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Adaptive Rejection of Stochastic and Deterministic Sinusoidal Disturbances with Unknown Frequency

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***Abstract*— In this paper, an adaptive control algorithm is proposed to reject unknown deterministic disturbances and minimize output variance. The proposed algorithm contains two adaptive control actions. One rejects a set of unidentified deterministic disturbances by an adaptive internal model with online frequency identification. The other minimizes the output variance using an adaptive finite impulse response filter. The stability and performance of the proposed algorithm are analyzed and demonstrated by simulation results.**

I. Introduction

A common approach to disturbance rejection control problem is to model disturbances as a result of an excitation signal passing through a disturbance model. These disturbances can be categorized as deterministic or stochastic. The stochastic disturbance is modeled as the output of stable system driven by stationary white noise, while the deterministic disturbance can be modeled as the output of a marginally stable system with an initial state or driven by a Dirac impulse.

If the disturbance model and plant model are both known Linear Time Invariant (LTI) systems, an LTI controller is often designed. The LQG/H_2 control [1] was proposed to minimize the steady-state output variance under stochastic disturbances to minimize H_2 norm of the closed-loop system. In order to reject deterministic disturbances, the feedback loop has to incorporate the disturbance model via the Internal Model Principle (IMP) [2], [3]. The objective then becomes both to minimize the H_2 norm of closed-loop system as well as achieve asymptotic deterministic disturbance rejection. The existence condition of optimal H_2 controller were addressed in [4]. Based on this result, quadratic terms were added to the usual LQG/Wiener-Hopf cost to ensure the existence of (unique) solutions [5]. In [6], an integral square output cost was added to LQG cost for tradeoffs between transient performance and H_2 disturbance rejection.

When the disturbance model is unknown or time-varying, adaptive control is often applied. In [7], [8], adaptive feedback control based on the IMP and Youla-parametrization was proposed for canceling sinusoidal disturbances with unknown frequency. In [9], [10], the disturbances were rejected by using an internal model structure with adaptive frequency in parallel with a feedback controller. An adaptive-Q scheme in [11], [12] was proposed to minimize the effects of stochastic disturbances improving the performance of the existing controller. This scheme was implemented successfully in Hard Disk Drive (HDD) control [13]. In [14], [15], adaptive disturbance observers were designed to estimate the unknown frequency. In [16], an adaptive feed-forward controller based on phase-locked loop structure was proposed to reject the unknown sinusoidal disturbance. Despite the different structures, they are essentially the same. The equivalence between adaptive feed-forward control and internal model principle was shown in [17]. In [18], a periodic disturbance was rejected by using repetitive control with online periodic estimation. These methods have been applied to optical and hard disk drives systems [19], active noise control systems [20], and vibration suppression systems [8], [21], [22].

The Adaptive Inverse Control (AIC) is another scheme for stochastic disturbance rejection [23], [24] which minimizes the Least-Mean-Square (LMS) values of the plant output. Adaptive controllers, based on LMS adaptive filters, are numerically stable and efficient, but typically converge slowly. The AIC scheme based on Recursive Least Square (RLS) [25] filters minimizes least square values of the plant output was reported in [26], since RLS filters offer the potential of significantly faster adaptation. For better numerical stability, inverse QR-RLS algorithms [25] or the lattice algorithms [27] have been used. Either inverse QR-RLS or lattice RLS algorithms show significant performance improvement over conventional RLS implementations [13], [28].

Using AIC structure, it is possible to handle both stochastic and deterministic disturbances. In [29], an fixed internal model was integrated in AIC to minimize H_2 norm of closed-loop system as well as reject the deterministic periodic disturbances with known frequency. In this paper, we further extend the work in [29] by assuming the frequency of deterministic disturbance is unknown and/or time-varying. An adaptive internal model is embedded to estimate the unknown or time-varying deterministic sinusoidal disturbances. The frequency estimation is achieved through an Adaptive Notch Filter (ANF). The ANF proposed in [30] used a minimal number of parameters equal to the number of sinusoidal components. The results of [31] confirmed the effectiveness of this algorithm for stationary processes and the performance of this algorithm for non-stationary signals was analyzed in [32]. Based on the same structure, [33], [34] proposed the direct frequency estimation algorithms for ANF.

The remainder of this paper is organized as follows: Section 2 formulates the problem as a constrained H_2

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norm minimization problem. In section 3, cascaded notch filters are studied and used to construct a particular solution with ZPETC compensation. H_2 norm minimization can then be achieved by using Youla-parametrization. In section 4, adaptive notch filter is used to estimate the frequency of deterministic sinusoidal disturbance online, and updates the controller accordingly. The convergence, stability and performance is also analyzed in this section. Section 5 presents simulation results to show the effectiveness of the proposed approach. The paper will be concluded in section 6 and some future research will be discussed.

II. Problem Formulation

Consider the adaptive inverse control (AIC) scheme for disturbance rejection as shown in Figure 1.

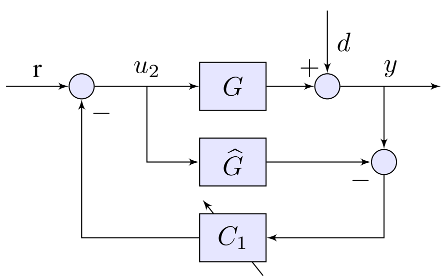


Fig. 1. Block Diagram of Adaptive Control for Disturbance Rejection

Assume both G and $\hat{G}$ are stable and causal, where G and $\hat{G}$ represent the actual plant and identified plant model, respectively. If not, a closed-loop system with a stabilizing controller will be considered as the plant. The plant model $\hat{G}(z^{-1})$ can be represented as

$$\hat{G}(z^{-1}) = z^{-d}\frac{B(z^{-1})}{A(z^{-1})} \tag{1}$$

where d is defined as delay steps and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \tag{2}$$
$$B(z^{-1}) = b_0 + a_1 z^{-1} + \cdots + a_m z^{-m} \tag{3}$$

where $b_0 \neq 0$ and $n \geq m$.

The disturbance $d(t)$ consists of both stochastic $d_1(t)$ and deterministic $d_2(t)$ signals, which can be modeled as

$$\begin{aligned} d(t) &= d_1(t) + d_2(t) \\ &= W_s(q^{-1})w(t) + W_d(q^{-1})\delta(t) \end{aligned} \tag{4}$$

where $w(t)$ is a stationary zero mean white process and $\delta(t)$ is the Dirac impulse. W_s and W_d are the unknown disturbance models, where W_s is assumed to be stable and W_d has roots on the unit disk. The control objective is to have $E[y(t)]$ approaching zero asymptotically and to minimize the H_2 norm of the closed-loop transfer function between $d(t)$ and $y(t)$.

The relationship between disturbance d and output y can be shown to be,

$$y = \frac{1 - C_1\hat{G}}{1 + C_1(G - \hat{G})}d \tag{5}$$

Using the small gain theorem [35], if $\hat{G}$ accurately represents the true closed-loop system G, then the system shown in Figure 1 is guaranteed to be stable with any stable controller C_1. If $G = \hat{G}$, then Equation (5) reduces down to

$$y = (1 - C_1\hat{G})d \tag{6}$$

The feedback control design can then be formulated as a model matching problem: given a stable $\hat{G}$, find a stable C_1, such that the error is minimized in H_2 sense, i.e.,

$$\min_{C_1 \in RH_\infty} \|1 - C_1\hat{G}\|_2 \tag{7}$$

To guarantee that the expected value of the measurement $E[y(t)]$ approaches zero asymptotically, the feedback loop must contain the internal model that captures the dynamics of deterministic disturbance model W_d [2], [3]. Choose D such that

$$e_2(t) = D(q^{-1})d_2(t) \tag{8}$$
$$\lim_{t\to\infty} e_2(t) = 0 \tag{9}$$

Then, it is straightforward to let the transfer function

$$1 - C_1\hat{G} = RD \tag{10}$$
$$RD + C_1\hat{G} = 1 \tag{11}$$

Assuming $\hat{G}$ and D are coprime, (11) is guaranteed to have at least one solution. Additionally from Youla-parameterization [35], we know that if (R_{10}, C_{10}) is a solution pair to (11) then all solutions can be characterized as $R_1 = R_{10} - Q\hat{G}$, $C_1 = C_{10} + QD$ where Q is a design parameter constrained to be stable. The H_2 optimization problem in (7) can be reformulated as

$$\min_{\substack{Q \in RH_\infty \\ Dd = 0}} \|(R_{10} - Q\hat{G})D\|_2 \tag{12}$$

III. Adaptive Control for Disturbance Rejection

A. Cascaded notch filters

In order to reject the deterministic disturbance $d_2(t)$ shown in (4), D is constructed using cascaded Notch Filters (NF). The cascaded NF is defined as follows

$$D(z^{-1}) = \prod_{k=1}^{p} \frac{1 - 2\beta_k \cos\omega_k z^{-1} + \beta_k^2 z^{-2}}{1 - 2\rho_k \cos\omega_k z^{-1} + \rho_k^2 z^{-2}} \tag{13}$$
$$= \prod_{k=1}^{p} D_k(z^{-1}) \tag{14}$$

where p is the number of harmonics, w_k is the frequency of sinusoidal disturbances, and ρ_k and β_k are *contraction factors* with $0 \ll \rho_k < \beta_k \leq 1$. It is easy to verify that $De_2 \to 0$ if $\beta_k = 1$. Figure 2 shows the frequency response magnitude with different choices of ρ, where $\beta = 1$.

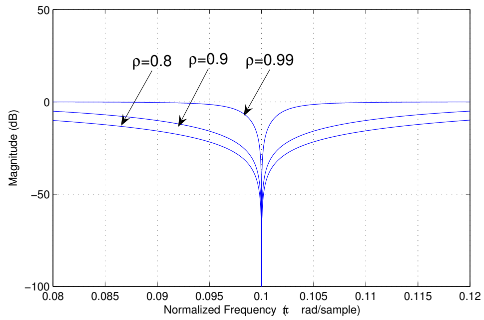


Fig. 2. Frequency response magnitude with different ρ

Equation (14) approach ideal notches as $\rho \rightarrow 1$, i.e.

$$\begin{cases} D(e^{-j\omega}) \approx 1 & \texttt{if } \omega \neq \omega_k \\ D(e^{-j\omega}) = 0 & \texttt{if } \omega = \omega_k \end{cases} \tag{15}$$

B. A particular solution

It is necessary to construct an initial solution pair (R_{10}, C_{10}) in order to apply 12, that satisfies the Diophantine equation

$$C_{10}\hat{G} + R_{10}D = 1 \tag{16}$$

In general, the Diophantine equation can be solved numerically by forming a Sylvester matrix [36] or through state space methods. In this paper, we propose a simple method to construct (R_{10}, C_{10}) using zero-phase-error-tracking (ZPETC) type feed-forward controllers [37].

ZPETC corrects the phase of any stable plant to zero phase using a simple feed-forward control. Given the stable plant shown in (1), factor B as:

$$B = B^{+}B^{-} \tag{17}$$

where B^{+}, B^{-} represent stable and unstable zeros of the stable plant, respectively. The ZPETC compensator F_{ZPC} is defined as

$$F_{ZPC} = \gamma z^{-(n_u+d)} \frac{A(z^{-1})B^{-}(z)}{B^{+}(z^{-1})} \tag{18}$$

where γ is a positive real design parameter, and n_u is the number of unstable zeros.

By cascading the ZPETC compensator (18) and the NF (14), (R_{10}, C_{10}) can be designed as follows:

$$C_{10} = \frac{F_{ZPC}(1-D)z}{D + z^{-(n_u+d-1)}\gamma B^{-}(z)B^{-}(z^{-1})(1-D)} \tag{19}$$

$$R_{10} = \frac{1}{D + z^{-(n_u+d-1)}\gamma B^{-}(z)B^{-}(z^{-1})(1-D)} \tag{20}$$

The stability of closed-loop system will be shown in the following Lemma:

Lemma 1: Given any stable plant G, shown in (1), there exists (R_{10}, C_{10}) such that if γ is small enough to satisfy the following condition

$$\max_{\omega_k} |1 - \gamma e^{-j(n_u+d-1)\omega_k} B^{-}(e^{-j\omega_k})B^{-}(e^{j\omega_k})| < \frac{1}{1+\delta} \tag{21}$$

where $\delta > 0$ is a small constant. So given any plant, the closed-loop system is guaranteed to be stable when γ is chosen to be small enough.

Proof: The denominator of closed-loop system can be rewritten as

$$1 + (1 - \gamma z^{-(n_u+d-1)} B^{-}(z^{-1})B^{-}(z))(D-1) \tag{22}$$

where H is a stable filter with $|D-1| \leq 1+\delta$, and $B^{-}(z^{-1})B^{-}(z)$ is also a stable filter. So, according to small gain theorem [35], if Equation (21) holds, the closed-loop system is stable. ■

C. Adaptive control for disturbance rejection

Assuming disturbance model W_s is unknown and D is fixed, the adaptive control can be used. The H_2 optimization in (12) will identify the disturbance model W_s implicitly, i.e., direct adaptive algorithm. The online identification of D will be introduced in the next section.

IV. Adaptive Control for Disturbance Rejection with Adaptive Internal Model

From (19) and (20), (R_{10}, C_{10}) are functions of internal model with unknown frequency ω_k. In this proposed approach, the frequency ω_k is estimated online and D, R_{10} and C_{10} are updated accordingly. The online frequency estimation is implemented using a Adaptive Notch Filter (ANF).

A. Adaptive Notch Filter

The online frequency estimation algorithm for the notch filter can be found in [30], [33], [34] and will be reviewed in this section.

Let $d(t)$ be the noise corrupted measurement of sinusoidal signal defined by

$$d(t) = \sum_{k=0}^{p} B_k \sin(\omega_k t + \phi_k) + d_1(t) \tag{23}$$

The objective is to apply the notch filter shown in Equation (14) to cancel the sinusoidal signals with unknown frequencies ω_k, magnitudes B_k and phase shifts ϕ_k. Let

$$\epsilon(t, \hat{\omega}_k) = D\left(\hat{\omega}_k(t-1), z^{-1}\right) d(t) \tag{24}$$

where $\epsilon(t, \hat{\omega}_k)$ is the filtered output of the input disturbance d through the filter D. To estimate the frequency ω_k, define the cost function as

$$\hat{\omega}_k = \arg\min_{\hat{\omega}_k} \frac{1}{M} \sum_{t=1}^{M} \epsilon^2(t, \hat{\omega}_k) \tag{25}$$

where M denotes the number of data points.

A modified gradient function $\psi_i(t)$ is derived to be

$$\psi_i(t) \quad \approx \quad \psi_i(t-1) = -\frac{\partial \epsilon(t)}{\partial \omega_i(t)} \tag{26}$$

With the modified gradient function $\psi(t)$, the recursive frequency estimation algorithm can be proposed. For compactness, define

$$\begin{aligned}\omega(t) &= [\omega_1(t),\ \omega_2(t),\ \cdots,\ \omega_p(t)]^T \\ \Psi(t) &= [\psi_1(t),\ \psi_2(t),\ \cdots,\ \psi_p(t)]^T\end{aligned}$$

The algorithm is summarized as follows:

Initialization: $\hat{\omega}(0)$, $P(0)$, $\alpha(1)$, λ_0, λ_∞, λ, β,ρ;

Main loop: for $t = 1, 2, \cdots$

- Step 1: Prediction error $\epsilon(t)$ computation:

$$\epsilon(t) = D\left(\hat{\omega}(t-1), z^{-1}\right) d(t) \tag{27}$$

- Step 2: Computing $\Psi(t)$:

$$\psi_i(t) = \left[\frac{-2\beta \sin\hat{\omega}_i(t-1)z^{-1}}{A_0(\hat{\omega}_i(t-1), \beta z^{-1})} - \frac{-2\rho \sin\hat{\omega}_i(t-1)z^{-1}}{A_0(\hat{\omega}_i(t-1), \rho z^{-1})}\right]\epsilon(t) \tag{28}$$

- Step 3: Parameter updating:

$$K(t) = \frac{P(t-1)\Psi(t)}{\lambda + \Psi(t)P(t-1)\Psi(t)} \tag{29}$$

$$P(t) = [P(t-1) - K(t)\Psi(t)P(t-1)]/\lambda \tag{30}$$

$$\hat{\omega}(t) = [\hat{\omega}(t-1) + K(t)\epsilon(t)] \tag{31}$$

- Step 4: Derive *posteriori* prediction error $\bar{\epsilon}(t)$:

$$\bar{\epsilon}(t) = D\left(\hat{\omega}(t), z^{-1}\right) y(t) \tag{32}$$

With this adaptive notch filter, the frequencies of the disturbances are estimated online and the internal model is tuned to reject those disturbances. This structure is discussed in the next section.

B. Adaptive control with adaptive internal model

Figure 3 shows the block diagram of proposed adaptive control scheme. The adaptive internal model D estimates the frequencies of the deterministic sinusoidal disturbances and update C_{10} and R_{10} accordingly at each time step. In all, there are two adaptive channels: 1) the adaptive internal model for unknown deterministic sinusoidal rejection. 2) the adaptive control for H_2 norm minimization, i.e., stochastic disturbance rejection.

1) Convergence and Stability: Since there are two adaptive channels in the proposed scheme, they may interfere with each other and prevent the convergence. However, because the adaptive notch filter converges much faster than adaptive control channel, the adaptive controller can be considered as LTI with respect to the ANF. Therefore the two adaptive channels will both converge.

According to small gain theorem, the closed-loop is stable if both the filter Q and the notch filter are stable. By implementing the Q filter as an FIR filter, Q is guaranteed to be stable. According to Equation (14), the poles of notch filter are chosen to be $\rho e^{\pm j\omega_0}$ with $\rho < 1$, which is always within unit disk of z-plane forcing the closed-loop system always stable.

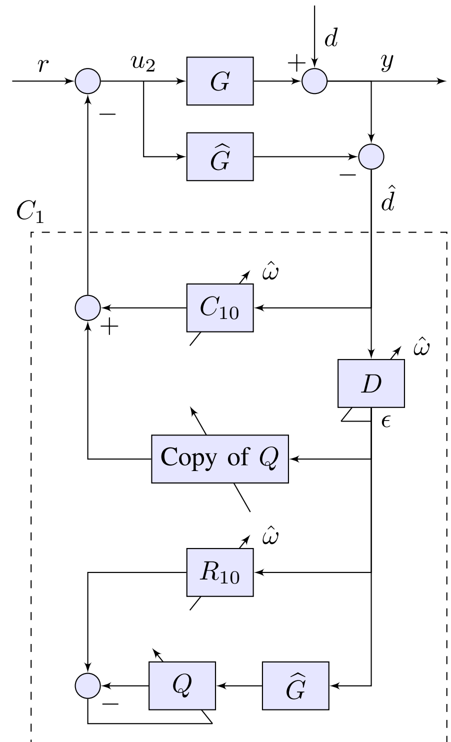


Fig. 3. Block Diagram of Adaptive Control with Adaptive Internal Model

2) Performance: The performance of the proposed method will be affected by the performance of adaptive notch filter. According to [31], we have,

$$\omega^* = \omega_0 + O((1-\rho)^2) \tag{33}$$

where $\omega^* = \lim_{t\to\infty}\hat{\omega}(t)$. The frequency estimation is biased but can be reduced quadratically by choosing ρ approaching 1. It is also interesting to note that although there is only one minimum point in Equation (25) when $M \to \infty$, it will certainly have local minimal points with finite M. The use of time-varying ρ can prevent convergence to a local minimum point. On the other hand, the bandwidth of the notch filter is also determined by ρ [30]:

$$\mathtt{BW} = \pi(1-\rho) \tag{34}$$

The bandwidth of notch filter determines the horizon of frequency tracking. Thus ρ represents the tradeoff between estimation accuracy and tracking performance of the adaptive notch filter.

V. Simulation results

The proposed algorithm will be applied to a linear motor plant. The identified model of the plant is

$$P(z) = \frac{-0.0028567(z-2.518)}{(z-.99631)(z-1.00244)} \tag{35}$$

The model $P(z)$ closely matches the expected simple free body double integrator Newtonian model

$$F = m\ddot{x} \tag{36}$$

Since the open-loop plant is unstable, a PD controller is applied to stabilize the plant.

$$C(z) = \frac{14.2z - 13}{z} \tag{37}$$

making the estimated closed-loop system

$$\hat{G}(z) = \frac{-0.040565(z - 2.58)(z - 0.9155)}{(z - 0.1015)(z^2 - 1.938z + 0.9438)} \tag{38}$$

where $n_u = 1$ and $d = 1$ as shown in (20). The disturbance in the simulation comprises of two sinusoidal signals at 60Hz and 120Hz and white noise passing through a high order stable IIR filter W_s, i.e.,

$$d(t) = 2sin(2\pi 60) + sin(2\pi 120) + W_s(q^{-1})w(t) \tag{39}$$

Figure 4 shows the online frequency estimation with ANF from $\hat{d}$. When there are two sinusoids, the ANF converges to frequency with larger magnitude, i.e., the ANF has a global convergence property, which is discussed in the previous section.

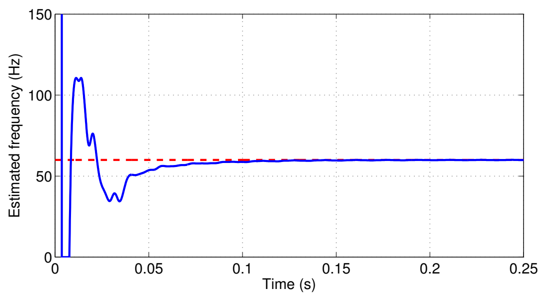


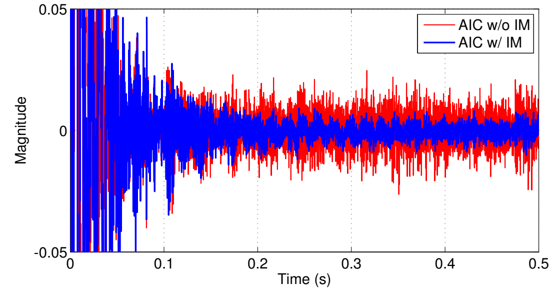
Fig. 4. Online Frequency Estimation from $\hat{d}(t)$, which converges to 60Hz

Figure 5 shows the performance comparison between conventional AIC structure and the proposed method with the adaptive internal model. In both methods, the filter length of Q is chosen to be 20 and the coefficients are updated by inverse QR based RLS algorithms [25]. In the proposed algorithm, D comprises of a second-order IIR filter, and C_{10} and R_{10} is chosen to be a 6th-order IIR filter. As seen in Figure 6, in order for the conventional AIC structure to get comparable performance, the filter length of Q has to be above 50. From Figure 5(b) and 5(c), one may find that the deterministic disturbances are rejected by both method, however, the proposed method shows better overall performance.

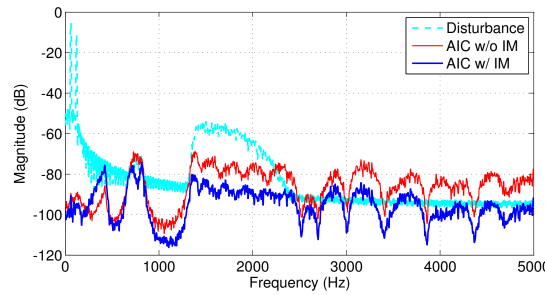
Figure 6 shows the RMS error with different adaptive filter lengths. From this figure, we may see that: 1) All three curves show improved performance with longer filter length. 3) The proposed method will achieve similar performance with lower filter length in Q and reduce the computational cost of coefficient update.

VI. Conclusion

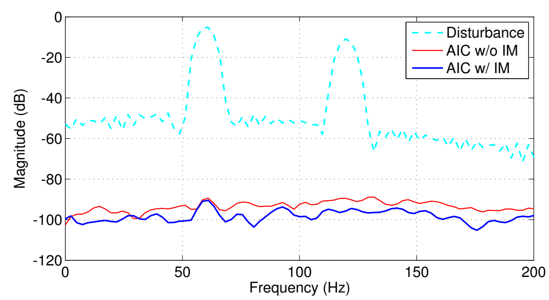
The proposed adaptive control scheme is able to minimize the output variance by using a recursive least squares adaptive FIR filter, which implicitly identifies the disturbance stochastic dynamics. At the same time, the control



(a) Time Domain Position Error: AIC w/o IM RMS = 0.0071, AIC w/ IM RMS = 0.0025



(b) PSD of Position Error, the disturbance signal, shown in dashed line, consists of colored noise and deterministic disturbances at 60Hz and 120Hz



(c) PSD of Position Error(Zoom in). The deterministic disturbance are rejected by both methods.

Fig. 5. Simulation Results Comparison of Position Error of AIC structure: w/ and w/o IM

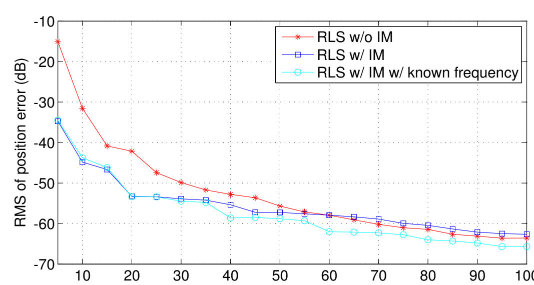


Fig. 6. Performance Comparison with Different Filter Lengths

is able to reject sinusoidal disturbances, where the unknown frequencies are estimated by another on-line identification algorithm. The simulation results show that the adaptive internal model for rejecting harmonic disturbances is able to achieve good performance with low order adaptive FIR filters. To achieve similar performance without the internal model the adaptive FIR filter would require to be of high order. Therefore, the proposed algorithm is more efficient in real-time implementation than the conventional minimum variance adaptive control.

REFERENCES

[1] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard h_2 and h_∞ control problems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 831–847, August 1989.

[2] E. J. Davison and A. Goldenberg, "Robust control of a general servomechanism problem: the servo compensator," *Automatica*, vol. 11, pp. 461–471, 1975.

[3] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, pp. 457–465, 1976.

[4] A. Iftar, "Optimal solution to the servomechanism problem for system with stochastic and deterministic disturbances," *Int. J. of Control*, vol. 51, pp. 1327–1341, 1990.

[5] M. Silveira and G. Correa, "h_2-optimal control of linear system with tracking/disturbance rejection constraints," *Trans. of ASME: J. of Dynamic Systems, Measurement, and control*, vol. 55, no. 5, pp. 1115–1139, 1992.

[6] A. G. Sparks and D. S. Bernstein, "Optimal rejection of stochastic and deterministic disturbances," *Int. J. of Control*, vol. 119, pp. 140–143, 1997.

[7] F. B. Amara, P. T. Kabamba, and A. G. Ulsoy, "Adaptive sinusoidal disturbance rejection in linear discrete-time systems - part i: Theroy," *Trans. of ASME: J. of Dynamic Systems, Measurement, and control*, vol. 121, pp. 648–654, 1999.

[8] I. D. Landau, A. Constantinescu, and D. Rey, "Adaptive narrow band disturbance rejection applied to an active suspension - an internal model principle approach," *Automatica*, vol. 41, pp. 563–574, 2005.

[9] L. J. Brown and Q. Zhang, "Identification of periodic signals with uncertain frequency," *IEEE Trans. Signal Processing*, vol. 51, pp. 1538–1545, June 2003.

[10] L. J. Brown and Q. Zhang, "Periodic disturbance cancellation with uncertain frequency," *Automatica*, vol. 40, pp. 631–637, 2004.

[11] T. T. Tay, I. Mareels, and J. B. Moore, *High Performance Control*. Boston, MA: Birkhauser, 1998.

[12] T. T. Tay and J. B. Moore, "Enhancement of fixed controllers via adaptive-q disturbance estimate feedback*," *Automatica*, vol. 27, no. 1, pp. 39–53, 1991.

[13] K. Krishnamoorthy and T.-C. Tsao, "Adaptive-q with lqg stabilization feedback and real time computation for disk drive servo control," in *Proc. of American Control Conf.*, (Boston, MA), June 2004.

[14] R. Marino, G. L. Santosuosso, and P. Tomei, "Robust adaptive compensation of biased sinusoidal disturbances with unknown frequency," *Automatica*, vol. 39, pp. 1755–1761, 2000.

[15] Z. Ding, "Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model," *Automatica*, vol. 39, pp. 471–479, 2000.

[16] M. Bodson and S. C. Douglas, "Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency," *Automatica*, vol. 33, no. 12, pp. 2213–2221, 1997.

[17] W. Messner and M. Bodson, "Design of adaptive feedforward algorithms using internal model equivalence," *Int. J. of Adaptive Control and Signal Processing*, vol. 9, pp. 199–212, 1995.

[18] T.-C. Tsao, Y.-X. Qian, and M. Nemani, "Repetitive control for asymptotic tracking of periodic signals with an unknown period," *ASME J. Dyn. Syst., Meas., Control*, vol. 122, pp. 364–369, June 2000.

[19] A. Sacks, M. Bodson, and P. Khosla, "Experimental results of adaptive periodic disturbance cancellation in a high performance magnetic disk drive," *ASME J. Dyn. Syst., Meas., Control*, vol. 118, pp. 416–424, 1996.

[20] M. Bodson, J. S. Jensen, and S. C. Douglas, "Active noise control for periodic disturbances," *IEEE Trans. Contr. Syst. Technol.*, vol. 9, no. 1, pp. 200–205, 2001.

[21] S. R. Hall and N. M. Wereley, "Performance of higher harmonic control algorithms for helicopter vibration reduction," *Journal of Guidance, Control and Dynamics*, vol. 16, no. 4, pp. 793–797, 1993.

[22] G. Hillerstrom, "Adaptive suppression of vibrations: a repetitive control approach," *IEEE Trans. Contr. Syst. Technol.*, vol. 4, no. 1, pp. 72–78, 1996.

[23] B. Widrow and K. S. Sin, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[24] B. Widrow and E. Walach, *Adaptive Inverse Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[25] A. H. Sayed, *Fundamentals of Adaptive Filtering*. New York, NY: Wiley, 2003.

[26] R. Horowitz and B. Li, "Adaptive control for disk file actuators," in *Proc. of the 34st IEEE Conf. on Decision and Control*, (New Orleans, LA), pp. 655–660, 1995.

[27] S.-B. Jiang and J. S. Gibson, "An unwindowed multichannedl lattice filter with orthogonal channels," *IEEE Trans. Signal Processing*, vol. 43, no. 12, pp. 2831–2842, 1995.

[28] N. O. P. Arancibia, T.-C. Tsao, and J. S. Gibson, "Adaptive tuning and control of a hard disk drive," in *Proc. of the the American Control Conference*, (New York City, NY), pp. 1526–1531, July 2007.

[29] N. O. P. Arancibia, C.-Y. Lin, T.-C. Tsao, and J. S. Gibson, "Adaptive-repetitive control of a hard disk drive," in *Proc. of the 46th IEEE Conf. on Decision and Control*, (New Orleans, LA), pp. 4519–4524, December 2007.

[30] A. Nehorai, "A minimal parameter adaptive notch filter with constrainted poles and zeros," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp. 983–996, August 1985.

[31] P. Stoica and A. Nehorai, "Performance analysis of an adaptive notch filter with constrained poles and zeros," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 911–919, June 1988.

[32] P. Handel and A. Nehorai, "Tracking analysis of an adaptive notch filter with constrained poles and zeros," *IEEE Trans. Signal Processing*, vol. 42, pp. 281–291, February 1994.

[33] B.-S. Chen, T.-Y. Yang, and B.-H. Lin, "Adaptive notch filter by direct frequency estimation," *Signal Processing*, vol. 27, pp. 161–176, 1992.

[34] G. Li, "A stable and efficient adaptive notch filter for direct frequency estimation," *IEEE Trans. Signal Processing*, vol. 45, pp. 2001–2009, August 1997.

[35] K. Zhou and J. C. Doyle, *Essentials Of Robust Control*. NY: Prentice-Hall, 1998.

[36] K. J. Åström and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*. New Jersey: Prentice-Hall, 1997.

[37] M. Tomizuka, "Zero-phase error tracking algortithm for digital control," *ASME J. Dyn. Syst., Meas., Control*, vol. 109, pp. 65–68, March 1987.