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Unknown Input and Sensor Fault Estimation Using Sliding-Mode Observers

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Abstract— **Sliding-mode observers are used to construct unknown input estimators. Then, these unknown input estimators are combined with sensor fault estimation schemes into one architecture that employs two sliding-mode observers for simultaneously estimating the plant's actuator faults (part of the unknown input) and detecting sensor faults. Closed form expressions are presented for the estimates of unknown inputs and sensor faults. A benchmark example of a controlled inverted pendulum system from the literature is utilized in the simulation study. The study shows that the observers analyzed in this paper generate good estimates of the unknown input and sensor faults signals in noisy environments for nonlinear plants.**

I. Introduction

Sliding-mode observers for dynamic systems with unknown inputs have recently found applications in robust detection and reconstruction of actuator and sensor faults [1], [2], [3], [4] and fault detection and isolation (FDI) [5]. In particular, the purpose of the FDI scheme is to generate an alarm when a fault, such as a component malfunction, develops in the process being monitored and to identify the location of the fault.

In almost all of the above referenced papers, the authors use the Edwards-Spurgeon sliding mode observer to estimate sensor or actuator faults. One of the objectives of this paper is to demonstrate the feasibility of other sliding mode unknown input observation schemes for fault detection. In this paper, sliding-mode observers are used to construct unknown input estimators. The unknown input could represent a combination of actuator faults and unmodeled system dynamics and uncertainties. The design of sliding-mode observer based sensor fault estimators is also considered. In the proposed schemes, Utkin's [6] sliding-mode observer and the sliding-mode observer proposed by the authors in [7] are employed.

In Yan and Edwards [3], actuator fault detection schemes are proposed for nonlinear systems using the Edwards-Spurgeon sliding-mode observer. Sensor fault detection schemes are advanced in [1], [4]. In these schemes, when considering sensor faults, bounded uncertainties can be present, however, it is assumed that there are no actuator faults. In [1], the proposed method for estimating sensor faults is based on filtering the faulty plant output. In [4], the fault signal to be estimated is augmented with the plant state vector. Then, the Edwards-Spurgeon sliding-mode observer is designed for the augmented system.

In this paper, the proposed unknown input and sensor fault estimation schemes are combined into one architecture that employs two sliding-mode observers for simultaneously estimating the plant's actuator faults (part of the unknown input) and detecting sensor faults. In this architecture, a designer can use two of the same type of observers considered in this paper or their combination. In this scheme, an estimate of the unknown input is combined with the output of the second sliding-mode observer to obtain an estimate of the sensor fault. As in Edwards and Spurgeon [8] and Edwards et al. [9], the sliding-mode observers used in this study feed back the output observation error through discontinuous terms that induce a sliding motion in the state observation error space.

The efficacy of the proposed designs is tested on the controlled inverted pendulum system from Edwards and Spurgeon [8]. In the observers' design, the linearized model of the above plant is used, however, in the simulations, the nonlinear model is employed to demonstrate the feasibility of the proposed architectures. The study includes a demonstration of the robustness of the methods by using the nonlinear plant model in the simulations and by adding zero mean uniform and Gaussian noise to the input and output signals of the system.

II. Design Model

The estimator designs are based on a linearized model of a given nonlinear plant model. The linear model used in the design has the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}_1 + \boldsymbol{B}_d\boldsymbol{u}_d + \boldsymbol{B}_a\boldsymbol{f}_a \qquad (1)$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}, \qquad (2)$$

where $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, the input matrix $\boldsymbol{B}_1 \in \mathbb{R}^{n\times m_1}$, $\boldsymbol{B}_d \in \mathbb{R}^{n\times m_d}$, $\boldsymbol{B}_a \in \mathbb{R}^{n\times m_a}$, and the output matrix $\boldsymbol{C} \in \mathbb{R}^{p\times n}$. The vector function $\boldsymbol{u}_1$ is the plant's control input. The vector function $\boldsymbol{u}_d = \boldsymbol{u}_d(t, \boldsymbol{x}, \boldsymbol{u}_1)$ may model lumped uncertainties or nonlinearities in the plant, as well as input disturbance. The vector function $\boldsymbol{f}_a$ models the actuator fault. It is assumed that the model parameters $\boldsymbol{A}$, $\boldsymbol{B}_1$, $\boldsymbol{B}_d$, $\boldsymbol{B}_a$, and $\boldsymbol{C}$ are known. Note that the plant's inputs $\boldsymbol{u}_d$ and $\boldsymbol{f}_a$ are assumed to be unknown. The two unknown inputs are combined into one and defined to be $\boldsymbol{u}_2$. That is,

$$\boldsymbol{u}_2 = \begin{bmatrix} \boldsymbol{u}_d^\top & \boldsymbol{f}_a^\top \end{bmatrix}^\top.$$

Correspondingly, the unknown input matrix is formed as,

$$\boldsymbol{B}_2 = \begin{bmatrix} \boldsymbol{B}_d & \boldsymbol{B}_a \end{bmatrix},$$

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where $\boldsymbol{B}_2 \in \mathbb{R}^{n\times m_2}$. Taking the above notation into account, the design model (1) is represented as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}_1 + \boldsymbol{B}_2\boldsymbol{u}_2, \tag{3}$$

It is also assumed that $\boldsymbol{u}_2$ is bounded, that is, there exists a nonnegative real number, ρ, such that $\|\boldsymbol{u}_2\| \le \rho$. Next, it is also assumed that the pair $(\boldsymbol{A}, \boldsymbol{C})$ is observable and that the uncertainty distribution matrix, $\boldsymbol{B}_2$, satisfies the condition

$$\operatorname{rank} \boldsymbol{C}\boldsymbol{B}_2 = \operatorname{rank} \boldsymbol{B}_2 = m_2, \tag{4}$$

that is, $\boldsymbol{C}\boldsymbol{B}_2$ is a full column rank matrix, which implies that $p \ge m_2$. Furthermore, it is also assumed that rank $\boldsymbol{C} = p$.

III. Construction of Unknown Input Estimators

In this section, the design of two different unknown input estimators is considered. The unknown input estimates are obtained by processing the output signal of a nonlinear component of the observer used to construct the unknown input estimator.

A. Utkin's Sliding-Mode Observer Based Unknown Input Estimator

In [10], Utkin's sliding-mode observer was applied to estimate the states of systems with unknown inputs. In this section, the aforementioned design is used to construct an unknown input estimator. To proceed, the uncertain model given by (1) and (2) is transformed into new coordinates. This transformation was suggested by Luenberger [11, page 305] and it has the form,

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{C} \end{bmatrix} \boldsymbol{x} = \boldsymbol{T}\boldsymbol{x} = \begin{bmatrix} \tilde{\boldsymbol{x}}_1 \\ \boldsymbol{y} \end{bmatrix}, \tag{5}$$

where the submatrix $\boldsymbol{N} \in \mathbb{R}^{(n-p)\times n}$ is such that $\det \boldsymbol{T} \neq 0$. The transformed system has the form

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}}_1 \\ \dot{\boldsymbol{y}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}}_1 \\ \boldsymbol{y} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_{11} \\ \boldsymbol{B}_{12} \end{bmatrix} \boldsymbol{u}_1 + \begin{bmatrix} \boldsymbol{B}_{21} \\ \boldsymbol{B}_{22} \end{bmatrix} \boldsymbol{u}_2. \tag{6}$$

Note that $\boldsymbol{C}\boldsymbol{B}_2 = \boldsymbol{B}_{22}$. Thus, $\boldsymbol{B}_{22}$ is a full column rank matrix and therefore there exists a matrix $\boldsymbol{B}_{22}^{\dagger}$ such that $\boldsymbol{B}_{22}^{\dagger}\boldsymbol{B}_{22} = \boldsymbol{I}_{m_2}$. Next, the following transformation,

$$\begin{bmatrix} \bar{\boldsymbol{x}}_1 \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{n-p} & -\boldsymbol{B}_{21}\boldsymbol{B}_{22}^{\dagger} \\ \boldsymbol{O} & \boldsymbol{I}_p \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}}_1 \\ \boldsymbol{y} \end{bmatrix}, \tag{7}$$

is applied to the model given by (6) to obtain

$$\begin{bmatrix} \dot{\bar{\boldsymbol{x}}}_1 \\ \dot{\boldsymbol{y}} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{x}}_1 \\ \boldsymbol{y} \end{bmatrix} + \begin{bmatrix} \bar{\boldsymbol{B}}_{11} \\ \bar{\boldsymbol{B}}_{12} \end{bmatrix} \boldsymbol{u}_1 + \begin{bmatrix} \boldsymbol{O} \\ \bar{\boldsymbol{B}}_{22} \end{bmatrix} \boldsymbol{u}_2, \tag{8}$$

where $\bar{\boldsymbol{B}}_{22} \in \mathbb{R}^{p\times m_2}$. The assumption that the pair $(\boldsymbol{A}, \boldsymbol{C})$ is observable implies that the pair $(\bar{\boldsymbol{A}}_{11}, \bar{\boldsymbol{A}}_{21})$ is also observable and hence we can select a matrix $\bar{\boldsymbol{L}}_1$ so that the matrix $(\bar{\boldsymbol{A}}_{11} - \bar{\boldsymbol{L}}_1\bar{\boldsymbol{A}}_{21})$ has all its eigenvalues in prescribed locations, symmetric with respect to the real axis, in the open left-half plane. For systems modeled by (8), an observer of the form

$$\begin{bmatrix} \dot{\hat{\boldsymbol{x}}}_1 \\ \dot{\hat{\boldsymbol{y}}} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{x}}_1 \\ \hat{\boldsymbol{y}} \end{bmatrix} + \begin{bmatrix} \bar{\boldsymbol{B}}_{11} \\ \bar{\boldsymbol{B}}_{12} \end{bmatrix} \boldsymbol{u}_1 + \begin{bmatrix} \bar{\boldsymbol{L}}_1 \\ \boldsymbol{I}_p \end{bmatrix} \boldsymbol{v}, \tag{9}$$

can be constructed. The nonlinear injection function $\boldsymbol{v}$ is given by $\boldsymbol{v} = M\,\mathbf{sign}\,(\boldsymbol{e}_y)$, where $\boldsymbol{e}_y = \boldsymbol{y} - \hat{\boldsymbol{y}}$ and the gain $M > 0$ is a design parameter. For other forms of the injection function, the reader is referred to [10]. It is important to emphasize that the vector $\boldsymbol{e}_y = \boldsymbol{y} - \hat{\boldsymbol{y}}$ is available, so it can be used in the observer synthesis. Let $\boldsymbol{e}_1 = \bar{\boldsymbol{x}}_1 - \hat{\boldsymbol{x}}_1$. Then subtracting (8) from (9), we obtain

$$\begin{bmatrix} \dot{\boldsymbol{e}}_1 \\ \dot{\boldsymbol{e}}_y \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_y \end{bmatrix} - \begin{bmatrix} \bar{\boldsymbol{L}}_1 \\ \boldsymbol{I}_p \end{bmatrix} \boldsymbol{v} + \begin{bmatrix} \boldsymbol{O} \\ \bar{\boldsymbol{B}}_{22} \end{bmatrix} \boldsymbol{u}_2. \tag{10}$$

The error system is transformed using

$$\begin{bmatrix} \tilde{\boldsymbol{e}}_1 \\ \boldsymbol{e}_y \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{n-p} & -\bar{\boldsymbol{L}}_1 \\ \boldsymbol{O} & \boldsymbol{I}_p \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_y \end{bmatrix}. \tag{11}$$

The error system in the new coordinates takes the form

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{e}}}_1 \\ \dot{\boldsymbol{e}}_y \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{A}}_{11} & \tilde{\boldsymbol{A}}_{12} \\ \tilde{\boldsymbol{A}}_{21} & \tilde{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{e}}_1 \\ \boldsymbol{e}_y \end{bmatrix} - \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{I}_p \end{bmatrix} \boldsymbol{v} + \begin{bmatrix} -\bar{\boldsymbol{L}}_1\bar{\boldsymbol{B}}_{22} \\ \bar{\boldsymbol{B}}_{22} \end{bmatrix} \boldsymbol{u_2}, \tag{12}$$

where $\tilde{\boldsymbol{A}}_{11} = \bar{\boldsymbol{A}}_{11} - \bar{\boldsymbol{L}}_1\bar{\boldsymbol{A}}_{21}$, $\tilde{\boldsymbol{A}}_{12} = \bar{\boldsymbol{A}}_{12} - \bar{\boldsymbol{L}}_1\bar{\boldsymbol{A}}_{22} + \tilde{\boldsymbol{A}}_{11}\bar{\boldsymbol{L}}_1$, and $\tilde{\boldsymbol{A}}_{22} = \bar{\boldsymbol{A}}_{22} + \bar{\boldsymbol{A}}_{21}\bar{\boldsymbol{L}}_1$. Note that by design, the matrix $\tilde{\boldsymbol{A}}_{11}$ is asymptotically stable. To proceed, the equivalent control method [8, p. 7] is used to obtain an expression for an estimate of the unknown input. It is assumed that the error system (12) is in sliding along $\boldsymbol{e}_y = \boldsymbol{0}$. Therefore, $\dot{\tilde{\boldsymbol{e}}}_1 = \boldsymbol{0}$ and $\dot{\boldsymbol{e}}_y = \boldsymbol{0}$. Taking the above into account we obtain, $\tilde{\boldsymbol{e}}_1 = \tilde{\boldsymbol{A}}_{11}^{-1}\bar{\boldsymbol{L}}_1\bar{\boldsymbol{B}}_{22}\boldsymbol{u}_2$ and $\tilde{\boldsymbol{A}}_{21}\tilde{\boldsymbol{e}}_1 - \boldsymbol{v}_{\text{eq}} + \bar{\boldsymbol{B}}_{22}\boldsymbol{u}_2 = \boldsymbol{0}$, where $\boldsymbol{v}_{\text{eq}}$ is the "equivalent control" that induces a sliding-mode. Solving the above system of equations for $\boldsymbol{u}_2$ yields the following estimate for $\boldsymbol{u}_2$,

$$\boxed{\boldsymbol{u}_2 \approx \left(\left(\boldsymbol{I}_p + \tilde{\boldsymbol{A}}_{21}\tilde{\boldsymbol{A}}_{11}^{-1}\bar{\boldsymbol{L}}_1\right)\bar{\boldsymbol{B}}_{22}\right)^{\dagger}\boldsymbol{v}_{\text{eq}}} \tag{13}$$

B. A Sliding-Mode Observer [7] Based Unknown Input Estimator

In [7], a sliding-mode observer was designed to estimate the states of systems with unknown inputs. In this section, the aforementioned design is used to construct an unknown input estimator. A Lyapunov second method approach is taken to synthesize and analyze the sliding-mode observer for the system modeled by (1) and (2). Let $\hat{\boldsymbol{x}}$ be an estimate of $\boldsymbol{x}$ and let $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ denote the state estimation error. The observability of $(\boldsymbol{A}, \boldsymbol{C})$ implies the existence of a matrix $\boldsymbol{L} \in \mathbb{R}^{n\times p}$ such that the matrix $(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})$ has prescribed (symmetric with respect to the real axis) eigenvalues in the open left-half plane. Because $(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})$

is asymptotically stable, there is a $\boldsymbol{P} = \boldsymbol{P}^\top > 0$ such that $(\boldsymbol{A} - \boldsymbol{LC})^\top \boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} - \boldsymbol{LC}) < 0$, and for some $\boldsymbol{F} \in \mathbb{R}^{m_1 \times p}$, $\boldsymbol{FC} = \boldsymbol{B}_2^\top \boldsymbol{P}$. The last condition is needed to ensure the realizability of the state estimator. Necessary and sufficient conditions for the existence of the triple of matrices $(\boldsymbol{L}, \boldsymbol{F}, \boldsymbol{P})$ such that the above two conditions are satisfied are given in [7]. To proceed, define the injection function

$$\boldsymbol{E}(\boldsymbol{e}, \kappa) = \begin{cases} \kappa \frac{\boldsymbol{FCe}}{\|\boldsymbol{FCe}\|_2} & \text{for} \quad \boldsymbol{FCe} \neq \boldsymbol{0} \\ \boldsymbol{0} & \text{for} \quad \boldsymbol{FCe} = \boldsymbol{0}, \end{cases}$$

where $\kappa \geq 0$ is a design parameter. Note that if the plant is single-input, then $\boldsymbol{E}(\boldsymbol{e}, \kappa) = \kappa \operatorname{sign}(\boldsymbol{FCe})$. Using the arguments from [12], it can be shown that the state $\hat{\boldsymbol{x}}$ of the dynamic system

$$\dot{\hat{\boldsymbol{x}}} = (\boldsymbol{A} - \boldsymbol{LC})\hat{\boldsymbol{x}} + \boldsymbol{B}_2 \boldsymbol{E}(\boldsymbol{e}, \kappa) + \boldsymbol{Ly} + \boldsymbol{B}_1 \boldsymbol{u}_1, \qquad (14)$$

for $\kappa \geq \rho$, is an asymptotic estimate of the state $\boldsymbol{x}$ of the system described by (1) and (2), that is,

$$\lim_{t \to \infty} \boldsymbol{e}(t) = \lim_{t \to \infty} (\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)) = \boldsymbol{0}.$$

The differential equation describing the estimation error is

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}} - \dot{\hat{\boldsymbol{x}}} = (\boldsymbol{A} - \boldsymbol{LC})\boldsymbol{e} - \boldsymbol{B}_2 \boldsymbol{E}(\boldsymbol{e}, \kappa) + \boldsymbol{B}_2 \boldsymbol{u_2}. \qquad (15)$$

Since $\boldsymbol{B}_2$ has full rank, we have the approximation

$$\boxed{\boldsymbol{u}_2 \approx \boldsymbol{E}(\boldsymbol{e}, \kappa)} \qquad (16)$$

IV. Estimating Sensor Faults

In this section, the problem of detecting faults in the output channels i.e. sensor faults is analyzed. It is assumed that the plant output has the form

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{f}_o, \qquad (17)$$

where the function $\boldsymbol{f}_o$ models the sensor fault. The Utkin sliding-mode observer and the sliding-mode observer from [7] are used to construct sensor fault detectors.

A. Utkin's Sliding-Mode Observer Based Sensor Fault Estimator

First, the Utkin's sliding-mode observer based sensor fault estimator is designed. Taking into account that now the plant output contains the fault $\boldsymbol{f}_o$ as given by (17), the error dynamics given by (10) take the form,

$$\begin{bmatrix} \dot{\boldsymbol{e}}_1 \\ \dot{\boldsymbol{e}}_y \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{A}}_{11} & \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{21} & \bar{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_y \end{bmatrix} - \begin{bmatrix} \bar{\boldsymbol{A}}_{12} \\ \bar{\boldsymbol{A}}_{22} \end{bmatrix} \boldsymbol{f}_o - \begin{bmatrix} \bar{\boldsymbol{L}}_1 \\ \boldsymbol{I}_p \end{bmatrix} \boldsymbol{v} + \begin{bmatrix} \boldsymbol{O} \\ \bar{\boldsymbol{B}}_{22} \end{bmatrix} \boldsymbol{u}_2 + \begin{bmatrix} \boldsymbol{O} \\ \dot{\boldsymbol{f}}_o \end{bmatrix}. \qquad (18)$$

Using (11), the error dynamics become,

$$\begin{bmatrix} \dot{\tilde{\boldsymbol{e}}}_1 \\ \dot{\boldsymbol{e}}_y \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{A}}_{11} & \tilde{\boldsymbol{A}}_{12} \\ \tilde{\boldsymbol{A}}_{21} & \tilde{\boldsymbol{A}}_{22} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{e}}_1 \\ \boldsymbol{e}_y \end{bmatrix} - \begin{bmatrix} \tilde{\boldsymbol{A}}_{12} \\ \tilde{\boldsymbol{A}}_{22} \end{bmatrix} \boldsymbol{f}_o - \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{I}_p \end{bmatrix} \boldsymbol{v} + \begin{bmatrix} -\bar{\boldsymbol{L}}_1 \bar{\boldsymbol{B}}_{22} \\ \bar{\boldsymbol{B}}_{22} \end{bmatrix} \boldsymbol{u_2} + \begin{bmatrix} -\bar{\boldsymbol{L}}_1 \dot{\boldsymbol{f}}_o \\ \dot{\boldsymbol{f}}_o \end{bmatrix}. \qquad (19)$$

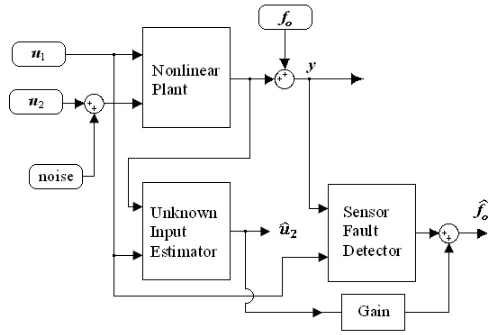

Fig. 1. A block diagram of combined unknown input estimator and sensor fault detector.

Assuming that the system is in sliding along $\boldsymbol{e}_y = \boldsymbol{0}$, we have $\dot{\tilde{\boldsymbol{e}}}_1 = \boldsymbol{0}$ and $\dot{\boldsymbol{e}}_y = \boldsymbol{0}$. Taking the above into account we obtain,

$$\tilde{\boldsymbol{e}}_1 = \tilde{\boldsymbol{A}}_{11}^{-1} \tilde{\boldsymbol{A}}_{12} \boldsymbol{f}_o + \tilde{\boldsymbol{A}}_{11}^{-1} \bar{\boldsymbol{L}}_1 \bar{\boldsymbol{B}}_{22} \boldsymbol{u}_2 + \tilde{\boldsymbol{A}}_{11}^{-1} \bar{\boldsymbol{L}}_1 \dot{\boldsymbol{f}}_o \qquad (20)$$

and

$$\boldsymbol{0} = \tilde{\boldsymbol{A}}_{21} \tilde{\boldsymbol{e}}_1 - \tilde{\boldsymbol{A}}_{22} \boldsymbol{f}_o - \boldsymbol{v}_{\text{eq}} + \bar{\boldsymbol{B}}_{22} \boldsymbol{u}_2 + \dot{\boldsymbol{f}}_o. \qquad (21)$$

Let $\boldsymbol{A}_{\text{feq}} = \left(\tilde{\boldsymbol{A}}_{22} - \tilde{\boldsymbol{A}}_{21} \tilde{\boldsymbol{A}}_{11}^{-1} \tilde{\boldsymbol{A}}_{12}\right)^\dagger$. Combining (20) and (21), and performing some simple manipulations, the expression for the estimate of the sensor fault in terms of the "equivalent" error injection term is obtained as,

$$\boldsymbol{f}_o \approx -\boldsymbol{A}_{\text{feq}} \boldsymbol{v}_{\text{eq}} + \boldsymbol{A}_{\text{feq}} \left(\boldsymbol{I}_p + \tilde{\boldsymbol{A}}_{21} \tilde{\boldsymbol{A}}_{11}^{-1} \bar{\boldsymbol{L}}_1\right) \left(\dot{\boldsymbol{f}}_o + \bar{\boldsymbol{B}}_{22} \boldsymbol{u}_2\right). \qquad (22)$$

In the above, the generalized inverse is taken instead of the usual inverse because the matrix $\left(\tilde{\boldsymbol{A}}_{22} - \tilde{\boldsymbol{A}}_{21} \tilde{\boldsymbol{A}}_{11}^{-1} \tilde{\boldsymbol{A}}_{12}\right)$ may be singular, as it is the case in our numerical experiment example in the following section.

It should be noted that in (22), we do not have access to $\boldsymbol{u}_2$ since it is unknown. We can, however, obtain an estimate of the unknown input, denoted $\hat{\boldsymbol{u}}_2$, using the architecture described in Section III. Furthermore we also assume that $\boldsymbol{f}_o$ is slow varying so $\dot{\boldsymbol{f}}_o \approx 0$.

The proposed unknown input and sensor fault estimation schemes are combined into one architecture that employs two sliding-mode observers for simultaneously estimating the plant's actuator faults (part of the unknown input) and detecting sensor faults. The first observer is used to construct an estimate of the unknown input (containing disturbances and actuator faults). The second sliding-mode observer is designed to detect sensor faults and it uses the estimates of the unknown input. A block diagram of this architecture is shown in Figure 1. The following estimate is obtained,

$$\boxed{\boldsymbol{f}_o \approx -\boldsymbol{A}_{\text{feq}} \left(\boldsymbol{v}_{\text{eq}} - \left(\boldsymbol{I}_p + \tilde{\boldsymbol{A}}_{21} \tilde{\boldsymbol{A}}_{11}^{-1} \bar{\boldsymbol{L}}_1\right) \bar{\boldsymbol{B}}_{22} \hat{\boldsymbol{u}}_2\right)}, \qquad (23)$$

which is adequate for simple disturbances as the simulations will demonstrate.

B. A Sliding-Mode Observer [7] Based Sensor Fault Estimator

The design of the sliding-mode observer [7] based sensor fault estimator is now considered. Using (3), (14) and (17), we obtain

$$\dot{\boldsymbol{e}}_y = \boldsymbol{C}\dot{\boldsymbol{e}} + \dot{\boldsymbol{f}}_o = \boldsymbol{C}(\boldsymbol{A}-\boldsymbol{L}\boldsymbol{C})\boldsymbol{e} + \boldsymbol{C}\boldsymbol{B}_2(\boldsymbol{u}_2 - \boldsymbol{E}(\boldsymbol{e},\kappa)) - \boldsymbol{C}\boldsymbol{L}\boldsymbol{f}_o + \dot{\boldsymbol{f}}_o.$$

It is assumed that $\boldsymbol{E}(\boldsymbol{e},\kappa)$ is selected so that the above system is in sliding along $\boldsymbol{F}\boldsymbol{e}_y = \boldsymbol{0}$ and that $\|\boldsymbol{e}\|$ is small. This implies the following expression for the "equivalent" $\boldsymbol{E}$,

$$\boldsymbol{0} = -\boldsymbol{F}\boldsymbol{C}\boldsymbol{B}_2\boldsymbol{E}_{\text{eq}} - \boldsymbol{F}\boldsymbol{C}\boldsymbol{L}\boldsymbol{f}_o + \boldsymbol{F}\boldsymbol{C}\boldsymbol{B}_2\boldsymbol{u}_2 + \boldsymbol{F}\dot{\boldsymbol{f}}_o. \quad (24)$$

An estimate of the fault $\boldsymbol{f}_o$ can be obtained as

$$\boldsymbol{f}_o \approx -(\boldsymbol{F}\boldsymbol{C}\boldsymbol{L})^{\dagger}(\boldsymbol{F}\boldsymbol{C}\boldsymbol{B}_2)\boldsymbol{E}_{\text{eq}} + (\boldsymbol{F}\boldsymbol{C}\boldsymbol{L})^{\dagger}\left(\boldsymbol{F}\boldsymbol{C}\boldsymbol{B}_2\boldsymbol{u}_2 + \boldsymbol{F}\dot{\boldsymbol{f}}_o\right). \quad (25)$$

It is assumed that $\boldsymbol{f}_o$ is slow varying so $\dot{\boldsymbol{f}}_o \approx 0$. An estimate of the unknown input $\boldsymbol{u}_2$ using the architecture from Section III is used to obtain the following sensor fault estimate,

$$\boxed{\boldsymbol{f}_o \approx -(\boldsymbol{F}\boldsymbol{C}\boldsymbol{L})^{\dagger}(\boldsymbol{F}\boldsymbol{C}\boldsymbol{B}_2)(\boldsymbol{E}_{\text{eq}} - \hat{\boldsymbol{u}}_2)} \quad (26)$$

V. Numerical Experiments

A. Plant and Noise Models

The model used to test the performance of the estimators is a pendulum-balancer from [9]. The modeling equations are: $(M+m)\ddot{x} + F_x\dot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = u$ and $J\ddot{\theta} + F_\theta\dot{\theta} - mlg\sin\theta + ml\ddot{x}\cos\theta = 0$. The system parameter values are given in Table I. The state variables are as in [9],

TABLE I

Parameters for the inverted pendulum system [8, p. 119].

	M	m	J	l	F_x	F_θ	g
Values	3.2	0.535	0.062	0.365	6.2	0.009	9.807
Units	kg	kg	kg·m^2	m	kg/s	kg·m^2	m/s^2

that is; $x_1 = x$, $x_2 = \theta$, $x_3 = \dot{x}$, and $x_4 = \dot{\theta}$. The system is linearized about the origin giving

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.9333 & -1.9872 & 0.0091 \\ 0 & 36.9771 & 6.2589 & -0.1738 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \boldsymbol{B}_a = \boldsymbol{B}_d = \begin{bmatrix} 0 \\ 0 \\ 0.3205 \\ -1.0095 \end{bmatrix},$$

and

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

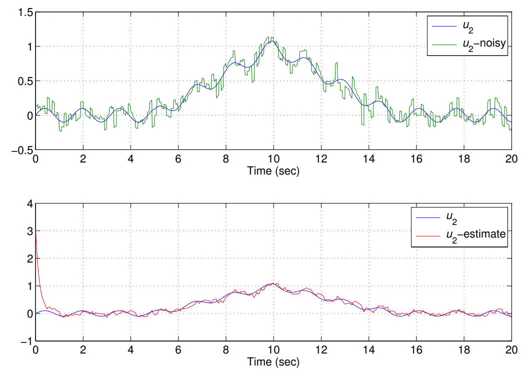

Fig. 2. Estimation of the unknown input corrupted by the Gaussian noise using Utkin's observer.

The linearized plant model is used to construct the unknown input and sensor fault estimators. However, in the simulations, our designs are tested on the nonlinear plant model. Since the plant is unstable, a stabilizing feedback controller using the state estimates, $\hat{\boldsymbol{x}}$, is used to stabilize it, $\boldsymbol{u}_1 = -\boldsymbol{k}\hat{\boldsymbol{x}}$, where

$$\boldsymbol{k} = \begin{bmatrix} -41.2181 & -171.6711 & -43.1215 & -29.3803 \end{bmatrix}.$$

B. Unknown Input Estimation

The unknown input for the system has the form,

$$u_2(t) = 0.1\sin(4t) + \max\left\{0, 1 - \frac{|t-10|}{5}\right\},$$

where the disturbance is modeled as a sinusoid with frequency 4 radians per second and the actuator fault is the triangle function defined above. A nonzero initial state $\boldsymbol{x}(0) = [0.10,\ -0.05,\ 0.15,\ 0.05]^{\top}$ is used in all the simulations. A Gaussian noise model with zero mean and variance 0.01 at the unknown input and independent zero mean Gaussian with variance 0.001 is introduced at each measurement channel. The sample rate of the noise is 0.1 second. The estimated unknown input is obtained by filtering the signal obtained from (16) or (13). The filter used is a first order lowpass filter with the transfer function

$$H(s) = \frac{1}{\tau s + 1} \quad \text{where} \quad \tau = 0.15.$$

It can be seen from Figure 2 that Utkin's observer yields good estimates of the unknown input in the presence of Gaussian noise. This unknown input estimator also performs equally well in the presence of uniform noise.

Next, the unknown input estimator constructed using the sliding-mode observer from [7] is used to estimate the unknown input. This observer also estimates the unknown input exceptionally well in the presence of Gaussian noise as can be seen from Figure 3.

C. Sensor Fault Estimation

In this section, simulations involving the response of sensor fault detectors, constructed using the Utkin and the observer from [7] are presented. The unknown input for

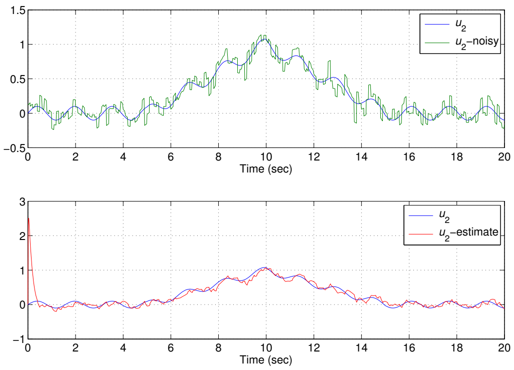

Fig. 3. Estimation of the unknown input corrupted by the Gaussian noise using observer from [7].

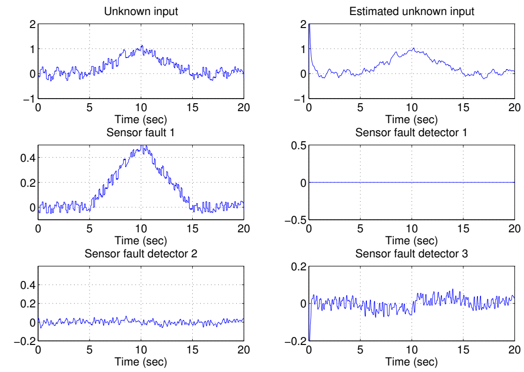

Fig. 4. Response of detectors to fault on the first output channel with uniform noise using Utkin's observer.

our system contains both bounded disturbances and actuator faults and is given as

$$u_2(t) = 0.1\sin(4t) + \max\left\{0, 1 - \frac{|t-10|}{5}\right\}.$$

The sensor fault is modeled as

$$f_o(t) = 0.5\max\left\{0, 1 - \frac{|t-10|}{5}\right\}. \qquad (27)$$

and is corrupted by noise that is uniform on $[-0.05,\ 0.05]$. The sample rate of the noise is 0.1 second. In the estimation of the sensor fault, the output of the nonlinear injection term is filtered using a first order lowpass filter with $\tau = 0.3$.

Using Utkin's observer, when the fault is on output channel 1, which is also corrupted by uniform measurement noise, the responses of the three detectors are shown in Figure 4. It can be seen that the detectors are not able to pick up the fault since the matrix $\boldsymbol{A}_{\text{feq}}$ in (23) has zero first row, which makes $f_{o1} = 0$. However, it can be seen that the detector on channel 3 approximates the negative derivative of the fault signal. It is interesting to note that in a similar experiment performed by Edwards-Spurgeon in [8], [9], the detector on channel 3 constructed using their observer also estimated the negative derivative of the sensor fault.

The detectors are able to easily detect the sensor faults affecting channel 2 and 3, respectively, as can be seen in Figures 5 and 6. It can also be seen from the above figures that the estimates of the unknown input are very close to the actual signal. In summary, it can be said that the Utkin observer based fault detector has the ability to detect and isolate the sensor faults, that is, the fault on channel 2 is detected by the second detector and the fault on channel 3 is detected by the third detector.

Using the observer from [7], when the fault is on the first output channel corrupted by uniform noise, the response of the detectors are shown in Figure 7. It can be seen that once again the fault affecting channel 1 cannot be successfully estimated by the detectors. However, the sensor faults affecting channel 2 and 3, respectively, are picked up in both cases by the detectors of channel 3 as seen in Figures 8

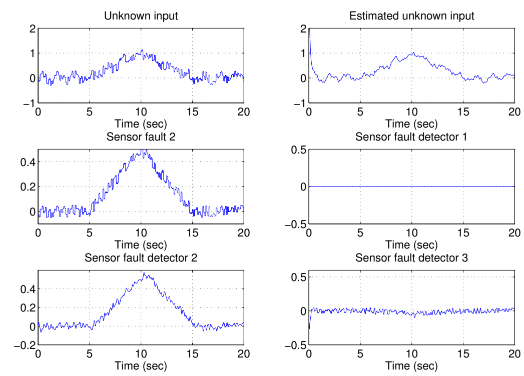

Fig. 5. Response of detectors to fault on the second output channel with uniform noise using Utkin's observer.

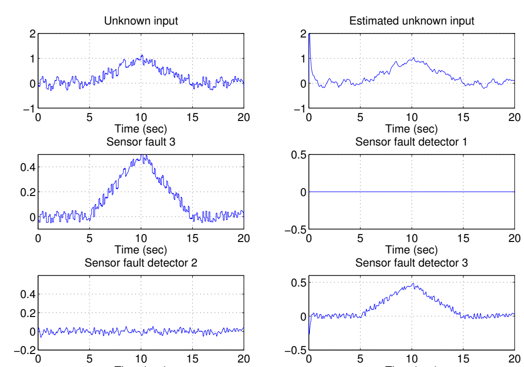

Fig. 6. Response of detectors to fault on the third output channel with uniform noise using Utkin's observer.

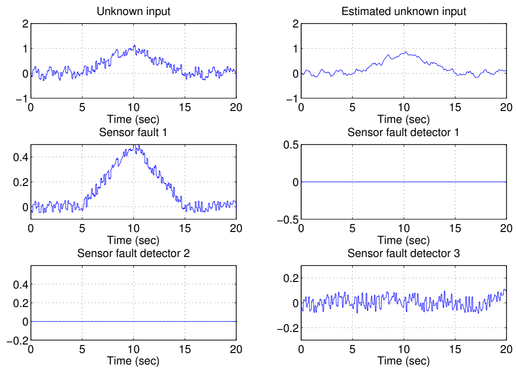

Fig. 7. Response of detectors to fault on the first output channel with uniform noise using observer from [7].

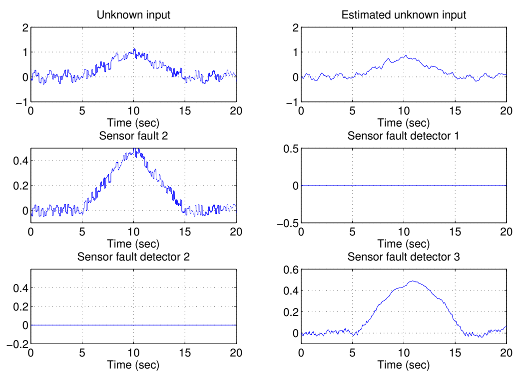

Fig. 8. Response of detectors to fault on the second output channel with uniform noise using observer from [7].

and 9. It can also be seen from the above figures that the estimates of the unknown input are very close to the actual signal. Since $\boldsymbol{F}$ has the form $[0 \ \ 0 \ \ 0.3205]$, the faults are always picked up by the detectors on channel 3, thereby making it difficult to distinguish between faults in different channels as is also noted in [8, p. 153].

VI. Conclusions

In this paper, unknown input estimators and sensor fault detectors based on the Utkin observer analyzed in [10] and the sliding-mode unknown input observer proposed in [7] were considered. The construction of the observers requires that the matrix rank condition, $\text{rank}\, \boldsymbol{B}_2 = \text{rank}\, \boldsymbol{CB}_2$, be satisfied. That is, the first Markov parameter from the unknown input to the output must be full rank. If the above condition is not satisfied, the methods recently proposed by Kalsi et al. [13], Tan et al. [2], or by Tan and Edwards [14] can be used to construct the unknown input and sensor fault detectors. An interesting open problem is to apply the architectures presented in this paper to construct robust sensor and fault isolation schemes proposed by Chen and Patton [15, pp. 80–81]. Another open problem is to extend the approach advanced in this paper to uncertain nonlinear systems.

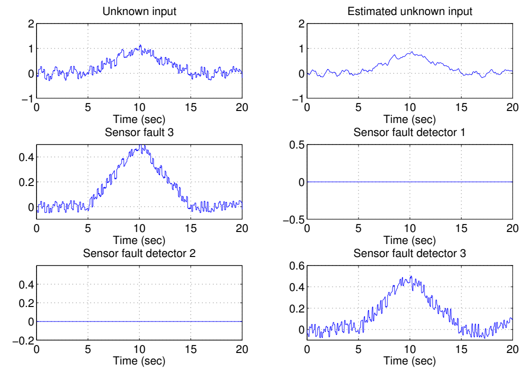

Fig. 9. Response of detectors to fault on the third output channel with uniform noise using observer from [7].

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