2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Control of Robotic Manipulators with Input/Output Delays

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Abstract— **Input/output delays in a control system can pose significant impediments to the stabilization problem. Recently, passivity based control has emerged as a promising approach to guarantee delay independent stability of passive systems with delays in the input-output channel. In this paper we study two problems in motion control of rigid robots with input/output delays. The first problem is the classical set-point control problem of rigid robots where we demonstrate that the use of the scattering variables can stabilize an otherwise unstable system for arbitrary constant time delays. The second problem we address is that of stabilizing a rigid robot with external stiffness and input /output delays using bounded output feedback. Employing the the scattering transformation, and by encoding the inputs and outputs in the scattering variables, we show that the mechanical system can be asymptotically stabilized independent of the input-output delays. The proposed algorithms are numerically verified on a two-degree-of-freedom manipulator.**

I. Introduction

In this paper we study the problem of motion control of rigid robots when there are time delays in their input-output channel. In the last three decades, several control schemes [24] have been developed for control of robots. Starting with the work of [25], passivity-based control [19] has been a fruitful methodology for control design of robotic systems. Several control design have been presented in the literature [18], [12] where the controller and the mechanical system can be represented as a negative feedback interconnection of passive systems. Invoking the fundamental passivity theorem [5], it is then possible to guarantee passivity of the closed loop system. Under additional assumptions, stability of the closed loop system can also be established.

The problem of bilateral teleoperation [1], [17], a classical problem in robot control, highlighted the deleterious effect of time delays on the stability of the closed loop system. This problem has received widespread attention and several results [8] have been developed to address the network delay and lossy nature of the communication network. However, input/output delays may manifest in a robotic control system from many other sources, for example processing delays in visual systems [4] or from communication between different computers on a single humanoid robot [20]. It is well known [21] that guaranteeing stability of a control system with input delays is a challenging problem. In this paper we exploit the passivity property of robotic systems to study two problems in motion control of robots with input/output delays.

The issue of time delay instability in dissipative systems has been studied by several authors [13], [16], [6], [10], [3], [15], [26], [14], [2], [20], [22]. Scattering or the wave-variable representation, which was developed in [1], [17] for guaranteeing stability of bilateral teleoperators, has emerged as a novel tool for studying network control systems [13], [10], [3], [2], [20], [22]. The basic idea in these results is to use the scattering variables to guarantee passivity of the communication block, thereby creating a passive two-port network between a passive plant, communication and the passive controller. Furthermore, time-varying gains [11], dependent on the maximum rate of the delay, can be additionally added in the communication path to guarantee stability independent of the time-varying delays [3], [2].

In this paper we first study the problem of set-point control in rigid robots with input delays and which are constrained to move in the horizontal plane. Using simulations on a two-degree-of-freedom system we show that if the classical PI (with velocities as outputs) is used and there are small input delays, then the closed loop becomes unstable. However, if the scattering transformation is used to encode the input-output variables for the robot and the controller, then stability is recovered independent of the time delays, The second problem we address is that of stabilizing a robotic system, interacting with a stiff environment, using bounded output feedback. Using the scattering variables together with a static controller, asymptotic stabilization of the closed loop system is demonstrated.

The outline of the paper is as follows. A brief background on the general concept of passivity and a description of the robot dynamics is presented in Section II. This is followed by the two main results in Section III. Section IV describes the details of the numerical simulations and the results are summarized in Section V.

II. Background

The concept of passivity is one of the most physically appealing concepts of system theory [23] and, as it is based on input-output behavior of an system, is equally applicable to both linear and nonlinear systems. Most of the ideas presented in this section are adapted from [9]. Consider a dynamical system represented by the state space model

$$\dot{x} = f(x,u) \quad (1)$$
$$y = h(x) \quad (2)$$

where f: $R^n \times R^p \rightarrow R^n$ is locally Lipschitz, h: R^n R^p is continuous, $f(0,0)=0$, $h(0)=0$ and the system has the

This research was supported by a faculty startup grant from the University of Maryland, College Park.

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same number of inputs and outputs.

Definition The dynamical system (1)-(2) is said to be passive if there exists a continuously differentiable non-negative definite scalar function $S(x)$: $R^n \rightarrow R$ (called the storage function) such that

$$u^T y \geq \dot{S}(x), \qquad \forall (x,u) \in R^n \times R^p$$

Moreover, the system is said to be

- strictly passive if $u^T y \geq \dot{S}(x) + D(x)$ for some positive definite function $D(x)$
- lossless if $u^T y = \dot{S}(x)$
- input strictly passive if $u^T y \geq \dot{S}(x) + u^T \psi(u)$, where $u^T \psi(u) > 0$ for some function ψ and $\forall u \neq 0$
- output strictly passive if $u^T y \geq \dot{S}(x) + y^T \rho(y)$, where $y^T \rho(y) > 0$ for some function ρ and $\forall y \neq 0$

Following [24], in the absence of friction and disturbances, the Euler-Lagrange equations of motion for an n-degree-of-freedom robotic system in the horizontal plane are given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = -\tau_s + \tau_e = \tau_t \qquad (3)$$

where $q(t) \in R^n$ is the vector of generalized configuration coordinates, $\tau_s(t) \in R^n$ is motor torque acting on the system, $\tau_e(t) \in R^n$ is the external torque acting on the system, $M(q) \in R^{n\times n}$ is the positive definite inertia matrix and $C(q,\dot{q})\dot{q} \in R^n$ is the vector of Coriolis/Centrifugal forces. The above equations exhibit certain fundamental properties due to their Lagrangian dynamic structure [24].

- **Property 1:** The matrix $M(q)$ is symmetric positive definite and there exists a positive constant m such that $mI \leq M(q)$.
- **Property 2:** Under an appropriate definition of the matrix C, the matrix $\dot{M}$ - $2C$ is skew-symmetric

Moreover, it is well known that the robot dynamics are passive [24] with

$$S(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} \qquad (4)$$

as the storage function and $(\tau_t(t), \dot{q}(t))$ as the input-output pair. The passivity property of the robot dynamics has led to constructive control designs for the robot manipulators. Specifically, several robot control algorithms can be reformulated as a negative feedback interconnection of two passive systems [12]. Observing Figure 1, the controller takes in the robot velocity as the input, and the output of the controller block is fed back to the robot as the desired control input. If the controller is input-output passive, then by the fundamental passivity theorem [5], the closed loop system formed by the robot dynamics and the controller is passive.

We briefly review the set point problem for robotic manipulators when there are no delays in the input-output path. The controller dynamics are given as

$$\text{Controller} = \begin{cases} \dot{x}_c = u_c = \dot{q} \\ y_c = K_P u_c + K_I(x_c - q_d) \end{cases} \qquad (5)$$

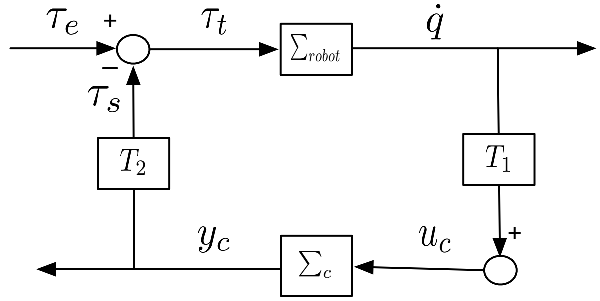

Fig. 1. A negative feedback interconnection of the robot dynamics and the controller

It is to be noted that the controller dynamics (5) are input strictly passive with $S_c(x_c) = \frac{1}{2}K_I(x_c - q_d)^T(x_c - q_d)$ as

$$\begin{aligned} \dot{S}_c(x_c) &= K_I(x_c - q_d)^T \dot{x}_c \\ &= (y_c - K_P u_c)^T u_c = y_c^T u_c - K_P u_c^T u_c \end{aligned}$$

where $K_P, K_I > 0$ are the controller gains. Assuming $\tau_e(t) \equiv 0$, the control input to the robot is given as $\tau_s(t) = y_c(t)$. The sum of the storage functions of the robot and the controller defines a positive definite storage function for the system and is given by

$$S(x_c, \dot{q}) = \frac{1}{2}(\dot{q}^T M(q)\dot{q} + K_I(x_c - q_d)^T(x_c - q_d))$$

It is then possible to show [12], [18] that the derivative of this storage function along system trajectories is given by

$$S(x_c, \dot{q}) = -K_P u_c^T u_c = -K_P \dot{q}^T \dot{q} \leq 0$$

Thus all signals in the closed loop system are bounded. Invoking Lasalle's Invariance principle [12], all bounded trajectories converge to the largest invariant set where $\dot{q}(t) \equiv 0$ and consequently $\lim_{t\rightarrow\infty}(q(t) - q_d) = 0$ provided $x_c(t_0) = q(t_0)$. Asymptotic stability of the closed loop system follows from the above discussion.

III. Main Results

In this section we first study the set point problem for mechanical systems with input/output delays. The controller dynamics are then given as

$$\text{Controller} = \begin{cases} \dot{x}_c = u_c \\ y_c = K_P u_c + K_I(x_c - q_d) \end{cases} \qquad (6)$$

where $u_c(t) = \dot{q}(t - T_1)$ and furthermore the control input to the robot is given as $\tau_s(t) = y_c(t - T_2)$ where T_1, T_2 are the constant, heterogeneous time delays between the robot and the controller. It is possible to show via simulations (see Section IV) that if the delays in the input-output path are non-neglibible, then the system can be easily rendered unstable for modest values of the controller gains.

Let $x(t) = [x_c(t) \;\; \dot{q}(t)]^T$ and denote by x_t the state of the system. Denote by $\mathscr{C} = \mathscr{C}([-h,0], R^{2n})$, the Banach space of continuous functions mapping the interval $[-h,0]$ into R^{2n}, with the topology of uniform convergence. Define $x_t = x(t + \phi) \in \mathscr{C}, -h < \phi < 0$ as the state of the system [7]. We assume

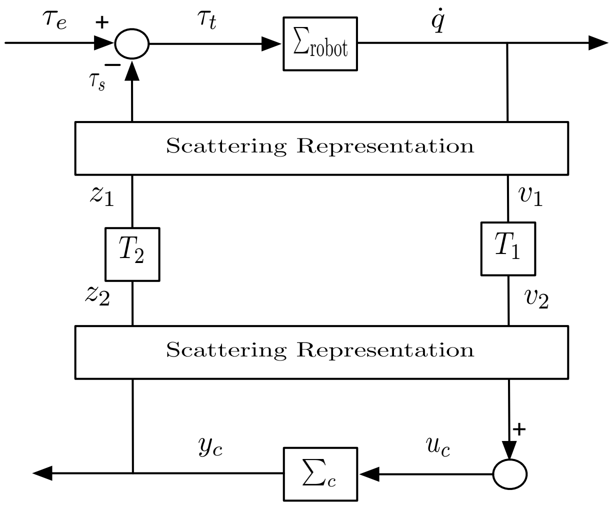

Fig. 2. A negative feedback interconnection of the robot dynamics and the controller

in this note that $x(\phi) = \eta(\phi), \eta \in \mathscr{C}$ and that all signals belong to $\mathscr{L}_{2e}$, the extended $\mathscr{L}_2$ space.

With the aim of stabilizing the closed loop system, instead of transmitting the joint velocities and input torques directly, the scattering variables are transmitted across the communication channel

$$\begin{array}{ll} v_1 = \frac{1}{\sqrt{2b}}(\tau_s + b\dot{q}) & ; \; z_1 = \frac{1}{\sqrt{2b}}(\tau_s - b\dot{q}) \\ v_2 = \frac{1}{\sqrt{2b}}(y_c + bu_c) & ; \; z_2 = \frac{1}{\sqrt{2b}}(y_c - bu_c) \end{array} \tag{7}$$

where $b > 0$ is a constant. The proposed architecture is demonstrated in Figure 2. The transmission equations between the robot and the controller can be written as

$$\begin{array}{rl} z_1(t) = & z_2(t - T_2) \\ v_2(t) = & v_1(t - T_1) \end{array} \tag{8}$$

The controller dynamics for this system are described by (6), however $u_c \neq \dot{q}(t - T_1)$ but is derived from the scattering representation (7) and the transmission equations (8).

The first claim in the paper follows

Theorem 3.1: Consider the closed loop system described by (3), (6), (7) and (8).

1) The closed loop system is input-output passive with $(\tau_e, \dot{q})$ as the input-output pair.
2) If $\tau_e(t) \equiv 0$ and $K_P = b$, then the signals $\dot{q}(t)$ and $x_c(t) - q_d$ are asymptotically stable.

Proof: Consider a positive semi-definite storage functional for the system as

$$S(x_t) = \frac{1}{2}(\dot{q}^T M(q)\dot{q} + K_I(x_c - q_d)^T (x_c - q_d)) + \frac{1}{2}(\int_{t-T_1}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2}^{t} ||z_2(\tau)||^2 d\tau)$$

The derivative of the storage function yields

$$\begin{aligned} \dot{S}(x_t) &= \dot{q}^T(-C(q,\dot{q})\dot{q} - \tau_s + \tau_e) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} \\ &+ K_I(x_c - q_q)^T \dot{x}_c + \frac{1}{2}(||v_1||^2 - ||v_1(t - T_1)||^2 + ||z_2||^2 \\ &- ||z_2(t - T_2)||^2) \\ &= (-\tau_s + \tau_e)^T \dot{q} + y_c^T u_c - K_P u_c^T u_c + \frac{1}{2}(||v_1||^2 - ||z_1||^2 \\ &+ ||z_2||^2 - ||v_2||^2) \\ &= (-\tau_s + \tau_e)^T \dot{q} + y_c^T u_c - K_P u_c^T u_c + \tau_s^T \dot{q} - u_c^T y_c \\ &= \tau_e^T \dot{q} - K_P u_c^T u_c \end{aligned} \tag{9}$$

From the above calculations it is evident that the closed loop system is passive with $(\tau_e, \dot{q})$ as the input-output pair.

To prove the second claim, note that with $\tau_e(t) \equiv 0$,

$$\dot{S}(x_t) = -K_P u_c^T u_c \leq 0 \tag{10}$$

Therefore, the storage function is bounded which implies that signals $\dot{q}, x_c(t) \in \mathscr{L}_\infty$. Using the scattering variables (7) and the transmission equations (8), the relationship between the various power variables can be written as

$$\begin{aligned} y_c(t) + bu_c(t) &= \tau_s(t - T_1) + b\dot{q}(t - T_1) \\ y_c(t - T_2) - bu_c(t - T_2) &= \tau_s(t) - b\dot{q}(t) \end{aligned}$$

Using (6) in the above equation yields

$$\begin{aligned} (b + K_P)u_c(t) + K_I(x_c - q_d) &= \tau_s(t - T_1) + b\dot{q}(t - T_1) \\ (K_P - b)u_c(t - T_2) + K_I(x_c(t - T_2) - q_d) &= \tau_s(t) - b\dot{q}(t) \end{aligned}$$

Choosing $K_P = b$, the above equations can be rewritten as

$$2bu_c(t) + K_I(x_c(t) - q_d) = \tau_s(t - T_1) + b\dot{q}(t - T_1) \tag{11}$$

$$K_I(x_c(t - T_2) - q_d) = \tau_s(t) - b\dot{q}(t) \tag{12}$$

Using (12) and the fact that $x_c(t), \dot{q}(t)$ are bounded signals, we get that $\tau_s(t) \in \mathscr{L}_\infty$. Using this result in (11) yields the boundedness of $u_c(t)$. Observing the robot dynamics (3) with $\tau_e(t) \equiv 0$ and using Property 1 gives us that $\ddot{q}(t) \in \mathscr{L}_\infty$. Differentiating (12), we then get that $\dot{\tau}_s(t)$ is bounded and furthermore differentiating (11) we have that the signal $\dot{u}_c(t)$ is bounded.

Integrating (9) (with $\tau_e(t) \equiv 0$) and letting $t \to \infty$ we get that $u_c(t) \in \mathscr{L}_2[0, \infty)$. It is well known [24] that a square integrable signal with a bounded derivative approaches the origin, and thus $\lim_{t\to\infty} u_c(t) = 0$. Delaying the transmission equation (12) by T_1 and subtracting from (11) we get that

$$2bu_c(t) + K_I(x_c(t) - x_c(t - T_1 - T_2)) = 2b\dot{q}(t - T_1)$$

Taking the limit $t \to \infty$ on both sides we get that

$$\lim_{t\to\infty} 2bu_c(t) + \lim_{t\to\infty} K_I(x_c(t) - x_c(t - T_1 - T_2)) = \lim_{t\to\infty} 2b\dot{q}(t - T_1)$$

Noting that $\lim_{t\to\infty} u_c(t) = 0$ yields

$$\begin{aligned} \lim_{t\to\infty} K_I(x_c(t) - x_c(t - T_1 - T_2)) &= \lim_{t\to\infty} 2b\dot{q}(t - T_1) \\ \lim_{t\to\infty} K_I \int_{t-T_1-T_2}^{t} \dot{x}_c(\tau) d\tau &= \lim_{t\to\infty} 2b\dot{q}(t - T_1) \\ \lim_{t\to\infty} K_I \int_{t-T_1-T_2}^{t} u_c(\tau) d\tau &= \lim_{t\to\infty} 2b\dot{q}(t - T_1) \end{aligned}$$

The last equation gives us that $\lim_{t\to\infty} \dot{q}(t) = 0$. Therefore, the robot velocity is asymptotically stable independent of the time delay.

Differentiating the robot dynamics (3), it can be shown that $\dddot{q}(t) \in \mathscr{L}_\infty$. This observation coupled with the fact the $\lim_{t\to\infty} \dot{q}(t) = 0$, and invoking Barbalat's lemma [9] yields that $\lim_{t\to\infty} \ddot{q}(t) = 0$. Therefore, from (3), $\lim_{t\to\infty} \tau_s(t) = 0$. Taking limits on both sides of the transmission equation (12) implies that $\lim_{t\to\infty}(x_c(t-T_2) - q_d) = 0$. As q_d is a constant reference, we have $\lim_{t\to\infty}(x_c(t) - q_d) = 0$, and hence the signal $(x_c(t) - q_d)$ is asymptotically stable. ■

We next study the problem of stabilization of the mechanical system, when the external forcing term $\tau_e(t) = -Kq; K > 0$, and using bounded input torques. The external term, for example, may represent the interaction force with a stiff environment. We consider the case when there may be constant time delays in the input-output path. As before, these delays may be caused due to control over a communication network or may be inherent in the system structure. Using (3), the system dynamics can be rewritten as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + Kq = -\tau_s \tag{13}$$

Let $x(t) = [q(t) \quad \dot{q}(t)]^T$ and denote by x_t the state of the system. The controller output, in the absence of any delay compensation, is given as

$$y_c \quad = \quad K_P \tanh(\dot{q}(t-T_1)) \tag{14}$$

where T_1 is the output delay and $\tanh(\cdot)$ is the hyperbolic tangent function which acts elementwise on the enclosed vector. The signal $\tanh(x) \le 1$, $\tanh(x)x > 0; \forall x \in R,\ x \neq 0$ and furthermore $\tanh(x) = 0 \iff x = 0$. The hyperbolic tangent function is used to guarantee that the controller output remains bounded. We assume that the constant K_p is selected so that

$$K_P \le u_{\max}$$

where $u_{\max}$ is the desired bound on the control torque. The control input to the mechanical system is given as $\tau_s(t) = y_c(t-T_2)$, where as before T_2 is the input delay. It can be demonstrated that the desired stabilization goal is not achieved even for small time delays in the input-output path (see Section IV).

It is to be noted that the linearized dynamics of the closed loop system formed by (13) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ m\dot{x}_2 &= -Kx_1 - \tau_s \end{aligned}$$

The above system is a simple oscillator and the stability problem due to input delays in this system was studied by [14]. The proposed control law in [14] was delay-dependent and we next propose a delay-independent control law to stabilize the nonlinear system described by (13).

Define a static bounded control as

$$y_c \quad = \quad K_P \tanh(u_c) \tag{15}$$

where as before, scattering variables (7) are used to stabilize the system. The next result demonstrates that the null solution of the closed loop system with input/output delays is asymptotically stable.

Theorem 3.2: Consider the closed loop system described by (13), (15), (7), and (8). Then the zero solution of closed loop system is asymptotically stable independent of the time delays and with bounded control inputs.

Proof: Consider a positive definite storage functional for the system as

$$S(x_t) = \frac{1}{2}(\dot{q}^T M(q)\dot{q} + Kq^T q) + \frac{1}{2}(\int_{t-T_1}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2}^{t} ||z_2(\tau)||^2 d\tau)$$

The derivative of this functional along system trajectories is given as

$$\begin{aligned}
\dot{S}(x_t) &= \dot{q}^T(-C(q,\dot{q}) - Kq - \tau_s) + K\dot{q}^T q + \frac{1}{2}(||v_1||^2 \\
&\quad -||v_1(t-T_1)||^2 + ||z_2||^2 - ||z_2(t-T_2)||^2) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} \\
&= -\dot{q}^T\tau_s + \frac{1}{2}(||v_1||^2 - ||v_1(t-T_1)||^2 + ||z_2||^2 - ||z_2(t-T_2)||^2) \\
&= -\dot{q}^T\tau_s + \frac{1}{2}(||v_1||^2 - ||v_2|^2 + ||z_2||^2 - ||z_1||^2) \\
&= -\dot{q}^T\tau_s + \dot{q}^T\tau_s - y_c^T u_c \\
&= -K_P u_c^T \tanh(u_c) \le 0
\end{aligned}$$

Thus the storage functional $S(x_t)$ is bounded and consequently $x_t \in \mathscr{L}_\infty$. Using the invariance principle for time delay systems [7], all bounded solutions asymptotically converge to the largest invariant set where $\dot{S}(x_t) \equiv 0$. Hence, using the above calculations and properties of the $\tanh(\cdot)$ function, it is evident that all solutions converge to the largest invariant set where $u_c(t) \equiv 0$.

Using the scattering variables (7) and the transmission equations (8) we have

$$\begin{aligned}
\tau_s(t-T_1) + b\dot{q}(t-T_1) &= K_P \tanh(u_c(t)) + bu_c(t) \\
\tau_s(t) - b\dot{q}(t) &= K_P \tanh(u_c(t-T_2)) - bu_c(t-T_2)
\end{aligned}$$

Delaying the second equation by T_1 and subtracting from the first equation yields

$$2b\dot{q}(t-T_1) = K_P(\tanh(u_c(t)) - \tanh(u_c(t-T_1-T_2)) + bu_c(t)) - bu_c(t-T_2-T_1)$$

Therefore, all solutions converge to the largest invariant set where $\dot{q}(t-T_1) \equiv 0$. Observing the robot dynamics (13), we conclude that $\dot{q}(t) \equiv 0 \Rightarrow \ddot{q}(t) \equiv 0 \Rightarrow q(t) \equiv 0$. Hence, the zero solution is asymptotically stable independent of the input-output time delay. ■

IV. Numerical Simulations

The results were simulated on a two-link revolute joint arm [24]. The dynamics of a two link robot, in the absence of gravitational forces, are given as

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_2\dot{q}_1 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 = \tau_1 \tag{16}$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 = \tau_2 \tag{17}$$

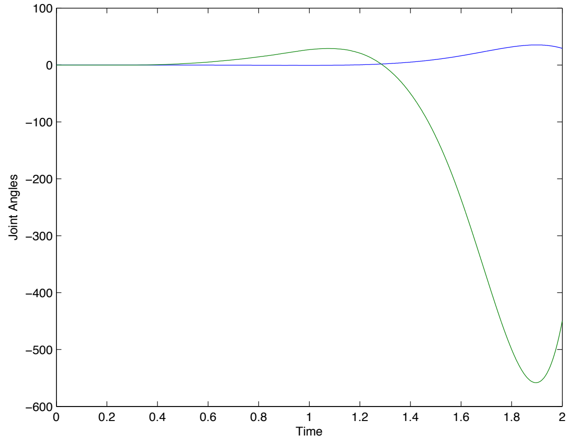

Fig. 3. The closed loop system is unstable due to time delays in communication

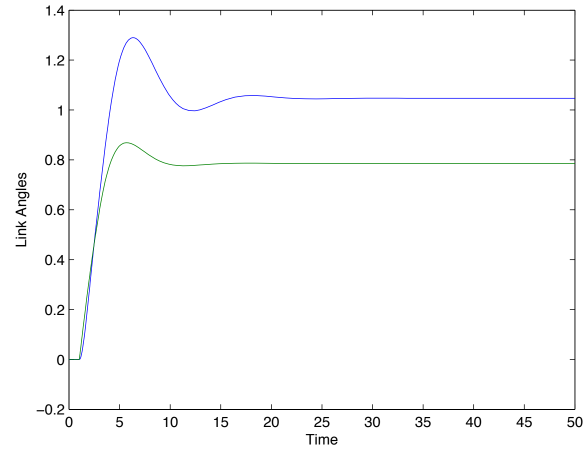

Fig. 4. If the scattering variables are used, the closed loop system is stable independent of the constant time delays

where the entries of the inertia matrix are given as

$$\begin{aligned} d_{11} &= m_1 l^2 c1 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos(q_2)) + I_1 + I_2 \\ d_{12} &= d_{21} = m_2(l_{c2}^2 + l_1 l_{c2} \cos(q_2)) + I_2 \\ d_{22} &= m_2 l_{c2}^2 + I_2 \end{aligned}$$

On the other hand, the $c_{121} = -m_2 l_1 l_{c2} sin(q_2) = h$ and $c_{221} = h, c_{112} = -h$. In the simulations, $m_1 = 7.848, m_2 = 4.49, I_1 = 0.176, I_2 = 0.0411, l_1 = 0.3, l_2 = 1, l_{c1} = 0.1554, l_{c2} = 0.0341$.

We first simulate the remote set point stabilization problem. The desired set point was chosen to be $q_d = [\frac{\pi}{3} \quad \frac{\pi}{4}]^T$. In the first simulation, the dynamical system described by (3) and (6) was studied with $K_P = 2, K_I = 1$ and $T_1 = 0, T_2 = 0.1s$ and therefore only input delay was assumed in the communication path. In the absence of any compensation for the time delay, as seen in Figure 3, the time delay renders the closed loop system unstable. However, when the scattering variables, as described in Theorem 3.1, are used and the time delay is increased to $T_1 = T_2 = 1s$, the closed loop system is stable independent of the constant time delays and the joint angles are driven to the desired configuration as shown in Figure 4.

We next simulate the stabilization scheme for the robot interacting with a stiff environment using delayed output feedback and bounded controls. The upper bound on the control input was given as $u_{\max} \leq 3$ and hence $K_P = 2$. In the absence of any delay compensation, using the bounded control input described by (14), the closed system is unstable as seen in Figure 5. However, when the scattering variables are used to encode the inputs and outputs of the mechanical system and the controller, then as seen in Figure 6, the zero solution for the closed loop system is asymptotically stable.

V. Conclusions

In this paper we studied two problems in motion control of rigid robots with input/output delays. The set-point

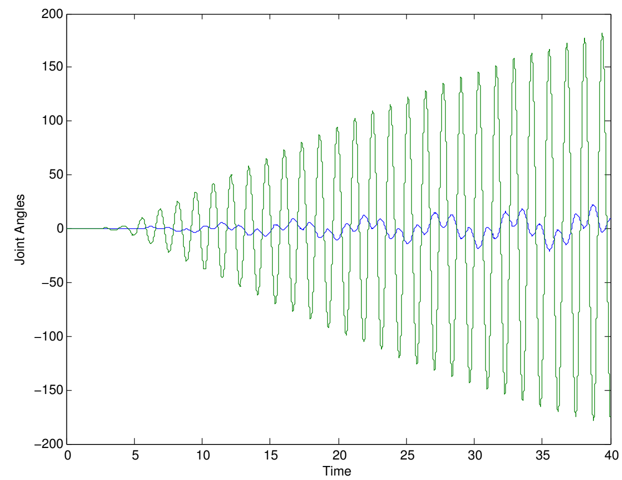

Fig. 5. The closed loop system is unstable due to time delays in the input channel

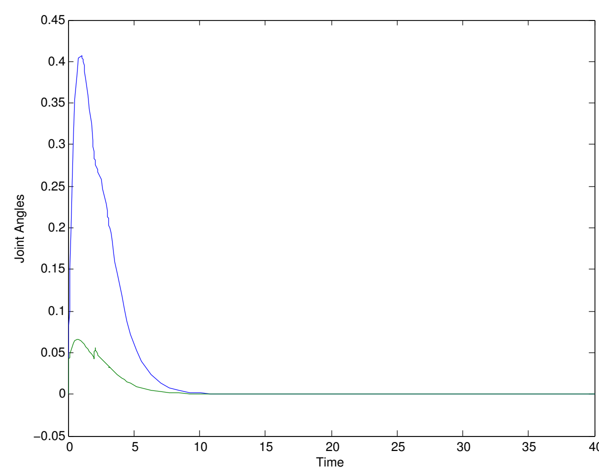

Fig. 6. When the scattering variables are used, the zero solution is asymptotically stable independent of the constant time delays

control problem for a robot, constrained to move in the horizontal plane, was first addressed. It was demonstrated via simulations on a two-degree-of-freedom manipulator that the classical algorithm can render the closed loop system unstable in the presence of input delays. However, using the scattering variables between the controller and the robot can stabilize an otherwise unstable system for arbitrary constant time delays. The problem of stabilizing a rigid robot, in contact with a stiff environment, using bounded output feedback was also studied. It was demonstrated that by encoding the inputs and outputs in the scattering variables, the mechanical system can be asymptotically stabilized independent of the input-output delays. The proposed algorithm provides a constructive methodology for stabilizing passive nonlinear oscillators [14] with input delays using bounded output feedback. The proposed schemes were also numerically verified on a two-degree-of-freedom manipulator.

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