2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Robust Linear Output Feedback Control of a Synchronous Generator

Hebertt Sira-Ramírez

Abstract— **In this article we propose a robust linear output feedback control scheme for the regulation and trajectory tracking tasks of the load angle variable in a widely used, nonlinear, single synchronous generator model. The proposed linear feedback scheme is based on the use of a classical linear feedback controller and a suitably extended high gain linear observer; aiding the linear feedback controller, in two important tasks: 1) accurate estimation of the input-output system model nonlinearities, 2) accurate estimation of the unmeasured phase variables associated with the load angle variable (shaft angular speed deviation, and shaft angular acceleration). These two key pieces of information are used in the proposed feedback controller to a) cancel, as a lumped unstructured time-varying term, the influence of the nonlinearities and b) devise a proper linear output feedback based on the approximate estimates of the phase variables. The robustness of the scheme is tested against a three phase short circuit of significant duration. The proposed, observer-based, feedback controller requires knowledge of only two constant parameters of the model. The closed loop responses are shown to be robust with respect to reasonable deviations of these parameters from their nominal values.**

I. INTRODUCTION

Asymptotic estimation of external, unstructured, perturbation inputs, with the aim of exactly, or approximately, canceling their influences at the controller stage, has been treated in the existing literature under several headings. The outstanding work of professor C.D. Johnson in this respect, under the name of *Disturbance Accommodation Control* (DAC), dates from the nineteen seventies (see [11]). Ever since, the theory and practical aspects of DAC theory have been actively evolving, as evidenced by the survey paper by Johnson [13]. The theory enjoys an interesting and useful extension to discrete-time systems, as demonstrated in the book chapter [12]. In a recent article, by Parker and Johnson [17], an application of DAC is made to the problem of decoupling two nonlinearly coupled linear systems. An early application of disturbance accommodation control in the area of Power Systems is exemplified by the work of Mohadjer and Johnson in [16], where the operation of an interconnected power system is approached from the perspective of load frequency control.

A closely related vein to DAC is represented by the sustained efforts of the late Professor Jingqing Han, summarized in the posthumous paper, Han [9], and known as: *Active Disturbance Estimation and Rejection* (ADER). The numerous and original developments of Prof. Han, with many laboratory and industrial applications, have not been translated into English and his seminal contributions remain written in Chinese (see the references in [9]). Although the main idea of observer-based disturbance estimation, and subsequent cancelation via the control law, is similar to that advocated in DAC, the emphasis in ADER lies, mainly, on *nonlinear* observer based disturbance estimation, with necessary developments related to: efficient time derivative computation, practical relative degree computation and non-linear PID control extensions. The work, and inspiration, of Professor Han has found interesting developments and applications in the work of Professor Z. Gao and his colleagues (see [7], [8], also, in the work by Sun and Gao [21] and in the article by Sun [22]). In a recent article, a closely related idea, proposed by Prof. M. Fliess and C. Join in [6], is at the core of *Intelligent PID Control*(IPIDC). The mainstream of the IPIDC developments makes use of the Algebraic Method and it implies to resort to first order, or at most second order, *non-phenomenological* plant models. The interesting aspect of this method resides in using suitable algebraic manipulations to locally deprive the system description of the effects of nonlinear uncertain additive terms and, via further special algebraic manipulations, to efficiently identify time-varying control gains as piece-wise constant control input gains (see [5]). An entirely algebraic approach for the control of synchronous generator was presented in Fliess and Sira-Ramírez, [19].

In this article, we advocate, within the context of the angular deviation trajectory control for a single synchronous generator model, the use of approximate, yet accurate, state dependent disturbance estimation via linear Generalized Proportional Integral (GPI) observers. GPI observers are the dual counterpart of GPI controllers, developed by M. Fliess *et al.* in [4]. A high gain GPI observer naturally includes a, self-updating, lumped, time-polynomial model of the nonlinear state-dependent perturbation; it estimates it and delivers the time signal to the controller for on-line cancelation while simultaneously estimating the phase variables related to the measured output. The scheme is, however, approximate since only a small as desired re-construction error is guaranteed at the expense of high, noise-sensitive, gains. The on-line approximate estimation is suitably combined with linear, estimation-based, output feedback control with the appropriate, on-line, disturbance cancelation. The many similarities and the few differences with the DAC and ADER techniques probably lie in 1) the fact that we do not discriminate between *exogenous* (i.e., external) unstructured perturbation inputs and *endogenous* (i.e., state-dependent) perturbation inputs in the nonlinear input-output model. These perturbations are all lumped into

This work was supported by Cinvestav-IPN

H. Sira-ramírez is with Cinvestav-IPN, AV. IPN, No. 2508. Departamento de Ingeniería Eléctrica, Sección de Mecatrónica, Col. San Pedro Zacatenco, AP 14700, 07360 México, D.F., México hsira@cinvestav.mx

a simplifying time-varying signal that needs to be linearly estimated. Notice that plant nonlinearities generate time functions that are *exogenous* to any observer and, hence, algebraic loops are naturally avoided 2) We emphasize the natural possibilities of *differentially flat systems* in the use of linear disturbance estimation and linear output feedback control with disturbance cancelation (For the concept of flatness see Fliess *et al.* [3]) and the book [20].

This article is organized as follows: Section II presents an introduction to linear control of nonlinear systems via (high-gain) GPI observers and suitable linear controllers feeding back the phase variables related to the output function. The single synchronous generator model in the form a *swing* equation, is described in Section III. Here, we formulate the reference trajectory tracking problem under a number of information restrictions about the system. In this section we propose the linear observer-linear controller output feedback control scheme for lowering the deviation angle of the generator. Section IV is devoted to present some simulations regarding the closed loop performance of the proposed linear feedback control scheme. In that section we carry out a robustness test regarding the response to a three phase short circuit. We also carry an evaluation of the performance of the control scheme under significant variations of the two control gain parameters required for an exact cancelation of the gain. The last section presents the conclusions and suggestions for further work.

II. Linear GPI observer-based control of nonlinear systems

Consider the following perturbed nonlinear single-input single input-output, smooth, nonlinear system,

$$y^{(n)} = \psi(t,y,\dot{y},...,y^{(n-1)}) + \phi(t,y)u + \zeta(t) \qquad (1)$$

The unperturbed system, ($\zeta(t) \equiv 0$) is evidently flat, as all variables in the system are expressible as differential functions of the flat output y. We assume that the exogenous perturbation $\zeta(t)$ is uniformly absolutely bounded, i.e., it an L_∞ scalar function. Similarly, we assume that for all bounded solutions, $y(t)$, of (1), obtained by means of suitable control input u, the additive, endogenous, perturbation input, $\psi(t,y(t),\dot{y}(t),...,y^{(n-1)}(t))$, viewed as a time signal is uniformly absolutely bounded. We also assume that the nonlinear gain function $\phi(t,y(t))$ is L_∞ and uniformly bounded away from zero, i.e., there exists a strictly positive constant μ such that

$$\inf_t |\phi(t,y(t))| \geq \mu \qquad (2)$$

for all smooth, bounded solutions, $y(t)$, of (1) obtained with a suitable control input u. Although the results below can be extended when the input gain function ϕ depends on the time derivatives of y, we let, motivated by the synchronous generator case study to be presented, ϕ to be an explicit function of time and of the measured flat output y. This is equivalent to saying the $\phi(t,y(t))$ is perfectly known.

We have the following formulation of the problem:

Given a desired flat output reference trajectory, $y^*(t)$, devise a linear output feedback controller for system (1) so that regardless of the endogenous perturbation signal $\psi(t,y(t),\dot{y}(t),...,y^{(n-1)}(t))$ and of the exogenous perturbation input $\zeta(t)$, the flat output y tracks the desired reference signal $y^*(t)$ even if in an approximate fashion. This approximate character specifically means that the tracking error, $e(t) = y - y^*(t)$, and its first, n, time derivatives, globally asymptotically exponentially converge towards a small as desired neighborhood of the origin in the reference trajectory tracking error phase space.

The solution to the problem is achieved in an entirely linear fashion if one conceptually considers the nonlinear model (1) as the following <u>linear</u> perturbed system

$$y^{(n)} = v + \xi(t) \qquad (3)$$

where $v = \phi(t,y)u$, and $\xi(t) = \psi(t,y(t),\dot{y}(t),...,y^{(n-1)}(t)) + \zeta(t)$.

Consider the following preliminary result:

Proposition 1: The unknown perturbation vector of time signals, $\xi(t)$, in the simplified tracking error dynamics (3), is *observable* in the sense of Diop and Fliess (see [2])).

Proof The proof of this fact is immediate after writing (3) as

$$\xi(t) = y^{(n)} - v = y^{(n)} - \phi(t,y)u \qquad (4)$$

i.e., $\xi(t)$ can be written in terms of the output vector y, a finite number of its time derivatives and the control input u. Hence, $\xi(t)$ is observable.

Remark 2: This means, in particular, that if $\xi(t)$ is bestowed with an exact linear model; an exact asymptotic estimation of $\xi(t)$ is possible via a linear observer. If, on the other hand, the linear model is only approximately locally valid, then the estimation obtained via a linear observer is asymptotically convergent towards an equally approximately locally valid estimate.

We assume that the perturbation input $\xi(t)$ may be locally modeled as a $p-1$-th degree time polynomial z_1 plus a residual term, $r(t)$, i.e.,

$$\xi(t) = z_1 + r(t) = a_0 + a_1 t + \cdots + a_{p-1}t^{p-1} + r(t), \text{ for all } t \qquad (5)$$

The time polynomial model, z_1, (also called: a Taylor polynomial) is invariant with respect to time shifts and it defines a family of $p-1$ degree Taylor polynomials with arbitrary real coefficients. We incorporate z_1 as an internal model of the additive perturbation input (see [11]). The perturbation model z_1 will acquire a *self updating* character when incorporated as part of a linear asymptotic observer whose estimation error is forced to converge to a small vicinity of zero. As a consequence of this, we may safely assume that the self-updating residual function, $r(t)$, and its time derivatives, say $r^{(p)}(t)$, are uniformly absolutely bounded. To precisely state this, let us denote by y_j an estimate of $y^{(j-1)}$ for $j = 1,...,n$. We have the following general result:

Theorem 3: The GPI observer-based dynamical feedback controller:

$$u = \frac{1}{\phi(t,y)}\left[[y^*(t)]^{(n)} - \sum_{j=0}^{n-1}\left(\kappa_j[y_j - (y^*(t))^{(j)}]\right) - \hat{\xi}(t)\right] \quad (6)$$

$$\begin{aligned}
\hat{\xi}(t) &= z_1 \\
\dot{y}_1 &= y_2 + \lambda_{p+n-1}(y - y_1) \\
\dot{y}_2 &= y_3 + \lambda_{p+n-2}(y - y_1) \\
&\vdots \\
\dot{y}_n &= v + z_1 + \lambda_p(y - y_1) \\
\dot{z}_1 &= z_2 + \lambda_{p-1}(y - y_1) \\
&\vdots \\
\dot{z}_{p-1} &= z_p + \lambda_1(y - y_1) \\
\dot{z}_p &= \lambda_0(y - y_1)
\end{aligned} \quad (7)$$

asymptotically exponentially drives the tracking error phase variables, $e_y^{(k)} = y^{(k)} - [y^*(t)]^{(k)}$, $k = 0,1,..,n-1$ to an arbitrary small neighborhood of the origin, of the tracking error phase space, which can be made as small as desired from the appropriate choice of the controller gain parameters $\{\kappa_0,...,\kappa_{n-1}\}$. Moreover, the estimation errors: $\tilde{e}^{(i)} = y^{(i)} - y_i$, $i = 0,...,n-1$ and the perturbation estimation error: $z_m - \xi^{m-1}(t)$, $m = 1,...,p$ asymptotically exponentially converge towards a small as desired neighborhood of the origin of the reconstruction error space which can be made as small as desired from the appropriate choice of the controller gain parameters $\{\lambda_0,...,\lambda_{p+n-1}\}$.

Proof The proof is based on the fact that the estimation error $\tilde{e}$ satisfies the perturbed linear differential equation

$$\tilde{e}^{(p+n)} + \lambda_{p+n-1}e^{(p+n-1)} + \cdots + \lambda_0\tilde{e} = r^{(p)}(t) \quad (8)$$

Since $r^{(p)}(t)$ is assumed to be uniformly absolutely bounded then there exists coefficients λ_k such that $\tilde{e}$ converges to a small vicinity of zero, provided the roots of the associated characteristic polynomial in the complex variable s:

$$s^{p+n} + \lambda_{p+n-1}s^{p+n-1} + \cdots + \lambda_1 s + \lambda_0 \quad (9)$$

are all located deep into the left half of the complex plane. The further away from the imaginary axis, of the complex plane, are these roots located, the smaller the neighborhood of the origin, in the estimation error phase space, where the estimation error $\tilde{e}$ will remain ultimately bounded (see Kailath [14]). Clearly, if $\tilde{e}$ and its time derivatives converge to a neighborhood of the origin, then $z_j - \xi^{(j)}$, $j = 1,2,...$, also converge towards a small vicinity of zero.

The tracking error $e_y = y - y^*(t)$ evolves according to the following linear perturbed dynamics

$$e_y^{(n)} + \kappa_{n-1}e_y^{(n-1)} + \cdots + \kappa_0 e_y = \xi(t) - \hat{\xi}(t) \quad (10)$$

Choosing the controller coefficients $\{\kappa_0,\cdots,\kappa_{n-1}\}$, so that the associated characteristic polynomial

$$s^n + \kappa_{n-1}s^{n-1} + \cdots + \kappa_0 \quad (11)$$

exhibits its roots sufficiently far from the imaginary axis in the left half portion of the complex plane, the tracking error, and its various time derivatives, are guaranteed to converge asymptotically exponentially towards a vicinity of the tracking error phase space. Note that, according to the observer expected performance, the right hand side of (10) is represented by a uniformly absolutely bounded signal already evolving on a small vicinity of the origin. For this reason the roots of (11) may be located closer to the imaginary axis than those of (9). A rather detailed proof of this theorem may be found in the article by Luviano *et al.* [15]

Remark 4: The proposed GPI observer (7) is a high gain observer which is prone to exhibiting the "peaking" phenomena at the initial time. We use a suitable "clutch" to smooth out these transient peaking responses in all observer variables that need to be used by the controller. This is accomplished by means of a factor function smoothly interpolating between an initial value of zero and a final value of unity. We denote this clutching function as $s_f(t) \in [0,1]$ and define it in the following (non-unique) way

$$s_f(t) = \begin{cases} 1 & \text{for } t > \varepsilon \\ \sin^q\left(\frac{\pi t}{2\varepsilon}\right) & \text{for } t \leq \varepsilon \end{cases} \quad (12)$$

where q is a suitably large positive even integer.

III. LINEARLY CONTROLLING THE SINGLE SYNCHRONOUS GENERATOR MODEL

A. The single synchronous generator model

Consider the swing equation of a synchronous generator, connected to an *infinite bus*, with a series capacitor connected with the help of a thyristor bridge (See Hingorani [10]),

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= P_m - b_1x_2 - b_2x_3\sin(x_1) \\
\dot{x}_3 &= b_3(-x_3 + x_3^*(t) + u + \zeta(t))
\end{aligned} \quad (13)$$

x_1 is the load angle, considered to be the measured output. The variable, x_2, is the deviation from nominal, synchronous, speed at the shaft, while x_3 stands for the admittance of the system. The control input, u, is usually interpreted as a quantity related to the fire angle of the switch. $\zeta(t)$ is an unknown, external, perturbation input. The static equilibrium point of the system, which may be parameterized in terms of the equilibrium position for the angular deviation, $\bar{x}_1$, is given by,

$$x_1 = \bar{x}_1,\ \bar{x}_2 = 0,\ \bar{x}_3 = \bar{x}_3^*(t) = \frac{P_m}{b_2\sin(\bar{x}_1)} \quad (14)$$

We assume that the system parameters, b_2, and, b_3, are known. The constant quantities P_m, b_1 and the time varying quantity, $x_3^*(t)$, are assumed to be completely unknown.

B. Problem formulation

It is desired to have the load angular deviation, $y = x_1$, track a given reference trajectory, $y^*(t) = x_1^*(t)$, which remains bounded away from zero, independently of the unknown system parameters and in spite of possible external system disturbances (such as short circuits in the three *phase line, setting, momentarily, the mechanical power, P_m, to zero), and other unknown, or un-modeled, perturbation inputs comprised in $\zeta(t)$.*

C. Main results

The unperturbed system in (13) is *flat*, with flat output given by the load angle deviation $y = x_1$. Indeed, all system variables are differentially parameterizable in terms of the load angle and its time derivatives. We have:

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ x_3 &= \frac{P_m - b_1\dot{y} - \ddot{y}}{b_2\sin(y)} \\ u &= -\frac{b_1\ddot{y} + y^{(3)}}{b_3 b_2 \sin(y)} - \frac{P_m - b_1\dot{y} - \ddot{y}}{b_3 b_2 \sin^2(y)}\dot{y}\cos(y) \\ &\quad + \frac{P_m - b_1\dot{y} - \ddot{y}}{b_2\sin(y)} - x_3^*(t) \end{aligned} \tag{15}$$

The perturbed input-output dynamics, devoid of any zero dynamics, is readily obtained with the help the control input differential parametrization (15). One obtains the following simplified, perturbed, system dynamics, including $\zeta(t)$, as:

$$y^{(3)} = -[b_3 b_2 \sin(y)]\,u + \xi(t) \tag{16}$$

where $\xi(t)$ is given by

$$\begin{aligned} \xi(t) &= -b_1\ddot{y} + b_3\left(P_m - b_1\dot{y} - \ddot{y}\right)\left(1 - \frac{\dot{y}\cos(y)}{b_3\sin(y)}\right) \\ &\quad - b_3 b_2 \sin(y)\left(x_3^*(t) + \zeta(t)\right) \end{aligned} \tag{17}$$

We consider $\xi(t)$ as an unknown but uniformly absolutely bounded disturbance input that needs to be on-line estimated by means of an observer and, subsequently, canceled from the simplified system dynamics via feedback in order to regulate the load angle variable y towards the desired reference trajectory $y^*(t)$. It is assumed that the gain parameters b_2 and b_3 are known.

The problem is then reduced to the trajectory tracking problem defined on the perturbed third order, predominantly, linear system (16) with measurable state dependent input gain and unknown, but uniformly bounded, disturbance input. We propose the following estimated state feedback controller with a smoothed (i.e., "clutched") disturbance cancelation term, $z_{1s}(t) = s_f(t)z_1(t)$, and smoothed estimated phase variables $y_{js} = s_f(t)y_j(t)$, $j = 1,2,3$ with $s_f(t)$ as in equation (12) with a suitable ε value.

$$\begin{aligned} u &= -\frac{1}{b_3 b_2 \sin(y)}\Big[(y^*(t))^{(3)} - k_2(y_{3s} - \ddot{y}^*(t)) \\ &\quad - k_1(y_{2s} - \dot{y}^*(t)) - k_0(y - y^*(t)) - z_{1s}\Big] \end{aligned}$$

The corresponding variables, y_3, y_2 and z_1, are generated by the following linear GPI observer:

$$\begin{aligned} \dot{y}_1 &= y_2 + \lambda_5(y - y_1) \\ \dot{y}_2 &= y_3 + \lambda_4(y - y_1) \\ \dot{y}_3 &= -(b_3 b_2 \sin(y))\,u + z_1 + \lambda_3(y - y_1) \\ \dot{z}_1 &= z_2 + \lambda_2(y - y_1) \\ \dot{z}_2 &= z_3 + \lambda_1(y - y_1) \\ \dot{z}_3 &= \lambda_0(y - y_1) \end{aligned} \tag{18}$$

where y_1 is the redundant estimate of the output y, y_2 is the shaft velocity estimate and y_3 is the shaft acceleration estimate. The variable z_1 estimates the perturbation input $\xi(t)$ by means of a local, self updating, polynomial model of third order, taken as an internal model of the state dependent additive perturbation affecting the input-output dynamics (16).

The clutched observer variables z_{1s}, y_{2s} and y_{3s} are defined by

$$\theta_s = s_f(t)\theta, \quad s_f(t) = \begin{cases} \sin^8(\frac{\pi t}{2\varepsilon}) & \text{for } t \le \varepsilon \\ 1 & \text{for } t > \varepsilon \end{cases} \tag{19}$$

with θ_s being either z_{1s}, y_{2s} or y_{3s}

The reconstruction error system is obtained by subtracting the observer model from the perturbed simplified linear system model. We have, letting $\tilde{e} = e_1 = y - y_1$, $e_2 = \dot{y} - y_2$, etc.

$$\begin{aligned} \dot{e}_1 &= e_2 - \lambda_5 e_1, \ \dot{e}_2 = e_3 - \lambda_4 e_1, \ \dot{e}_3 = \xi(t) - z_1 - \lambda_3 e_1 \\ \dot{z}_1 &= z_2 + \lambda_2(y - y_1) \\ \dot{z}_2 &= z_3 + \lambda_1(y - y_1) \\ \dot{z}_3 &= \lambda_0(y - y_1) \end{aligned} \tag{20}$$

The reconstruction error, $\tilde{e} = e_1 = y - y_1$, is seen to satisfy the following linear, perturbed, dynamics

$$\tilde{e}^{(6)} + \lambda_5\tilde{e}^{(5)} + \lambda_4\tilde{e}^{(4)} + \cdots + \lambda_1\dot{\tilde{e}} + \lambda_0\tilde{e} = \xi^{(3)}(t) \tag{21}$$

Choosing the gains $\{\lambda_5, \cdots, \lambda_0\}$ so that the roots of the characteristic polynomial,

$$p_o(s) = s^6 + \lambda_5 s^5 + \lambda_4 s^4 + \cdots + \lambda_1 s + \lambda_0, \tag{22}$$

are located deep into the left half of the complex plane, it follows from the bounded input, bounded output stability theory that the trajectories of the reconstruction error $\tilde{e}$ and those of its time derivatives $\tilde{e}^{(j)}$, $j = 1,2,...$ are uniformly ultimately bounded by a disk, centered at the origin in the reconstruction error phase space, whose radius can be made arbitrarily small as the roots of $p_o(s)$ are pushed further to the left of the complex plane.

The closed loop tracking error dynamics satisfies

$$e_y^{(3)} + \kappa_2 e_y^{(2)} + \kappa_1\dot{e}_y + \kappa_0 e_y = \xi(t) - z_{1s} \tag{23}$$

The difference, $\xi(t) - z_{1s}$, being arbitrarily small after some time, produces a reference trajectory tracking error, $e_y = y - y^*(t)$, that also asymptotically exponentially converges towards a small vicinity of the origin of the tracking error phase space.

The characteristic polynomial of the predominant linear component of the closed loop system may be set to have poles placed in the left half of the complex plane at moderate locations

$$p_c(s) = s^3 + \kappa_2 s^2 + \kappa_1 s + \kappa_0 \qquad (24)$$

IV. SIMULATION RESULTS

A. A desired rest-to-rest maneuver

It is desired to smoothly lower the load angle, $y_1 = x_1$, from an equilibrium value of $y = 1$ [rad] towards a smaller value, say, $y = 0.6$ [rad] in a reasonable amount of time, say, $T = 5$ [s], starting at $t = 5$ [s] of an equilibrium operation characterized by (see Bazanella *et al.* [1] and Pai [18])

$$x_1 = 1,\ x_2 = 0,\ x_3 = 0.8912$$

We used the following parameter values for the system

$$b_1 = 1,\ b_2 = 21.3360,\ b_3 = 20$$

We set the external perturbation input, $\zeta(t)$, as the time signal,

$$\zeta(t) = 0.005 e^{(\sin^2(3t)\cos(3t))}\cos(0.3t)$$

The observer parameters were set in accordance with the following desired characteristic polynomial $p_o(s)$ for the, predominantly, linear reconstruction error dynamics. We set $p_o(s) = (s^2 + 2\zeta_o \omega_{no} s + \omega_{no}^2)^3$, with

$$\zeta_o = 1, \quad \omega_{no} = 20$$

The controller gains $\kappa_2, \kappa_1, \kappa_0$ were set so that the following closed loop characteristic polynomial, $p_c(s)$, was enforced on the tracking error dynamics,

$$p_c(s) = (s^2 + 2\zeta_c \omega_{nc} s + \omega_{nc}^2)(s + p_c)$$

with

$$p_c = 3,\ \omega_{nc} = 3,\ \zeta_c = 1$$

The trajectory for the load angle, $y^*(t)$, was set to be

$$y^*(t) = \bar{x}_{1,\text{initial}} + (\rho(t,t_1,t_2))(\bar{x}_{1,\text{final}} - \bar{x}_{1,\text{initial}})$$

with $\rho(t,t_1,t_2)$ being a smooth Bèzier polynomial achieving a smooth rest-to-rest trajectory for the nominal load angle $y^*(t)$ from the initial equilibrium value $y^*(t_1) = \bar{x}_{1,\text{initial}} = 1$ [rad] towards the final desired equilibrium value $y^*(t_2) = \bar{x}_{1,\text{final}} = 0.6$ [rad]. We set $t_1 = 5.0$ [s], $t_2 = 10.0$ [s]; $\varepsilon = 3.0$

The interpolating polynomial $\rho(t,t_1,t_2)$, is of the form:

$$\begin{aligned}\rho(t) \quad = \quad & \tau^8 \left[r_1 - r_2\tau + r_3\tau^2 - r_4\tau^3 + r_5\tau^4 \right. \\ & \left. - r_6\tau^5 + r_7\tau^6 - r_8\tau^7 + r_9\tau^8 \right]\end{aligned}$$

with,

$$\tau = \frac{t - t_1}{t_2 - t_1}$$

The choice,

$$\begin{aligned} r_1 &= 12870,\ r_2 = 91520,\ r_3 = 288288 \\ r_4 &= 524160,\ r_5 = 600600,\ r_6 = 443520 \\ r_7 &= 205920,\ r_8 = 54912,\ r_9 = 6435 \end{aligned}$$

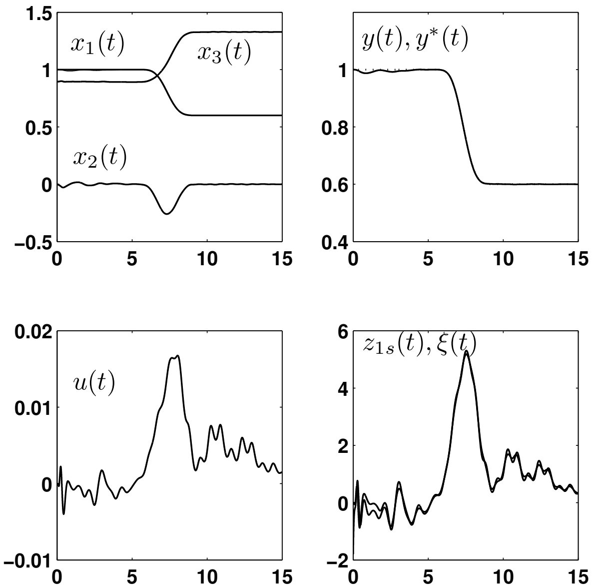

Fig. 1. Performance of GPI observer based linear controller for load angle rest-to-rest trajectory tracking in a perturbed synchronous generator.

renders a time polynomial which is guaranteed to have enough derivatives being zero, both, at the beginning and at the end of the desired rest to rest maneuver.

Figure 1 depicts the closed loop performance of the proposed GPI observer based linear output feedback controller for the forced evolution of the synchronous generator load angle trajectory following a desired rest to rest maneuver.

B. Robustness with respect to controller gain mismatches

We simulated the behavior of the closed loop system when the gain parameters product, b_3b_2, is not precisely known and the controller is implemented with an estimated (guessed) value of this product, denoted by $\widehat{b_2b_3}$, and set to be $\widehat{b_2b_3} = \kappa b_2 b_3$. We determined that κ is a positive factor ranging in the interval $[0.95, \infty]$. However, if we allow independent estimates of the parameters in the form $\hat{b}_2 = \kappa_{b2} b_2$ and $\hat{b}_3 = \kappa_{b3} b_3$, we found that a larger robustness interval of mismatches is allowed by satisfying the empirical relation $\kappa_{b2}\kappa_{b3} \geq 0.95$. The assessment was made in terms of the proposed rest to rest maneuver and possible simulations look about the same.

C. Robustness with respect to sudden power failures

We simulated an un-modeled sudden three phase short circuit occurring at time $t = 2$ [s]. The power failure lasts for $t = 0.2$ [s]. Figure 3 depicts the performance of the GPI observer based controller in the rapid transient occurring during the recovery of the prevailing equilibrium conditions.

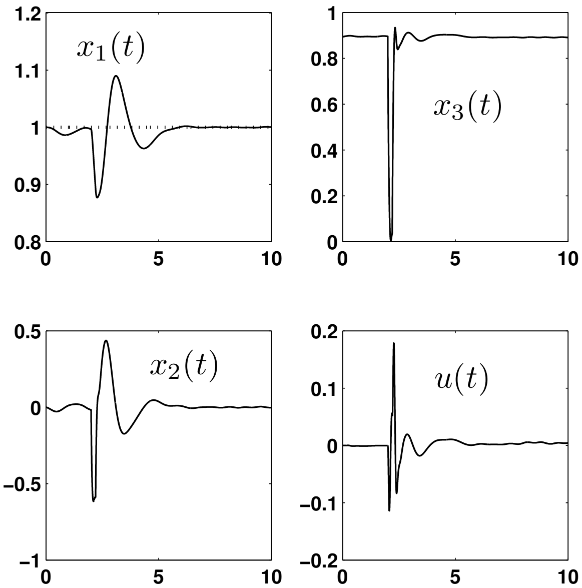

Fig. 2. Performance of GPI observer based controller under a sudden loss of power at t=2 [sec] during 0.2 [sec].

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this article, we have explored, within the context of the angular deviation trajectory control for a nonlinear single synchronous generator model, the use of approximate, yet accurate, state-dependent disturbance estimation, and simultaneous state estimation, via linear Generalized Proportional Integral (GPI) observers aided by a linear output feedback controller. The overall scheme is, however, approximate since only small as desired reconstruction and reference trajectory tracking errors are guaranteed, at the expense of high, noise-sensitive, observer-controller gains. Simulations were provided where the robustness of the proposed control method is assessed with respect to external perturbations and crucial control gain parameters. We also experimented with a significant temporary short circuit condition in order to assess the recovery features of the proposed controller.

B. Future Works

GPI observer-based linear control of nonlinear systems is naturally fit for differentially flat systems provided the flat output is measurable. The approach may be extended to discrete systems, to multi-variable continuous, or discrete, systems and even to nonlinear systems exhibiting known input, or output, time delays. The fundamental restriction of unavailable flat outputs remains to be fully explored. In this respect, the minimum-phase restriction seems to be natural. These topics, and other related limitations, need to be explored in the future and we propose them as topics for further development.

VI. ACKNOWLEDGMENTS

The author gratefully acknowledges the contribution of Conacyt-México under Research Project 60877-Y .

REFERENCES

[1] A. S. Bazanella, P. Kokotovic, and A. Silva, "A dynamic extension for L_gV controllers", *IEEE Transactions on Automatic Control*, Vol. 44, No. 3, 1999.
[2] S. Diop and M. Fliess, "Nonlinear observability, identifiability and persistent trajectories", *36th IEEE Conference on Decision and Control*, Brighton, U.K., December 1991.
[3] M. Fliess and J. Levine and Ph. Martin and P. Rouchon, "Flatness and defect of non-linear systems: introductory theory and examples", *International Journal of Control*, Vol. 61, No. 6, pp. 1327-1361, 1995.
[4] M. Fliess, R. Marquez, E. Delaleau, and H. Sira-Ramírez, "Correcteurs proportionnels integraux généralisés," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 7, pp. 2341, 2002.
[5] M. Fliess and H. Sira-Ramírez, "An algebraic framework for linear identification" *ESAIM, Control, Optimization and Cal- culus of Variations*, Vol 9, pp. 151-168, 2003.
[6] M. Fliess and C. Join, "Intelligent PID controller," in *16th Mediterranean Conference on Control and Automation*, Ajaccio, Corsica, France, 2006
[7] Z. Gao, Y. Huang, and J. Han, "An alternative paradigm for control system design," *40th IEEE Conference on Decision and Control*, vol. 5, pp. 45784585, 2001.
[8] Z. Gao, "Active disturbance rejection control: A paradigm shift in feedback control system design", *American Control Conference* pp. 23992405, 2006.
[9] J. Han, "From pid to active disturbance rejection control," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 3, pp. 900906, March 2009.
[10] N. G. Hingorani and L. Gyugyi, *Understanding FACTS*, IEEE Press, Piscataway, N.J. 2000.
[11] C. D. Johnson, "Accommodation of external disturbances in linear regulator and servomechanism problems," *IEEE Transactions on Automatic Control*, vol. AC-16, no. 6, December 1971.
[12] C. D. Johnson, "Discrete-Time Disturbance- Accommodating Control Theory," in C. T. Leondes, *Control and Dynamic Svstems: Advances in Theory and ilpplications*, Vol. 18, pp. 224-315, Academic Press NY, 1982.
[13] C. D. Johnson, "Real-time disturbance-observers; origin and evolution of the idea. Part 1: The early years," *40th Southeastern Symposium on System Theory*. New Orleans, LA, USA, March 16-18, 2008.
[14] T. Kailath, *Linear Systems*. Information and System Science Series, Prentice-Hall, Upper Saddle River, N.J. 1979,
[15] A. Luviano-Juárez and J. Cortés-Romero and H. Sira-Ramírez, "Synchronization of Chaotic Oscillators by means of GPI Observers", *International Journal of Bifurcations and Chaos in Applied Science and Engineering*, Vol. 20, No. 5, pp. 1509-1517, 2010.
[16] M. Mohadjer and C. D. Johnson "Power system control with disturbance-accommodation", *The 22nd IEEE Conference on Conference on Decision and Control*, San Antonio, Texas, December 14-16, 1983.
[17] G. A. Parker and C. D. Johnson "Decoupling Linear Dynamical Systems Using Disturbance Accommodation Control Theory" *41st Southeastern Symposium on System Theory*, Tullahoma, TN, USA, March 15-17, 2009.
[18] M. A. Pai, *Energy Function Analysis for Power System Stability*, Kluwer Academic Publishers, 1989.
[19] H. Sira-Ramírez and M. Fliess, "On the output feedback control of a synchronous generator", *43d. IEEE Conference on Decision and Control*, Bahamas, December 2004.
[20] H. Sira-Ramírez and S. Agrawal, *Differentially flat systems*, Marcel Dekker Inc., New York, N.Y. 2004,
[21] B. Sun and Z. Gao, "A DSP-based active disturbance rejection control design for a 1-kW H-bridge DCDC power converter," *IEEE Transactions on Industrial Electronics*, vol. 52, no. 5, pp. 12711277, 2005.
[22] D. Sun, "Comments on Active Disturbance Rejection Control" *IEEE Transactions on Industrial Electronics*, Vol. 54, NO. 6, pp. 3428-3429, December 2007