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FIR System Identification using Higher-Order Statistics

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***Abstract*—In this paper, new approaches for the identification of FIR systems using HOS are proposed. The unknown model parameters are obtained using optimization algorithms. In fact, the proposed method consists first in defining an optimization problem and second in using an appropriate algorithm to resolve it. Moreover, we develop a new method for estimating the order of FIR Models using only the output cumulants. The results presented in this paper illustrate the performance of our methods and compare them with a range of existing approaches.**

I. INTRODUCTION

FINITE Impulse Response (FIR) models have found applications in many fields, such as signal processing and control, since they can fit any complex stable system [1]. Moreover, they do not require any structure to be selected, but they need only the model order to be identified.

For the identification of FIR systems, two main problems must be considered: one, identification of model order, and, two, identification of model parameters.

Considerable work has been done in the area of model parameters identification [2], [3], [4], which consist in using second order statistics. However, these statistics are phase blind and sensible to additive Gaussian noise. Thus, they are incapable to identify the nonminimum phase systems and their performances degrade when the output is noisy. To overcome these problems, other approaches were proposed and consist in using higher-order statistics (HOS) [2], [3], [4], which present several interesting properties such as: robustness to additive Gaussian noise, ability to preserve phase and detection of nonlinearities. Therefore, HOS based methods are very useful in dealing with non-Gaussian and nonminimum phase linear systems as well as non linear systems. The proposed methods for the identification of FIR system parameters using HOS can be classified in three categories of solutions: optimization based solutions [3], [4], closed-form solutions [5]–[8], and linear algebra solutions [5], [8]–[15]. Only the linear algebra solutions are considered in this paper, since they have been found to be more robust to measurement noise and they deliver estimated parameters with a lower variance.

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The linear algebra methods consist in constructing a system of equations which link the FIR model parameters to the output cumulants and in using the least squares approaches to solve the obtained system. But, most of the existing methods are characterized by a redundant unknown vector (i.e. whose elements are related to each other). Consequently, the obtained solution is suboptimal because the least squares approach assumes that the parameters are independent which they are not [3]. To overcome this problem, we have presented a solution which consists in solving the obtained system using least squares approaches under the constraint of dependency between the estimated parameters [16]. In this paper, we propose two methods which allow to avoid the problem of redundancy. They treat the obtained system as a nonlinear set of equations.

The linear algebra methods assume that the model order is either a priori known or arbitrarily chosen. However, successful identification of FIR model parameters requires exact knowledge of the model order. This is one of the areas of research in which several efforts have been devoted in the past [5], [6], [17]–[19]. A new approach is also proposed for determining the order of FIR model using only the output cumulants.

This paper is organized as follows. Section II presents the model and its assumptions. Section III recalls the principal relations linking HOS to FIR model parameters. Moreover, it presents in a unified way the main methods of FIR model identification using HOS. In section IV, we propose new solutions to overcome the problem of redundancy. A new method to identify a FIR model order is developed in section V. Results of simulations are illustrated in the last section.

II. MODEL AND ASSUMPTIONS

In the following, we address the problem of estimating the order q and the parameters of a discrete, causal, stationary, minimum or nonminimum phase FIR system described by:

$$x(n)=\sum_{i=0}^{q}h(i)\,e(n-i) \qquad (1)$$

where $\{e(n)\}$ is the input sequence, $\{h(i)\}$ are the impulse response coefficients, q is the order of the FIR system and $\{x(n)\}$ is the non observable output sequence.

The observed output process $\{y(n)\}$ is given by:

$$y(n)=x(n)+v(n) \qquad (2)$$

where $\{v(n)\}$ is the noise sequence.

Model (1) is assumed to be exponentially stable with $h(0)=1$.

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The following assumptions are assumed to be verified:

A1. The input sequence *{e(n)}* is an independent and identically distributed zero mean, non-Gaussian, stationary process.

A2. The additive noise *v(n)* is assumed to be an independent and identically distributed Gaussian process, independent of *e(n)* and with unknown variance.

III. FIR PARAMETER IDENTIFICATION METHODS

We present the main linear algebra methods using only HOS. This presentation allows, firstly, to compare the considered methods in terms of numerical complexity, and secondly, to show that the use of all available cumulants increases the redundancy in the parameters vector. However, the comparison between the performances of the different methods can be made through simulations, as well as by computing an asymptotic lower bound for the variance of the estimated parameters based on results presented in [20].

A. Basic relations

We describe the main general relationships between cumulants and impulse response coefficients. From these relations, we can unify most of the linear HOS based methods proposed in the literature.

Relation 1

Brillinger and Rosenblatt showed that the mth order cumulants of *{x(n)}* can be expressed as a function of impulse response coefficients h(i) as follows [21]:

$$C_{m,x}(\tau_1,\tau_2,\ldots,\tau_{m-1})=\gamma_{m,e}\sum_{i=s_1}^{s_2}h(i)\,h(i+\tau_1)\ldots h(i+\tau_{m-1}) \quad (3)$$

The methods presented in [6], [7] are derived from this relation**.**

Relation 2

This Relation was introduced for the first time in [22], and then used in [14]. It relates the mth and nth order cumulants to FIR system parameters:

$$\sum_{i=0}^{q}h(i)C_{m,x}(\tau_1-i,\ldots,\tau_{m-n+1}-i,\tau_{m-n+2},\ldots,\tau_{m-1})=\frac{\gamma_{m,e}}{\gamma_{n,e}} \quad (4)$$
$$\sum_{i=0}^{q}h(i)h(\tau_2-\tau_1+i)\ldots h(\tau_{m-n+1}-\tau_1+i)C_{n,x}(\tau_1-i,\tau_{m-n+2},\ldots,\tau_{m-1})$$

Relation (4) is the basis of the Giannakis and Mendel [10], Tugnait [8], [15], Alshebeili [5] and Srinivas & Hari, [13]–[14] methods.

Relation 3

Stogioglou and McLaughlin proposed a relation between different slices of the same order cumulants m [18]:

$$\sum_{i=0}^{q}h(i)\left[\prod_{k=1}^{r}h(i+\tau_k)\right]C_{m,x}(\beta_1,\ldots,\beta_r,i+\alpha_1,\ldots,i+\alpha_{m-r-1})=$$
$$\sum_{j=0}^{q}h(j)\left[\prod_{k=1}^{r}h(j+\beta_k)\right]C_{m,x}(\tau_1,\ldots,\tau_r,j+\alpha_1,\ldots,j+\alpha_{m-r-1})$$
$$\text{where } 1\le r\le m-2 \quad (5)$$

Relation (5) was used by Tugnait in [8] to modify the method of Giannakis & Mendel [10].

Relation 4

This relation is presented in [16], [23]. It links the mth and nth order cumulants of the FIR system output.

$$\sum_{i=0}^{q}h(i)\left[\prod_{k=1}^{m-s-1}h(i+\tau_k)\right]C_{n,x}(\beta_1,\ldots,\beta_{n-s-1},i+\alpha_1,\ldots,i+\alpha_s)= \quad (6)$$
$$\frac{\gamma_{n,e}}{\gamma_{m,e}}\sum_{j=0}^{q}h(j)\left[\prod_{k=1}^{n-s-1}h(j+\beta_k)\right]C_{m,x}(\tau_1,\ldots,\tau_{m-s-1},j+\alpha_1,\ldots,j+\alpha_s)$$

where *s* is an arbitrary integer number satisfying :

$1\le s\le min[m,n]-2$

The methods presented in [23] and [11] are derived from this relation**.**

B. The main parameters identification methods

The linear algebra methods use the following steps to identify the FIR model parameters:

- Constructing a system of equations having the following form:

$$A\theta=b \quad (7)$$

where *A, b*: respectively matrix and vector of cumulants
θ: the unknown parameters vector.

- Solving the obtained system using the least squares approach.

$$\theta=\left(A^TA\right)^{-1}A^T.b \quad (8)$$

- Extracting the FIR parameter

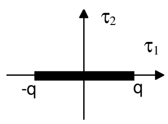
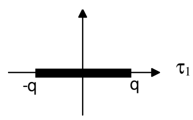
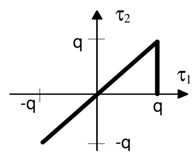
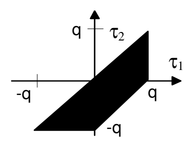
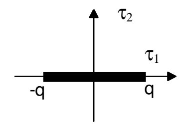
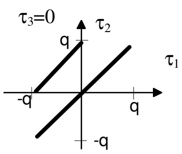
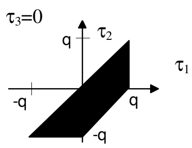
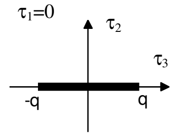
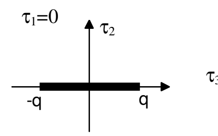
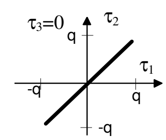
The best FIR parameters estimation methods use third and fourth orders cumulants only. They are presented in a unified way in table I. This unification contains the basic relations, the used HOS information, the effective estimated parameters and the dimension of the equations system.

According to table I, we can deduce the following:

- The unknown vector is redundant in all presented methods. Consequently, they treat the unknown parameters as independent in order to simplify the identification algorithm [3].
- Methods 1, 2 and 5 are based on the same basic relation and use the same information of the third order cumulants, but they only differ in the information of the fourth order cumulants. The increase of slice number of fourth order cumulants improves the estimation quality, but it introduces a redundancy in the unknown vector.
- Methods 3 and 4 are based on the same relation and use the same information of the fourth order cumulant, but they differ in the information of the third order cumulants. In fact, method 3 uses two slices. However, Method 4 uses all third order cumulants. The unknown vector is redundant for these two methods.
- Methods 2 and 4 can be considered among the best algorithms because they use the third and fourth order cumulants and not autocorrelation. Consequently, they give consistent estimates in the presence of colored Gaussian noise. Moreover, they exploit nearly all the available statistics information which is expected to improve the quality of estimation [3]. For this reason, only these methods are to be considered in from now on.

TABLE I

FIR PARAMETERS ESTIMATION METHODS BASED ON FOURTH AND THIRD ORDER CUMULANTS.

	Method 1	Method 2	Method 3	Method 4	Method 5
	[13]–[14]	[13]	[11]	[23]	[12]
General Rel.	relation 2 $\tau_3 = 0$ $m = 4$ & $n = 3$	relation 2 $\tau_3 = 0$ $m = 4$ & $n = 3$	relation 4 $\tau_1 = \tau_2 = 0$ $m = 4$ & $n = 3$	relation 4 $\tau_1 = \tau_2 = 0$ $m = 4$ & $n = 3$	relation 2 $\tau_3 = 0$ $m = 4$ & $n = 3$
Variations	• $\tau_2 = \tau_1$ $-q \le \tau_1 \le 2q$ • $\tau_2 = \tau_1 + q$ $-q \le \tau_1 \le q$	$-q \le \tau_1 \le 2q$ $\tau_1 \le \tau_2 \le r$ $r = \min[\tau_1 + q,\ 2q]$	• $\beta_1 = 0$ $-2q \le \alpha_1 \le q$ • $\beta_1 = q$ $-q \le \alpha_1 \le q$	$-q \le \beta_1 \le q$ $-q \le i + \alpha_1 \le q$ $-q \le \beta_1 - \alpha_1 - i \le q$ $0 \le i + \beta_1 \le q \quad i \in [0, q]$	$\tau_2 = \tau_1$ $-q \le \tau_1 \le 2q$
Estimated parameters	$\{h(i)\}_{i=1,\dots,q}, \varepsilon_{4,3}h(q)$ *and* $\{\varepsilon_{4,3}h^2(i)\}_{i=0,\dots,q}$	$\{h(i)\}_{i=1,\dots,q}$ *and* $\{\varepsilon_{4,3}h(i)h(j)\}_{0\le i\le j\le q}$	$\{h^3(i)\}_{i=1,\dots,q}, \varepsilon_{3,4}h(q)$ *and* $\{\varepsilon_{3,4}h^2(i)\}_{i=0,\dots,q}$	$\{h(i)h(j)\}_{1\le i\le j\le q}, \{h(i)\}_{i=1,\dots,q}$ *and* $\{\varepsilon_{4,3}h^3(i)\}_{i=0,\dots,q}$	$\{h(i)\}_{i=1,\dots,q}, \varepsilon_{4,3}$ and $\{\varepsilon_{4,3}h^2(i)\}_{i=0,\dots,q}$
$C_{3,x}(\tau_1,\tau_2)$					
$C_{4,x}(\tau_1,\tau_2,\tau_3)$					
Dim. A	$[5q+2, 2q+2]$	$[(5q^2+7q+2)/2,$ $(q^2+5q+2)/2]$	$[5q+2, 2q+2]$	$[(5q^2+7q+2)/2,$ $(q^2+5q+2)/2]$	$[3q+1, 2q+1]$

IV. PROPOSED SOLUTION FOR REDUNDANCY PROBLEM

The selected methods use the least squares approach to solve a system of equations characterized by a redundant vector of unknown parameters. Mathematically, this approach is not suitable since the obtained system is nonlinear and must be treated as an optimization problem. The objective of all optimization problems is to find a minimum or maximum objective function value. In fact, we must define the optimization problem firstly, and secondly use an appropriate algorithm to solve it.

A. *The considered optimization problems*

Three optimization problems can be considered in our case:

Problem 1

This problem is defined as follows:

$$P1: \begin{cases} \min\limits_{\theta} \|b - A\theta\|^2 \\ subject\ \ to\ \ g(\theta) = 0 \end{cases}$$

It consists in minimizing a least squares function which includes a constraint to take the interdependence of the estimated parameters into account.

These non linear equality constraints are inspired from the redundancy in the estimated vector.

Example

We illustrate the use of this problem in the case of method 2 (SRH) and method 4 (ABM) methods which are characterized by $\frac{q^2+3q}{2}$ non linear constraints.

For q=2, θ and $g(\theta)$ are given by:

$$\theta_{SRH} = [h(1)\quad h(2)\quad \varepsilon_{4,3}\quad \varepsilon_{4,3}h(1)\quad \varepsilon_{4,3}h(2)\quad \varepsilon_{4,3}h^2(1)\quad \varepsilon_{4,3}h(1)h(2)\quad \varepsilon_{4,3}h^2(2)]^T$$

$$\theta_{ABM} = [\varepsilon_{4,3}\quad \varepsilon_{4,3}h^3(1)\quad \varepsilon_{4,3}h^3(2)\quad h(1)\quad h(2)\quad h^2(1)\quad h(1)h(2)\quad h^2(2)]^T$$

$$g_{SRH}(\theta) = \begin{cases} \theta(1)\theta(3) - \theta(4) = 0 \\ \theta(2)\theta(3) - \theta(5) = 0 \\ \theta(1)\theta(4) - \theta(6) = 0 \\ \theta(1)\theta(5) - \theta(7) = 0 \\ \theta(2)\theta(5) - \theta(8) = 0 \end{cases} \text{ and } g_{ABM}(\theta) = \begin{cases} \theta(2) - \theta(1)\theta^3(4) = 0 \\ \theta(3) - \theta(1)\theta^3(5) = 0 \\ \theta(6) - \theta^2(4) = 0 \\ \theta(7) - \theta(4)\theta(5) = 0 \\ \theta(8) - \theta^2(5) = 0 \end{cases}$$

This approach improves the estimation performance, but still conserves the redundancy between the elements of the unknown vector.

Problem 2

The optimization problem P2 can be presented without non linear equality constraints:

$$P2: \begin{cases} \min\limits_{x} \|b - \varphi(x)\|^2 \\ where\ \ \varphi(x) = A\theta(x) \end{cases}$$

with $x = [\varepsilon_{4,3}\quad h(1)\quad h(2) \dots h(q)]^T$, $dim(x) = (q+1,1)$

Example

For SRH and ABM methods, the vector $\theta(x)$ is given by the following when q=2:

$$\theta_{SRH}(x)=[x(2) \quad x(3) \quad x(1) \quad x(1)x(2) \quad x(1)x(3) \quad x(1)x^2(2) \quad x(1)x(2)x(3) \quad x(1)x^2(3)]^T$$

$$\theta_{ABM}(x)=[x(1) \quad x(1)x^3(2) \quad x(1)x^3(3) \quad x(2) \quad x(3) \quad x^2(2) \quad x(2)x(3) \quad x^2(3)]^T$$

Problem P2 is a non-convex problem, therefore cannot guaranty that it's a global minimum. To overcome the local minimum problem we proposing a third optimization approach.

Problem 3

The problem 3 consists of taking P2 and framing the unknown parameters:

$$P3:\begin{cases} \min\limits_x \|b-\varphi(x)\|^2 \\ where\ \varphi(x)=A\theta(x) \\ subject\ \ to\ x_{min} \le x \le x_{\max} \end{cases}$$

We can use the results of an explicit solution to initialize $x_{\min}$ and $x_{\max}$.

B. The used algorithms

- Problems 1 and 3 define an optimization problem with constraints. These problems can be solved using the sequential quadratic programming (SQP) which is one of the best methods for solving nonlinear constrained problems particularly when a high degree of nonlinearity is present [24].
- Problem 2 is an unconstrained problem. Several optimization methods can be used to solve this problem such as the methods of Gauss-Newton, Newton-Raphson, quasi-Newton, gradient methods and Levenberg-Marquardt method. A comparative study of Gradient descent algorithm, Gauss–Newton algorithm and Newton–Raphson algorithm methods is presented in [25]. It can be mentioned that the quasi-Newton BFGS method is more often adopted to solve the unconstrained optimization problem because it presents good convergence performance [26]. Consequently, we propose the use of quasi-Newton BFGS method to solve problem P2.

V. Model Order Identification

The presented methods for the identification of FIR model parameters assume that the model order is priory known or selected by a higher-level 'wrapper' selection algorithm. However, the model order identification is an important problem toward the objective of system identification. In fact, this problem presents an area of research in which several efforts have been devoted in the past. In fact, several algorithms have been proposed in the literature for the identification of a model order which can be divided in two families: statistical test approach and parameters estimation approach. The statistical test approach consists in searching the smallest integer for which the third order cumulants are zero. The methods belonging to this family use only one slice of third order cumulants. On the other hand, the parameters estimation approachs require an identification of the parameter in order to determine the model order. Consequently, their computational complexity is very expensive. This section proposes a new method for the identification of FIR model order which belongs to the statistical test family. It uses all the available statistics information in order to improve the estimation quality.

A. The existing methods

We recall the main existing methods which are based on relation 1 for *n=3*:

$$C_{3,x}(\tau_1,\tau_2)=\sum_k h(k)h(k+\tau_1)h(k+\tau_2)$$

For a FIR model of order *q*, the third-order cumulant equals zero for either one or both of its arguments is greater or equal to $q+1$. Consequently, it is very easy to determine the order of the FIR model *q*, by testing in a statistical sense, the smallness of a third-order cumulant such as $C_{3,y}(q+1,0)$. This idea has been exploited in [17]. Two methods based on visual inspection and statistical tests are suggested. Obviously, the first method is impractical, while the second depends on statistics of a single random variable. In fact, the authors of [19] were suggested to compute the effective rank of the following cumulants matrix using the SVD which yields the effective order of the FIR model.

$$M_p=\begin{bmatrix} C_{3,y}(0,0) & C_{3,y}(0,1) & \cdots & C_{3,y}(0,p) \\ C_{3,y}(0,1) & \cdots & C_{3,y}(0,p) & 0 \\ \vdots & .\cdot^{\cdot} & \ddots & \vdots \\ C_{3,y}(0,p) & 0 & \cdots & 0 \end{bmatrix} where\ p>q$$

For a set of noisy data, the model order selected by this approach may be undetermined or overdetermined, depending on selection of the threshold. To improve the robustness of the order selection, they proposed a combination of the SVD and the product of diagonal entries (PODE) test.

B. The proposed approach

The above methods give correct results when cumulants are known. However, in practice, the cumulants are estimated from data. Consequently, the exact cumulants are unknown and they can not be estimated correctly. In fact, we propose another formulation of this method which uses more cumulants. This formulation is based on the following cumulants matrix:

$$M_p=\begin{bmatrix} C_{3,y}(0,0) & C_{3,y}(0,1) & \cdots & C_{3,y}(0,p) \\ C_{3,y}(1,0) & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ C_{3,y}(p,0) & 0 & \cdots & C_{3,y}(p,p) \end{bmatrix}$$

For a FIR model of order *q*, matrix $M_p\ (for\ p>q)$ has the following form:

$$M_p = \begin{bmatrix} C_{3,y}(0,0) & \cdots & C_{3,y}(0,q) & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ C_{3,y}(q,0) & \ldots & C_{3,y}(q,q) & \vdots & \ddots & \vdots \\ 0 & \cdots & \ldots & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & 0 & \ldots & 0 \end{bmatrix}$$

In fact, the order of a FIR model can be easily deduced using the index of the first null row (or column) in matrix M_p. But, this solution depends on the threshold selection [17], [19]. To overcome this problem and to improve the quality of estimation, we suggest the use of the following normalized confidence variable: $\overline{\lambda_{M(p)}}(k) = \frac{\lambda_{M(p)}(k)}{max(\lambda_{M(p)})}$ where

$$\lambda_{M_p}(k) = \sum_{l=1}^{k}\left(\sum_{t}\left(M_{p(t,l)}\right)^2\right),$$

$M_{p(t,l)}$ is t^{th} row and l^{th} column element of M_p.

It is easy to show that the normalized variable $\overline{\lambda_{M(p)}}(k)$ converge to one for $k \geq q+1$ if $p \geq q+1$.

Remark

The proposed method use only third order cumulants. Consequently, it yields consistent estimation when the output is contaminating by Gaussian colored noise. In addition, it presents the advantages of not depending on threshold selection as in the methods of [5] and [18].

VI. SIMULATION RESULTS

We now present simulation example to illustrate the performance of the proposed approaches for model order and parameters identification of FIR systems.

A. The simulated model

$$\begin{cases} x(k) = e(k) + 0.9e(k-1) - 2.2e(k-2) \\ + 1.25e(k-3) - 1.5e(k-4) + 0.98e(k-5) + 0.36e(k-6) \\ - 2.5e(k-7) + 0.7e(k-8) + 1.37e(k-9) + 0.9e(k-10) \\ zeros: -2.2600\ , 1.0581 \pm j\,0.2081, 0.6519 \pm j\,0.8378, \\ -0.2330 \pm j\,1.0667, -0.3746 \pm j\,0.4019, -0.8447 \\ v(k) = w(k) + 0.5w(k-1) - 0.25w(k-2) + 0.5w(k-3) \end{cases}$$

B. The simulation conditions

The simulations are performed under the following conditions:

- The input signal *e(n)* is zero mean, exponentially distributed and independent and identically distributed sequence with $\gamma_{2,e}=1$, $\gamma_{3,e}=2$, $\gamma_{4,e}=6$.
- The additive colored noise *v(n)* is simulated as the output of MA(*p*) model deriving by a Gaussian sequence *w(n)*.
- The Signal to Noise Ratio (SNR) is defined as:

$$SNR_{(dB)} = 10\log_{10}\left[\frac{E\{x^2(n)\}}{E\{v^2(n)\}}\right]$$

- The parameter were obtained from 50 Monte Carlo runs, where *N* data are used to estimate the third and the fourth order cumulants.

The mean (μ), the standard deviation (σ), the mean square error (ρ) and the normalized mean square error (NMSE) values are considered to study the performance of each method.

C. Model order estimation

In this section, we present simulation results illustrating the use of the proposed method for the identification of model order. For comparison, the methods presented in [17] and [19] were also implemented using the same data. We observe from the obtained results, presented in table II, that the SVD and the $|C_{3y}(q,0)|$ methods [17] and [19] give results having very high standard deviation (σ) very high. Consequently, the model order identified by these approaches may be undetermined or overdetermined, depending on the selection of the threshold. However, the proposed method yields the correct model order for the three considered examples (see also Figure 1).

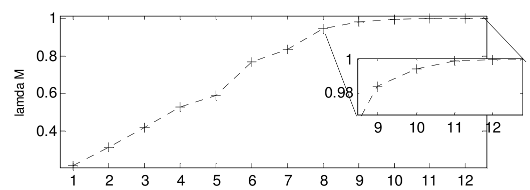


Fig 1. Model order identification, N=10000, SNR=10dB

D. FIR model identification

This section presents simulation studies allowing comparing the performance of the following approaches for the identification of the FIR model:

- The least squares approach;
- The sequential quadratic programming (SQP) to solve problems P1 and P3 [24];
- Quasi-Newton BFGS method to solve P2 [26].

Figure 2 shows the obtained results which lead to the following observations:

- The performance of all the methods improves when SNR increases.
- PR2 yields bad results in terms of standard deviations and mean square errors for model 1 which is a second model order. This is, however, expected since the Problem P2 is a non-convex problem and consequently cannot guaranty that it's a global minimum.
- The PR1 approach, generally, outperforms that of PR2.
- For all the values of the considered SNR, the mean, the standard deviation, the mean square error and the normalized mean square error (NMSE) values are much lower in the PR3 method than in the LSA, PR1 and PR2 methods.
- It can be noted from the above presented numerical results and the previous observations made that the performance of the proposed method (PR3) shows a significant improvement even in a system having a large order and an output which is contaminated by noise with higher variance.

TABLE II
MODEL ORDER IDENTIFICATION N=10000, 50 MONTE CARLO RUNS

q	SVD[19]		$\left\|C_{3y}(q,0)\right\|$ [17]		$\overline{\lambda_M}(k)$ $(k=q-1)$	
	μ	σ	μ	σ		
0	63.1813	6.9421	42.7459	5.1882	*0.2194*	*0.22*
1	41.3507	5.0155	23.1515	2.6258	*0.3138*	*0.31*
2	30.0840	3.4178	0.8538	2.3312	*0.4189*	*0.42*
3	28.6629	3.2768	12.5579	2.3196	*0.5259*	*0.53*
4	22.6964	2.9391	0.1165	1.9257	*0.5911*	*0.59*
5	19.9910	2.9765	11.7702	2.4743	*0.7702*	*0.77*
6	9.0428	1.9309	11.2979	2.5216	*0.8317*	*0.83*
7	3.9171	1.9572	12.5712	2.2344	*0.9419*	*0.94*
8	2.5943	1.2479	12.2311	1.8757	*0.9844*	*0.98*
9	1.2172	0.7955	3.8956	1.9588	*0.9945*	*0.99*
10	0.6906	0.5543	2.1114	1.6380	0.9999	1.00
11	0.3197	0.2908	0.1935	1.7740	0.9999	1.00
12	0.1675	0.1745	0.1348	1.8514	1.0000	1.00
13	0.1008	0.1351	0.2455	1.3443	1.0000	1.00

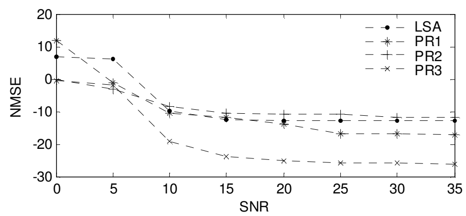


Fig 2. NMSE variation for ABM method, N=10000

VII. CONCLUSION

In this paper, we have addressed the problem of identification of FIR models using HOS. In fact, we have presented the main linear algebra methods. This presentation shows that the use of all available cumulants increases the redundancy in the parameters vector. To overcome this problem, we have suggested considering the obtained system as a set of nonlinear equations which can be solved using optimization algorithms. In fact, we have developed three optimization problems. The first one consists in minimizing a least squares function which includes a constraint to take the interdependence of estimated parameters into account. The second approach concerns the resolution of a nonlinear problem without constraints. The last one represents a nonlinear optimization problem with inequality constraints. Moreover, a new algorithm for identifying the model order for FIR model is presented. Numerical simulation results are presented to demonstrate the performance of the proposed approaches.

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