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An Analysis and Synthesis of Internal Model Principle Type Controllers

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Abstract— **This paper presents a method to synthesize internal model principle type controllers for rejection of sinusoidal disturbances. The frequencies of the disturbance are not required to be of rational ratios of the sampling interval or each other, thus addressing both periodic and aperiodic signals. The control synthesis utilizes positive feedback around notch filters to create the internal model and employs stable inversion of the plant dynamics to achieve closed loop stability. An example and simulation on a general electromechanical motion control system are presented to demonstrate the effectiveness of the proposed method.**

I. INTRODUCTION

Internal Model Principle (IMP) type controllers are based on the well known Internal Model Principle [1], which places the exogenous signal generating dynamics in the feedback path between the external input and the output to be regulated. By doing so, the controller achieves asymptotic convergence. The ubiquitous PID controller is an IMP type controller with an integral action internal model. Repetitive control [2] is another IMP type controller that deals with periodic exogenous signals and has been successfully applied to many areas, including hard disk drives [3], optical disk drives [4], non-circular metal cutting [5], industrial robots [6] and PWM inverters and rectifiers [7].

In a digital repetitive control system, a periodic signal generator $1/(1 - z^N)$ is included in the feedback loop to generate an infinitely large feedback gain at the periodic signal's fundamental frequency and its harmonics. Thus, periodic signals can be tracked or rejected asymptotically provided that the resulting closed loop system is stable. The high order, marginally stable internal model inserted in the feedback loop makes the task of designing a controller for closed loop stability challenging. In [8], a prototype discrete-time repetitive controller design was proposed using Zero-Phase Error Tracking Control (ZPETC) technique. A zero-phase low-pass filter $Q(z)$ was introduced to this structure [5], [9] to improve robust stability. The tradeoff between robustness stability and disturbance rejection performance in this formulation is simplified to the design of the Q filter.

The repetitive control based on the previously mentioned internal model has some limitations. Firstly, the internal model includes all harmonics of the periodic signal, i.e. all frequencies that are integer multiples of the fundamental frequency. In some cases the disturbance signal may be aperiodic, including multiple frequencies that are not commensurate with the Fourier harmonic frequencies. Secondly, the convergence rate of repetitive control largely depends on the internal model's delay length, which corresponds to the period length. Meaning when the signal period is long, the convergence transient is slow. Thirdly, the delay length N must be an integer. Although interpolation [10] and least square based approximation [11] have been used, the performance is still compromised by approximations.

In [12], a sinusoidal internal model was proposed to reject a sinusoidal disturbance. In [13], the internal model of a sinusoidal signal, a second order peak filter, was designed and plugged into a servo loop in parallel with an existing controller. The control design for multiple frequencies involve solving high order Diophantine equations and do not enjoy the simplicity of the prototype repetitive control design based on plant inversion. Other adaptive feed-forward control algorithms with sinusoidal regressors were proposed in [14] and shown to be equivalent to the IMP. These realizations of the parameter adaptation algorithms, based on least means squares (LMS) or recursive least squares (RLS) are more complex than the IMP-type feedback controllers.

In this paper, we construct internal models of sinusoidal signals whose frequencies are not required to be of rational ratios so that both periodic and aperiodic signals are addressed. The internal models are created using positive feedback loops wrapping around any number of cascaded notch filters, where the width and depth of each notch can be independently tuned. Using the internal model construction, we formulate the closed loop control design problem as one similar to the prototype repetitive control design, where the stable inversion of the plant model, such as the ZPETC, is applied to satisfy a simple and transparent stability condition. The transient and steady state performance, as well as robust stability of the proposed method are also analyzed. The proposed synthesis is then applied to a motion control example.

The remainder of this paper is organized as follows: Section 2 presents the problem formulation and design objective. Section 3, constructs the internal model using cascaded notch filters and positive feedback loops. Section 4 establishes the stability conditions and designs inversion based compensator, the Zero Phase Error Tracking Compensator, to achieve closed loop stability. Section 5 presents a motion control example and simulation results with the conclusions given in Section 6.

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II. Problem Formulation

Consider the feedback control system shown in Figure 1. where P represents the plant model and C_1 is a preexisting

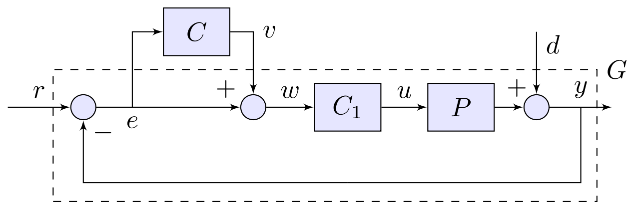

Fig. 1. The proposed internal model principle type controller C is plug-in to an existing feedback loop

stabilizing controller, which satisfies some predetermined performance requirements. Define

$$d(t) = \sum_{k=1}^{p} M_k \sin(\omega_k t + \phi_k) \tag{1}$$

where $d(t)$ is the disturbance signal characterized by multiple sinusoidal signals of known frequencies ω_k. In this paper, the additional controller C will be designed to reject these disturbances.

From Figure 1, the controller C will be designed based on the closed loop system G defined as

$$G(z^{-1}) = \frac{PC_1}{1+PC_1} = z^{-d}\frac{B(z^{-1})}{A(z^{-1})} \tag{2}$$

where d is represents a known delay, and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \tag{3}$$
$$B(z^{-1}) = b_0 + a_1 z^{-1} + \cdots + a_m z^{-m} \tag{4}$$

where $b_0 \neq 0$ and $n \geq m$.

The design objective is to find $C(z^{-1})$ which assures the asymptotic stability and asymptotic disturbance rejection,

$$\lim_{t\to\infty} e(t) = 0 \tag{5}$$

III. Synthesis of IMP Type Control

In order to reject the sinusoidal disturbances shown in (1). the feedback loop has to incorporate the disturbance model, using the Internal Model Principle (IMP) [1], [15]. The internal model can be implemented using a positive feedback loop wrapped around a digital filter $L(z^{-1})$ as shown in Figure 2.

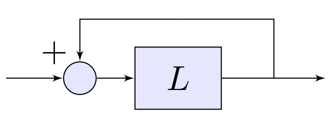

Fig. 2. Discrete internal model construction with positive feedback loop wrapping around a digital filter $L(z^{-1})$

If the frequency response of $L(z^{-1})$ satisfies

$$\begin{cases} L(e^{-j\omega}) = 1 & \texttt{if } \omega = \omega_k \\ L(e^{-j\omega}) \approx 0 & \texttt{if } \omega \neq \omega_k \end{cases} \tag{6}$$

the feedback loop will generate infinity gain when $\omega = \omega_k$. Placing this internal model (Figure 2) in a feedback control loop, it is possible to reject the sinusoidal disturbances at those frequencies while keeping the original system almost unchanged at the remaining frequencies.

In this paper, we will construct filter L through cascaded Notch Filters (NF). A cascaded NF can be described as

$$H = \prod_{k=1}^{p} \frac{1 - 2\beta_k \cos\omega_k z^{-1} + \beta_k^2 z^{-2}}{1 - 2\rho_k \cos\omega_k z^{-1} + \rho_k^2 z^{-2}} = \prod_{k=1}^{p} H_k \tag{7}$$

where p is the number of notches and the w_k's are the frequencies at which the notches are placed. ρ_k and β_k are *contraction factors* with $0 \ll \rho_k < \beta_k \leq 1$. When $\beta_k = 1$ is chosen, the output of the filter will be zero. Figure 3 shows

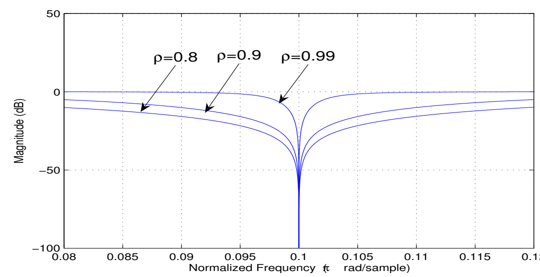

Fig. 3. Magnitude of frequency response of example H with different ρ with $\beta = 1$. BW rejection becomes narrower as $\rho \to 1$

the frequency response magnitude for one notch of $H(z^{-1})$ with different selections of ρ and fixed $\beta = 1$. Equation (7) approaches ideal notches as $\rho \to 1$. The bandwidth (-3dB) of notch filter is roughly determined by the contraction factor:

$$\texttt{BW}_k \approx \pi(1-\rho_k) \tag{8}$$

Now define

$$L(z^{-1}) = 1 - H(z^{-1}) \tag{9}$$

making the internal model $\widetilde{D}^0$ become

$$\widetilde{D}^0 = \frac{L}{1-L} = \frac{1-H}{H} \tag{10}$$

ensuring that $\widetilde{D}^0(e^{-j\omega})$ has infinite gain when $\omega = \omega_k$ and almost zero gain when $\omega \neq \omega_k$.

Given this internal model, a general form of internal model type controller can be described by

$$C = \widetilde{D}^0 F = \widetilde{D}^0 \frac{R}{S} \tag{11}$$

where F or R/S represents the rest of the controller not captured by $\widetilde{D}^0$. By using this controller C in the system described in Figure 1, we set the closed-loop characteristic equation K equal to zero to find

$$K = SA(1-L) + z^{-d}BLR = 0 \tag{12}$$

where (S, R) should be chosen so that K is asymptotically stable, i.e., the roots of K are inside the unit disk of z-plane. Equation (12) is the Diophantine Equation and the existence of solutions for S and R given K is guaranteed if $A(1-L)$ and $z^{-d}BL$ are coprime [16].

In general, the Diophantine equation (12) can be solved numerically by forming a Sylvester matrix [16] or through state space methods. However, the control design for multiple frequencies involve solving high order Diophantine equations. If we combine Equations (10) and (11), a sufficient stability condition for the closed-loop system shown in Figure 4 can be derived from Nyquist stability criterion or small gain theorem [17]

$$\|L(1-FG)\|_\infty < 1 \tag{13}$$

and the problem becomes H_∞ norm model matching problem, which has been discussed in detail [18].

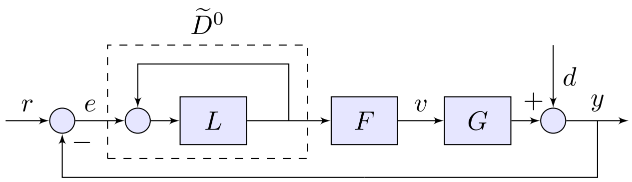

Fig. 4. IMP type control system for stability analysis

One simple way to solve the stable plant inversion problem in (13) is to apply the zero-phase-error-tracking feed-forward control (ZPETC) [19]. Given the stable plant shown in (2), factor B into

$$B = B^+B^- \tag{14}$$

where B^+, B^- represent stable and unstable zeros, respectively. The ZPETC causal compensator F_{ZPC} is defined as follows,

$$F_{ZPC} = \gamma z^{-(n_u+d)} \frac{A(z^{-1})B^-(z)}{B^+(z^{-1})} \tag{15}$$

where γ is a positive real design parameter, and n_u is the number of unstable zeros.

Combining internal model (10) and ZPETC compensator (15), the IMP type controller C is becomes:

$$C = \widetilde{D}^0 F_{ZPC} \tag{16}$$

The stability of closed-loop system will be shown in the following Lemma:

Lemma 1: Given any stable plant G, shown in (2), there exists a feedback controller C, as shown in (16), such that there always exists a small enough γ such that

$$\max_{\omega_k} |1-\gamma e^{-j(n_u+d)\omega_k} B^-(e^{-j\omega_k})B^-(e^{j\omega_k})| < \frac{1}{1+\delta} \tag{17}$$

where $\delta > 0$ is a small constant. The closed-loop system is always stabilizable given a small enough γ.

Proof: The denominator of closed-loop system can be rewritten as

$$1-(1-\gamma z^{-(n_u+d)}B^-(z^{-1})B^-(z))L \tag{18}$$

where H is a stable filter with $|L| \leq 1+\delta$, and $B^-(z^{-1})B^-(z)$ is also a stable filter. So, according to small gain theorem [17], if Equation (17) holds, the closed-loop system is stable. ■

Remark 1: For non-minimum phase systems, $n_u + d > 1$ and the phase delay $z^{-(n_u+d)}$ in (17) forces γ to be samll to ensure closed-loop system stability. In order to achieve larger stability range of γ, it is necessary for (10) to generate more phase lead steps.

An effortless extension is to define a new $\widetilde{D}^1$

$$\widetilde{D}^1 = \frac{Lz}{1-L} = \frac{(1-H)z}{H} \tag{19}$$

to include a phase lead, which still remains causal and allows for a larger stability range. To introduce even more phase lead, the structure of internal model can be modified by up sampling the notch filter. Define

$$\widetilde{H}^m = \prod_{k=1}^{p} \frac{1-2\beta_k\cos(m\omega_k)z^{-m}+\beta_k^2 z^{-2m}}{1-2\rho_k\cos(m\omega_k)z^{-m}+\rho_k^2 z^{-2m}} = \prod_{k=1}^{p} \widetilde{H}_k^m \tag{20}$$

where $m = n_u + d$, and the internal model can then be defined as

$$\widetilde{D}^m = \frac{(1-\widetilde{H}^m)z^m}{\widetilde{H}^m} \tag{21}$$

It can be seen that $\widetilde{D}^m$ is able to provide $n_u + d$ step look ahead to compensate the delay of F_{ZPC}. However, using this notch filter as shown in (20) will generate $m-1$ unwanted notches due to aliasing. Figure 5 shows an example of this

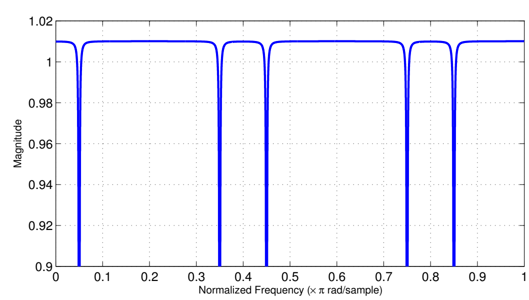

Fig. 5. Aliasing of Notch filter $\widetilde{H}^5$, which is supposed to have notch only at 0.05π. The notch is duplicated at $0.4\pi \pm 0.05\pi$ and $0.8\pi \pm 0.05\pi$

phenomenon. The position of notches can be calculated to be

$$\omega_{k,i} = \left| \left(\omega_k \pm \frac{2\pi i}{m}\right) \mod(2\pi) \right| \tag{22}$$

where $i \in \mathbb{Z}$, and $0 \leq i \leq \lfloor m/2 \rfloor$. To remove the $m-1$ unwanted notches, define the new filter L^m as

$$L^m = \widetilde{L}^1 \widetilde{L}^{m-1} \tag{23}$$

where $\widetilde{L}^m = 1 - \widetilde{H}^m$. As such, $\widetilde{L}^1$ masks the unwanted peaks of $\widetilde{L}^{m-1}$. Then H^m can be calculated to be

$$H^m = \widetilde{H}^1 + \widetilde{H}^{m-1} - \widetilde{H}^1\widetilde{H}^{m-1} \tag{24}$$

the feedback control is then

$$C = D^m F_{ZPC} \tag{25}$$

and the stability is addressed by the following Lemma:

Lemma 2: Given any stable plant G, which is shown in (2), there exists a feedback controller C, as shown in (25), such that if γ is small enough to satisfy the following condition

$$\max_{\omega_k} |1 - \gamma B^-(e^{-j\omega_k}) B^-(e^{j\omega_k})| < \frac{1}{1+\delta} \tag{26}$$

then, the closed-loop system is stable.

Proof: The proof is similar to Lemma 1 and thus omitted. ■

IV. Analysis of IMP Type Control

A. Transient performance

Consider the difference equation of second order IIR notch filter:

$$\begin{aligned} y(t) = \quad & x(t) - 2\beta_k \cos\omega_k x(t-1) + \beta^2 x(t-2) + \\ & 2\rho\cos\omega_k y(t-1) - \rho^2 y(t-2) \end{aligned} \tag{27}$$

Given initial state $y(-2)$ and $y(-1)$, the free response of notch filter converges as fast as ρ_k^t. A smaller ρ_k ensures a faster transient. For cascaded NF, the transient is determined by largest pole contraction factor, i.e., $\max_k\{\rho_k\}$.

B. Steady state performance

The steady state performance of proposed scheme is summarized in the following Theorem.

Theorem 1: Given stable plant shown in (2) and feedback controller C shown in (25), if Lemma 2 holds and $\beta_k = 1$, then the error signal due to disturbances (1) converges to zero asymptomatically.

Proof: The proof is due to IMP and hence omitted here. ■

C. Robust stability

Suppose plant with uncertainty $G_{\boldsymbol{\Delta}}$ is described by the following multiplicative form [17]

$$G_{\boldsymbol{\Delta}} = (1 + \Delta_r W_r) G \tag{28}$$

Where G is the nominal plant model; $W_r(z) \in \mathcal{RH}_\infty$ is uncertainty weighting function, which is specified to bound the uncertainties; $\Delta_r \in \mathcal{RH}_\infty$ is a variable transfer function with $\|\Delta_r\|_\infty \le 1$.

Assume that the nominal stability of closed-loop system is satisfied, i.e., the controller in (25) satisfies Lemma 2. Let $\epsilon > 0$ be a small constant, define

$$\delta_k^1 = |H^m(e^{-j\omega_k})| \tag{29}$$

$$\delta_k^2 = |H^m(e^{-j(\omega_k - \epsilon)})| \tag{30}$$

$$\delta_k^3 = \max_\omega |H^m(e^{-j\omega})| - 1 \tag{31}$$

From Figure 6, it is easy to verify that

$$\begin{cases} \delta_k^1 < |H^m(e^{-j\omega})| < \delta_k^2 & \texttt{if } |\omega - \omega_k| < \epsilon \\ \delta_k^2 \le |H^m(e^{-j\omega})| \le 1 + \delta_k^3 & \texttt{if } |\omega - \omega_k| \ge \epsilon \end{cases} \tag{32}$$

and we have

$$\begin{cases} 1 - \delta_k^2 < |1 - H^m(e^{-j\omega})| < 1 + \delta_k^2 & \texttt{if } |\omega - \omega_k| < \epsilon \\ 0 < |1 - H^m(e^{-j\omega})| < \delta_k^4 & \texttt{if } |\omega - \omega_k| \ge \epsilon \end{cases} \tag{33}$$

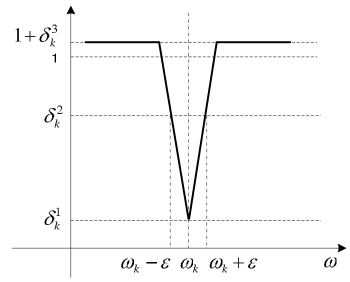

Fig. 6. Frequency Response of H^m

The robust stability of system with multiplicative uncertainty is shown as follows [17]

Lemma 3: Given $G_{\boldsymbol{\Delta}}$ shown in (28) and let C be a stabilizing controller for the nominal plant P, then the closed-loop is stable for all $\|\Delta_r\|_\infty \le 1$ if and only if $\|TW_r\|_\infty \le 1$, where $T = I - (I + PC)^{-1}$.

The robust stability of proposed approach is shown as follows,

Theorem 2: Consider $G_{\boldsymbol{\Delta}}$ shown in (28) and feedback controller C shown in (25), if Lemma 2 holds and one of the following conditions is satisfied

$$|W_r(e^{j\omega_k})| \le \frac{1 - \delta_k^2}{1 + \delta_k^2} \tag{34}$$

$$\frac{1 - \delta_k^2}{1 + \delta_k^2} < |W_r(e^{j\omega_k})| < \frac{\delta_k^1 + (1 - \delta_k^2)\gamma b(\omega_k)}{(1 + \delta_k^2)\gamma b(\omega_k)} \tag{35}$$

where $b(\omega_k) = B^-(e^{j\omega_k}) B^-(e^{-j\omega_k})$, then, the closed-loop system is stable for all $\|\Delta_r\|_\infty \le 1$

Proof: From Equation (2) and (25), the complementary sensitivity function of closed-loop system is

$$T = \frac{(1 - H^m)\gamma B^-(z) B^-(z^{-1})}{H^m + (1 - H^m)\gamma B^-(z) B^-(z^{-1})} \tag{36}$$

Substituting (32) and (33) into (36), we have

$$|T(e^{-j\omega})| < \frac{(1 + \delta_k^2)\gamma b(\omega_k)}{\delta_k^1 + (1 - \delta_k^2)\gamma b(\omega_k)}, \quad |\omega - \omega_k| < \epsilon \tag{37}$$

$$|T(e^{-j\omega})| < \frac{\delta_k^4 \gamma b(\omega_k)}{\delta_k^2 + \delta_k^4 \gamma b(\omega_k)}, \quad |\omega - \omega_k| \ge \epsilon \tag{38}$$

If $|\omega - \omega_k| \ge \epsilon$, it is reasonable to assume $\delta_k^4 \gamma b(\omega_k) \ll \delta_k^2$, when γ is small enough, i.e., $|T(e^{-j\omega})| \ll 1$.

$$\max_{|\omega - \omega_k| \ge \epsilon} |T(e^{-j\omega}) W_r(e^{-j\omega})| \ll 1 \tag{39}$$

If $|\omega - \omega_k| < \epsilon$ and Equation (37) holds,

$$\frac{1}{|W_r(e^{-j\omega})|} \ge \frac{1 + \delta_k^2}{1 - \delta_k^2} \ge \frac{(1 + \delta_k^2)\gamma b(\omega_k)}{\delta_k^1 + (1 - \delta_k^2)\gamma b(\omega_k)} > |T(e^{-j\omega})| \tag{40}$$

$$\max_{|\omega - \omega_k| < \epsilon} |T(e^{-j\omega}) W_r(e^{-j\omega})| < 1 \tag{41}$$

If $|\omega - \omega_k| < \epsilon$ and Equation (38) holds,

$$\frac{1}{|W_r(e^{-j\omega})|} > \frac{(1 + \delta_k^2)\gamma b(\omega_k)}{\delta_k^1 + (1 - \delta_k^2)\gamma b(\omega_k)} > |T(e^{-j\omega})| \tag{42}$$

$$\max_{|\omega-\omega_k|<\epsilon} |T(e^{-j\omega})W_r(e^{-j\omega})| < 1 \quad (43)$$

Combining Equation (39),(41) and (43), we have

$$\|TW_r\|_\infty < 1 \quad (44)$$

According to Lemma 3, the closed-loop is stable for all $\|\Delta_r\|_\infty \leq 1$. ■

Remark 2: In Equation (38), the condition will be valid only if $\delta_k^1 > 0$, which means $D^m(e^{-j\omega_k}) \neq 0$. So, the results in Theorem 2 suggest that for those frequencies where the unmodeled dynamics $|W_r| \geq 1$, the error signal can not be completely canceled. It reflects the trade-offs between robustness and performance in the proposed scheme.

Remark 3: For the non-minimum phase plant, the ZPETC compensator F_{ZPC} will generate high gain at high frequency range, which allows only for very small γ from Lemma 2 and Theorem 2. To remedy this, a zero-phase low-pass filter $M(z, z^{-1})$ can be introduced to suppress the high gain at high frequencies. This can be accomplished by replacing γ with $\gamma M(z, z^{-1})$, and let $m = n_u + d + n_m$, where n_m is the order of $M(z, z^{-1})$.

D. Relationship with prototype repetitive control

The prototype repetitive control based on ZPETC was presented in [8]. The internal model in prototype repetitive control is

$$H_r(z^{-1}) = 1 - Q(z, z^{-1})z^{-N} \quad (45)$$

where $Q(z, z^{-1})$ is zero-phase low-pass filter. Assume the order of $Q(z, z^{-1})$ is high enough so that it approaches an ideal low-pass filter, i.e.,

$$\begin{cases} Q(\omega) \approx 1 & \texttt{if } \omega \leq \omega_c \\ Q(\omega) \approx 0 & \texttt{if } \omega > \omega_c \end{cases} \quad (46)$$

where ω_c is the cut-off frequency of $Q(z, z^{-1})$. The internal model shown in Equation (45) can be factorized as follows,

$$H_r = (1 - z^{-1}) \prod_{k=1}^{s} \left(1 - 2\cos\left(\frac{2k\pi}{N}\right) z^{-1} + z^{-2}\right) \quad (47)$$

where

$$s = \arg\max_{1\leq k\leq N} \frac{2k\pi}{N} \leq \omega_c$$

This can be considered as a special case of H_s^m with $\beta_k = 1$, $\rho_k = 0$, and $\omega_k = 2k\pi/N$.

E. Real-time implementation

Figure 7 shows the real-time implementation of proposed controller C, where a positive feedback loop is used to construct internal model with better numerical properties, and $L_m = 1 - H_m$.

Note that in prototype repetitive control [8], L_m is replaced by the delay z^-m, where m is the signal period. The internal model for the repetitive control can also be obtained by up sampling an integrator to the signal's period length.

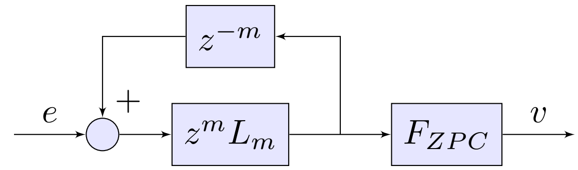

Fig. 7. Realtime implementation of C, the positive feedback loop provides m-step phase lead to make $F_{ZPC}G$ zero-phase

V. Application to Motion Control Problems

Consider a typical servo motor driven motion control system. Using the parameters of an existing AC brushless servo motor shown in [20], [18] and sampling time $T_s = 0.0005$ second, the zero-order hold discrete transfer function is shown as follows

$$\frac{B_p}{A_p} = \frac{5.276 \times 10^{-5}(z + 1.239)(z - 0.0886)(z + 0.0122)}{(z - 1.000)^2(z - 0.0316)(z - 0.00013)} \quad (48)$$

Before applying the proposed controller, a stabilizing feedback controller B_c/A_c is used:

$$\frac{B_c}{A_c} = \frac{2221.8818(z - 0.8051)}{z - 0.2802} \quad (49)$$

and the closed-loop transfer function becomes

$$\begin{aligned} G &= \frac{B}{A} = \frac{B_c B_p}{A_c A_p + B_c B_p} \\ &= \frac{0.11723(z+1.239)(z-0.8051)(z-0.08859)(z+0.01223)}{(z-0.5064)(z-0.05496)(z+0.006098)(z^2-1.639z+0.74)} \end{aligned} \quad (50)$$

which has one step delay and one unstable zero at -1.239, making $n_u = 1$ and $d = 1$.

This first example attempts to reject two sinusoidal disturbances at 60Hz and $60\sqrt{3}$Hz with unity amplitudes. Using Equation (7), (25), the controller is designed, where $\rho_k = 0.9$, $\beta_k = 1$, $m = n_u + d = 2$. According to Lemma 2, the closed-loop system is stable if (17) is satisfied. Here we choose $\gamma = 1.5$, $|1-\gamma B^-(e^{-j2\pi 60})B^-(e^{j2\pi 60})| = 0.4869 < 1$ and $|1 - \gamma B^-(e^{-j2\pi\sqrt{3}})B^-(e^{j2\pi 60\sqrt{3}})| = 0.4637 < 1$. Figure 8 shows the Bode plot of sensitivity function $S = 1/(1+CG)$. The total closed-loop sensitivity function will be $S_{tol} = S/(1 + C_1 P)$. The time domain simulation in Figure 9 shows that the tracking error converges at 0.03 seconds and the steady state error converges to zero asymptotically.

Another simulation compares the proposed method with the prototype repetitive control presented in [8]. In both methods, ZPETC compensator is used. The disturbance in this simulation comprises 50Hz, 100Hz and 150Hz sinusoids with unity amplitude. Figure 10 shows the time domain simulation comparison. In our proposed method, γ is chosen to be 1.5. In repetitive control, the delay N is chosen to be 40 since the sampling frequency is 2kHz, learning gain k_r to be 0.5. It is shown that the proposed method converges faster than the repetitive control. In repetitive control, the control signal for periodic disturbance is only updated every N steps. If the disturbance signal period is long, the convergence will be slow.

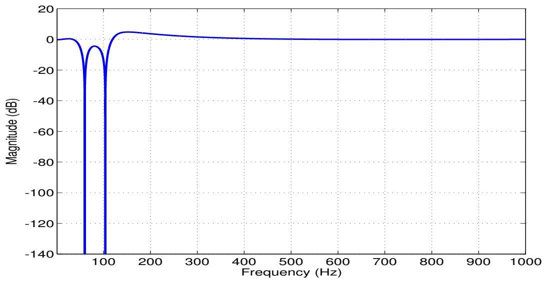

Fig. 8. Sensitivity function $S = 1/(1+CG)$, which rejects disturbances at 60Hz and $60\sqrt{3}$Hz and does not change the performance at other frequency range achieved by existing controller C_1. $\beta_k = 1$ and $\rho_k = 0.9$, where $k = 1, 2$

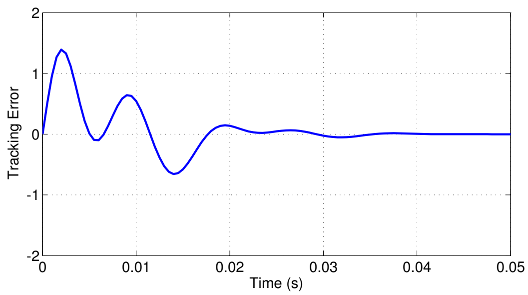

Fig. 9. Time domain simulation: the transient of tracking error signal. The disturbance signal consists of 60Hz and $60\sqrt{3}$ Hz sinusoids with amplitude 1

VI. Conclusion

The novelties presented in this paper for synthesizing internal model principle type controller include the following:

(1) The construction of the internal model utilizes a positive feedback loop of a "unity" filter L described in (6) that is complementary to regular notch filters;

(2) By this construction, the closed loop stability can be achieved by any stable inversion, or the simpler ZPETC, of the plant dynamics;

(3) The forward and feedback paths of the positive feedback loop formed by the internal model are arranged to account for plant delays and unstable zeros.

(4) Up-sampling of the internal model is proposed to allow compensation of larger number of delays and unstable zeros.

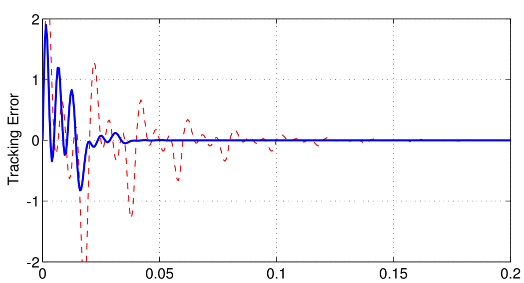

Fig. 10. Tracking error transient comparison: repetitive control (red dashed line) converges within 0.2 seconds and proposed method (blue solid line) converges within 0.02 seconds

Bearing some similarities to the approach to the design of the so called prototype repetitive control, the method discussed in this paper is more general than the repetitive control. In this design both aperiodic and periodic signals consisting multiple harmonics can be rejected. Transient convergence can be made faster than repetitive control when fewer harmonics are included in the internal model. These features are demonstrated by the simulation results.

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