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Feedback control of dissipative PDE systems in the presence of uncertainty and noise using extended Kalman filter

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***Abstract*— This article focuses on dynamic output feedback and robust control of quasi linear parabolic partial differential equations (PDE) systems with time-varying uncertain variables. Especially processes that are described by dissipative PDEs are considered. The states of the process required for designing controllers are dynamically estimated from limited number of noisy process measurements employing an Extended Kalman filter. The issue of utilizing these estimated states in a robust controller to achieve the desired process objective is investigated. The controller design needs to address both model uncertainty and sensor noise. The methodology is employed on an representative example wherein the desired objective is to stabilize an unstable operating point in a catalytic rod, where an exothermic reaction occurs. A finite dimensional robust controller, utilizing dynamically estimated states, is used to successfully stabilize the process to an open-loop unstable steady-state.**

I. INTRODUCTION

Transport reaction process are characterized by the coupling of chemical reactions with significant convection, diffusion and dispersion phenomena. Examples include plug flow reactors, packed bed reactors, rapid thermal chemical vapor deposition reactors, metallorganic vapor phase epitaxy in semiconductor manufacturing and various other fluid dynamic systems. Mathematical descriptions of these transport-reaction processes [8] can be derived from dynamic conservation equations and usually involve highly dissipative (typically parabolic) partial differential equation (PDEs) systems.

The feedback control issue of such processes is nontrivial owing to the spatially distributed mathematical descriptions of their dynamics; as a result the state in the corresponding control problem is infinite dimensional (when presented in appropriate functional spaces). A traditional approach to feedback control of these PDE systems involves finding a finite dimensional approximation of the original infinite dimensional system by finite discretization of the underlying PDEs to yield a set of ordinary differential equations (ODEs) which are subsequently used in designing feedback controllers for the processes [5], [6]. This method typically requires a high dimensionality in the resulting ODE system in order to capture the PDE dynamics accurately, thus leading to high dimensionality in the resulting controllers.

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Other model reduction techniques recognize the fact that the eigenspectrum of the spatial differential operators in the parabolic PDE systems can be partitioned into a finite-dimensional slow subspace and a infinite-dimensional fast subspace [20]. In other words, the long-term dynamics of the dissipative PDEs are finite-dimensional and therefore a finite dimensional model would accurately capture the dynamics of the original infinite dimensional problem. One such approach is to use Galerkin's method, wherein the solution of the system is expanded using the eigenfunctions of the spatial differential operator. This approach yields a system of ODEs that accurately describes the dominant (slow) modes of the PDEs, which can be subsequently used in the design of feedback controllers [2], [1], [4], [18], [17].

The concepts of inertial manifold [20], [12] and approximate inertial manifold [3], [9], [21] were also used to derive lower order differential algebraic equation (DAE) systems which capture the dominant dynamics of quasi-linear PDEs. The derived DAE systems were then used for the synthesis of nonlinear low-dimensional output feedback controllers which enforced stability and output tracking in the closed-loop system. However these results do not consider process uncertainty during the design of the feedback controllers. To address this issue adaptive state-feedback controllers for these PDE systems were also developed [10]. In [7] a robust feedback controller was designed based on static output feedback for processes described by quasi-linear PDE systems with slowly varying uncertain parameters. The designed controllers enforced feedback stability and robustness to the system. However, this and other related results [8], [11] do not utilize the available process model for better estimation of the state of the reduced order model (computed using Galerkin's method) by designing a dynamic observer due to limitations in the proof of closed-loop behavior guarantees. Moreover, for situations wherein the available process measurements are corrupted by sensor noise, the used static observer may yield an inaccurate prediction of the states of the reduced order model. In these situations dynamic estimation of the system states yield better results, as they utilize the closed loop system outputs while estimating the states of the system, even if only local results can be guaranteed.

As the above model reduction methodologies transform PDE systems in to ODE system, a wealth of available resources [19] in dynamic estimation of these ODE system states can be utilized. Kalman filter (KF) [15] is one such dynamic state estimation tool that provides an optimal estimate of the system states of a linear system by propagating the mean & covariance of the system states through time. For nonlinear systems, various other formulations of nonlinear Kalman filters are available [14]. Extended Kalman filter (EKF), a straightforward extension of KF, is one such formulation that has been heavily used as a nonlinear filtering tool in the literature. However, this estimation tool and other such methods are not known to work well in presence of model uncertainties.

In the present work the problem of dynamic estimation and control of a PDE system with slowly varying uncertainties using limited noisy measurement data, is investigated. The proposed method employs Galerkin's method and a combination of feedback linearization and Lyapunov methods along with an EKF. The quality of state estimates that results when using EKF along with a robust controller is investigated. We present an application of the above approach to control temperature in a catalytic rod where an exothermic reaction is taking place. A finite dimensional robust controller is coupled with dynamic output feedback to stabilize the process to an open-loop unstable steady-state. This work is at the investigation stage of such combinations, without presentations of formal guarantee of closed-loop behavior.

II. Mathematical preliminaries

We focus on the problem of feedback control of spatially distributed processes described by highly dissipative PDEs with the following state-space description:

$$\begin{aligned} \frac{\partial x}{\partial t} &= \mathcal{A}x + b(z)u + f(x) + W(x, r(z)\theta(t)), \\ y_c &= \int_\Omega c(z)\, x\, dz, \\ y_m &= \int_\Omega s(z)\, x\, dz + v(t). \end{aligned} \tag{1}$$

subject to the mixed-type boundary conditions:

$$\begin{aligned} C_1 x(\alpha,t) + D_1 \frac{\partial x}{\partial z}(\alpha,t) - R_1 = 0 \\ C_2 x(\beta,t) + D_1 \frac{\partial x}{\partial z}(\beta,t) - R_2 = 0 \end{aligned} \tag{2}$$

and the following initial condition

$$x(z,0) = x_0(z). \tag{3}$$

In the above PDE system, $x(z,t) \in \mathbb{R}^n$ denotes the vector of state variables, $y_c \in \mathbb{R}^k$ denotes the vector of controlled outputs, t is the time, $y_m \in \mathbb{R}^{n_{mo}}$ denotes the vector of measured outputs, $z \in \Omega \subset \mathbb{R}$ is the spatial coordinate and $\Omega = [\alpha, \beta]$ is the domain of definition of the process. $\mathcal{A}$ is a highly dissipative, linear spatial differential operator, $f(x)$ is a nonlinear vector function which is assumed to be sufficiently smooth with respect to its arguments, $c(z)$ is a known smooth vector function of z which is determined by the desired performance specifications in the domain Ω and $s(z)$ is a known smooth vector function of z which is determined by the location and type of measurement sensors (e.g., point or distributed sensing). $W(x, r(z)\theta(t))$ is a nonlinear vector function, $\theta(t)$ denotes uncertain process parameters or exogenous disturbances, $r(z)$ is a known smooth function of z that specifies the position of action of the uncertain variables on Ω, $v \sim \mathcal{N}(0,R)$ is a gaussian white noise sequence of intensity Q. $u = [u_1, u_2, \cdots, u_k] \in \mathbb{R}^k$ denotes the vector of manipulated inputs, $b(z) \in \mathbb{R}^{n\times k}$ is a known smooth matrix function of z of the form $[b_1(z), b_2(z), \cdots, b_k(z)]$, where $b_i(z)$ describes how the i^{th} control action $u_i(t)$ is distributed in the spatial domain Ω. C_1, D_1, C_2, D_2, R_1,R_2 are nonlinear vector functions which are assumed to be sufficiently smooth and $x_0(z)$ is a smooth vector function of z. We assume that for a given set of initial and boundary conditions the system of Eqs. 1-3 has a unique solution. We formulate the problem in the space of square integrable functions $L_2[\Omega]$ and employ the following definition for the norm:

$$(\phi_1, \phi_2) = \int_\Omega \phi_1^*(z)\phi_2(z)dz,\ ||\phi_1||_2 = (\phi_1, \phi_1)^{1/2} \tag{4}$$

where $\phi_1, \phi_2 \in L_2[\Omega]$ and * denotes the complex conjugate transpose.

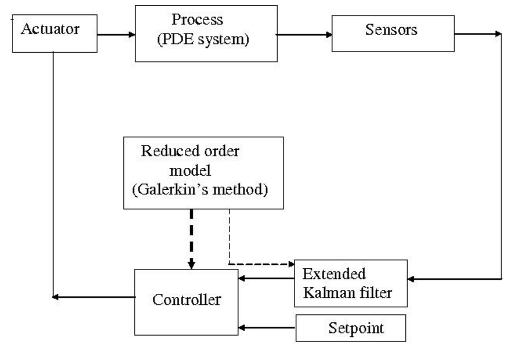

Fig. 1. Process operation block diagram under proposed methodology

III. Problem formulation and solution methodology

In this section our objective is to present an outline of the steps of the proposed dynamic output feedback control methodology for processes that are described by the system of Eqs.1-3. The control problem is formulated as the one of deriving a feedback control law $u(t) = G(\hat{x}(t))$, where $G(\hat{x}(t))$ is any nonlinear function of an estimate of x(t), such that the closed-loop system is stabilized within a neighborhood of the desired set point. Without loss of generality, we assume the setpoint is $x(z,t) = 0$. The steps of the proposed methodology to achieve the above task are:

1) We use the eigenfunctions of the spatial differential operator $\mathcal{A}$ to derive finite-dimensional approximations of the infinite-dimensional PDE system of Eqs.1-3 by using Galerkin's method.
2) Employ Extended Kalman Filter to provide estimates of the states of the resulting finite-dimensional approximation, using information obtained from limited noisy measurement sensors.
3) Design a robust controller using the state estimates to drive the PDE system Eqs.1-3 within a neighborhood of the desired setpoint.

A block diagram elucidating the above steps is presented in Fig. 1. The following subsections briefly describe each of the above steps.

A. Derivation of finite dimensional approximations using Galerkin's method

We employ the spectral eigenfunctions of the operator $\mathcal{A}$ to derive finite-dimensional approximations of the original infinite-dimensional PDE system of Eq.1 by using Galerkin's method. To simplify the notation, without loss of generality we consider the system of Eq.1 with $n = 1$. In principle, $x(z,t)$ can be represented as an infinite weighted sum of a complete set of eigenfunctions $\phi_k(z)$. We can obtain an approximation $x_N(z,t)$, by truncating the series expansion of $x(z,t)$ up to order N, as follows:

$$x_N(z,t) = \sum_{k=1}^{N} a_k(t)\phi_k(z) \overset{N\to\infty}{\longrightarrow} x(z,t) = \sum_{k=1}^{\infty} a_k(t)\phi_k(z) \tag{5}$$

where $a_k(t)$ is a time-varying coefficient called the mode of the system. The eigenfunctions are obtained from the solution of the eigenfunction eigenvalue problem of the operator $\mathcal{A}$

$$\mathcal{A}\phi = \lambda\phi. \tag{6}$$

We assume that the eigenfunction problem can be solved analytically. Furthermore, we order the eigenfunction-eigenvalue pairs such that

$$\lambda_1 \geq \lambda_2 \geq \cdots \lambda_n \geq \cdots$$

A known property of highly dissipative PDEs is that the eigenspectrum of the operator $\mathcal{A}$ can be partitioned into a finite size of ones that are close to the imaginary axis and an infinite size set of eigenvalues that lies in the left half plane. Furthermore there is a separation between the "slow" and "fast" eigenvalues

$$\lambda_N/\lambda_{N+1} = O(\varepsilon)$$

where ε is small number. This implies that the long term dynamics of the process can be accurately described by a finite dimensional approximation and there is a time-scale separation between the slow dynamics and the fast highly-stable ones. Thus substituting the expansion of Eq.5 into Eq.1, multiplying the PDE with the eigenfunctions, $\phi(z)$, and integrating over the entire spatial domain (i.e., taking inner product in $L_2[\Omega]$ with the eigenfunctions), the following N-th order system of ODEs is obtained.

$$\begin{aligned}
-\sum_{k=1}^{N}\dot{a}_k\Big(\int_\Omega \phi^*(z)\phi_k(z)dz\Big) + \int_\Omega \phi^*(z)\mathcal{A}\sum_{k=1}^{N}a_k(t)\phi_k(z)dz \\
+\int_\Omega \phi^*(z)b(z)udz + \int_\Omega \phi^*(z)f\Big(\sum_{k=1}^{N}a_k(t)\phi_k(z)\Big)dz \\
+\int_\Omega \phi^*(z)W\Big(\sum_{k=1}^{N}a_k\phi_k(z), r(z)\theta(t)\Big)dz = 0, \\
y_m = \int_\Omega s(z)\Big(\sum_{k=1}^{N}a_k\phi_k(z), r(z)\theta(t)\Big)dz.
\end{aligned}$$

The resulting ODE system along with the measurement equation is written in the following compact form

$$\begin{aligned} \dot{a} &= \mathcal{F}(a) + Gu + W_s(a,\theta) \\ y_m &= \Phi\ a + v \end{aligned} \tag{7}$$

where $\mathcal{F}$, $\mathcal{W}_s$ are vector functions of appropriate dimensions and $\mathcal{G}$, Φ are matrix functions of appropriate dimensions. We note that the only information assumed to be available about the model uncertainty term, $\mathcal{W}_s$, is a time varying bounding function $c_0(t)$ that captures the size of uncertain terms.

The availability of measurement sensors are often restricted and the measurement available from them tend to be noisy; estimation techniques to predict the system states in Eq.7 are required. Since the state equation (Eq.7) is nonlinear we use a nonlinear filtering tool called Extended Kalman filter to estimate the system states.

IV. EXTENDED KALMAN FILTER

Given a linear model of the system along with noisy output measurements, the Kalman filter (KF) [15] provides an optimal estimate of the system states. KF operates by propagating the mean & covariance of the system states through time. For nonlinear systems, various formulations of nonlinear Kalman filters are available [14]. Extended Kalman filter (EKF) is one such formulation which has been heavily used as a nonlinear filtering tool in the literature. In EKF, the state equations of the system (Eq.7) are linearized at each time step and the states are estimated by using these linearized state equations in the KF. The relevant state equation and measurement equation from the above section is

$$\begin{aligned} \dot{a} &= \mathcal{F}(a) + Gu + w \\ y_m &= \Phi a + v \end{aligned} \tag{8}$$

$$w \sim \mathcal{N}(0,Q); v \sim \mathcal{N}(0,R) \tag{9}$$

Where w, v are Gaussian zero-mean white noise sequences with intensities Q and R; represent process noise and measurement noise, respectively. We note that the process noise is introduced in the above equation to act as a cover for model uncertainty, as EKF doesn't accommodate model uncertainty directly; rather the issue is implicitly addressed through the definitions of process noise and measurement noise. Because, in this work we employed a continuous time version of EKF we present a brief outline of the EKF; a detailed overview can be found elsewhere [19].

1) Initialize the state estimates $\hat{a}$ and the error covariance P.
2) Linearize the state equation at the current state estimate to obtain the following partial derivative matrix.

$$A = \frac{\partial \mathcal{F}(a)}{\partial a}\bigg|_{\hat{a}}$$

And solve the following Riccati equation for the error covariance matrix,

$$\dot{P} = AP + PA^T + Q - P\Phi^T R^{-1}\Phi P$$

and compute the Kalman gain matrix: $K = P\Phi^T R^{-1}$. Use the computed Kalman gain matrix to get an revised estimate of the states a, by solving the following equation.

$$\dot{\hat{x}} = \mathcal{F}(\hat{a}) + Gu + K[y - \Phi\hat{a}] \quad (10)$$

Remark 4.1: We note that EKF is not known to produce accurate estimates for systems with model uncertainties. Since we are using EKF in combination with a robust controller, which explicitly accounts for model uncertainty, we can expect the effect of model uncertainty on the state estimates obtained from EKF to be minimal.

Remark 4.2: To further refine our assumption concerning w term so that it more accurately reflects $W_s(a,\theta)$, in the future we will consider sigma-point filter [14] for the proposed controller design.

V. Design of Robust controller using EKF state estimates

We employ a robust feedback controller to design a dynamic output feedback controller for the system of Eqs.1-3 using the state estimates of Eq.7 obtained using EKF. In [7] robust state & output feedback controllers were synthesized via Lyapunov's direct method for quasi linear parabolic PDEs with slowly varying uncertain variables. Under certain assumptions, the designed controllers enforced closed loop stability and attenuated (asymptotically) the effect of process uncertainties on the output. In their work the authors have not utilized the available process model to provide better state estimates; the issue of measurement noise was not considered. We are currently investigating the impact of a combination of dynamic output feedback (using the state estimates obtained from EKF) and the robust controller on the closed loop performance of the system. We will briefly present the structure of the employed robust controller; a detailed overview can be found in [7]. The dynamic robust feedback control law used is of the following generic form:

$$u = d(q) + Q(q)\hat{v} + r(q,t), \quad (11)$$

where $d(q)$, $r(q,t)$ are vector functions, $Q(q)$ is a matrix, q is the vector of measured outputs and $\hat{v}$ is a vector function of the external reference inputs and their time derivatives. The component $d(q) + Q(q)\hat{v}$ in the controller is responsible for output tracking and is based on differential geometry; the component $r(q,t)$ is responsible for the asymptotic attenuation of the effect of the uncertain variables on the outputs of the closed-loop slow system and is designed using Lyapunov arguments. Note that for $r \equiv 0$, the controller of Eq.11 attains the form of feedback linearizing controllers of [13], [16].

VI. Application to diffusion reaction process

In this section, we use the above methodology to stabilize an unstable steady state of a typical diffusion-reaction process with a time varying uncertainty. Specifically, we consider a zero-th order exothermic reaction $A \rightarrow B$ taking place on a thin catalytic rod. The temperature of the rod is adjusted by means of an actuator (by cooling the rod) located along the length of the rod. Assuming that the reactant A is present in excess, the spatial profile of the dimensionless temperature of the rod is described by the following parabolic PDE.

$$\begin{aligned}\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial z^2} + \beta_{T,n}(\mathrm{e}^{-\gamma/(1+x)} - \mathrm{e}^{-\gamma}) + \dots \\ + \beta_U(b(z)u(t) - x) + \mathrm{e}^{-\gamma/(1+x)}\theta(t)\end{aligned} \quad (12)$$

Subject to the following boundary condition and initial conditions:

$$x(0,t) = 0,\ x(\pi,t) = 0,\ x(z,0) = 0.05 \quad (13)$$

The dimensionless rod temperature is given as $x = \frac{T-T_0}{T_0}$, where T is the temperature of the reactor in ^0K and T_0 is the reference temperature used. The domain of this process is $\Omega = [0,\pi]$; z is the spatial coordinate along the axis of the rod, $\beta_{T,n}$ denotes the nominal dimensionless heat of reaction, γ denotes the dimensionless activation energy, β_U denotes the dimensionless heat transfer coefficient, $u(t)$ denotes the magnitude of actuation, $\theta(t)$ denotes a time varying uncertainty in the dimensionless heat of reaction and $b(z)$ accounts for the spatial profile of the actuator. A spatially distributed actuation with $b(z) = \sqrt{(2/pi)}sin(z)$ was considered. The nominal values of the parameters were $\beta_{T,n} = 50$, $\gamma = 4$, and $\beta_U = 2$. In this numerical study, the slowly varying uncertainty in the process model (Eqs.12-13) is assigned to be $\theta(t) = \beta_{T,n}sin(0.524t)$.

A. Estimator implementation

We now present the effectiveness of EKF by estimating the open loop profile of Eq.12 with no process uncertainty under a Gaussian white measurement noise $v \sim N(0, 0.03)$ and the process is simulated until time $t_{final} = 15$. The value of the parameters used are: initial error covariance $P_0 = 0.15I$, where I is an identity matrix of appropriate dimensions. We assume the availability of two noisy measurement sensors at positions $L/4$ and $3L/4$ on the catalytic rod of length L. The performance of the estimator was evaluated by calculating the 2-norm of the estimation error (between the states predicted using EKF and actual states evaluated from numerical simulation of Eqs.12-13) and trace of the error covariance matrix P, using Monte-Carlo simulations. Fig. 2 presents the average of 50 Monte-Carlo simulations, it can be observed that the trace matrix P and the estimation error very rapidly converges very close to zero. In other words the performance of the estimator improves as it gets more information from the sensors.

B. Controller implementation

Fig. 3 presents the open-loop evolution of the PDE (with process uncertainty) for $u(t) = 0$. It can be observed that the open-loop process behavior is unstable even though the process noise and its derivatives, $\theta(t)$ and $\dot{\theta}(t)$, are kept small. The initial operating point $x(z,0) = 0$ is therefore an unstable one. The control objective in this case is to design a dynamic output feedback controller that stabilizes the rod temperature to the spatially open-loop unstable steady state.

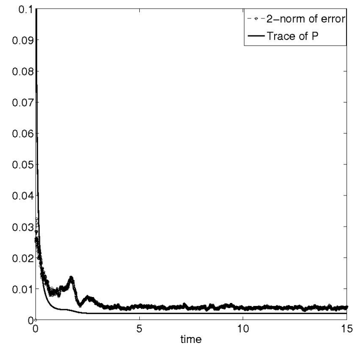

Fig. 2. Temporal profile of the 2-norm of the error between true and estimated states and the trace of P obtained during the open-loop operation of Eq.12

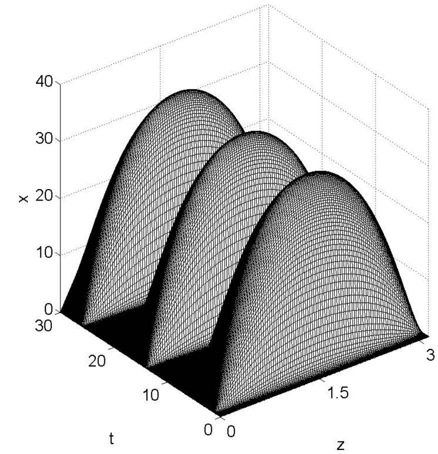

Fig. 3. Open-loop profile of Eq.12 with measurement noise

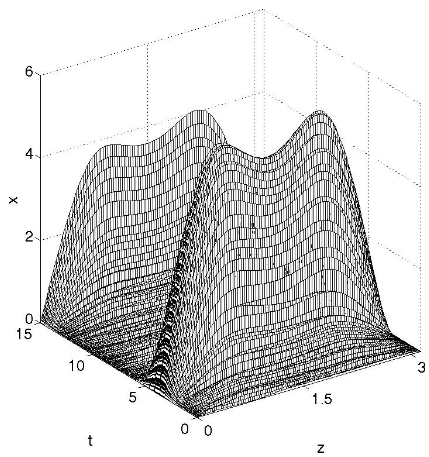

Fig. 4. Estimated surface profile of Eq.12 in closed loop with controller of Eq.11 with $r \equiv 0$.

We utilize the dynamic output robust feedback controller based on the reduced order system Eq.7 to achieve the above objective. In the present case study the reduced order system was obtained using a truncated series expansion of x, using $N = 11$ eigenfunctions. Subsequently, the reduced order model was utilized to design the robust controller of the generic form of Eq.11. The expression for one of the parameters responsible for accounting model uncertainty ($r(\hat{a},t)$) is presented below

$$r(\hat{a},t) = -\chi \frac{\hat{a}}{|\hat{a}| + \Lambda} \int_0^{\pi} \phi_1(z) e^{\frac{-\gamma}{(1+\Sigma_{i=1}^{N} \hat{a}_i(t)\phi_i(z))}} dz \tag{14}$$

where χ, Λ are adjustable control parameters and $\phi_1(z)$ is the first eigenfunction of the spatial differential operator in Eq.12. It should be noted that the controlled output chosen to stabilize the process (as the first eigenvalue gives us the desired separation required in Eq.III-A) was the first mode i.e $y_c = \hat{a}_1$. Also note that the expression for nonlinear time varying bounding function which captures the size of the uncertain terms in the system

$$c_0(\hat{a},t) = \int_0^{\pi} \phi_1(z) e^{\frac{-\gamma}{(1+\Sigma_{i=1}^{N} \hat{a}_i(t)\phi_i(z))}} dz$$

is explicitly used in the formulation of $r(\hat{a},t)$. The control parameters for this case study were set at $\chi = 1.2$ and $\Lambda = 0.01$.

In Fig. 4, we present the closed loop performance of Eq.12 using EKF and controllers designed with $r \equiv 0$. The closed loop performance clearly is unacceptable as the controller does not account for model uncertainty; moreover state estimates by EKF are not reliable in this case as EKF is sensitive to errors due to the unaccounted model dynamics. The closed loop performance of the PDE system (Eq. 12) using the robust controllers is presented in Fig. 5. The performance of robust controller far exceeds the performance of controllers designed using feedback linearization as it explicitly accounts for the process uncertainty.

The performance of the closed loop estimator is evaluated using Monte-Carlo simulations (Fig. 6). It is observed that as the process evolves, the trace of the error covariance matrix relaxes to zero and the estimation error remains bounded very close to zero. We also observe that as the robust controller accounts for model uncertainty the estimates from EKF become reliable. Fig. 7 presents the temporal profile of control action used. The chattering observed in the control action is due to measurement noise present in the sensors. This is confirmed by simulating the process under no measurement noise and again stabilizing the open-loop unstable operating point of the process using the designed robust controller. The smooth control action obtained (used) in this simulation is presented in Fig. 8.

VII. Conclusions

The issue of utilizing dynamically estimated states (obtained from EKF using limited noisy process measurements) in a robust controller, that addresses model uncertainty, was investigated. We initially found a finite dimensional approximation of the PDE system employing Galerkin's method, then an EKF was designed to estimate the system states from the available noisy measurement data. Employing these estimated states along with a robust controller resulted in a reliable estimation of the system states, in presence of model uncertainty and simultaneously achieved the neces-

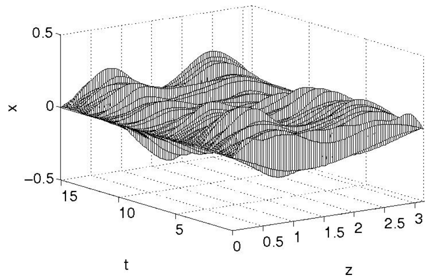

Fig. 5. Closed-loop estimated surface profile of Eq.12 using robust controller of Eq.11.

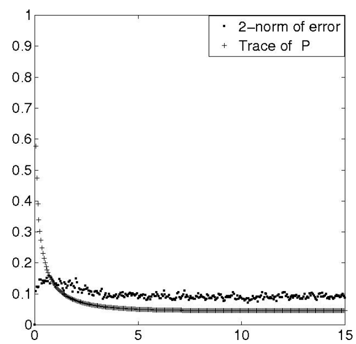

Fig. 6. Temporal profile of the 2-norm of the error between true and estimated states and the trace of P obtained during the closed-loop operation of Eq.12 with the robust controller

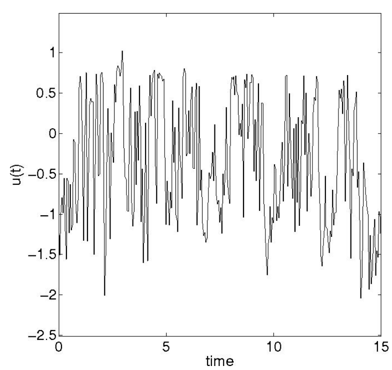

Fig. 7. Control action needed to stabilze Eq.12 using robust controller; with measurement noise.

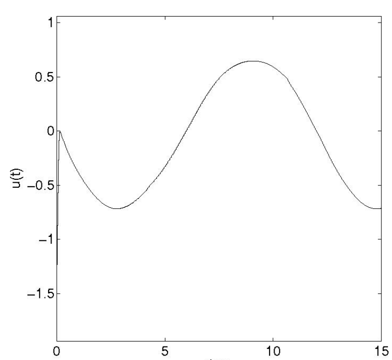

Fig. 8. Control action needed to stabilze Eq.12 calculated using robust controller; no measurement noise.

sary control objective. The methodology was applied to a representative example wherein the control objective was to control temperature in a catalytic rod where an exothermic reaction occurs. It was observed from numerical simulations that the 2-norm of the estimation error asymptotically goes to zero as more measurements from the process was made available to the estimator. The robust dynamic output feedback controller, using the states estimated through EKF, was found to successfully stabilize the process around an open-loop unstable steady-state.

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