Control Strategies for a Distributed Mass Damper System

Tat S. Fu and Erik A. Johnson

Abstract-Recent developments of a distributed mass damper (DMD) system integrate structural and environmental control systems for buildings. External shading fins are used as mass dampers such that they can (i) control building energy consumption by adjusting the fins and, thus, the amount of sunlight coming into the building and (ii) control structural movements by dissipating energy with the dampers during strong motions due to wind or earthquakes. Shading fins are placed along the height of the building, distributing the mass along the building instead of being concentrated in a few locations like traditional tuned mass dampers (TMDs). This eliminates any large damper mass on the top of the building that can be a structural and architectural challenge to design. The DMD system is formulated, simulated and analyzed with passive, active and semiactive control strategies. The passive DMD is shown to be as effective in response mitigation as a conventional TMD; active and semiactive strategies give further improvements. The building energy consumption using the movable shading fins is also briefly presented in this paper.

I. INTRODUCTION

A new type of mass damper system is studied to integrate structural and environmental control systems for buildings. The main objective is to provide a means to improve buildings both structurally and environmentally with a Shading Fin Mass Damper (SFMD) system (Fig. 1). By examining the interdependence and exploiting the synergy of structural and environmental controls, it is possible to move closer to the goal of sustainable buildings. External shading fins affect the sunlight entering the building and, therefore, the internal temperature and lighting conditions. In the SFMD system, the fins are moveable (normal to the building face) and/or rotatable, allowing the fins positions to be adjusted for greater control and minimization of energy consumption. Further, the fins can act as tuned mass dampers (TMDs) that move and dissipate energy during strong structural motions such as those due to earthquakes and strong winds. The synergy of the integrated system comes not only from its individual functions but also its

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Fig. 1. A shear structure with a VSDD brace control system.

utilization. Structural control systems are used only during the infrequent recurrence of strong motions, whereas environmental control systems are in constant use to provide continuous comfort for building occupants. The proposed integrated system would perform environmental control most of the time and switch to structural control when needed, thus providing a synergistic dual-purpose system to improve building energy efficiency and enhance structural and life safety. This system follows other environmental/ structural synergy examples such as the Sendagaya INTES Building in Tokyo in which two ice thermal storage tanks serve both as thermal sinks (cooled at night when power costs are lower) and as hybrid mass dampers to control transverse and torsional motion [1].

Since shading fins are placed along the height of a building, the dampers are placed on each floor instead of concentrated in a few locations like traditional TMDs. This distributed mass damper (DMD) system is more difficult for engineers to design due to the very large number of individual dampers, but can be less disruptive for architectural design because there is no massive damper at the top of, or elsewhere in, the building. Massive dampers can disturb the continuation of floor plans and are problematic at the top of buildings where the space and views may be more valuable. Further, weight constrains how massive a damper can be placed at the top of the building. Since the DMD system distributes the weight throughout the building rather than concentrating it, a larger



Fig. 2. TMD, MTMD, and DMD systems.

total damper weight can be implemented with the DMD system. The greatest challenge in designing a DMD system is the large number of damper parameters (*i.e.*, mass, stiffness and damping coefficients for passive control, and controller for active/semiactive control); optimizing these parameters will provide an effective and robust structural control system.

II. BACKGROUND

Passive energy dissipation systems use some type of damping device to dissipate energy from a structure to reduce vibration induced by various natural hazards. Of all passive systems, mass dampers are one of the most utilized. A mass damper is a secondary mass, attached to a (usually much larger) primary mass, to affect the dynamic response of the primary mass. The tuned mass damper (TMD) was initially suggested by Frahm in 1909 [2] and later studied by many others to reduce vibration of the primary system by tuning the TMD stiffness and damping coefficients to specific natural frequencies of the primary system.

A TMD can only suppress the response of a primary system in a narrow frequency band and, therefore, is ineffective for excitation in other frequency ranges. Thus, multiple tuned mass damper (MTMD) systems have been introduced to increase robustness by tuning MTMDs to multiple and/or wider frequency bands, compensating for uncertainties in building natural frequencies and so forth (e.g., [3]-[6]). Many studies concentrate multiple dampers in one floor or use just a single degree-of-freedom model of the primary system (e.g., [7], [8]); others have had dampers on several floors of a multistory building (e.g., [9], [10]). In contrast, the proposed distributed mass damper (DMD) system herein has a shading fin damper on each and every floor so that energy and environmental control is effective. Figure 2 illustrates the difference between the single TMD, a single-floor MTMD and a DMD. In the SFMD system depicted in Fig. 1, each fin has multiple degrees-of-freedom and is connected to two consecutive floors. The DMD

system analyzed herein is a simpler model, as shown in the rightmost part of Fig. 2, where the mass (fin) is just connected to a single floor and has only a single degree-of-freedom.

Active mass driver (AMD) systems can improve the performance of TMDs by adding an active force element in the TMD. The AMD system was first proposed by Chang and Soong [11] by applying a control force between the structure and the mass damper. Herein, an active distributed mass damper (ADMD) will be studied and compared with the passive DMD.

Additionally, semiactive dampers are also considered for the DMD system. Semiactive control can effectively reduce vibration by altering the characteristics of the dampers depending on the responses of the structure. Controllable fluid dampers, such as magnetorheological (MR) fluid dampers, use less energy compared to fully active devices such as hydraulic actuators. Various studies (*e.g.*, [12]–[15]) showed the effectiveness of semiactive control for vibration reduction. A recent study by Scruggs [17] examines regenerative damping forces between multiple semiactive dampers distributed throughout a structure; this study offers great insights into semiactive control for multiple damper control systems similar to the DMD system.

III. STRUCTURAL MODEL

The equations of motions for an n-story structure with a DMD can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{1}\ddot{x}_{g} + \mathbf{f}$$
(1)

where mass matrix **M** and stiffness matrix **K** are given by

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & \cdots & 0 \\ -m_1^d & m_1^d & 0 & 0 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & -m_2^d & m_2^d & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ & & & & & & & \\ 0 & 0 & 0 & 0 & \cdots & -m_n^d & m_n^d \end{bmatrix}$$
(1a)
$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_1^d & -k_2 & 0 & 0 & \cdots & 0 \\ 0 & k_1^d & 0 & 0 & 0 & 0 \\ -k_2 & 0 & k_2 + k_3 & -k_2^d & -k_3 & 0 \\ 0 & 0 & 0 & k_2^d & 0 & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \dots & 0 & -k_{n-1} & 0 & k_n & -k_n^d \\ 0 & & 0 & 0 & 0 & 0 & k_n^d \end{bmatrix}$$
(1b)

C takes a form similar to **K**; x_g is the ground displacement, $\mathbf{x} = [x_1 \quad x_1^d \quad x_2 \quad x_2^d \quad \cdots \quad x_n \quad x_n^d]^T$; **f** is the external force vector of the system (*e.g.*, wind); and **1** is a column vector of ones. Here, m_i and m_i^d are the masses of the *i*-th floor and of the damper attached to the *i*-th floor, respectively; k_i and k_i^d are the stiffness coefficients of the *i*-th story and between the *i*-th floor and the *i*-th damper, respectively; and x_i and x_i^d are the *i*-th floor displacement relative to the ground and the *i*-th damper displacement relative to the *i*-th floor, respectively.

The state space representation of (1) is

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{1} & \mathbf{M}^{-1} \end{bmatrix} \begin{bmatrix} \ddot{x}_{g} \\ \mathbf{f} \end{bmatrix}$$
(2)

or $\dot{z} = Az + Bu$, where **u** is the system input vector. With only an earthquake load, **f** is zero and **u** depends only on the ground acceleration.

In the study described herein, a 20-story linear shearbuilding model is used (*i.e.*, n = 20), with identical floors of mass $m_i = 368.18$ Mg (810 kips), identical stories of stiffness $k_i = 561.77$ MN/m (38400 kips/ft), and a damping ratio of 5%. The fundamental frequency (first mode) of this is 0.476 Hz (a period of 2.10 sec).

Two different classes of earthquake excitations are used herein. First, a Kanai-Tajimi stochastic model of earthquake ground motion [18] is used, with which the passive and active response statistics can be computed in closed form by solving a Lyapunov equation that is much faster than time history response simulation. The Kanai-Tajimi excitation is a low-pass filtered Gaussian white noise, using a filter of the form

$$F(s) = \frac{2\zeta_g \omega_g s + \omega_g^2}{s^2 + 2\zeta_g \omega_g s + \omega_g^2}.$$
 (3)

The Kanai-Tajimi parameters herein are be $\omega_g = 17 \text{ s}^{-1}$ and $\zeta_g = 0.3$ to represent four historical ground motions (1940 El Centro, 1968 Hachinohe, 1995 Kobe and 1994 Northridge) [15]. The second class of earthquake excitations is a suite of several historical earthquakes (for brevity, only the results from the 1994 Northridge are reported herein).

IV. PASSIVE CONTROL

The DMD system is analyzed by simulating a 20-story linear shear-building model. 20 dampers, one per floor, result in 60 parameters over which to optimize: the mass and stiffness of each mass damper (relative to the story at which it is attached) and the damping ratios computed as if the mass dampers were each an isolated structure. The DMD parameters are chosen to minimize the sum of the mean square interstory drift response to the Kanai-Tajimi ground motion.

While finding the global minimum is not trivial, a pattern search [19]–[20] optimization method shows that (local) near-optimal DMD designs significantly reduce structural vibration (Fig. 3). The amount of reduction is similar to the performance of a conventional TMD system. The initial DMD system in Fig. 3 is chosen by studying the effect of single TMDs on each floor. The increasing damper mass ratio on higher floors is because the TMDs have been shown to be more effective when placed on higher floors. The near-optimal designs in Fig. 3 further increase the damper masses on higher floors at different rates; however, the variation in damper masses may be difficult to implement and/or build. Thus, several sub-optimal DMD system designs with fixed damper masses were also studied so as to be simpler for design without compromising structural benefits; Fig. 4 shows the results of one such optimization that is only marginally sub-optimal. Clearly, the dampers in the top two-thirds of the structure are tuned to frequencies in the vicinity of the natural frequency of the structure alone; the dampers in the first six stories, on the other hand, are tuned around the second and third frequencies of the structure alone.

V. ACTIVE CONTROL

Three active control strategies (Fig. 5) are studied here as a comparison with the passive DMD system:





Fig. 4. Sub-optimal DMD systems.



Fig. 5. Active (ACT), active mass driver (AMD), and active DMD (ADMD) systems

- active control (ACT): *n* actuators, one attached in each story of the structure; no mass dampers
- active mass driver (AMD): one actuator attached between the roof and the roof-mounted mass damper
- active distributed mass dampers (ADMD): *n* actuators, one attached between each floor and its corresponding mass damper

The linear quadratic regulator (LQR) is used to determine the appropriate control force for the active systems, minimizing the cost function

$$J = E[\mathbf{y}^{\mathrm{T}}\mathbf{Q}\mathbf{y} + \mathbf{u}_{\mathrm{c}}^{\mathrm{T}}\mathbf{R}\mathbf{u}_{\mathrm{c}}]$$
(3)

where y is the output vector (floor drifts, accelerations and damper displacements), \mathbf{u}_c is the vector of control forces, and Q and R are response and control weighting matrices.

Figure 6 shows the cost and benefit of the active systems (ACT, AMD and ADMD) along with the passive systems (TMD and DMD). The different active systems are obtained by varying the weighting matrices \mathbf{Q} and \mathbf{R} for each type of active control, whereas the passive systems are from varying damper stiffness, damping terms and/or masses (for the DMD system only). Only active strategies that use RMS control force less than 6% of the weight of the primary building are considered. (Allowing larger forces will make the ACT strategy the most capable, but would require significant structural modification to distribute such large forces into the structure.)





Among the control strategies presented in Fig. 6, the AMD and ADMD systems best reduce the structural motions. The AMD systems can best reduce the interstory rifts, whereas the ADMD systems can best reduce the absolute accelerations though generally at larger cost (larger damper displacements and control force) than the AMD systems.

VI. SEMIACTIVE CONTROL

In the SFMD system, the actuators are designed to protract/retract the mass dampers and rotate the fins simultaneously. By mechanically linking the rotations of the fins to the movements of the mass dampers, only one set of actuators is needed on each SFMD to control all the necessary motions. The advantage of such system is cost reduction and utilization of the actuators. The actuators are used frequently for shading and seldom for reducing vibration. The adjustment for shading is not as demanding as structural control since the sun moves gradually. Thus, small actuators are likely to be used for the SFMDs, supplemented with semiactive dampers for vibration control. Two semiactive strategies are studied herein: a conventional clipped-optimal control strategy, and one using gain scheduling.

Unlike the active control that can both inject energy into and dissipate energy from the system, semiactive dampers can only dissipate energy. A clipped-optimal strategy has been used to derive a suitable control scheme for semiactive DMD system [12]–[13]. In the clipped-optimal strategy, assuming f_d is desired control force determined from a standard optimal controller (LQR herein), the semiactive force can be expressed as

$$f_{\rm SA} = \begin{cases} f_{\rm d}, & f_{\rm d} \dot{x}_{\rm d} < 0 & (f_{\rm d} \text{ would dissipate energy}) \\ 0, & \text{otherwise} & (f_{\rm d} \text{ would inject energy}) \end{cases}$$
(4)

where \dot{x}_d is the velocity across the damper. Using this clipping strategy, the semiactive force can be caused to act on the system; often, low pass filters will also be applied to the control force to mimic the smoothing caused by internal dynamics of the dampers and their corresponding electronics.

Since semiactive dampers are only active for dissipative forces (or $f_d \dot{x}_d < 0$ in (4)), they are less effective (though more robust) compared to active dampers that exert forces continuously. In a multiple damper system like the DMD system, the effectiveness of the semiactive dampers can be improved via gain scheduling. Gain scheduling is one of the most popular approaches for nonlinear control designs [21]. The nonlinear system is first divided into different regions in which the system behaves (more) linearly; a linear controller is designed for each region so that the corresponding controller gives optimal performance in that region. By "scheduling" these controllers at appropriate points of the nonlinear system, the system can be controlled effectively.

Gain scheduling can be applied to the *n*-DOF semiactive DMD (SADMD) system in the following way. For any given time step, there are m active dampers and n-minactive dampers, depending on (4). With *n* semiactive dampers, there are 2^n different control gains since each semiactive damper can be active or inactive. At each time step, knowing the velocities across the dampers, each control gain can be categorized as feasible (all n forces are dissipative) or infeasible (one or more of the n dampers would exert a non-dissipative force). Since more dampers being active may have a larger impact on the system response, it will be assumed here that the SADMD system is most effective by maximizing m (the number of active dampers) in each time step. Thus, of all feasible control gains, the one with the largest number of active dampers is used; in the case that multiple control gains have the maximum number of active dampers, the gain with more dampers at higher floors is preferred because dampers in the higher floors are often seen to have a larger performance improvement.

The behavior of the SADMD system excited by the Kanai-Tajimi earthquake model is simulated with conventional clipped-optimal control and with gain scheduling. Since semiactive dampers require simulations for nonlinear behavior that typically require larger computing power than the Lyapunov solutions of the linear systems, a simplified 5DOF primary system [22] is studied here instead of the 20DOF primary system. Figure 7 shows a comparison of the responses of the uncontrolled, passive active and both semiactive strategies. The SADMD system with gain scheduling outperforms the conventional clippedoptimal SADMD system in reducing both interstory drifts and accelerations. In fact, the SADMD system also bests the active control (ADMD) system in drift reduction but not in accelerations. Although the improvement is small in this specific example, larger improvements are expected when there are large gaps between the active and semiactive control systems. A more thorough cost and benefit analysis is needed to assess the performance of the control systems (active, semiactive, and semiactive with gain scheduling) in terms of structural and damper responses.







Fig. 8. Energy comparison for static and moveable fins.

VII. ENVIRONMENTAL CONTROL

Using eQuest [23] — a building energy simulation program — a three-story office building model is used to analyze the effects of shading fins. The building model is 150 ft long and 90 ft wide, located in Los Angeles (hot climate), with windows only on the longer sides of the building facing East and West. Vertical shading fins are placed on the eastern and western windows, 5 ft apart across the entire length (150 ft) of the windows. There are no windows on the northern and southern faces of the building because (*i*) the sun shines on the northern façades of buildings only a very small fraction of the year in Los Angeles, and (*ii*) overhangs are typically more suitable than vertical fins for southern façades.

Stationary shading fins, perpendicular to the windows, are studied initially to observe the effects of fin length (1 ft, 2 ft, 3 ft, 5 ft); in warm climates like Los Angeles, longer fins are seen to be the most effective in reducing overall energy consumption because they block the dominant solar heat gain. Using the longer 5 ft fins, the effects of three fin orientations are studied: 45° North, perpendicular, and 45° South). The 45° North case allows sunlight entering the building from North while the southern 45° case allows sunlight from South. Simulations show that northern 45° fins outperform other fin orientations, primarily because northern sunlight only occurs in early morning and late afternoon when the temperatures are fairly cool, and the fins block midday sunlight when the temperatures are warm.

From the initial study on stationary fins, orientations have larger effects on the energy cost compared to different lengths. Actively rotating shading fins are then studied using eQuest. The fins are rotated to one of the three prefixed orientations (perpendicular, 45° North, 45° South) to affect the amount of sunlight entering the building. When heating loads are needed, the fins are rotated to the orientation that would allow the most sunlight depending on the current sun path; similarly, the fins are rotated to block sunlight when the building is too warm. The result, graphed in Fig. 8, shows improvement in energy consumption using the actively rotating fins compared to stationary fins at different orientations.

VIII. CONCLUSION

This paper proposes a synergy system between structural and environmental controls through integrating shading fins and mass dampers. The resulting distributed mass damper (DMD) system can significant reduce structural motions passively and actively when subject to earthquake excitation. Using a pattern search method, the passive DMD system is optimized on the damper parameters to reduce interstory drifts. The movable shading fins require actuators that can also be used for active controls; thus, it is also demonstrated that designing the control force for the active DMD system using an LQR methodology can considerably reduce structural motions. In addition to active control, since the actuators required for moving the shading fins may be small and too weak for full active DMD control, semiactive control schemes are investigated for the DMD system. Using gain scheduling, the authors improve the performance of semiactive dampers on the DMD system.

This same synergy system has also been simulated for building energy profiles. The movable shading fins adjust the amount of sunlight exposure on the building and thus affecting the cooling, heating and lighting loads. Simulations of a 3 story office building with movable fins show saving of 18.5% in electricity and about 20% cost in energy consumption. The joint benefit structurally and environmentally synergizes the shading fin mass damper system by addressing two distinct building concerns with an integrated solution.

Further research is ongoing to optimize the parameters of the active and semiactive DMD system for performance and robustness. The adaptability of the DMD system to different types of excitations will also be addressed. Cost and benefit analysis of the SFMD system will be studied accounting both the structural and environmental effect during the life cycle of the building. Additionally, the more complicated attachments of multi-degree-of-freedom shading fins, as shown in Fig. 1, will be studied. Finally, investigations are needed to study the stability and performance of the gain scheduled semiactive DMD.

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