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Impact Dynamics Based Control of Compass Gait Biped

A. K. Kamath and N. M. Singh

***Abstract*— In this paper we investigate the control of compass gait biped based on its impact dynamics. We use the Receding Horizon Control (RHC) strategy to develop an active control law so as to mimic the passive gait. Our results shows that this control strategy not only mimics the passive gait but can also stabilize it for those initial conditions, which make the passive gait unstable.**

I. INTRODUCTION

Since the pioneering work of McGeer, many researchers have studied the mechanical biped with or without knees, powered only by gravity, [1] - [3]. These passive gaits exhibit stable limit cycles only for very shallow slopes. The biped studied in [3], show that a passive gait exhibits extreme sensitivity to slope and shows period doubling bifurcation leading to chaos as the ground slope is changed from 3 to 5.

Spong [4], introduced a potential energy shaping controller that ensures the closed-loop system is invariant under the slope changing action which is referred to as the Controlled Symmetry. He showed for a three dimensional n- degrees of freedom biped that by changing the ground slope defines a group action on the configuration manifold of the system and both kinetic energy and impact dynamics are invariant under this group action. Hence by compensating just the potential energy of the system, invariance of the passive limit cycles can be achieved.

The compass gait biped considered here, is a hybrid dynamical system, where the transition stage of the biped is characterized by continuous change in energy while the impact stage is characterized by discontinuous change in energy. This paper focuses on developing a control strategy based on impact dynamics of the biped. It is our belief that the human brain optimizes the control effort needed to perform a particular task. The brain first develops an intention to perform a task.This intention is nothing but the reference trajectory that needs to be tracked. As the person performs the task, the brain optimizes the control effort needed and continuously tracks this reference trajectory and stabilizes around it. RHC strategy uses this same philosophy. RHC uses a reference trajectory, which it tracks and stabilizes by optimizing the control. The past inputs and outputs are feedback to the controller which uses this information along with the reference trajectory to solve the optimization problem. The controller provides the best current and future control inputs out of which the current control action is implemented.

The main contribution of this paper is the use of RHC strategy on the impact dynamics of the compass gait biped.Through simulations, we show that, we can not only mimic a stable passive gait but also stabilize the gait which would be unstable in the absence of control.

II. THE COMPASS GAIT BIPED

We follow the model and notations from Goswami, et.al. (1997). The compass gait biped, shown below, is equivalent to a double pendulum with point masses mH and m concentrated at the hip and legs. The configuration of the gait is determined by the support angle, θ_s, and nonsupport angle, θ_{ns}. The dynamic equations, from Goswami, et. al. (1997), are those of a 2-DOF robot and can be written as:

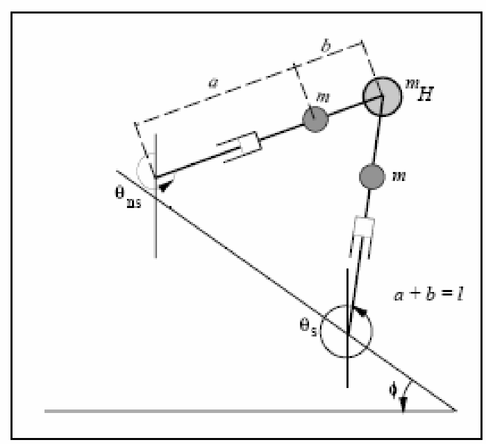

Figure 1: Model of a compass-like biped robot

$$M(\theta)\ddot{\theta} + N(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Su \qquad (1)$$

Where $M(\theta) isa2x2matrix, N(\theta,\dot{\theta})$ is 2x2 matrix with the centrifugal co-efficient, $G(\theta)$ is a 2x1 with gravitational torques and S is a 2x3 matrix, which selects the actuator torques.$\theta = [\theta_{ns}, \theta_s]$ is a vector of joint angles (see fig(1)) and u is a vector of joint torques. These torques appear at the hip and ankle, and are assumed to be identically zero in case of the passive biped. In this case, equation 1 becomes

A. K. Kamath is lecturer with Department of Electrical Engineering, Veermata Jijabai Technological Institute, Matunga, Mumbai 400019,India. atulkamath84@gmail.com

N. M. Singh is Professor with the Department of Electrical Engineering, Veermata Jijabai Technological Institute, Matunga, Mumbai 400019,India. nmsingh59@gmail.com

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$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+G(\theta)=0 \tag{2}$$

The matrices $M(\theta), N(\theta,\dot{\theta}) and G(\theta)$ are given as,

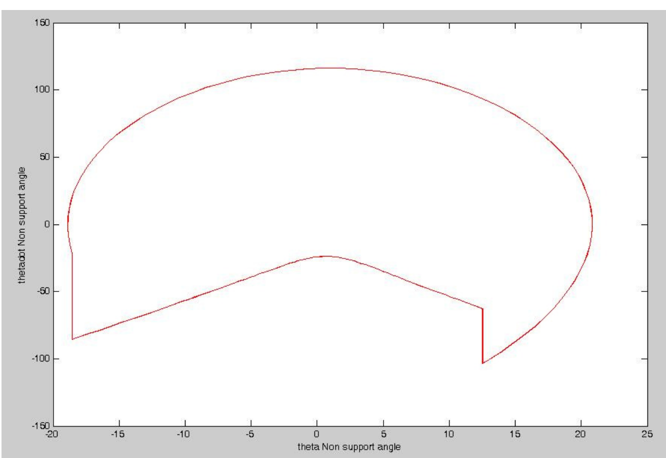
$$M(\theta)=\begin{bmatrix} mb^2 & -m\ell b\cos(\theta_s-\theta_{ns}) \\ -m\ell b\cos(\theta_s-\theta_{ns}) & (m_H+m)\ell^2+ma^2 \end{bmatrix}$$

$$N(\theta,\dot{\theta})=\begin{bmatrix} 0 & m\ell b\sin(\theta_s-\theta_{ns})\dot{\theta}_s \\ -m\ell b\sin(\theta_s-\theta_{ns})\dot{\theta}_{ns} & 0 \end{bmatrix}$$

$$G(\theta)=\begin{bmatrix} mb\sin(\theta_{ns}) \\ -(m_H\ell+ma+m\ell)\sin(\theta_s) \end{bmatrix}$$

Where $\ell=a+b$

Assuming a perfectly inelastic collision at foot contact, an instantaneous change in angular velocity results in a loss of kinetic energy while total angular momentum is conserved. A limit cycle results when the velocities after impact equal the initial velocities and the loss of kinetic energy at impact equals the change in potential energy during the step. For a given distribution of masses and leg lengths, and a given ground slope a stable limit cycle may exist as shown below. The limit cycles are typically determined from the momentum equations using a numerical search procedure (Goswami, et. al. 1998).



Fig(2): Stable Limit Cycle for a 3 slope.

Ideally, during transition, two things happen simultaneously, the swing leg touches the ground and the support leg leaves the ground. For an inelastic no-sliding collision of the robot foot with the ground the robots angular momentum during the collision is conserved. This allows us to linearly relate the post-impact and the pre-impact angular velocities of the robot in the following way

$$\dot{\theta}^{+}=H(\alpha)\dot{\theta}^{-} \tag{3}$$

where $\dot{\theta}^{-} and \dot{\theta}^{+}$ are angular velocities just before and after the transition.

Also, $H(\alpha)=Q(\alpha)^{-1}P(\alpha)$

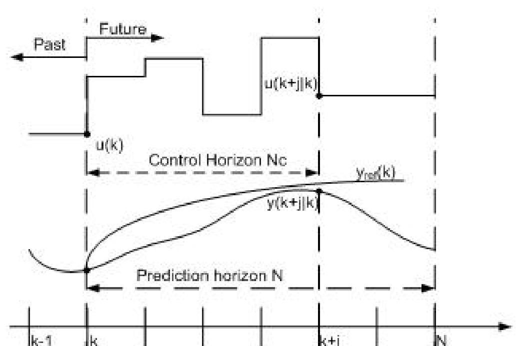
where

$$P(\alpha)=\begin{bmatrix} (m_H\ell^2+2m\ell^2)\cos(2\alpha) & \\ -mab-2mb\ell\cos(2\alpha) & -mab \\ -mab & 0 \end{bmatrix}$$

$$Q(\alpha)=\begin{bmatrix} mb^2-mb\ell\cos(2\alpha) & (m\ell^2+ma^2+m_H\ell^2) \\ -mb\ell\cos(2\alpha) & \\ mb^2 & -mb\ell\cos(2\alpha) \end{bmatrix}$$

III. Receding Horizon Control Strategy

In general, the model predictive control problem is formulated as solving on-line a finite horizon open-loop optimal control problem subject to system dynamics and constraints involving states and controls. Figure 3 shows the basic principle of model predictive control. Based on measurements obtained at time t, the controller predicts the future dynamic behavior of the system over a prediction horizon N and determines (over a control horizon $N_c \leq N$) the input such that a predetermined open-loop performance objective functional is optimized. If there were no disturbances and no model-plant mismatch, and if the optimization problem could be solved for infinite horizons, then one could apply the input function found at time $t=0$ to the system for all times $t \geq 0$. However, this is not possible in general. Due to disturbances and model-plant mismatch, the true system behavior is different from the predicted behavior. In order to incorporate some feedback mechanism, the open-loop manipulated input function obtained will be implemented only until the next measurement becomes available. The time difference between the recalculation/measurements can vary, however often it is assumed to be fixed, i.e the measurement will take place every Ts sampling time-units. Using the new measurement at time t+Ts, the whole procedure - prediction and optimization - is repeated to find a new input function with the control and prediction horizons moving forward.



Fig(3): Receding Horizon Principle.

Suppose a linear, discrete-time, state-space model of the plant is given in the form

$$x(k+1) = Ax(k) + Bu(k) \quad (4)$$

$$y(k) = C_y x(k) \quad (5)$$

$$z(k) = C_z x(k) \quad (6)$$

where x is an n_x-dimensional state vector, u is an n_u-dimensional input vector,y is an n_y-dimensional vector of measured outputs and z is an n_z-dimensional vector of outputs which are to be controlled, either to particular set-points, or to satisfy some constraints, or both. The components in y and z may overlap, and may be the same that is, all the controlled outputs could as well be measured. We will assume that $y = z$, and we will then use C to denote both C_y and C_z.

Problem: $\min_u J_k(x(k), u)$

$$J_k(x(k), u) = \| Y(k) - Y_{ref}(k) \|_Q^2 + \| \Delta U(k) \|_R^2 \quad (7)$$

Where Y_{ref} denote given set point, Q(j) and R(j) denote positive definite, symmetric weighting matrices. Q(j) penalizes the tracking error while R(j) penalizes for control efforts required.Notice here that we are solving a unconstrained optimization problem.

RHC Algorithm is given as follows:

1) Obtain measurements/estimates of the states of the system
2) Compute an optimal input signal by minimizing a given cost function over a certain prediction horizon in the future using a model of the system
3) Implement the first part of the optimal input signal until new measurements/estimates of the state are available
4) Continue with 1.

IV. Impact Dynamics based Control of Biped

The receding horizon control strategy is applied to the impact dynamics of the gait i.e. the transition stage of the gait. So, the transition equation is modified as follows:

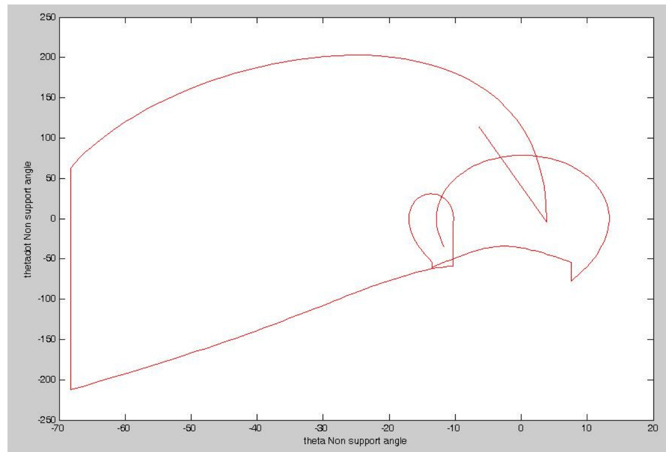
$$\dot{\theta}(k+1) = A\dot{\theta}(k) + Bu_{RHC} \quad (8)$$

where k is the kth impact of the swing foot with the ground. $\dot{\theta}(k) and \dot{\theta}(k+1)$ are the angular velocities before and after impact. u_{RHC} is the optimal control found by minimizing the cost function.

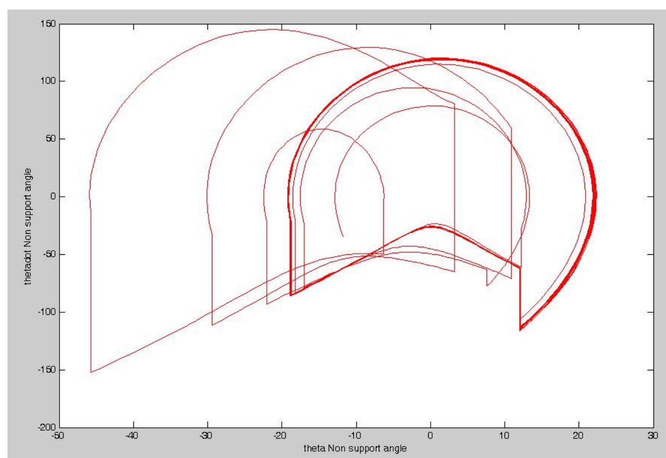
No control is applied in the swing stage and hence u = 0. Therefore the swing stage dynamics follows eqn (2).

V. Matlab Simulation and Results

Fig (1) shows the stable limit cycle for a 3 slope. Fig (2) shows the passive gait is unstable with the initial conditions $z0 = [-11.713867; 9.713867; -34.688622; -9.347404]$. Fig (3) shows how the gait stabilizes when MPC control strategy is used starting from the above initial conditions. It eventually converges to the stable limit cycle.

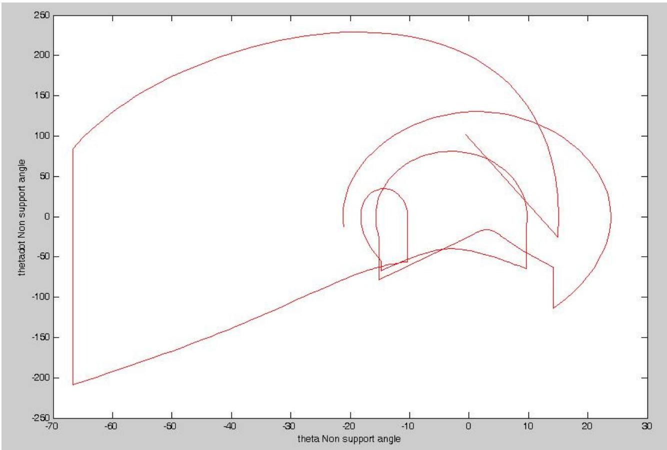


Fig(4): Unstable passive gait.

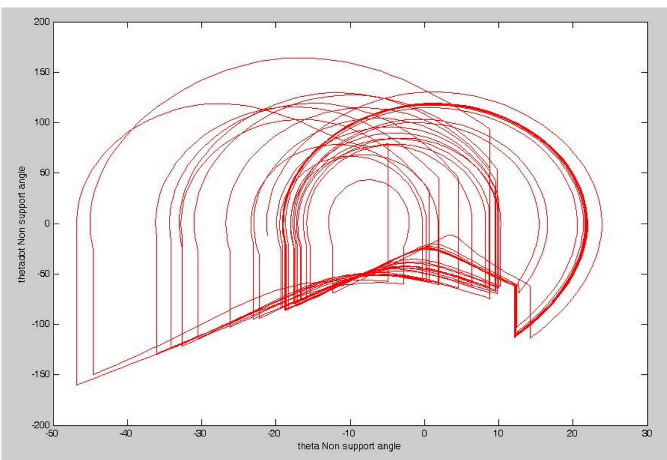


Fig(5): Gait stabilizes under the influence of active control.

Another example using initial conditions $z0 = [-21.12699; 13.12699; -12.388625; -65.279246]$ is shown below.



Fig(6): Unstable passive gait.



Fig(7): Gait stabilizes under the influence of active control.

VI. CONCLUSION

RHC successfully stabilizes the gait around the stable limit cycle of a passive gait. But there are conditions which cannot be stabilized as these conditions either makes the initial step size too small or very large which makes the gait unstable even before it can complete a cycle.

Using RHC, we can not only mimic a passive gait corresponding to a particular slope but can also stabilize the gait by starting with those initial conditions that make the passive gait unstable.

The convergence of the compass gait to its stable limit cycle takes more time using RHC algorithm. By using PI controller along with RHC we can make faster convergence possible.

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