

Control Objectives for Seismic Simulators

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Abstract—Seismic simulators are designed to recreate the effects of seismic activity on structural and geotechnical systems. The performance objective in recreating the effects of a particular ground motion on a test article is typically for the seismic simulator to track the pre-recorded motion of a particular historical earthquake. The reference signal is often the earthquake acceleration, as the dynamic loads imparted upon the structure are proportional to this acceleration. During seismic simulations, interactions between the test article and the actuators may be significant; the inertia forces of the test article may not be much less than the actuator force capacity. In such cases, the table motion is not an exogenous input to the test article, but is an aspect of the response of these two coupled systems. This study examines the suitability of actuator acceleration tracking accuracy as the performance objective for seismic simulators by examining the dynamics of linearized models for coupled systems of actuators and test articles. A test metric related to the test system inertia and the test article inertia indicates the degree to which accurate accelogram tracking may not recreate the desired prototype responses.

I. INTRODUCTION

For dynamic testing of light (< 5 ton) structures subjected to non-stationary base accelerations, and for experimentation in structural control in particular, small-scale uni-axial servo-hydraulic seismic simulators have become popular [1], [2], [3], [4], [5], [6], [7], [8], [9]. A number of notable new large-scale seismic simulator facilities have recently been commissioned [10], [11], [12], [13]. These systems all feature servo-hydraulic actuators, stabilized and controlled by a feedback control system. Commercial servovalve controllers commonly implement variants of PID error feedback although other approaches have been applied [8], [14], [15], [16], [17], [18], [19], [20], [21].

Earthquake-excited structural responses are typically expressed in terms of the motion of structural elements with respect to their foundation. The seismic excitation for such models is the acceleration of the foundation. In an earthquake, the *acceleration* of the foundation is proportional to the dynamic load subjected to each mass. The acceleration tracking error of seismic simulators is therefore commonly viewed as the salient performance metric. Acceleration tracking is challenging, in part, because the PID servovalve controllers typically operate on displacement error feedback signals. The acceleration response of the simulator table is related to the displacement command through the dynamics of the servovalve-actuator-structure system as well as the

dynamics of the servovalve controller. Feedforward compensation of these dynamics and off-line tuning of PID gains require accurate models for the closed-loop system. Another challenge is related to the fact that large test specimens can be more massive than the simulator table. When the inertia of the test structure is not insignificant in comparison to the inertia of the simulator table, the simulator table is driven by both the actuator and by the test structure.

Analyses of the interactions between the actuator, test structure, and simulator foundation system have been experimentally confirmed [1], [5], [6]. Using Laplace-domain analyses and experimentation, the effects of controller gains on these interactions were also investigated. Others have combined state-variable modeling with dynamic programming methods to develop optimal table controllers which reduce phase errors over broad frequency ranges [8], [17].

Command signal manipulation is a common strategy for improving shaking table performance. Combined feedback/feed-forward control systems have been shown to perform better than feedback-only controllers; [8], [22]. Transfer function iteration methods, in which an actuator command signal is determined through repeated testing, has been used in industries [23] and research labs [3] for vibration control testing.

Using the derivative-of-acceleration as a feed-back signal has reduced actuator response times [24]. An adaptive minimal control synthesis (MCS) method has been applied to large shaking tables to address property changes in the test specimen and the shaking table during the test [21]. MCS is model independent, and was shown to improve the actuator acceleration tracking for a uni-directional sinusoidal reference in low-to-medium frequency ranges.

The goal of the present study is to investigate the effect of interactions between the actuator and the test structure on the fidelity of the experimental simulation. In contrast to previous work addressing the actuator acceleration tracking error, the fidelity of the simulation in this study is quantified by the error between the simulated response of the test structure interacting with a relatively light simulator platform and the response of the test structure excited by accelerations of the truly massive (and presumably rigid) foundation, soil and bedrock. The performance weights used to design the linear state-feedback controller balance earthquake tracking accuracy, structural response tracking accuracy, and the control effort. It is proposed that the test fidelity can be significantly enhanced by including structural response tracking in the weighted performance function. The important caveat of this assertion is that seismic simulator tests that endeavor to only reproduce the earthquake acceleration will not correctly

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simulate the desired structural response. This deleterious effect is exacerbated by massive structural models.

II. MODELING

A one-dimensional representation of the coupled actuator-structure system is shown in Fig. 1.

A. Actuator Dynamics

The flow-pressure relationship for servovalves is non-linear due largely to turbulent flow and Bernoulli effects [25]. For small variations about a nominal operating point, however, the servovalve is conveniently described with the linearized expression

$$Q = K_q x_v - K_c p_l \quad (1)$$

where Q is the volumetric flow rate into the cylinder, x_v is the valve spool position, p_l is the load pressure, K_q is the flow gain, and K_c is the flow-pressure coefficient. The linearization is normally made for the valve in its centered position ($x_v = 0$) in which case the flow into the cylinder, Q , and the flow-pressure coefficient should be zero [25]. A non-zero flow-pressure coefficient in this case can model leakage in the valve and the cylinder piston, and has the effect of adding damping to the actuator dynamics. While valve responses exhibit a zero-order hold due to coil inductance, in this study the valve position x_v is assumed to respond to the servovalve input current i_s with a first order delay,

$$\dot{x}_v = -\frac{1}{T_v} x_v + \frac{K_v}{T_v} i_s(t), \quad (2)$$

where T_v is the time constant of the valve, and K_v is the valve gain. The linearized dynamic equations of state for an actuator can be written

$$\frac{d}{dt} \begin{bmatrix} p_l \\ x_a \\ \dot{x}_a \\ x_v \end{bmatrix} = \begin{bmatrix} -K_c \frac{4\beta}{V} & 0 & -A_p \frac{4\beta}{V} & K_q \frac{4\beta}{V} \\ 0 & 0 & 1 & 0 \\ \frac{A_p}{m_a} & -\frac{K_a}{m_a} & -\frac{c_a}{m_a} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_v} \end{bmatrix} \begin{bmatrix} p_l \\ x_a \\ \dot{x}_a \\ x_v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_a} \\ 0 \end{bmatrix} f_s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_v}{T_v} \end{bmatrix} i_s \quad (3)$$

where the state vector includes the load pressure, p_l , the actuator position, x_a , the actuator velocity, \dot{x}_a and the valve spool position, x_v [1]. The actuator has a piston area A_p and is filled with a volume V of hydraulic fluid with a bulk modulus of β . The stiffness, mass, and damping

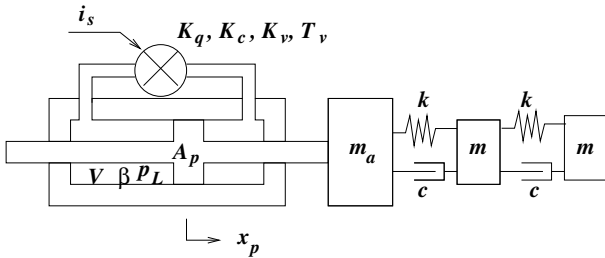


Fig. 1. One dimensional representation of the coupled actuator-structure system.

of the actuator load, are k_a , m_a , and c_a , and f_s is the interaction force between the structure and the actuator mass. Equation (3) for the actuator dynamics will be represented by the system

$$\dot{x}_A = A_A x_A + E_A f_s + B_A u, \quad (4)$$

$$y_A = \ddot{x}_a = C_A x_A, \quad (5)$$

and the transfer function $G_A(s)$, where the servovalve current i_s is the control u and the actuator acceleration \ddot{x}_a is the output y_A . If $k_a = 0$, which is the case in seismic simulators, the system G_A includes an integrator.

B. Structural Dynamics

The dynamics of a test structure represented by a series of n_m spring-mass-damper systems ($n_m = 2$ in Fig. 1), is

$$\frac{d}{dt} \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} 0_{n_m \times n_m} & I_{n_m} \\ -M_r^{-1} K_r & -M_r^{-1} C_r \end{bmatrix} \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix} + \begin{bmatrix} 0_{n_m \times 1} \\ -1_{n_m \times 1} \end{bmatrix} \ddot{x}_a, \quad (6)$$

where x_r is a vector of n_m displacements with respect to the actuator position x_a , M_r is a diagonal mass matrix, K_r is a tri-diagonal stiffness matrix and C_r is a tri-diagonal damping matrix. Equation (6) for the structural dynamics is asymptotically stable and will be represented by the system

$$\dot{x}_S = A_S x_S + B_S y_A \quad (7)$$

$$y_S = f_s = [k \ 0_{1 \times n_m - 1} \ c \ 0_{1 \times n_m - 1}] x_S = C_S x_S, \quad (8)$$

and the transfer function $G_S(s)$.

C. Disturbance Model

The “disturbance” is a filtered white noise process, \tilde{y}_A , used as a target acceleration record representative of earthquake ground accelerations.

$$\frac{d}{dt} \begin{bmatrix} x_w \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2\zeta_g \omega_g \end{bmatrix} \begin{bmatrix} x_w \\ \dot{x}_w \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \quad (9)$$

$$\dot{x}_W = A_W x_W + B_W w, \quad (10)$$

where w is the exogenous standard white noise process. The target earthquake ground acceleration is

$$\tilde{y}_A = [0 \ 2\zeta_g \omega_g] x_W = C_W x_W \quad (11)$$

This “disturbance” filter, $G_W(s)$, is strictly proper and positive real.

D. Weighted Performance

The weighted performance includes the actuator acceleration tracking error, the structural response tracking error, and the control input,

$$z = \begin{bmatrix} q_1 (y_A - \tilde{y}_A) \\ q_2 (f_s - \tilde{f}_s) \\ r u \end{bmatrix} = E_1 x + E_2 u \quad (12)$$

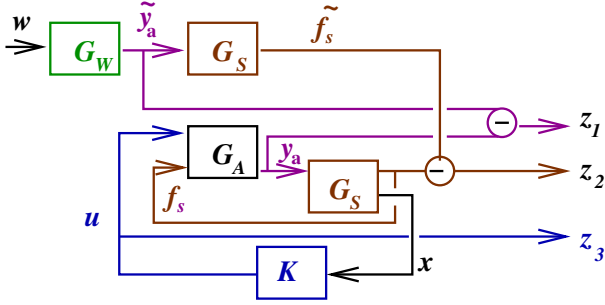


Fig. 2. Block diagram of the seismic simulator control system.

where \tilde{f}_s is the structural force resulting from the target earthquake acceleration, $\tilde{f}_s = G_S \tilde{y}_A$, and the scalar weights are q_1 for the actuator acceleration tracking, q_2 for the structural response tracking, and r for the actuator effort.

E. Closed Loop System

The closed-loop system is illustrated in Fig. 2, in which the static (LQR) gain matrix K multiplies the states of the system, x , to compute the servovalve current, u . The dynamics matrix of the coupled system is given by

$$A = \begin{bmatrix} A_W & 0 & 0 & 0 \\ 0 & A_A & E_A C_S & 0 \\ B_S C_W & B_S C_A & A_S & 0 \\ B_S C_W & 0 & 0 & A_S \end{bmatrix}. \quad (13)$$

The first two states are x_W , the next four states are x_A , the following $2n_m$ states are x_A , and the last $2n_m$ states are \tilde{x}_S , the structural response resulting from the target earthquake acceleration. The control input matrix of the coupled system is given by

$$B = \begin{bmatrix} 0 \\ B_A \\ 0 \end{bmatrix}. \quad (14)$$

The static gain K is computed using a state weight matrix $R_1 = E_1^T E_1 + 5|\lambda_{\min}(E_1^T E_1)|I$, where

$$E_1 = \begin{bmatrix} -q_1 C_W & q_1 C_A & 0 & 0 \\ 0 & 0 & q_2 C_S & -q_2 C_S \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (15)$$

and a control weight matrix $R_2 = r^2$.

The closed-loop system is evaluated in terms of the transfer functions from w to $y_A - \tilde{y}_A$ and from w to $f_a - \tilde{f}_a$. These transfer functions have a realization

$$\tilde{G}(s) \sim \left[\begin{array}{c|c} A - BK & \tilde{D}_1 \\ \hline \tilde{E}_1 & 0 \end{array} \right], \quad (16)$$

where

$$\tilde{D}_1 = \begin{bmatrix} B_W \\ 0 \end{bmatrix}, \quad (17)$$

and

$$\tilde{E}_1 = \begin{bmatrix} -C_W & C_A & 0 & 0 \\ 0 & 0 & C_S & -C_S \end{bmatrix}. \quad (18)$$

TABLE I

PARAMETER VALUES USED IN THE NUMERICAL EXPERIMENTS.

| variable | value | units |
|--------------|----------------------------|----------------------|
| c_a | 1000 | N/m/sec |
| m_a | 180 | kg |
| k_a | 0 | N/m |
| A_p | 0.00237 | m ² |
| V | 0.00036 | m ³ |
| β | 1.4×10^8 | N/m ² |
| K_c | 1×10^{-16} | m ³ /Pa.s |
| K_q | 76.0 | m ² /sec |
| K_v | 5×10^{-4} | meter/amp |
| T_v | 0.010 | seconds |
| ω_g | $2\pi \cdot 1.1$ | rad/s |
| ζ_g | 1.4 | - |
| k_{ac} | $4\beta A_p^2/V$ | N/m |
| f_{ac} | $\sqrt{k_{ac}/m_a}/(2\pi)$ | Hz |
| ζ_{ac} | $c_a/(2\sqrt{m_a k_{ac}})$ | - |
| n_m | 2, 5, or 10 | - |
| μ | 0.1, 0.2, or 0.5 | - |
| f_1 | 7.0 | Hz |
| ζ_1 | 0.05 | - |
| m | $\mu m_a/n_m$ | kg |
| k | $(2\pi f_1)^2 m$ | N/m |
| c | $\zeta_1 2\sqrt{mk}$ | N/m/s |
| q_1 | 10^{-3} or 10^3 | - |
| q_2 | 10^{-3} or 10^3 | - |
| r | 1 | - |

III. NUMERICAL EXAMPLES

Numerical examples illustrate the effect of dynamic interaction between the actuator and the test structure on actuator acceleration tracking and structural response tracking. These numerical values correspond to a small scale seismic simulator. The actuator force capacity is rated at 50 kN. The actuator-table system has an oil-column resonant frequency, f_{ac} of 30 Hertz and an oil-column damping ratio ζ_{ac} of 1 percent. The total mass of the test structure ranges from ten percent to fifty percent of the table mass. The test structure mass is distributed equally among two, five, or ten lumped masses. The weights are adjusted to emphasize actuator acceleration tracking performance or structural response tracking performance. The results are presented in terms of transfer functions \tilde{G}_1 and \tilde{G}_2 from the standard Gaussian white noise "disturbance" to the tracking errors. The transfer function from w to $y_A - \tilde{y}_A$ (actuator acceleration tracking) is \tilde{G}_1 . The transfer function from w to $f_s - \tilde{f}_s$ (structural response tracking) is \tilde{G}_2 . The H_2 norm of these transfer functions are also tabulated for the various cases in Tables II and III. Plots of the transfer functions corresponding to these tables are shown in Fig's 3 and 4.

IV. RESULTS

Comparing Tables II and III, and comparing Fig's 3 and 4, it is clearly evident that while weighting the actuator acceleration tracking error alone results in good actuator acceleration tracking performance, the errors between the simulated structural response and true structural responses are substantial. These errors increase with the ratio μ of the mass of the test article to the mass of the seismic simulator. When the tracking error of the structural response

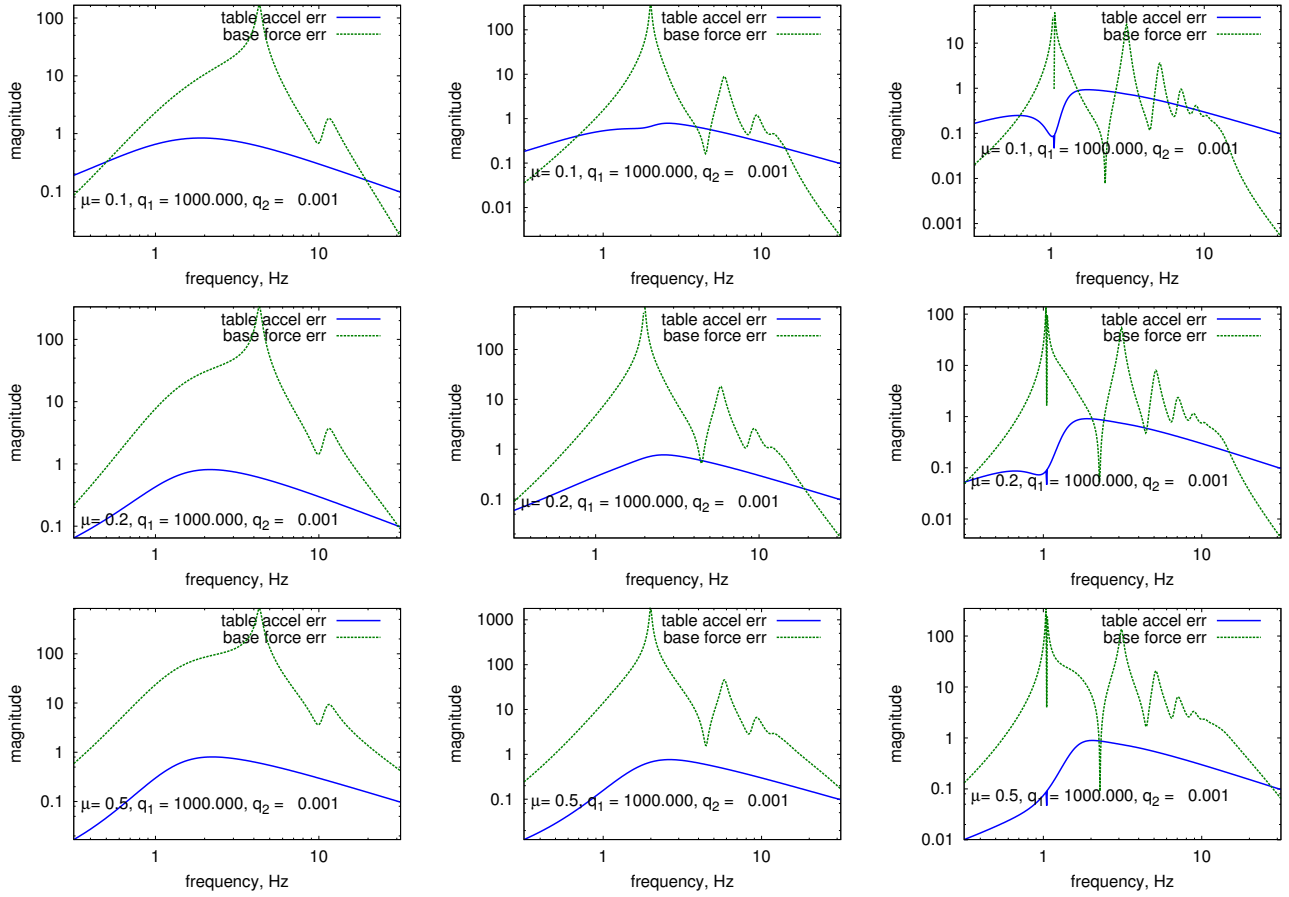


Fig. 3. Acceleration tracking transfer functions, \tilde{G}_1 (solid blue line), and structural response tracking functions, \tilde{G}_2 (dashed green line), for earthquake acceleration (\ddot{x}_a) tracking performance, $q_1 = 1000$, $q_2 = 0.001$

TABLE II

H_2 NORMS OF ACTUATOR ACCELERATION TRACKING TRANSFER FUNCTIONS, $\|\tilde{G}_1\|_2^2$ AND STRUCTURAL RESPONSE TRACKING FUNCTIONS, $\|\tilde{G}_2\|_2^2$ FOR EARTHQUAKE ACCELERATION (\ddot{x}_a) TRACKING PERFORMANCE, $q_1 = 1000$, $q_2 = 0.001$

| n_m | 2 | 5 | 10 |
|-------------|---|---|---|
| | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ |
| $\mu = 0.1$ | $7.4 : 10^4$ | $6.7 : 10^4$ | $7.2 : 10^2$ |
| $\mu = 0.2$ | $6.8 : 10^5$ | $6.2 : 10^5$ | $6.9 : 10^3$ |
| $\mu = 0.5$ | $6.5 : 10^6$ | $6.1 : 10^6$ | $6.6 : 10^4$ |

TABLE III

H_2 NORMS OF ACTUATOR ACCELERATION TRACKING TRANSFER FUNCTIONS, $\|\tilde{G}_1\|_2^2$ AND STRUCTURAL RESPONSE TRACKING FUNCTIONS, $\|\tilde{G}_2\|_2^2$ FOR STRUCTURAL RESPONSE (f_s) TRACKING PERFORMANCE, $q_1 = 0.001$, $q_2 = 1000$

| n_m | 2 | 5 | 10 |
|-------------|---|---|---|
| | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ | $\ \tilde{G}_1\ _2^2 : \ \tilde{G}_2\ _2^2$ |
| $\mu = 0.1$ | 9.1 : 40 | 8.9 : 0.5 | 8.3 : 12 |
| $\mu = 0.2$ | 9.5 : 650 | 9.0 : 4.7 | 7.7 : 84 |
| $\mu = 0.5$ | 10 : 160 | 10 : 6500 | 7.1 : 2500 |

is heavily weighted, there is a slight reduction in the actuator acceleration tracking error and a substantial reduction in the tracking error of the structural response.

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

Seismic simulators are intended to experimentally replicate the effects of earthquakes on test structures. The control objective of seismic simulators has traditionally been to match the acceleration of the simulator to a given earthquake ground acceleration record, because the dynamic forces acting upon the structure are proportional to these accelerations.

The mass of the test structure can be significant in comparison to the mass of the seismic simulator. In such cases, actuator forces and forces transmitted between the structure and the simulator both contribute to the simulator accelerations. Furthermore, in these cases the simulator acceleration should not be viewed as an independently controlled input to the test structure, but should instead be viewed as an aspect of the response of a coupled structure-actuator system.

Linearized models for the (nonlinear) hydraulic actuator behavior, and linear models for the structural behavior allow for frequency domain analysis. As expected, structure-actuator interactions effects are substantial when structural

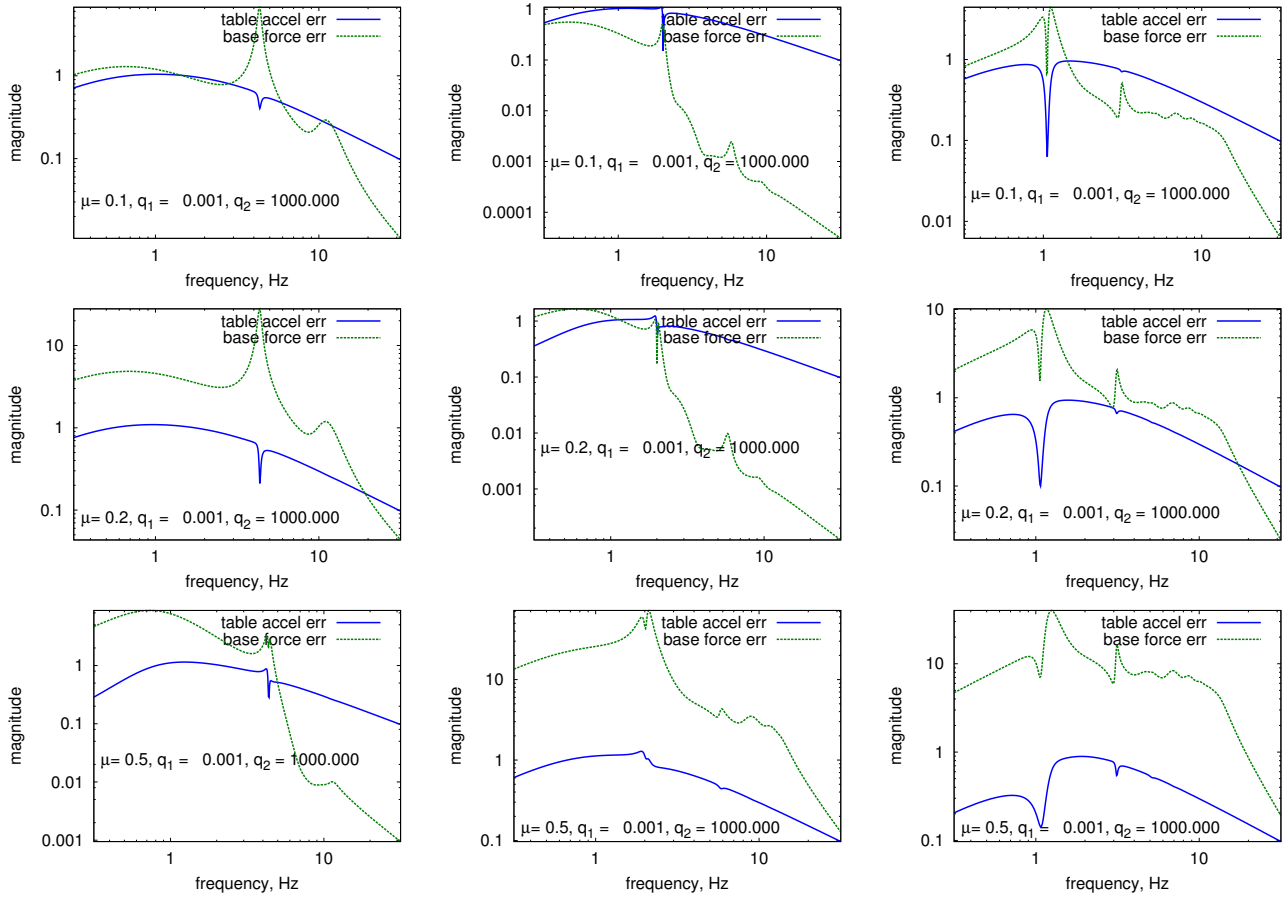


Fig. 4. Acceleration tracking transfer functions, \tilde{G}_1 (solid blue line), and structural response tracking functions, \tilde{G}_2 (dashed green line), for structural response (f_s) tracking performance, $q_1 = 0.001$, $q_2 = 1000$

response tracking is not considered, and test results in such cases could be misleading.

A state-feedback control that targets the reproduction of structural responses as well as earthquake accelerations alleviates this issue. Of course, a paradox lies in the fact that a test structure's dynamic behavior can not be known exactly prior to a test, but is needed in order to accurately control the test.

B. Future Work

The performance does not weight all of the states and $E_1^T E_1$ is not invertible. The term $a|\lambda_{\min}(E_1^T E_1)|$ is added to the diagonal of $E_1^T E_1$ in order to ensure that R_1 is invertible. Other approaches to adjusting R_1 should be investigated so that inconsequential states are not overly weighted.

Transient response analyses will be used to validate the model by establishing that state and output responses are within normal ranges. Time domain analysis will require a balanced reduction of the closed-loop dynamics in order to truncate very high-frequency poles. Transient dynamic analyses will more clearly illustrate how structure-actuator interaction effects can distort a test result.

These additional analyses will ultimately provide the experience required to state, in advance of a test, what the test's

expected fidelity might be, and when the data analysis must involve modeling of the coupled actuator-structure-control system.

Shaking amplitudes in large-scale tests are typically increased from low levels to ultimate levels, over a number of shakes. Such test protocols allow for Bayesian updating of the structural model with the intent of increasing the test fidelity with each shake.

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