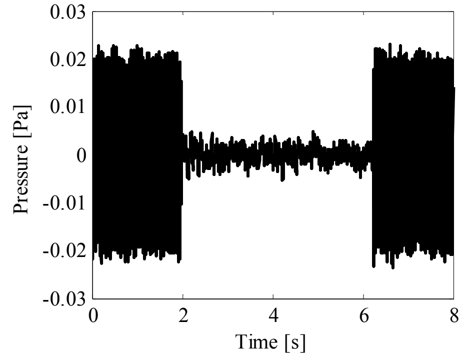

Fig. 9. Pressure measured for a fixed disturbance frequency of 120 Hz; the control sequence is off/on/off.

In Fig. 10 the behavior for a disturbance with a time-varying frequency is demonstrated. The disturbance is a sine sweep with a linearly increasing frequency from 80 Hz to 120 Hz in 10 seconds. The control systems also performed well for a sweep of 5 s duration. For even faster sweeps, the system remained stable but did not achieve satisfactory disturbance attenuation.

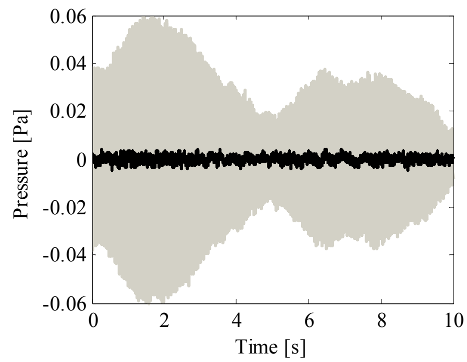

Fig. 10. Pressure measured in open loop (gray) and closed loop (black) for a sweep of 10 s duration, where the disturbance frequency increases linearly from 80 Hz to 120 Hz.

B. Active Vibration Control

The AVC test bed is schematically shown in Fig. 11. Two shakers (inertia mass actuators) are attached to a steel cantilever beam. One shaker acts as the disturbance source and the other shaker is used to counteract this disturbance. An accelerometer is used to measure the output signal.

A multisine test signal is used to identify the transfer function of the system. A sampling frequency of 1 kHz was chosen. The identified system is of 12th order and the controller of 15th order.

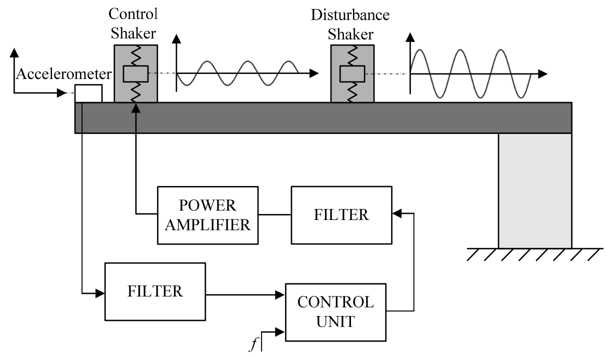

Fig. 11. AVC test bed.

Figs. 12 and 13 show the amplitude frequency responses of the open-loop and the closed-loop disturbance transfer functions for fixed frequencies of 90 Hz and 130 Hz for the AVC system. Real-time results for the rejection of constant disturbances of 90 Hz and 130 Hz are shown in Figs. 14 and 15.

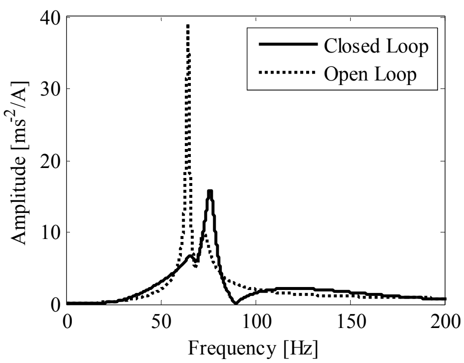

Fig. 12. Open-loop and closed-loop amplitude frequency responses for the AVC system; the closed loop transfer function is calculated for a fixed disturbance frequency of 90 Hz.

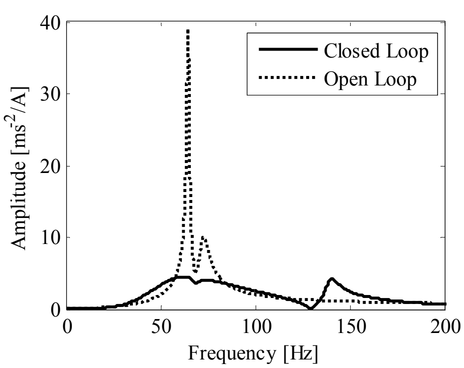

Fig. 13. Open-loop and closed-loop amplitude frequency responses for the AVC system; the closed loop transfer function is calculated for a fixed disturbance frequency of 130 Hz.

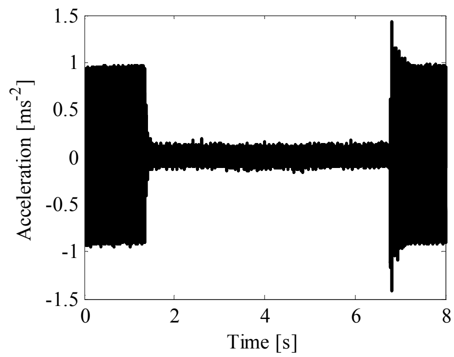

Fig. 14. Acceleration measured for a fixed disturbance frequency of 90; the control sequence is off/on/off.

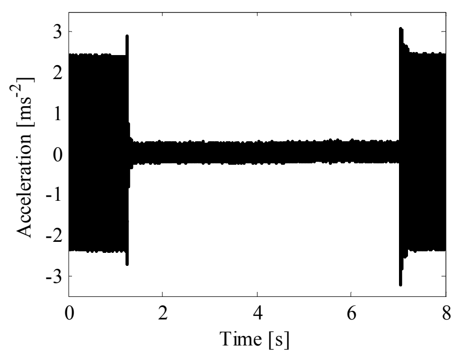

Fig. 15. Acceleration measured for a fixed disturbance frequency of 130; the control sequence is off/on/off.

In Fig. 16 the behavior for a disturbance with a time-varying frequency is demonstrated. The disturbance is a sine sweep with a linearly increasing frequency from 90 Hz to 130 Hz, respectively, in 10 seconds. The controller achieves an excellent disturbance rejection for constant and for varying disturbance frequencies.

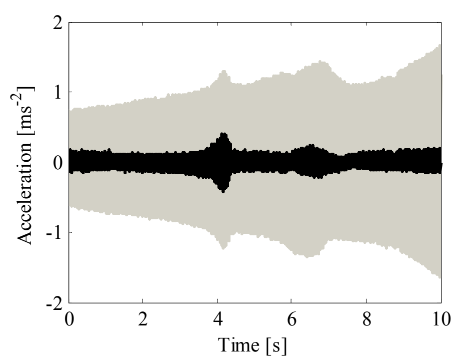

Fig. 16. Acceleration measured in open loop (gray) and closed loop (black) for a sweep of 10 s of duration, where the disturbance frequency increases linearly from 90 Hz to 130 Hz.

V. Conclusions

A discrete-time LPV gain-scheduling controller is presented as an approach to cancel a harmonic disturbance with time-varying, but known (measured) frequency based on the LFT-LPV gain scheduling method of [15]. To the best of the authors' knowledge, this design method has not been used for this control problem before.

The controller is designed is carried out by augmenting the plant with a weighting function that represents a harmonic disturbance and using the frequency as the gain scheduling variable. In this paper, only a single-frequency disturbance is considered. The method is used in [17] and [18] for the rejection of two and six frequencies, respectively.

The design in discrete-time avoids problems with controller implementation (discretization at each sampling instant or frequency distortion resulting from approximate discretization) and results in a controller that can be readily implemented. Experimental results demonstrate that excellent disturbance rejection is achieved even when the disturbance frequency changes fairly rapidly.

A major advantage of the algorithm seems to be that stability of the closed-loop system is guaranteed for all values of the scheduling variable (frequency) in the range specified in the design process. This is a favorable result compared to the usual adaptive filtering methods. Compared to approaches in which an observer-based controller is used and the observer gain is calculated on-line to guarantee stability [6], the computational effort of the LPV controller is much lower. The computational complexity of the LPV controller is about the same as for an observer-based controller where the observer gain is switched [2, 3] or interpolated [6], but these approaches do not guarantee stability.

The fact that the closed-loop system is stable would also allow for an application without measurements of the frequency. For example, an ANC headset is possible where the frequency to be rejected can be readjusted manually by the user to the dominant frequency that is present in a noisy environment.

Future work will focus on the rejection of multiple harmonics, alternative LPV controller structures and multi-input multi-output systems.

Acknowledgements

The authors wish to thank Sennheiser corporation for providing the active noise control headset and ContiTech Vibration Control corporation for providing actuators and sensors for the AVC test bed. Helpful discussions with Dr. Foudhaili (Sennheiser), Dr. Karkosch and Mr. Marienfeld (ContiTech Vibration Control) about ANC/AVC systems from an industrial perspective were also greatly appreciated.

References

[1] S. M. Kuo and D. R. Morgan, *Active noise control systems: Algorithms and DSP implementations*. New York: Wiley, 1996.

[2] C. Bohn, A. Cortabarria, V. Härtel and K. Kowalczyk. "Disturbance-observer-based active control of engine-induced vibrations in automotive vehicles," *Proceedings of the SPIE's 10th Annual International Symposium on Smart Structures and Materials*, 2003, Paper No. 5049-68.

[3] C. Bohn, A. Cortabarria, V. Härtel and K. Kowalczyk, "Active control of engine-induced vibrations in automotive vehicles using disturbance observer gain scheduling," *Control Engineering Practice*, vol. 12, pp. 1029-1039, Aug. 2004.

[4] P. L. Feintuch, N. J. Bershad and A. K. Lo, "A frequency-domain model for filtered LMS algorithms - Stability analysis, design, and elimination of the training mode," *IEEE Transactions on Signal Processing*, vol. 41, pp. 1518-1531, Apr. 1993.

[5] B. Francis and W. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, pp. 457-465, Sep. 1976.

[6] C. E. Kinney and R. A. de Callafon, "A comparison of fixed point designs and time-varying observers for scheduling repetitive controllers," *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007, pp. 2844-2849.

[7] G. H. Hostetter, "Fourier-analysis using spectral observers," *Proceedings of the IEEE*, vol. 68, pp. 284-285, Feb. 1980.

[8] C. E. Kinney and R. A. de Callafon, "An adaptive internal model-based controller for periodic disturbance rejection," *Proceedings of the 14th IFAC Symposium on System Identification*, 2006, pp. 273-278.

[9] C. E. Kinney and R. A. de Callafon, "Scheduling control for periodic disturbance attenuation," *Proceedings of the American Control Conference*, 2006, pp. 4788-4793.

[10] W. Heins, P. Ballesteros and C. Bohn, "Gain-scheduled state-feedback control for active cancellation of multisine disturbances with time-varying frequencies," *Proceedings of the 10th MARDiH Conference on Active Noise and Vibration Control Methods*, 2011, accepted for publication.

[11] H. Du and X. Shi, "Gain-scheduled H_∞ control for use in vibration suppression of system with harmonic excitation," *Proceedings of the American Control Conference*, 2002, pp. 4668-4669.

[12] H. Du, L. Zhang and X. Shi, "LPV technique for the rejection of sinusoidal disturbance with time-varying frequency," *IEE Proceedings on Control Theory and Applications*, vol. 150, pp. 132-138, Mar. 2003.

[13] P. Apkarian, "On the discretization of LMI-synthesized linear parameter-varying controllers," *Automatica*, vol. 33, pp. 655-661, April 1997.

[14] Ogata, K. *Discrete-time control systems*. 2nd ed. Englewood Cliffs: Prentice Hall, 1995.

[15] P. Apkarian and P. Gahinet, "A convex characterization of gain-scheduled H_∞ controllers," *IEEE Transactions on Automatic Control*, vol. 40, pp. 853-864, May 1995.

[16] J. H. Hong and D. S. Bernstein, "Bode integral constraints, colocation, and spillover in active noise and vibration control," *IEEE Transactions on Control Systems Technology*, vol. 6, pp. 111-120, Jan. 1998.

[17] P. Ballesteros and C. Bohn, "Disturbance rejection through LPV gain-scheduling control with application to active noise cancellation," *Proceedings of the 18th IFAC World Congress*, 2011, accepted for publication.

[18] X. Shu, P. Ballesteros and C. Bohn, "Active vibration control for harmonic disturbances with time-varying frequencies through LPV gain scheduling," *Proceedings of the 23rd Chinese Control and Decision Conference*, 2011, accepted for publication.