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# Necessary and Sufficient Conditions for Resultant Siphons to be controlled

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***Abstract*—Based on key resource subsets, a necessary and sufficient condition is proposed under which a resultant siphon can be always marked if its strict minimal siphons (SMS) are optimally controlled. The proposed condition is established by analyzing the structural characteristics and markings of the resource subnets in a class of Petri nets called L-S$^3$PR. When it is used in deadlock prevention policies, the number of monitors can be significantly reduced, thereby decreasing control implementation complexity and cost.**

***Index Terms*—Deadlock, manufacturing systems, Petri nets, siphon**

## I. Introduction

FOR a class of Petri nets called Systems of Simple Sequential Processes with Resources (S$^3$PR), Ezpeleta *et al.* [1] propose an approach where liveness is enforced by adding a monitor to every SMS. However, too many monitors need to be added, leading to a highly complex controlled Petri net. The number of monitors to be added is equal to the number of SMS in the net and the number of arcs added is generally much larger than that of monitors, particularly for large-scale Petri nets.

In fact, not all SMS have to be controlled via monitors. In other words, some monitors may be redundant. Many researchers have worked on the problem of redundant monitors and made remarkable progress [2-11]. In this paper, we focus on finding ways to solve this problem.

Li and Zhou [6], [7] pioneered in classifying SMS in a Petri net into two categories: elementary and dependent siphons. By making the former invariant-controlled in an S$^3$PR net, they prove that under some conditions, the latter can be always marked. In [8], they investigate the existence of dependent siphons and propose more general conditions under which a dependent siphon can be always marked. Based on the results of [8], Li and Zhao [9] claim that the controllability of dependent siphons in an ordinary Petri net is a special case of that in a generalized one. In their work, controllability condition of a dependent siphon is expressed in terms of the control depth variables of its elementary siphons. These methods significantly reduce the number of monitors, but a shortcoming of their methods is that they need to compute all the SMS beforehand. Some related work is reported in [10, 11]. In [11], Chao proposes the concept of basic and compound siphons. By controlling the basic siphons via monitors, he finds the conditions for a compound to be implicitly controlled. But his condition is also sufficient but not necessary.

By fully utilizing the structural information in a Petri net, Li and Zhou [12] propose a method to compute a set of elementary siphons in S$^3$PR based on resource circuits. They claim that any dependent siphon can be found through the composition of elementary ones that are derived from resource circuits. However, it remains unexplored to relax the controllability conditions of the resultant siphons. Similar works are reported by Xing *et al*. [13] and Wang *et al*. [14]. Based on resource circuits, this work for the first time studies the relationship between two SMS and their resultant siphon by analyzing the structural characteristics and markings of the resource subnets.

Given two SMS and their resultant siphon, this paper derives the controllability condition of the latter in an L-S$^3$PR. The new contributions of this paper include:

1) The concept of loop resource subset is proposed, which is important in establishing new results of the controllability conditions of an SMS;

2) Given two SMS and their resultant siphon, the concept of a key resource subset is proposed. It plays a critical role in deciding the controllability conditions of resultant siphons.

3) A necessary and sufficient condition under which a resultant siphon can be always marked if its SMS are optimally controlled is proposed and proved.

## II. Preliminaries

### A. *Petri Nets* [15], [16]

A Petri net is a 3-tuple $N= (P, T, F)$, where $P$ and $T$ are finite, nonempty, and disjoint sets. $P$ is a set of places, and $T$ is a set of transitions. The set $F\subseteq(P\times T)\cup(T\times P)$ is the incidence relation. Given a net $N= (P, T, F)$, and a node $x\in(P\cup T)$, ${}^{\bullet}x =\{y\in P\cup T|(y, x)\in F\}$ is the preset of $x$, while

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$x^{\bullet}=\{y\in P\cup T|(x, y)\in F\}$ is the post-set of $x$. The incidence matrix of $N$ is a matrix $[N]$: $P\times T\rightarrow Z$ indexed by $P$ and $T$ such that $[N](p, t)= -1$ if $p\in {}^{\bullet}t\backslash t^{\bullet}$; $[N](p, t)=1$ if $p\in t^{\bullet}\backslash {}^{\bullet}t$; otherwise $[N](p, t)=0$ for all $p\in P$ and $t\in T$. $N$ is called a state machine if $\forall t\in T$, $|{}^{\bullet}t|=|t^{\bullet}|=1$.

Let $N= (P, T, F)$ be a Petri net. A marking $M$ of $N$ is a mapping from $P$ to $\mathbb{N}$ where $\mathbb{N}= \{0, 1, 2, ...\}$. In general, we use multi-set notation $\sum_{p\in P} M(p)p$ to denote vector $M$, where $M(p)$ indicates the number of tokens in $p$ at $M$. For example, $M =[1, 2, 0, 0]^T$ is denoted by $M = p_1+2p_2$. $p$ is marked by $M$ if $M(p)>0$.

A transition $t$ is enabled at a marking $M$, denoted by $M[t>$, if $\forall p\in {}^{\bullet}t$, $M(p) > 0$. An enabled transition $t$ at $M$ can fire, resulting in a new marking $M'$, denoted by $M[t>M'$, where $M'(p) = M(p)+ [N](p, t)$. A sequence of transitions $\alpha=t_1 t_2 \ldots t_k$, $t_i \in T$, $i = 1, 2, \ldots, k$ is feasible from a marking $M$ if there exist $M_i[t_i>M_{i+1}$ and $i = 1, 2, \ldots, k$, where $M_1 = M$. In such a case, we use $M[\alpha>M_i$ to denote the case that $M_i$ is reachable from $M$ after firing a sequence of transitions $\alpha$. Let $R(N, M_0)$ denote the set of all reachable markings of $N$ from the initial marking $M_0$.

A $P$-vector is a column vector $I$: $P\rightarrow Z$ indexed by $P$ and a $T$-vector is a column vector $J$: $T\rightarrow Z$ indexed by $T$, where $Z$ is the set of integers. $I$ is a $P$-invariant if $I\neq\mathbf{0}$ and $I^T\bullet[N]=\mathbf{0}^T$ hold. $P$-invariant $I$ is a semiflow if every element of $I$ is non-negative. $||I||= \{p\in P\,|I(p)\neq 0\}$ is called the support of $I$.

A nonempty set $S\subseteq P$ is a siphon if ${}^{\bullet}S\subseteq S^{\bullet}$. A siphon is minimal if there is no siphon contained in it as a proper subset. A minimal siphon that does not contain the support of any $P$-invariant is called an SMS. A subset $S\subseteq P$ is marked by $M$ if at least one place in $S$ is marked by $M$. The sum of tokens in all places in $S$ is denoted by $M(S)$, where $M(S)=\sum_{p\in S} M(p)$.

A siphon $S$ is said to be controlled in a net system $(N, M_0)$ if $\forall M\in R(N, M_0)$, $M(S)>0$. $S$ is said to be optimally controlled in a net system $(N, M_0)$ if only the markings at which $S$ becomes unmarked are removed.

Let $N= (P, T, F)$ be a Petri net. A string $x_1,\ldots,$ and $x_n$ in $P\cup T$ is called a path of $N$ if $\forall i\in\{1, 2,..., n-1\}$, $x_{i+1}\in x_i^{\bullet}$. An elementary path from $x_1$ to $x_n$ is a path whose nodes are all different (except, perhaps, $x_1$ and $x_n$). It is called an elementary circuit if it is an elementary path and $x_1= x_n$.

A transition without any input place is called a source transition, and one without any output place is called a sink transition. Note that a source transition is unconditionally enabled, and that the firing of a sink transition consumes tokens, but does not produce any.

### B. L-$S^3PR$ [17]

*Definition 1:* A Linear $S^3$PR (L-$S^3$PR) is an ordinary Petri net $N = (P, T, F)$ such that:

(1) $P= P_A \cup P_0 \cup P_R$ is a partition such that

a) $P_0=\{p_0^1,...,p_0^k\}, k>0$, is the set of idle places.

b) $P_A=\bigcup_{i=1}^{k} P_A^i$ is the set of operation places, where $P_A^i\cap P_A^j=\varnothing$, for all $i\neq j$.

c) $P_R=\{r_1,...,r_n\}, n>0$, is the set of resource places.

(2) $T=\bigcup_{i=1}^{k} T^i$ is the set of transitions, where $T^i\cap T^j=\varnothing$, for all $i\neq j$.

(3) $\forall i\in\mathbb{N}_k=\{1,2,...,k\}$, the subnet $N^i$ generated by $\{p_0^i\}\cup P_A^i \cup T^i$ is a strongly connected state machine, such that every circuit contains $\{p_0^i\}$ and $\forall p\in P_A^i, |p^{\bullet}|=1$.

(4) $\forall i\in\mathbb{N}_k$, $\forall p\in P_A^i, \exists r\in P_R, {}^{\bullet\bullet}p\cap P_R=p^{\bullet\bullet}\cap P_R=\{r\}$ and $|{}^{\bullet\bullet}p\cap P_R|=1$.

(5) For $r\in P_R, H(r)=({}^{\bullet\bullet}r)\cap P_A$ is the set of operation places that use $r$ and are called holders of $r$.

(6) For $p\in P_A, ({}^{\bullet\bullet}p)\cap P_R=\{r_p\}$ where resource $r_p$ is called the resource used by $p$.

(7) $N$ is strongly connected.

*Definition 2*: Let $N = (P_A \cup P_0 \cup P_R, T, F)$ be an L-$S^3$PR. An initial marking $M_0$ is called an acceptable one for $N$ if 1) $\forall p\in P_0, M_0(p)\geq 1$; 2) $\forall p\in P_A, M_0(p)=0$; and 3) $\forall p\in P_R, M_0(p)\geq 1$.

## III. Controllability Condition

In this section, we first briefly introduce some fundamental concepts of resource circuits, loop resource subsets, and resource subnets. Based on them, the controllability condition of resultant siphons is discussed for L-$S^3$PR. In the remaining discussion, we assume that $N = (P_A \cup P_0 \cup P_R, T, F)$ is an L-$S^3$PR net with an acceptable initial marking.

*Proposition 1*: Let $S$ be an SMS in $(N, M_0)$ and $(N_1, M_1)$ be the net derived from $(N, M_0)$ by adding a monitor $V_S$. $S$ is optimally controlled if $V_S$ is added such that 1) $\forall p\in P_A \cup P_0 \cup P_R$, $M_1(p)= M_0(p)$; 2) $M_1(V_S)= M_0(S)-1$; and 3) $I= p_x+\ldots+p_y+V_S$ is a $P$-invariant of $(N_1, M_1)$ where $\{p_x,\ldots, p_y\}=(\bigcup_{r\in S\cap P_R} H(r))\setminus S$.

*Proof*: Similar to the proof of Proposition 1 in [18].

*Definition 3*: Let $\{r_1, r_2,\ldots, r_m\}\subseteq P_R$ $(m\geq 2)$ be a set of resources in $N$. An elementary circuit $C(r_1, t_1, r_2, t_2, \ldots, r_m, t_m)$ is called a resource circuit if 1) $\forall i\in\{1, 2,..., m\}$, $r_i\in {}^{\bullet}t_i$; 2) $\forall i\in\{2,..., m\}$, $r_i\in t_{i-1}^{\bullet}$; and 3) $r_1\in t_m^{\bullet}$.

We use $C^R= \{r_1, r_2, \ldots, r_m\}$ to denote the set of resources in a resource circuit $C$ in $N$.

*Definition 4*: Let $\mathbf{C}= \{C_1, C_2,\ldots, C_n\}$ be the set of resource circuits in $N$. The set of loop resource subsets $\mathbb{Q}\subseteq 2^{P_R}$ is recursively defined as follows: 1) $\forall C_i\in \mathbf{C}$, $C_i^R\in\mathbb{Q}$; 2) if $\Omega_1$, $\Omega_2\in\mathbb{Q}$, $\Omega_1\cap\Omega_2\neq\varnothing$, then $\Omega_{1,2}=\Omega_1\cup\Omega_2\in\mathbb{Q}$.

The net shown in Fig. 1(a) has three resource circuits: $C_1$ $(p_{11}, t_2, p_{12}, t_7)$, $C_2$ $(p_{12}, t_3, p_{13}, t_8)$, and $C_3$ $(p_{13}, t_4, p_{14}, t_9)$. Let

$\Omega_1 = C_1^R$, $\Omega_2 = C_2^R$, and $\Omega_3 = C_3^R$. Clearly, $\Omega_1 = C_1^R = \{p_{11}, p_{12}\}$, $\Omega_2 = C_2^R = \{p_{12}, p_{13}\}$, and $\Omega_3 = C_3^R = \{p_{13}, p_{14}\}$ are loop resource subsets. Since $\Omega_1 \cap \Omega_2 = \{p_{12}\} \neq \emptyset$ and $\Omega_2 \cap \Omega_3 = \{p_{13}\} \neq \emptyset$, then $\Omega_4 = \Omega_{1,2} = \Omega_1 \cup \Omega_2 = \{p_{11}, p_{12}, p_{13}\}$ and $\Omega_5 = \Omega_{2,3} = \Omega_2 \cup \Omega_3 = \{p_{12}, p_{13}, p_{14}\}$ are loop resource subsets as well. Similarly, so is $\Omega_6 = \Omega_{1,2,3} = \Omega_1 \cup \Omega_{2,3} = \Omega_{1,2} \cup \Omega_{2,3} = \{p_{11}, p_{12}, p_{13}, p_{14}\}$.

*Definition 5*: Let $\Omega = \{r_1, r_2, \ldots, r_m\} \subseteq P_R$ $(m \geq 2)$ be a subset of resources in *N*. Then $N_\Omega = (P_\Omega, T_\Omega, F_\Omega)$ is called a resource subnet of *N* generated by $\Omega$ if 1) $P_\Omega = \Omega$; 2) $T_\Omega = {}^\bullet\Omega$; and 3) $F_\Omega = F \cap [(P_\Omega \times T_\Omega) \cup (T_\Omega \times P_\Omega)]$. $T_{\Omega\text{-}source} \subseteq T_\Omega$ is defined as the set of source transitions in $N_\Omega$, and the set of source transitions related to a resource *r* is defined as $T^r_{\Omega-source} = {}^\bullet r \cap T_{\Omega\text{-}source}$.

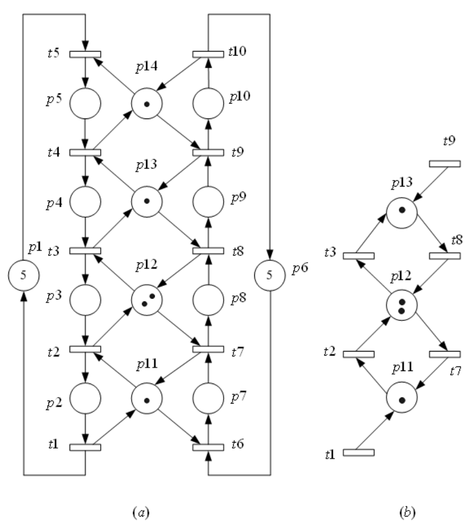


Fig. 1 (a) A marked L-S[3]PR $(N, M_0)$ and (b) Resource subnet $N_\Omega$ obtained from $(N, M_0)$

As shown in Fig. 1(a), $\Omega = \{p_{11}, p_{12}, p_{13}\}$ is a subset of resources in *N*. Then, resource subnet $N_\Omega = (P_\Omega, T_\Omega, F_\Omega)$ generated by $\Omega$ is shown in Fig. 1(b) with $P_\Omega = \Omega = \{p_{11}, p_{12}, p_{13}\}$ and $T_\Omega = \{t_1, t_2, t_3, t_7, t_8, t_9\}$. Trivially, $T_{\Omega\text{-}source} = \{t_1, t_9\}$, $T^{p11}_{\Omega-source} = \{t_1\}$, $T^{p12}_{\Omega-source} = \emptyset$, and $T^{p13}_{\Omega-source} = \{t_9\}$.

By using the proposed resource subnets, we can have the following lemmas.

*Lemma 3*: Let $\Omega = \{r_1, r_2, \ldots, r_m\} \subseteq P_R$ $(m \geq 2)$ be a subset of resources in *N*. *S* is a siphon if $S = \Omega \cup \bigcup_{t \in T_{\Omega-source}} ({}^\bullet t \cap P_A)$.

*Proof:* Refer to the work in [19]. ■

In what follows, we use $S_\Omega$ to denote the siphon derived from a resource subset $\Omega$, i.e., $S_\Omega = \Omega \cap \bigcup_{t \in T_{\Omega-source}} ({}^\bullet t \cap P_A)$.

*Lemma 4*: Let $\Omega = \{r_1, r_2, \ldots, r_m\} \in \mathbb{Q}$ $(m \geq 2)$ be a loop resource subset in *N*. Then, $S_\Omega$ is an SMS.

*Proof*: Similar to the proof of Theorem 10 in [14]. ■

*Definition 6*: Let $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ $(\alpha \neq \beta)$ be two siphons in *N*, where $\Omega_\alpha$, $\Omega_\beta$, and $\Omega_{\alpha,\beta} = \Omega_\alpha \cup \Omega_\beta$ are three resource subsets. $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ are composable if $\Omega_\alpha \cap \Omega_\beta \neq \emptyset$, $\Omega_\alpha \not\subset \Omega_\beta$, and $\Omega_\beta \not\subset \Omega_\alpha$. The resultant siphon by composing $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ is defined as

$$S_{\Omega_{\alpha,\beta}} = S_{\Omega_\alpha} \circ S_{\Omega_\beta} = \Omega_{\alpha,\beta} \cup \bigcup_{t \in T_{\Omega_{\alpha,\beta}\text{-}Source}} ({}^\bullet t \cap P_A).$$

As shown in Fig. 1(a), $\Omega_1 = \{p_{11}, p_{12}\}$, $\Omega_2 = \{p_{12}, p_{13}\}$, and $\Omega_{1,2} = \{p_{11}, p_{12}, p_{13}\}$ are three resource subsets.

$$S_{\Omega_1} = \Omega_1 \cup \bigcup_{t \in T_{\Omega_1-source}} ({}^\bullet t \cap P_A) = \{p_2, p_8, p_{11}, p_{12}\}.$$

$$S_{\Omega_2} = \Omega_2 \cup \bigcup_{t \in T_{\Omega_2-source}} ({}^\bullet t \cap P_A) = \{p_3, p_9, p_{12}, p_{13}\}.$$

$\Omega_1 \cap \Omega_2 = \{p_{12}\} \neq \emptyset$, $\Omega_1 \not\subset \Omega_2$, and $\Omega_2 \not\subset \Omega_1$. Hence $S_{\Omega_1}$ and $S_{\Omega_2}$ are composable and the resultant siphon by composing them is

$$S_{\Omega_{1,2}} = S_{\Omega_1} \circ S_{\Omega_2} = \Omega_{1,2} \cup \bigcup_{t \in T_{\Omega_{1,2}-source}} ({}^\bullet t \cap P_A) = \{p_2, p_9, p_{11}, p_{12}, p_{13}\}.$$

*Lemma 5*: Let $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ $(\alpha \neq \beta)$ be two composable siphons with $S_{\Omega_{\alpha,\beta}}$ being their resultant one in *N*. If $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ are SMS, so is $S_{\Omega_{\alpha,\beta}}$.

*Proof*: Similar to the proof of Theorem 11 in [14]. ■

*Lemma 6*: Let $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ $(\alpha \neq \beta)$ be two composable siphons with $S_{\Omega_{\alpha,\beta}}$ being their resultant one in *N*. If $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ are SMS, then there always exist two places $p_x$ and $p_y$ ($p_x \neq p_y$) such that $p_x \in S_{\Omega_\alpha} \setminus \Omega_\alpha$, $p_y \in S_{\Omega_\beta} \setminus \Omega_\beta$, and $p_x, p_y \notin S_{\Omega_{\alpha,\beta}}$.

*Proof:* Refer to the work in [19]. ■

*Definition 7*: Let $S_{\Omega_\alpha}$ and $S_{\Omega_\beta}$ ($\alpha \neq \beta$) be two composable SMS with $S_{\Omega_{\alpha,\beta}}$ being their resultant one in *N*. $D_{\alpha,\beta}(\Omega_\lambda) \subset \Omega_\lambda$ is called a key resource subset of $\Omega_\lambda$ if $D_{\alpha,\beta}(\Omega_\lambda) = \{r \mid r \in \Omega_\lambda, T^r_{\Omega_\lambda-source} \supset T^r_{\Omega_{\alpha,\beta}-source}, T^r_{\Omega_\lambda-source} \neq T^r_{\Omega_{\alpha,\beta}-source}\}$ ($\lambda = \alpha, \beta$). Key resource subsets denote the subsets of resource places whose source transition count is decreased after composing two SMS. Key resource subsets are key factors deciding the controllability condition of resultant siphons.

For example, $S_{\Omega_4} = \{p_2, p_9, p_{11}, p_{12}, p_{13}\}$ and $S_{\Omega_5} = \{p_3, p_{10}, p_{12}, p_{13}, p_{14}\}$ are SMS in Fig. 1(a) with $\Omega_4 = \{p_{11}, p_{12}, p_{13}\}$ and $\Omega_5 = \{p_{12}, p_{13}, p_{14}\}$. $S_{\Omega_{4,5}} = \{p_2, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}\}$ is the resultant one by composing $S_{\Omega_4}$ and $S_{\Omega_5}$, where $\Omega_{4,5} = \{p_{11}, p_{12}, p_{13}, p_{14}\}$. By Definition 5, the resource subnets that are generated by $\Omega_4, \Omega_5$, and $\Omega_{4,5}$ are shown in

Fig. 2.

Trivially, $T^{p13}_{\Omega_4-source} \supset T^{p13}_{\Omega_{4,5}-source}$ and $T^{p12}_{\Omega_5-source} \supset T^{p12}_{\Omega_{4,5}-source}$. Hence $D_{4,5}(\Omega_4)$ = {$p_{13}$} and $D_{4,5}(\Omega_5)$ = {$p_{12}$} are the respective key resource subsets of $\Omega_4$ and $\Omega_5$.

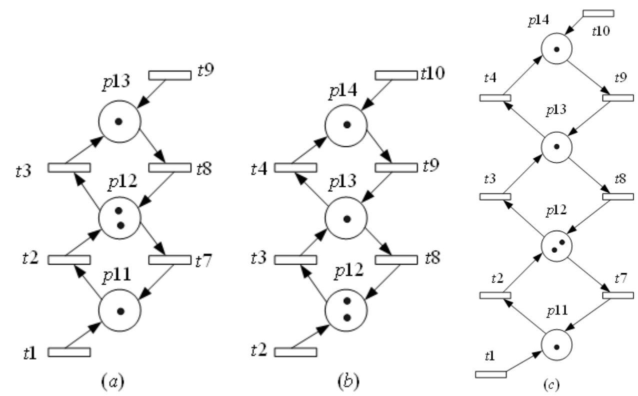


Fig. 2 Three resource subnets (*a*) $N_{\Omega_4}$, (*b*) $N_{\Omega_5}$, and (*c*) $N_{\Omega_{4,5}}$

*Lemma 7*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable siphons with $S_{\Omega_{1,2}}$ being their resultant one in *N*. If $S_{\Omega_1}$ and $S_{\Omega_2}$ are SMS, then $D_{1,2}(\Omega_1) \neq \varnothing$ and $D_{1,2}(\Omega_2) \neq \varnothing$.

*Proof:* Refer to the work in [19]. ■

Remark 1: Lemma 7 indicates that each of two composable SMS has at least one resource place whose source transition count is decreased after composing two SMS.

*Lemma 8* [26]: Let *S* be an SMS. Then, there exists an acceptable initial marking $M_0$ such that $\exists M \in R(N, M_0)$ : $M(S)$=0.

*Lemma 9*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. If $|D_{1,2}(\Omega_1) \cup D_{1,2}(\Omega_2)| \geq 2$, then there exists an acceptable initial marking $M_0$ such that $\exists M \in R(N, M_0)$ : $M(S_{\Omega_{1,2}})$=0, $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$.

*Proof:* Refer to the work in [19]. ■

Remark 2: Lemma 9 indicates that if the total number of key resources of $S_{\Omega_1}$ and $S_{\Omega_2}$ are larger than one, their resultant siphon $S_{\Omega_{1,2}}$ may be unmarked at a marking *M* where $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$.

For example, $S_{\Omega_4}$ = {$p_2, p_9, p_{11}, p_{12}, p_{13}$} and $S_{\Omega_5}$ = {$p_3, p_{10}, p_{12}, p_{13}, p_{14}$} are SMS in Fig. 1(a) with $S_{\Omega_{4,5}}$ ={$p_2, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}$} being their resultant one. The initial marking $M_0$ = [5, 0, 0, 0, 0, 5, 0, 0, 0, 0, 1, 2, 1, 1 ]$^T$. *M*= [3, 0, 1, 0, 1, 2, 1, 1, 1, 0, 0, 0, 0, 0 ]$^T$ is a marking reachable from $M_0$. According to Definition 7, $D_{4,5}(\Omega_4)$ = {$p_{13}$} and $D_{4,5}(\Omega_5)$ = {$p_{12}$}. Trivially, $M(S_{\Omega_1}) = 1$, $M(S_{\Omega_2}) = 1$, and $M(S_{\Omega_{1,2}})$ =0 with $|D_{4,5}(\Omega_4) \cup D_{4,5}(\Omega_5)| = 2$.

*Corollary 1*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be optimally controlled. $S_{\Omega_{1,2}}$ is not controlled if $|D_{1,2}(\Omega_1) \cup D_{1,2}(\Omega_2)| \geq 2$.

As shown in Fig. 3, two SMS $S_{\Omega_4}$ ={$p_2, p_9, p_{11}, p_{12}, p_{13}$} and $S_{\Omega_5}$ = {$p_3, p_{10}, p_{12}, p_{13}, p_{14}$} in the net in Fig. 1 are optimally controlled via monitors $V_4$ and $V_5$ by Proposition 1. Trivially, their resultant siphon $S_{\Omega_{4,5}}$ ={$p_2, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}$} is not controlled with $|D_{4,5}(\Omega_4) \cup D_{4,5}(\Omega_5)| = 2$.

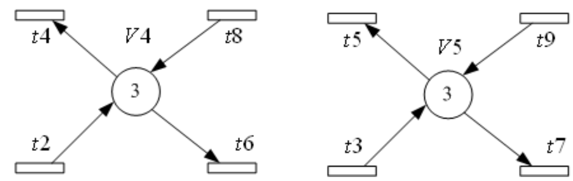


Fig. 3 Two monitors added to the net in Fig. 1(a)

*Lemma 10*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N* and $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$. If $M_0(r) \geq 2$, then there exists an acceptable initial marking $M_0$ such that $\exists M \in R(N, M_0)$ : $M(S_{\Omega_{1,2}})$=0, $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$.

*Proof*: Similar to the proof of Lemma 9. ■

Remark 3: Lemma 10 indicates that if two composable SMS $S_{\Omega_1}$ and $S_{\Omega_2}$ share the unique key resource *r* with $M_0(r) \geq 2$, their resultant siphon $S_{\Omega_{1,2}}$ may be unmarked at a marking *M* where $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$.

For example, $S_{\Omega_1}$ = {$p_2, p_8, p_{11}, p_{12}$} and $S_{\Omega_2}$ = {$p_3, p_9, p_{12}, p_{13}$} are SMS in Fig. 1(a) with $S_{\Omega_{1,2}}$ ={$p_2, p_9, p_{11}, p_{12}, p_{13}$} being their resultant one. The initial marking $M_0$ = [5, 0, 0, 0, 0, 5, 0, 0, 0, 0, 1, 2, 1, 1 ]$^T$. *M*= [3, 0, 1, 1, 0, 3, 1, 1, 0, 0, 0, 0, 0, 1 ]$^T$ is a marking reachable from $M_0$. According to Definition 7, $D_{1,2}(\Omega_1)$ = $D_{1,2}(\Omega_2)$ = {$p_{12}$}. Trivially, $M(S_{\Omega_1}) = 1$, $M(S_{\Omega_2}) = 1$, and $M(S_{\Omega_{1,2}})$ =0 with $M_0(p_{12}) = 2$.

*Corollary 2*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N* and $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$. Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be optimally controlled. $S_{\Omega_{1,2}}$ is not controlled if $M_0(r) \geq 2$.

As shown in Fig. 4, two SMS $S_{\Omega_1}$ = {$p_2, p_8, p_{11}, p_{12}$} and

$S_{\Omega_2}$ = {$p_3$, $p_9$, $p_{12}$, $p_{13}$} in the net in Fig. 1 are optimally controlled via monitors $V_1$ and $V_2$ by Proposition 1 where $D_{1,2}(\Omega_1)$ = $D_{1,2}(\Omega_2)$ = {$p_{12}$} with $M_0(p_{12}) = 2$ . Trivially, their resultant siphon $S_{\Omega_{1,2}}$ ={$p_2$, $p_9$, $p_{11}$, $p_{12}$, $p_{13}$} is not controlled.

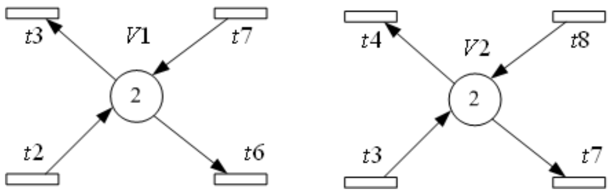


Fig. 4 Two monitors added to the net in Fig. 1(a)

*Lemma 11*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$ $\Rightarrow M(S_{\Omega_{1,2}}) \neq 0$ , if $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$ and $M_0(r) = 1$.

*Proof:* Refer to the work in [19]. ■

Remark 4: Lemma 11 indicates that if two composable SMS $S_{\Omega_1}$ and $S_{\Omega_2}$ share the unique key resource *r* with $M_0(r) = 1$, their resultant siphon $S_{\Omega_{1,2}}$ is marked at any marking *M* where $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$ .

For example, $S_{\Omega_2}$ = {$p_3$, $p_9$, $p_{12}$, $p_{13}$} and $S_{\Omega_3}$ = {$p_4$, $p_{10}$, $p_{13}$, $p_{14}$} are SMS in Fig. 1(a) with $S_{\Omega_{2,3}}$ ={$p_3$, $p_{10}$, $p_{12}$, $p_{13}$, $p_{14}$} being their resultant one. The initial marking $M_0$ = [5, 0, 0, 0, 0, 5, 0, 0, 0, 0, 1, 2, 1, 1 ]$^T$ and $D_{2,3}(\Omega_2)$ = $D_{2,3}(\Omega_3)$ = {$p_{13}$} with $M_0(p_{13}) = 1$ . Trivially, we cannot find a marking *M* reachable from $M_0$ such that $M(S_{\Omega_1}) \neq 0$ , $M(S_{\Omega_2}) \neq 0$ , and $M(S_{\Omega_{1,2}})$ =0.

*Corollary 3*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be (optimally) controlled. $S_{\Omega_{1,2}}$ is controlled if $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$ and $M_0(r) = 1$.

As shown in Fig. 5, two SMS $S_{\Omega_2}$ = {$p_3$, $p_9$, $p_{12}$, $p_{13}$} and $S_{\Omega_3}$ = {$p_4$, $p_{10}$, $p_{13}$, $p_{14}$} in the net in Fig. 1 are optimally controlled via monitors $V_2$ and $V_3$ by Proposition 1 where $D_{2,3}(\Omega_2)$ = $D_{2,3}(\Omega_3)$ = {$p_{13}$} with $M_0(p_{13}) = 1$ . Trivially, their resultant siphon $S_{\Omega_{2,3}}$ ={$p_3$, $p_{10}$, $p_{12}$, $p_{13}$, $p_{14}$} is controlled.

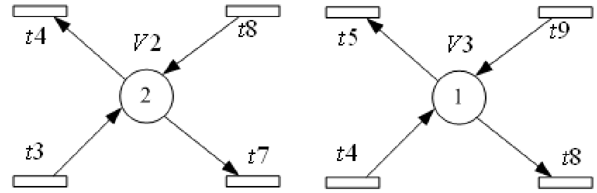


Fig. 5 Two monitors added to the net in Fig. 1(a)

*Theorem 1*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. If $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$ then $M(S_{\Omega_{1,2}}) \neq 0$ *iff* $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$ and $M_0(r) = 1$.

*Proof*: Straight forward from Lemmas 8, 9, 10, and 11. ■

Remark 5: Theorem 1 shows that if and only if two composable SMS $S_{\Omega_1}$ and $S_{\Omega_2}$ share the unique key resource *r* with $M_0(r) = 1$, their resultant siphon $S_{\Omega_{1,2}}$ is marked at any marking *M* where $M(S_{\Omega_1}) \neq 0$ and $M(S_{\Omega_2}) \neq 0$ .

*Theorem 2*: Let $S_{\Omega_1}$ and $S_{\Omega_2}$ be two composable SMS with $S_{\Omega_{1,2}}$ being their resultant one in *N*. If $S_{\Omega_1}$ and $S_{\Omega_2}$ are optimally controlled, then $S_{\Omega_{1,2}}$ is controlled *iff* $D_{1,2}(\Omega_1) = D_{1,2}(\Omega_2) = \{r\}$ with $M_0(r) = 1$.

*Proof*: Straight forward from Corollary 1, Corollary 2, and Theorem 1. ■

*Theorem 3*: Let $S_{\Omega_i}$ be optimally controlled for $\forall i \in \{1,2,\ldots,k\}$ and $S_{\Omega_{1,2,3,\cdots,k}} = S_{\Omega_1} \circ S_{\Omega_2} \circ S_{\Omega_3} \circ \cdots \circ S_{\Omega_k}$ $= (\cdots((S_{\Omega_1} \circ S_{\Omega_2}) \circ S_{\Omega_3}) \circ \cdots) \circ S_{\Omega_k}$ . $S_{\Omega_{1,2,3,\cdots,k}}$ is controlled if $D_{(1,2,\cdots i),i+1}(\Omega_{1,2,\cdots,i}) = D_{(1,2,\cdots i),i+1}(\Omega_{i+1}) = \{r_i\}$ with $M_0(r_i) = 1$ for $\forall i \in \{1,2,\ldots,k\text{-}1\}$.

*Proof*: Straight forward from Theorem 2. ■

Remark 6: Theorems 2 and 3 indicate that under some conditions, a resultant siphon is always controlled if its SMS are optimally controlled. Therefore, no monitor is needed to control such resultant siphons. In an L-S$^3$PR, there are many instances that the initial marking of each resource place is 1. Therefore, in a general case, utilizing Theorems 2 and 3 can reduce the number of monitors.

## IV. Conclusion

Given an L-S$^3$PR net model, the common deadlock prevention policies need to add a monitor to every SMS. These approaches have a problem that the supervisor can be highly complex when the number of SMS is very large. To minimize the number of SMS that need to be controlled, this paper proposes a sufficient and necessary condition under which the resultant siphon is always marked if its SMS are optimally controlled in an L-S$^3$PR. Future work includes extending the controllability conditions to more general classes of Petri nets and utilizing the newly derived controllability conditions in deadlock prevention policies.

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