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The Matching Coefficients PID Controller

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Abstract— **The problem of designing a PID controller is posed in a setting where a selected reference system presents the design requirements. This leads to a simple problem of equating coefficients of like powers in polynomials originating in the reference system transfer function, the transfer function of the system to be controlled as well as the PID coefficients. Effectively, an overdetermined system of equations in the PID coefficients results, which is solved in the minimum least squares sense. We refer to this controller as the Matching Coefficients PID (MC PID). The computation is very simple involving only basic high school mathematics. While there is no explicit criterion for a selection of a reference system that will guarantee closed loop stability, systematic approaches can be designed for modifying the reference system in these cases.**

I. INTRODUCTION

PID (Proportional, Integral, Differential) controllers for single input single output (SISO) as well as multiple input multiple output (MIMO) systems are the most common controllers in industry today. Thus, they are a very interesting and popular research topic, with many papers published every year. Dominant pole placement is guaranteed with PID controllers in [1], using a modified root locus method and a modified Nyquist plot for systems with time delays. Naturally, the guaranteed dominant pole placement may not guarantee specifications in terms of overshoot or settling time due to the effect of closed loop zeros. However, the methods are simple and easy to work with. Processes are approximated by a simple second order transfer function without regular zeros but with a time delay in [2]. The PID zeros are then used to cancel the poles of the second order system, leaving just the time delay, the PID integrator and a gain to be adjusted to set the closed loop poles. PID tuning methods have been popular, see e.g. [3] where the tuning is based on a P controlled step experiment. Another tuning method for high-order oscillatory systems is reported in [4]. A MIMO system is decomposed into equivalent single loops for design of multiloop PI/PID controllers in [5], where various other MIMO PID methods are also briefly discussed. An approximation is made such that each controller can be designed without knowledge of controllers in other loops.

Research on transfer function responses for continuous as well as discrete time [6],[7] has lead to several research topics. The general problem on how to optimize zero locations such as to get a system to track a reference system, is reported in [8] and [9]. Optimized zero locations are then applied in model reduction in [10]. An optimized PID controller has also been developed, tracking a given open loop reference system that effectively includes the design requirements for the resulting closed loop system. Then the squared difference between the open loop impulse or step response of the system we want to control and the reference system's open loop is minimized. The zero locations of the PID controllers or generalized PID controllers with more than two zeros are then optimized to get the best tracking of the design requirements, see [11]. Stability of the closed loop is not automatically inherited, however, by Parseval's theorem the difference in the frequency responses of the controlled system and the stable reference system is bounded, thus required phase and gain margins may be achieved by a simple gain reduction.

Recently, in [12], the optimized PID controller for a SISO system was expanded to an optimized MIMO PID controller. Most recently, this approach has been extended by adding an outer iteration procedure to the optimizer that allows the user to specify more precisely the nature of the design requirements for the resulting closed loop controlled system [13].

In this paper, we view the problem of PID design in yet another way, simply by equating coefficients of like powers in polynomials coming from the reference system transfer function, the transfer function of the system to be controlled as well as the PID coefficients. We state the problem in Section II. We then present the MC PID in continuous and discrete time in Sections III and IV, respectively, including examples. Finally, conclusions are discussed in Section V.

II. PROBLEM STATEMENT

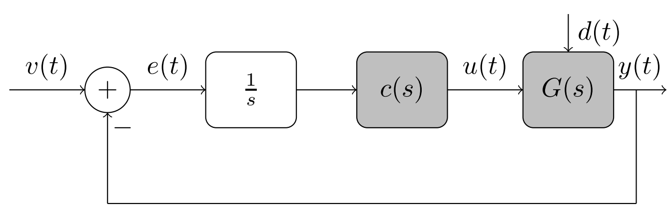

Fig. 1. A closed loop controlled SISO system.

Our approach is to control the closed loop system in Figure 1 in such a way that it behaves like the closed loop reference system in Figure 2. We are dealing with a SISO system where the transfer function for the system $G(s)$ is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b(s)}{a(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} \quad (1)$$

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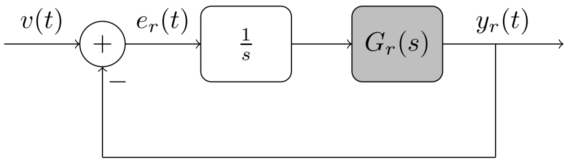

Fig. 2. A SISO reference system.

corresponding to the standard differential equation

$$\begin{aligned} &a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \cdots + a_0 y(t) \\ &= b_m u^{(m)}(t) + b_{m-1} u^{(m-1)}(t) + \cdots + b_0 u(t), \end{aligned} \tag{2}$$

where $a_n = 1$. The transfer function of a standard PID controller is given by

$$G_c(s) = \frac{c(s)}{s} = \frac{K_D s^2 + K_P s + K_I}{s}. \tag{3}$$

The closed loop transfer function is then given by

$$\frac{Y(s)}{V(s)} = \frac{\frac{c(s)b(s)}{sa(s)}}{1 + \frac{c(s)b(s)}{sa(s)}} = \frac{c(s)b(s)}{sa(s) + c(s)b(s)}. \tag{4}$$

The transfer function for the reference system is chosen such as to represent design requirements, the general form being

$$G_r(s) = \frac{b_r(s)}{sa_r(s)}. \tag{5}$$

The integrator assures a PID like behavior in closed loop in terms of input signal tracking and disturbance rejection. The closed loop of the reference system is given by

$$\frac{Y_r(s)}{V(s)} = \frac{\frac{b_r(s)}{sa_r(s)}}{1 + \frac{b_r(s)}{sa_r(s)}} = \frac{b_r(s)}{sa_r(s) + b_r(s)}. \tag{6}$$

We now wish to choose $c(s)$ such that the deviation between the closed loop controlled system and the closed loop reference system is in some sense minimal, i.e.

$$\frac{c(s)b(s)}{sa(s) + c(s)b(s)} \approx \frac{b_r(s)}{sa_r(s) + b_r(s)}. \tag{7}$$

Then

$$c(s)b(s)(sa_r(s) + b_r(s)) \approx b_r(s)(sa(s) + c(s)b(s)) \tag{8}$$

and hence

$$c(s)b(s)a_r(s) \approx b_r(s)a(s). \tag{9}$$

By matching the polynomial coefficients on each side of the expression, we obtain an overdetermined system of equations in the unknown coefficients of $c(s)$, which can be solved for by the method of least squares, e.g., by using Matlab's backslash command.

It is worth noting that the same expression is obtained by considering the deviation between the controlled open loop system and the open loop reference system, both with or without the integrator, namely

$$\frac{c(s)b(s)}{a(s)} \approx \frac{b_r(s)}{a_r(s)}. \tag{10}$$

Thus, the least squares solution of the open loop (10) is the same as the least squares solution of the closed loop (7). While this fact is no guarantee of stability, and stability can indeed be lost for highly underdamped reference systems as well as for close to unstable plants, it should be noted that the loss of stability is easily tested for and that systematic approaches can be designed for modifying the reference system in these cases, e.g., in a similar vein as proposed in [13].

Remark 1: A simple choice of the reference system is given by

$$G_r(s) = \frac{\omega_r^2}{s(s + 2\zeta_r \omega_r)} \tag{11}$$

which results in the the standard 2nd order system in closed loop

$$G_r^{CL}(s) = \frac{\omega_r^2}{s^2 + 2\zeta_r \omega_r s + \omega_r^2}. \tag{12}$$

Note that the reference system essentially states the design requirements of the system and may be chosen of a higher order if desirable. The derivations in this paper assume this simple choice of reference system. If, e.g., a settling time of approximately 4 seconds is required and less than 5% overshoot, one would choose $\omega_r = \sqrt{2}$ and $\zeta_r = 1/\sqrt{2}$ for the reference system.

III. Matching coefficients PID - continuous time

Returning to Eq. (9) and considering the simple reference system in Remark 1, we are dealing with an overdetermined system of equations of the form

$$\begin{aligned} &\left(K_D s^2 + K_P s + K_I\right)\left(b_m s^m + \cdots + b_1 s + b_0\right)\left(s + 2\zeta_r \omega_r\right) \\ &\approx \omega_r^2 \left(s^n + a_{n-1} s^{s-1} + \cdots + a_1 s + a_0\right). \end{aligned} \tag{13}$$

We further assume $m = n - 3$ such as to make the degrees of the polynomials on each side of the $\approx$ sign equal. If $m < n - 3$, we simply put the corresponding b-coefficients equal to zero. If $m > n - 3$, we must add high frequency dummy poles to the a-polynomial of the form $\frac{1}{s/N+1}$. We now define

$$\begin{aligned} &\begin{bmatrix} \phi_0 & \phi_1 & \cdots & \phi_{n-2} \end{bmatrix} \\ &= \begin{bmatrix} 2\zeta\omega_r & 1 \end{bmatrix} \begin{bmatrix} b_0 & b_1 & \cdots & 0 \\ 0 & b_0 & b_1 & \cdots \end{bmatrix} \end{aligned} \tag{14}$$

$$\phi_{3\times(n+1)} = \begin{bmatrix} \phi_0 & \phi_1 & \cdots & \phi_{n-2} & 0 & 0 \\ 0 & \phi_0 & \phi_1 & \cdots & \phi_{n-2} & 0 \\ 0 & 0 & \phi_0 & \phi_1 & \cdots & \phi_{n-2} \end{bmatrix} \tag{15}$$

and

$$\mathcal{C} = \begin{bmatrix} K_I & K_P & K_D \end{bmatrix}^T. \tag{16}$$

We thus have to find the least squares solution to the system

$$\phi^T \mathcal{C} \approx \omega_r^2 \begin{bmatrix} a_0 & a_1 & \cdots & a_{n-1} & 1 \end{bmatrix}^T, \tag{17}$$

which may, e.g., simply be solved by the backslash command in Matlab.

Remark 2: Disturbance rejection can in general be bad and in this case a slightly more complicated controller than a simple PID may be needed in order to reject a disturbance, e.g., into highly underdamped poles. A fairly simple way to deal with such a disturbance is to try to estimate its effects from the control signal $u(t)$ and measurements of the output signal $y(t)$ and use the estimation to correct the effect of the disturbance, i.e., we build a kind of a disturbance observer. We have

$$Y(s) = G_{p2}(s)D(s) + G_p(s)U(s)$$

where $G_p(s) = G_{p1}(s)G_{p2}(s)$ and the disturbance enters right after $G_{p1}(s)$ and right before $G_{p2}(s)$. We need to inject the disturbance correction right after the PID to become a part of the control signal entering the plant. In effect we need to produce

$$-\frac{\hat{D}(s)}{G_{p1}(s)} = A(U(s) - \frac{Y(s)}{G_p(s)})$$

where A is a constant to enhance the disturbance rejection. So we add an inner feedback loop $-\frac{A}{G_p(s)}$ from $Y(s)$ to the output of $G_c(s)$ and then we add $AU(s)$ to form $U(s)$ or equivalently multiply by $\frac{1}{1-A}$. This closed loop setup has the transfer function

$$\frac{Y(s)}{V(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

not affected by A. The transfer function from the disturbance to the output becomes

$$\frac{Y(s)}{D(s)} = (1 - A)\frac{G_{p2}(s)}{1 + G_c(s)G_p(s)}.$$

The open loop transfer function is still

$$\frac{Y(s)}{E(s)} = G_c(s)G_p(s)$$

so the root locus is not affected. Note that care must be taken in implementing $1/G_p(s)$ by padding it with high frequency poles of the form $1/(s/N+1)^\alpha$ as is done in a regular PID controller, where $\alpha = n - m$ denotes the relative degree of the plant.

Example 1: We first consider a highly underdamped 3rd order benchmark system from [14], with $\zeta = 0.2$ and $\omega_0 = 1$ in the underdamped part and a third pole at -2, given by

$$\frac{b(s)}{a(s)} = \frac{2}{(s+2)}\frac{1}{(s^2 + 0.2s + 1)}. \tag{18}$$

We choose a reference system

$$\frac{b_r(s)}{sa_r(s)} = \frac{1}{s(s+1)} \tag{19}$$

leading to a closed loop system having a damping coefficient of $\zeta_r = 0.5$ and $\omega_r = 1$. This results in PID coefficients given by

$$\begin{bmatrix} K_I & K_P & K_D \end{bmatrix} = \begin{bmatrix} 0.775 & 0.15 & 0.725 \end{bmatrix}. \tag{20}$$

The system is subject to a unit step input at time $t = 0$ and to a unit step disturbance at time $t = 20$. The disturbance hits the system after the $2/(s+2)$ part and before the $1/(s^2 + 0.2s + 1)$ part. The resulting closed loop control signal and output and the corresponding root locus are shown in Figure 3. The maximum control signal of 14.65 occurs in the very beginning. The PID controlled step response has a slightly smaller overshoot than the closed loop reference system and a slight continued oscillation. The closed loop poles of the reference system are given by $-0.5 \pm j0.87$. The closed loop poles of the PID controlled system are given by $-1.03 \pm j0.695, -0.074 \pm j1.002$, all maintaining similar imaginary parts as the reference system. All closed loop poles are labeled by $*$ on the corresponding root locus.

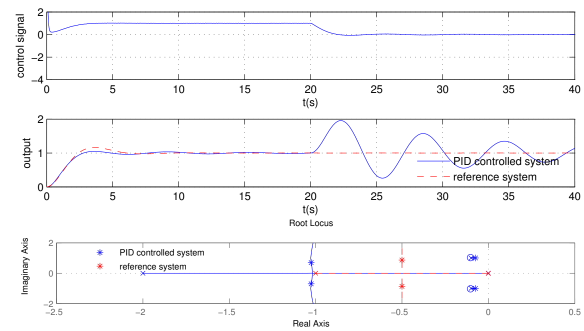

Fig. 3. Example 1. MC PID for a highly underdamped system - $\zeta_r = 0.5$ and $\omega_r = 1$.

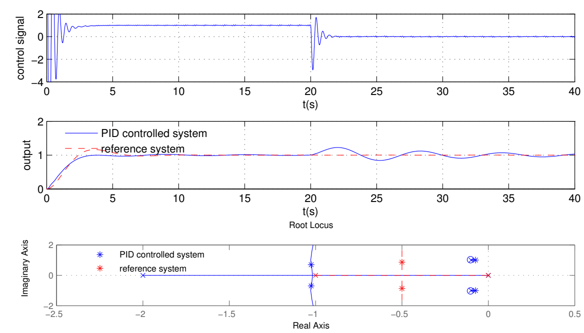

Fig. 4. Example 1. MC PID for a highly underdamped system with disturbance rejection - $A = 0.75$, $\zeta_r = 0.5$ and $\omega_r = 1$.

As noted, stability for the closed loop is not guaranteed, in particular a very underdamped reference system of $\zeta_r = 0.1$ results in an unstable PID controlled system. For this example, though, stability seems to be maintained for stable system poles.

Similar to [2], when the real system pole is the same as the reference system pole, the MC PID simply cancels the remaining system poles completely in a third order system with a constant numerator. Even in the case of a full cancellation of the underdamped system poles, the disturbance rejection is naturally still very bad, as the disturbance travels through the underdamped poles, before the cancelling zeros of the PID hit it.

Implementing the approach in Remark 2 with $A = 0.75$, but with the same reference system, leads to the results shown in Fig. 4. The disturbance is roughly $1/4$ of what it was before, the minimum control signal in the beginning

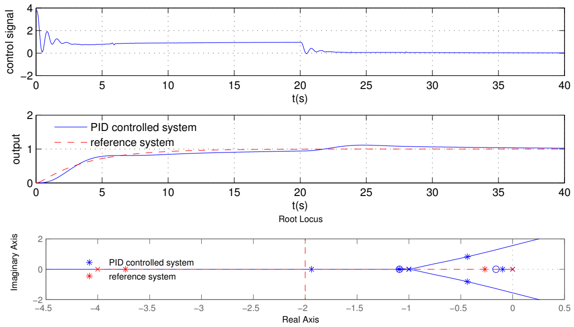

Fig. 5. Example 2. MC PID for a system with multiple equal poles with disturbance rejection - $A = 2/3$, $\zeta_r = 2$ and $\omega_r = 1$.

of the disturbance is -2.95. The activity in the control signal at the onset of the input step is naturally increased, having a first maximum of 58.6 and a first minimum of -15.27. Moving the $K_D s/(s/20+1)$ into the feedback from $Y(s)$ decreased the initial input activity significantly, but at a too high cost in the tracking of the reference system. Further, $1/G_p(s)$ was implemented by padding it with $1/(s/10+1)^3$.

Example 2: We now consider another challenging benchmark system from [14], given by

$$\frac{b(s)}{a(s)} = \frac{1}{(s+1)^4}. \tag{21}$$

We choose a reference system

$$\frac{b_r(s)}{s a_r(s)} = \frac{1}{s(s+4)} \tag{22}$$

leading to a closed loop system having a damping coefficient of $\zeta_r = 2$ and $\omega_r = 1$. This results in MC PID coefficients given by

$$\begin{bmatrix} K_I & K_P & K_D \end{bmatrix} = \begin{bmatrix} 0.176 & 1.253 & 1.0 \end{bmatrix}. \tag{23}$$

The system is subject to a unit step input at time $t = 0$ and to a unit step disturbance at time $t = 20$. The disturbance hits the system at the input.

The resulting closed loop control signal and output and the corresponding root locus are shown in Figure 5. The maximum control signal of 3.75 occurs in the very beginning. The PID controlled step response follows the reference system quite well. The disturbance rejection, implemented with $A = 2/3$, is also quite good. In this case $K_D s/(s/20+1)$ was implemented into the feedback from $Y(s)$. Further, $1/G_p(s)$ was implemented by padding it with $1/(s/10+1)^4$.

In this example, closed loop stability is obtained for a reference system with damping down to $\zeta_r = 0.2$ and when one of the system poles approaches zero at -0.2 the PID controlled system performance becomes very slow and has lost stability in -0.1.

IV. Matching coefficients PID - discrete time

Now consider the discrete time case with a transfer function

$$G(z) = \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_1 z + b_0}{z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0} \tag{24}$$

corresponding to the difference equation

$$\begin{aligned} &a_n y[k+n] + a_{n-1} y[k+n-1] + \cdots + a_0 y[k] \\ &= b_m u[k+m] + b_{m-1} u[k+m-1] + \cdots + b_0 u[k], \end{aligned} \tag{25}$$

where $a_n = 1$. In order to render a similar interpretation to the PID coefficients as in the continuous case we express the transfer function of the PID controller as

$$\frac{z}{z-1}\left(K_D\left(\frac{z-1}{z}\right)^2 + K_P\left(\frac{z-1}{z}\right) + K_I\right) = \frac{1}{z(z-1)}c(z) \tag{26}$$

where

$$c(z) = \left(K_2 z^2 + K_1 z + K_0\right) \tag{27}$$

with

$$\begin{bmatrix} K_2 \\ K_1 \\ K_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_I \\ K_P \\ K_D \end{bmatrix} \tag{28}$$

and we note that the matrix in Eq. (28) is the inverse of itself. Here we have replaced the differential operator in the continuous case, represented by s, by the backward difference operator ∇ in the discrete time case, represented by $1 - z^{-1} = \frac{z-1}{z}$.

Similarly, we express the reference system as

$$\begin{aligned} G_r(z) &= \frac{z}{z-1}\frac{\omega_r^2}{\frac{z-1}{z}+2\zeta_r\omega_r}\frac{1}{z} = \frac{z}{z-1}\frac{\omega_r^2/(1+2\zeta_r\omega_r)}{z-1/(1+2\zeta_r\omega_r)} \\ &= \frac{z}{z-1}\frac{b_r}{z+a_r} \end{aligned} \tag{29}$$

where we replace s by $\frac{z-1}{z}$ and add a delay, such that the reference system will have a relative degree of one. Further note that

$$\omega_r = \left(-\frac{b_r}{a_r}\right)^{1/2} \text{ and } \zeta_r = -\frac{a_r+1}{2\omega_r a_r} \tag{30}$$

and that $a_r \in]-1, 0[$ whenever $\zeta_r\omega_r \in]0, \infty[$. Proceeding in the same way as before we obtain

$$\frac{1}{z^2}c(z)b(z)(z+a_r) \approx b_r a(z) \tag{31}$$

or

$$\begin{aligned} &\left(K_2 z^2 + K_1 z + K_0\right)\left(b_m z^m + \cdots + b_1 z + b_0\right)(z+a_r) \\ &\approx b_r z^2\left(z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0\right). \end{aligned} \tag{32}$$

Here we assume $m = n-1$. Then, the original system has a relative degree of one and the degrees of the polynomials on each side of the $\approx$ sign are equal. Thus, the coefficients $\mathcal{C} = \begin{bmatrix} K_0 & K_1 & K_2 \end{bmatrix}^T$ are now found as the least squares solution to the system

$$\hat{\phi}^T\mathcal{C} \approx b_r \begin{bmatrix} 0 & 0 & a_0 & a_1 & \cdots & a_{n-1} & 1 \end{bmatrix} \tag{33}$$

where $\hat{\phi}$ is defined as ϕ in (15) except $\begin{bmatrix} 2\zeta_r\omega_r & 1 \end{bmatrix}$ is replaced by $\begin{bmatrix} a_r & 1 \end{bmatrix}$. We then close the loop using the integrator $\frac{z}{z-1}$ as shown in Figure 6. The delay in the PID-part is consistent with replacing the differential operator in the continuous case with the backward difference operator in the discrete case.

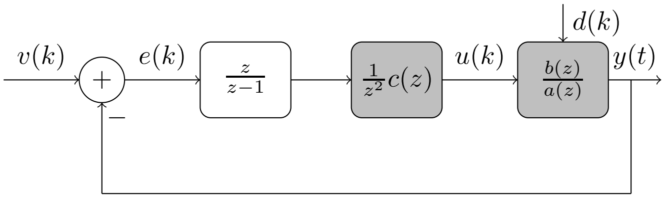

Fig. 6. A closed loop controlled SISO discrete time system.

Example 3: We consider an arbitrary unity DC gain discrete time system given by

$$\frac{b(z)}{a(z)} = \frac{0.5 \times 0.4 \times 0.3z^2}{(z-0.5)(z-0.6)(z-0.7)} \tag{34}$$

We choose a slow underdamped reference system with $a_r = -0.8$ and $b_r = 1$ corresponding to $\omega_r = 1.12$ and $\zeta_r = 0.11$. Thus

$$\frac{b_r}{z+a_r} = \frac{1}{z-0.8} \tag{35}$$

We compute the PID coefficients as in the continuous time case resulting in

$$\left[\begin{array}{ccc} K_0 & K_1 & K_2 \end{array}\right] = \left[\begin{array}{ccc} 4.43 & -16.71 & 16.65 \end{array}\right] \tag{36}$$

or alternatively

$$\left[\begin{array}{ccc} K_I & K_P & K_D \end{array}\right] = \left[\begin{array}{ccc} 4.36 & 7.85 & 4.43 \end{array}\right] \tag{37}$$

where we implement the PID as

$$\frac{z}{(z-1)}\frac{1}{z^2}c(z) = \frac{K_D(z-1)^2+K_P(z-1)z+K_Iz^2}{z(z-1)}. \tag{38}$$

The resulting closed loop step response and the corresponding root locus are shown in Figure 7, where all closed loop poles are labeled by $*$ on the corresponding root locus. The PID controlled step response follows the closed loop reference system very well. Reducing a_r down to -0.1 produced a stable but highly oscillating PID controlled system and stability was lost for $a_r = -0.08$. On the other side stability is lost for $a_r = -0.99$. The PID controlled system remains stable even if the system pole at 0.7 slides all the way up to 1.1, but then looses stability. Finally, selecting the reference pole as one of the system poles, promptly results in the PID canceling the other two system poles and a complete match of the two responses.

Example 4: We now consider the same discrete time system as in Example 3, but this time we choose a faster reference system with $a_r = -0.2$ and $b_r = 1$, corresponding to $\omega_r = 2.24$ and $\zeta_r = 0.89$. Then the reference system in the z domain is given by

$$\frac{b_r}{z+a_r} = \frac{1}{z-0.2}. \tag{39}$$

This results in MC PID coefficients given by

$$\left[\begin{array}{ccc} K_0 & K_1 & K_2 \end{array}\right] = \left[\begin{array}{ccc} 12.70 & -26.63 & 16.67 \end{array}\right] \tag{40}$$

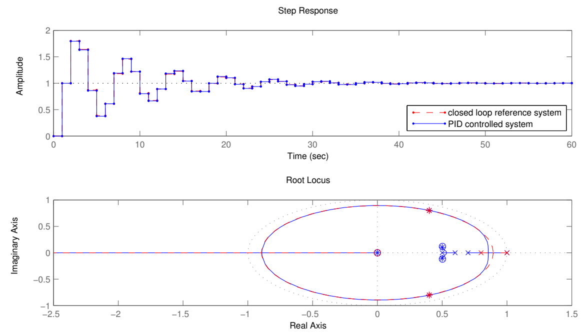

Fig. 7. Example 3. Discrete time MC PID for a slow underdamped reference system.

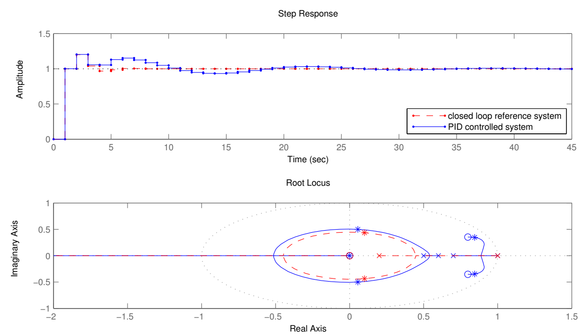

Fig. 8. Example 4. Discrete time MC PID for a faster reference system.

or

$$\left[\begin{array}{ccc} K_I & K_P & K_D \end{array}\right] = \left[\begin{array}{ccc} 2.75 & 1.23 & 12.70 \end{array}\right]. \tag{41}$$

The resulting closed loop step response and the corresponding root locus are shown in Figure 8, where all closed loop poles are labeled by $*$ on the corresponding root locus. The PID controlled step response follows the closed loop reference system quite well.

V. Conclusions

The problem of designing a PID controller is posed in a setting where a selected reference system presents the design requirements. This leads to a simple problem of equating coefficients of like powers in polynomials originating in the reference system transfer function, the transfer function of the system to be controlled as well as the PID coefficients. Effectively, an overdetermined system of equations in the PID coefficients results, which is solved in the minimum least square sense. We refer to this controller as the Matching Coefficients PID (MC PID). The computation is very simple involving basic high school mathematics only.

In addition, a disturbance rejection scheme based on a disturbance observer was set up in the continuous time, in particular to deal with a disturbance entering highly underdamped system poles. A similar disturbance rejection scheme may be set up in the discrete time case.

Several examples are presented showing excellent results. Simulation tests were also done demonstrating that stable closed loop systems were obtained until the reference system became highly underdamped, naturally resulting in degraded performance and eventually unstability. Some test were also done indicating stable well performing closed loops, even with slightly unstable system poles.

While there are no explicit criteria for a selection of a reference system that will guarantee closed loop stability, systematic approaches can be designed for modifying the reference system in these cases.

APPENDIX A: MATLAB CODE FOR COMPUTATION OF PID COEFFICIENTS IN EXAMPLE 1 - CONTINUOUS TIME:

```
a1=[1 2]'; a2=[1 0.2 1]';
a=conv(a1,a2); bra=omega^2*a;
beta1xnimpnus1=conv([2*zeta*omega 1 ],b);
beta3xnplus1=[beta1xnimpnus1 0 0; ...
 0 beta1xnimpnus1 0;0 0 beta1xnimpnus1]';
PID=beta3xnplus1\bra(end:-1:1);
KD=PID(3); KP=PID(2); KI=PID(1);
```

APPENDIX B: MATLAB CODE FOR COMPUTATION OF PID COEFFICIENTS $\begin{bmatrix} K_0 & K_1 & K_2 \end{bmatrix}$, $\begin{bmatrix} K_I & K_P & K_D \end{bmatrix}$ AND CLOSING OF LOOP IN EXAMPLE 3 - DISCRETE TIME:

```
b=0.5*0.4*0.3*poly([0 0]);
a=(poly([0.5 0.6 0.7]))';
ar=-0.8; br=1; bra=br*[0; 0; a];
hatphi=[conv([ar 1],b) 0 0; ...
0 conv([ar 1],b) 0;0 0 conv([ar 1],b)]';
k012=hatphi\bra(end:-1:1);
kPID=[1 1 1;0 -1 -2;0 0 1]*k012(end:-1:1);
cb=k012(1)*[0 0 b]+k012(2)*[0 b 0] ...
+k012(3)*[b 0 0];
cbPID=kPID(3)*([b 0 0]-2*[0 b 0]+ [0 0 b]) ...
+kPID(2)*([b 0 0]-[0 b 0])+kPID(1)*[b 0 0];
zminus1xzxa=[a' 0 0]-[0 a' 0];
OL=tf(cb,zminus1xzxa,1); LL=feedback(OL,1);
```

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REFERENCES

[1] Q.-G. Wang, Z. Zhang, K.J. Aström, L.S. Chek, "Guaranteed dominant pole placement with PID controllers," *Journal of Process Control,* 19, pp. 349-352, 2009.

[2] Q.-G. Wang, T.-H. Lee, H.-W. Fung, Q. Bi, Y. Zhang, "PID tuning for improved performance," IEEE Transactions on Control Systems Technology, vol. 7, no. 4, pp. 457-465, July 1999.

[3] M. Shamsuzzoha, S. Skogestad, "The setpoin overshoot method: A simple and fast closed-loop approach for PID tuning," *Journal of Process Control,* vol. 20, pp. 1220-1234, 2010.

[4] G.M. Malwatkar, S.H. Sonawane, L.M. Waghmare, "Tuning PID controllers for higher-order oscillatory systems with improved performance," *ISA Transactions,* vol. 48, pp. 347-353, 2009.

[5] H.-P. Huang, J.-C. Jeng, C.-H. Chiang, W. Pan, "A direct method for multi-loop PI/PID controller design," *Journal of Process Control,* 13, pp. 769-786, 2003.

[6] A.S. Hauksdóttir, *Analytic expressions of transfer function responses and choice of numerator coefficients (zeros)*, IEEE Transactions on Automatic Control, Vol. 41, No. 10, Oct. 1996, pp. 1482 - 1488.

[7] G. Herjólfsson, A.S. Hauksdóttir, S.Þ. Sigurðsson, *Closed Form Expressions of Linear Continuous- and Discrete Time Filter Responses*, NORSIG 2006, 7th Nordic Signal Processing Symposium, Reykjavík, Iceland, June 7-9, 2006.

[8] A.S. Hauksdóttir, *Optimal zero locations of continuous-time systems with distinct poles tracking first-order step responses*, IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications, Vol. 49, No. 5, May 2002, pp. 685 - 688.

[9] A.S. Hauksdóttir, *Optimal zero locations of continuous-time systems with distinct poles tracking reference step responses*, Dynamics of Continuous, Discrete, and Impulsive Systems, Series B: Applications & Algorithms, Vol. 11, 2004, pp. 353 - 361.

[10] G. Herjólfsson, B. Ævarsson, A.S. Hauksdóttir, S.Þ. Sigurðsson, *Closed Form $\mathcal{L}_2/\mathcal{H}_2$ Optimizing of Zeros for Model Reduction of Linear Continuous Time Systems*, International Journal of Control, Vol. 82, March 2009, pp. 555-570.

[11] A.S. Hauksdóttir, G. Herjólfsson, S.Þ. Sigurðsson, *Optimizing Zero Tracking and Disturbance Rejecting Controllers - The Generalized PID controller*, The 2007 American Control Conference, New York City, USA, July 11-13, 2007, pp. 5790 - 5795.

[12] H.I. Hinriksson, *An optimized MIMO PID controller*, MS thesis, Department of Electrical and Computer Engineering, University of Iceland, Reykjavík, Iceland, 2010.

[13] G. Herjólfsson, A.S. Hauksdóttir, S.Þ. Sigurðsson, *A Two-Stage Optimization PID Algorithm*, to be submitted.

[14] K.J. Aström, T. Hagglund, *Benchmark Systems for PID control*, in Proceedings of the IFAC workshop, Digital Control: Past, Present and Future of PID Control, Terassa, Spain, 2000, pp. 165-166.