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Stability of Uncertain Piecewise Affine Systems with Time-Delay

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***Abstract*— This paper addresses the problem of robust stability of piecewise affine (PWA) uncertain systems with unknown time-varying delay in the state. It is assumed that the uncertainty is norm-bounded and that upper bounds on the state delay and its rate of change are available. A set of linear matrix inequalities (LMI) is derived providing sufficient conditions for the stability of the system. These conditions depend on the upper bound of the delay. The main contributions of the paper are as follows. First, new delay-dependent LMI conditions are derived for the stability of PWA time-delay systems. Second, the stability conditions are extended to the case of uncertain PWA time-delay systems. Numerical examples are presented to show the effectiveness of the approach.**

I. INTRODUCTION

Continuous-time piecewise affine (PWA) systems have attracted considerable interest in the control literature in recent years [1], [2], [3], [4], [5], [6], [7], [8]. The theory of PWA systems has found important applications in CPU processing control [9], boost DC-DC converters [10] and aerospace [11], to name only a few. In brief, a PWA system consists of a set of affine subsystems (representing different operating conditions of a system, or an approximation of a complex nonlinear system) and a switching law that enables switching between different subsystems. It is to be noted that switching is also used in control to stabilize and regulate highly uncertain systems [12], [13], [14], [15].

Many practical systems, on the other hand, are subject to input and/or state delay. Examples of time-delay systems include power systems [16] and communication networks [17]. It is known that time-delay can cause poor performance or even instability if its effect is neglected in control design. The existing results for robust stability of time-delay systems can be categorized as delay independent and delay dependent results. Different delay independent robust stability criteria have been developed in [18] and [19]. Delay independent stability results are conservative in general because they do not take into account any available information on the delay. Delay dependent approaches for the systems subject to parameter uncertainty, on the other hand, are investigated in [20], [21], [22], [23], [24]. Stability analysis for switched systems with time-delay is provided in [25], [26], [27]. In [25], a common Lyapunov functional is used for robust stability analysis of switched uncertain time-delay systems with arbitrary switching. However, stability analysis using a common quadratic Lyapunov function is typically known to be conservative. In [27], sufficient conditions for exponential stability of linear time-delay systems with a class of switching signals is developed. To the best of the knowledge of the authors, however, the stability problem for PWA time-delay systems has only been addressed in [2], where a piecewise quadratic Lyapunov function is used to derive LMIs for stability analysis following the approach of [1]. Nevertheless, the important and practically relevant case of robust stability of PWA time-delay systems in presence of parametric uncertainty has not been considered in [2]. Furthermore, the affine term of the dynamics did not have a delay in that paper.

Based on the considerations of the previous paragraph, PWA uncertain systems with unknown time-delay are investigated in this paper, and LMI-based conditions for asymptotic stability are derived following the approach of [8]. It is assumed that the parameter uncertainties are norm bounded and that upper bounds on the time-varying delay and its rate of change are given. In order to reduce the conservatism of the results, piecewise quadratic Lyapunov functions are employed for stability analysis. The main contributions of this work are as follows. First, new delay dependent LMI conditions are derived for the stability of PWA time-delay systems. Second, the stability conditions are extended to the case of uncertain PWA time-delay systems.

This paper is organized as follows. The problem statement and formulation are given in Section II. The main result of the paper is provided in Section III, followed by robustness analysis in Section IV. Simulation results are presented in Section V. Finally, some concluding remarks are drawn in Section VI.

II. PROBLEM FORMULATION

Consider an uncertain piecewise affine system with time-delay described as

$$\begin{aligned}\dot{x}(t) = (A_i+\Delta A_i)x(t) + (A_{di}+\Delta A_{di})x(t-\tau(t)) + (a_i+\Delta a_i)\\ + (b_i+\Delta b_i)\ 1(t-\tau(t)), \quad x(t)\in X_i\end{aligned} \tag{1}$$

where $A_i, A_{di} \in R^{n\times n}$, $a_i,\ b_i \in R^n$, and $\{X_i\} \subseteq R^n$ form a partition of the state space into a number of open (possibly unbounded) polyhedral cells with pairwise empty intersection. The index set of the cells is denoted by $I=\{1,\ldots,M\}$. The set of cells that include the origin is denoted by $I_0 \subseteq I$, and its complement is represented by $I_1 = I/I_0$. It is assumed that $a_i = 0,\ \Delta a_i = 0,\ b_i = 0,\ \Delta b_i = 0$ for $i \in I_0$. In addition, $\Delta A_i,\ \Delta A_{di},\ \Delta a_i$ and Δb_i are norm-bounded uncertainties which will be defined later. Furthermore, $1(t)$ is the step function. In (1), $\tau(t)$ is a positive time-varying delay such that

$$0 \le \tau(t) \le h, \quad \dot{\tau}(t) \le d < 1 \tag{2}$$

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where h and d are known positive constants.

Assume the initial condition

$$x(\theta)=\phi(\theta), \qquad \theta\in[-h,\ 0]$$

for the system (1) such that $\phi(\theta)$ is a differentiable vector-valued initial function on $[-h,0],\ h>0$. Assume also $x(t)$ is a continuous piecewise C^1 function of time. Following [1], [8], the state space is partitioned based on $x(t)$ such that $x(t)\in\bigcup X_i$ as follows. Let $\bar{E}_i=\begin{bmatrix} E_i & e_i\end{bmatrix}$, (with $e_i=0$, $\forall i\in I_0$) such that

$$\bar{E}_i\begin{bmatrix} x(t)\\ 1\end{bmatrix}\geq 0 \qquad \forall x(t)\in X_i, i\in I \tag{3}$$

Let $\mathscr{N}_i$ denote the set of neighbouring cells that share a common facet with the cell X_i. The facet boundary between the cells X_i and X_k is contained in the set $\{x\in R^n \mid c_{ik}^T x(t)-d_{ik}=0\}$, where $c_{ik}\in R^n$, $d_{ik}\in R$, for all $i\in I$, $k\in\mathscr{N}_i$. Moreover, we use a parametric description of the boundaries as follows

$$\bar{X}_i\cap\bar{X}_k\subseteq\{l_{ik}+F_{ik}s|s\in R^{n-1}\} \tag{4}$$

for all $i\in I$, $k\in\mathscr{N}_i$, where $F_{ik}\in R^{n\times(n-1)}$ is a full rank matrix whose columns span the null space of c_{ik}^T, and $l_{ik}\in R^n$ is given by $l_{ik}=c_{ik}(c_{ik}^Tc_{ik})^{-1}d_{ik}$.

The main objective of this paper is to determine a set of computationally tractable conditions under which (1) is asymptotically stable. In the next section, a Lyapunov functional will be introduced to determine the stability of PWA systems.

III. Nominal Analysis

In this section, sufficient LMI conditions will be established for the stability of (1) without uncertainties. These conditions will then be extended to the systems with uncertainties in Section IV. To proceed further, we define the following matrices and sets

$$\bar{A}_i:=\begin{bmatrix} A_i & a_i\\ 0 & 0\end{bmatrix},\quad \bar{A}_{di}:=\begin{bmatrix} A_{di} & 0\\ 0 & 0\end{bmatrix},\quad \bar{b}_i:=\begin{bmatrix} b_i\\ 0\end{bmatrix}$$

$$\bar{\mathscr{A}}=\left\{\begin{bmatrix} A_j & a_j\\ 0 & 0\end{bmatrix},\ \forall j\in I\right\},\ \mathscr{A}=\{A_j,\ \forall j\in I\},\ \mathscr{E}=\{a_j,\ \forall j\in I_1\}$$

$$\bar{\mathscr{B}}=\left\{\begin{bmatrix} b_j\\ 0\end{bmatrix},\ \forall j\in I_1\right\},\qquad \mathscr{B}=\{b_j,\ \forall j\in I_1\}$$

$$\bar{\mathscr{A}_d}=\left\{\begin{bmatrix} A_{dj} & 0\\ 0 & 0\end{bmatrix},\ \forall j\in I\right\},\qquad \mathscr{A}_d=\{A_{dj},\ \forall j\in I\}$$

Note that system (1) without uncertainties can be rewritten as follows

$$\dot{\bar{x}}=\bar{A}_i\bar{x}(t)+\bar{A}_{di}\bar{x}(t-\tau(t))+\bar{b}_i\ 1(t-\tau(t)) \tag{5}$$

where $\bar{x}(t)=[x^T(t),\ 1]^T$, with $x(t)\in X_i$. We use the expression

$$\bar{x}(t-\tau(t))=\bar{x}(t)-\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds \tag{6}$$

Hence, considering (5), the equation (6) can be rewritten as

$$\begin{aligned}\dot{\bar{x}}(t)=&(\bar{A}_i+\bar{A}_{di})\bar{x}(t)+\bar{b}_i\ 1(t-\tau(t))-\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{j(s)}\bar{x}(s)\,ds\\ &-\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{dj(s)}\bar{x}(s-\tau(s))\,ds\\ &-\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{b}_{j(s)}\ 1(s-\tau(s))\,ds\end{aligned} \tag{7}$$

Note that $j(s)$ in (7) is a piecewise constant function which represents the index of the matrices $\bar{A}_{j(s)}\in\bar{\mathscr{A}}$, $\bar{b}_{j(s)}\in\bar{\mathscr{B}}$, $\bar{A}_{dj(s)}\in\bar{\mathscr{A}_d}$ at time s. In order to proceed further, the following well-known lemma is borrowed from [28].

Lemma 1: For any vectors or matrices z and y with appropriate dimensions and any symmetric matrix $P>0$, the following inequalities are satisfied:

$$\begin{aligned}-z^Ty-y^Tz\leq z^TPz+y^TP^{-1}y\\ z^Ty+y^Tz\leq z^TPz+y^TP^{-1}y\end{aligned} \tag{8}$$

Proof: See [28].

The following theorem presents sufficient conditions for the stability of the PWA system (5).

Theorem 1: Consider the symmetric matrices $\bar{U}_i, U_i$ and $\bar{W}_i, W_i$, which are composed of non-negative entries, and

$$\begin{bmatrix} H_i' & hP_iA_{di} & S_{31}+[0_{n\times n}\quad R_3+S_3]\\ * & -hM_{1i} & 0\\ * & * & \bar{S}_2-(1-d)\bar{R}+\begin{bmatrix}0_{n\times n} & 0_{n\times 1}\\ 0_{1\times n} & S_2+R_2\end{bmatrix}+\Pi\end{bmatrix}<0 \tag{9}$$

$$\Pi=\begin{bmatrix}0_{n\times(n+1)}\\ S_{32}\end{bmatrix}+\begin{bmatrix}0_{(n+1)\times n} & S_{32}^T\end{bmatrix}$$

$$\begin{bmatrix} hQ_i & \begin{bmatrix}hP_iA_{di}A_j & hP_iA_{di}a_j\end{bmatrix} & \begin{bmatrix}hP_iA_{di}A_{dj} & 0\end{bmatrix}\\ * & \bar{S}_1-\begin{bmatrix}0 & 0\\ 0 & hb_j^TM_{1i}b_j\end{bmatrix} & \bar{S}_3\\ * & \bar{S}_3^T & \bar{S}_2\end{bmatrix}\geq 0 \tag{10}$$

$$P_i-E_i^TU_iE_i>0, \qquad M_{1i}>0 \tag{11}$$

where

$$H_i'=P_i(A_i+A_{di})+(A_{di}+A_i)^TP_i+S_1+R_1+hQ_i+E_i^TU_iE_i \tag{12}$$

for any fixed $i\in I_0$ and for all $A_j\in\mathscr{A}$, $b_j\in\mathscr{B}$, $a_j\in\mathscr{E}$, $A_{dj}\in\mathscr{A}_d$, such that

$$\bar{S}_1=\begin{bmatrix}S_1 & S_3\\ S_3^T & S_2\end{bmatrix},\quad \bar{S}_3=\begin{bmatrix}S_{31}\\ S_{32}\end{bmatrix},\ S_1\in R^{n\times n},\ S_2\in R$$

$$S_3\in R^{n\times 1},\ S_{31}\in R^{n\times(n+1)},\ S_{32}\in R^{1\times(n+1)}$$

$$\bar{R}=\begin{bmatrix}R_1 & R_3\\ R_3^T & R_2\end{bmatrix},\ R_1\in R^{n\times n},\ R_2\in R,\ R_3\in R^{n\times 1}$$

satisfying

$$\begin{bmatrix}\bar{S}_1 & \bar{S}_3\\ \bar{S}_3^T & \bar{S}_2\end{bmatrix}>0, \qquad \bar{R}>0 \tag{13}$$

for $\bar{S}_1$, $\bar{S}_2$ and $\bar{S}_3$. Furthermore, let the following inequalities hold

$$\begin{bmatrix} \bar{H}_i' & \bar{P}_i & h\bar{P}_i\bar{A}_{di} & \bar{S}_3 \\ * & -\bar{M}_{1i} & 0 & 0 \\ * & * & -h\bar{M}_{2i} & 0 \\ * & * & * & \bar{S}_2-(1-d)\bar{R} \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} h\bar{Q}_i & h\bar{P}_i\bar{A}_{di}\bar{A}_j & h\bar{P}_i\bar{A}_{di}\bar{A}_{dj} \\ * & \bar{S}_1 - \begin{bmatrix} 0 & 0 \\ 0 & h\bar{b}_j^T\bar{M}_{2i}\bar{b}_j \end{bmatrix} & \bar{S}_3 \\ * & * & \bar{S}_2 \end{bmatrix} \geq 0 \tag{15}$$

$$\bar{P}_i - \bar{E}_i^T\bar{W}_i\bar{E}_i > 0, \qquad \bar{M}_{ki} > 0,\ k = 1,2 \tag{16}$$

for any fixed $i \in I_1$ and for all $\bar{A}_j \in \bar{\mathscr{A}}$, $\bar{b}_j \in \bar{\mathscr{B}}$, $\bar{A}_{dj} \in \bar{\mathscr{A}}_d$, where

$$\bar{H}_i' := \bar{P}_i(\bar{A}_i+\bar{A}_{di}) + (\bar{A}_{di}+\bar{A}_i)^T\bar{P}_i + \bar{S}_1 + \bar{R} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{b}_i^T\bar{M}_{1i}\bar{b}_i \end{bmatrix} + h\bar{Q}_i + \bar{E}_i^T\bar{U}_i\bar{E}_i \tag{17}$$

Assume also that for all $i \in I$ and $k \in \mathscr{N}_i$,

$$F_{ik}^T(P_i-P_k)F_{ik} = 0 \tag{18a}$$

$$F_{ik}^T(P_i-P_k)l_{ik} + F_{ik}^T(q_i-q_k) = 0 \tag{18b}$$

$$l_{ik}^T(P_i-P_k)l_{ik} + 2(q_i-q_k)^Tl_{ik} + (r_i-r_k) = 0 \tag{18c}$$

where $\bar{P}_i := \begin{bmatrix} P_i & q_i \\ q_i^T & r_i \end{bmatrix}$, for all $i \in I$. Under conditions (2), (3) and (9)-(18), every piecewise C^1 trajectory $x(t)$, governed by (5) for $t \geq 0$, tends to zero asymptotically in the absence of sliding modes.

Proof: Define the candidate Lyapunov-Krasovsky functional

$$\bar{V}_i = \bar{V}_{1i} + \bar{V}_{2i} + \bar{V}_{3i} \tag{19}$$

where, for $x(t) \in X_i$, $i \in I_1$

$$\bar{V}_{1i} = \bar{x}^T(t)\bar{P}_i\bar{x}(t) \tag{20a}$$

$$\bar{V}_{2i} = \int_{t-\tau(t)}^{t} \bar{x}^T(s)\bar{R}\bar{x}(s)ds \tag{20b}$$

$$\bar{V}_{3i} = h^{-1}\int_{-h}^{0}\int_{t+s}^{t} \begin{bmatrix} \bar{x}(\theta) \\ \bar{x}(\theta-\tau(\theta)) \end{bmatrix}^T \begin{bmatrix} \bar{S}_1 & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2 \end{bmatrix} \begin{bmatrix} \bar{x}(\theta) \\ \bar{x}(\theta-\tau(\theta)) \end{bmatrix} d\theta ds \tag{20c}$$

The conditions that guarantee the continuity of the Lyapunov function at the boundaries are give in (18a-c), and can be obtained using the same approach as the one in [8]. Note that the candidate Lyapunov functional is positive definite because of (16) and (13). Applying Leibnitz integral rule and using (2), the derivative of this Lyapunov functional is

$$\begin{aligned}\dot{\bar{V}}_i \leq\ & 2\bar{x}(t)^T\bar{P}_i\dot{\bar{x}}(t) + \bar{x}^T(t)\bar{R}\bar{x}(t) - \bar{x}^T(t-\tau(t))(1-d)\bar{R}\bar{x}(t-\tau(t)) \\ & + \bar{x}^T(t)\bar{S}_1\bar{x}(t) + \bar{x}^T(t-\tau(t))\bar{S}_2\bar{x}(t-\tau(t)) \\ & + 2\bar{x}^T(t)\bar{S}_3\bar{x}(t-\tau(t)) \\ & - h^{-1}\int_{t-\tau(t)}^{t} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix}^T \begin{bmatrix} \bar{S}_1 & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2 \end{bmatrix} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix} ds \end{aligned} \tag{21}$$

Substituting (7) in (21) leads to

$$\begin{aligned}\dot{\bar{V}}_i \leq\ & 2\bar{x}^T(t)\bar{P}_i(\bar{A}_i+\bar{A}_{di})\bar{x}(t) + 2\bar{x}^T(t)\bar{P}_i\bar{b}_i\ 1(t-\tau(t)) \\ & + \bar{x}^T(t)(\bar{S}_1+\bar{R})\bar{x}(t) + 2\bar{x}^T(t)\bar{S}_3\bar{x}(t-\tau(t)) \\ & + \bar{x}^T(t-\tau(t))(\bar{S}_2-(1-d)\bar{R})\bar{x}(t-\tau(t)) \\ & - 2\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{j(s)}\bar{x}(s)\,ds \\ & - 2\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{dj(s)}\bar{x}(s-\tau(s))\,ds \\ & - 2\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{b}_{j(s)}\ 1(s-\tau(s))\,ds \\ & - h^{-1}\int_{t-\tau(t)}^{t} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix}^T \begin{bmatrix} \bar{S}_1 & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2 \end{bmatrix} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix} ds \end{aligned} \tag{22}$$

Now, considering the positive-definite matrices $\bar{M}_{ki}$, $k = 1,2$, $i \in I_1$, using Lemma 1 and the inequalities (2), (22) yield

$$\begin{aligned}\dot{\bar{V}}_i \leq\ & 2\bar{x}^T(t)\bar{P}_i(\bar{A}_i+\bar{A}_{di})\bar{x}(t) + \bar{x}^T(t)\bar{P}_i\bar{M}_{1i}^{-1}\bar{P}_i\bar{x}(t) \\ & + \bar{b}_i^T\bar{M}_{1i}\bar{b}_i\ 1(t-\tau(t))\bar{x}^T(t)(\bar{S}_1+\bar{R})\bar{x}(t) \\ & + \bar{x}^T(t-\tau(t))(\bar{S}_2-(1-d)\bar{R})\bar{x}(t-\tau(t)) + 2\bar{x}^T(t)\bar{S}_3\bar{x}(t-\tau(t)) \\ & - 2\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{j(s)}\bar{x}(s)\,ds \\ & - 2\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\int_{t-\tau(t)}^{t}\bar{A}_{dj(s)}\bar{x}(s-\tau(s))\,ds \\ & + h\bar{x}^T(t)\bar{P}_i\bar{A}_{di}\bar{M}_{2i}^{-1}\bar{A}_{di}^T\bar{P}_i\bar{x}(t) + \int_{t-\tau(t)}^{t}\bar{b}_{j(s)}^T\bar{M}_{2i}\bar{b}_{j(s)}\ 1(s-\tau(s))\,ds \\ & - h^{-1}\int_{t-\tau(t)}^{t} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix}^T \begin{bmatrix} \bar{S}_1 & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2 \end{bmatrix} \begin{bmatrix} \bar{x}(s) \\ \bar{x}(s-\tau(s)) \end{bmatrix} ds \end{aligned} \tag{23}$$

Note that from (2), there always exists a symmetric positive semi-definite matrix $\bar{Q}_i$ such that

$$h\bar{x}^T(t)\bar{Q}_i\bar{x}(t) - \int_{t-\tau(t)}^{t}\bar{x}^T(t)\bar{Q}_i\bar{x}(t)ds \geq 0 \tag{24}$$

Define now

$$\bar{H}_i := \bar{P}_i(\bar{A}_i+\bar{A}_{di}) + (\bar{A}_{di}+\bar{A}_i)^T\bar{P}_i + \bar{S}_1 + \bar{R} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{b}_i^T\bar{M}_{1i}\bar{b}_i \end{bmatrix} + h\bar{Q}_i \tag{25a}$$

$$\bar{Z}_i := \begin{bmatrix} \bar{H}_i + \bar{P}_i\bar{M}_{1i}^{-1}\bar{P}_i + h\bar{P}_i\bar{A}_{di}\bar{M}_{2i}^{-1}\bar{A}_{di}^T\bar{P}_i & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2-(1-d)\bar{R} \end{bmatrix} \tag{25b}$$

$$\bar{Y}_{j(s)} := -\begin{bmatrix} h\bar{Q}_i & h\bar{P}_i\bar{A}_{di}\bar{A}_{j(s)} & h\bar{P}_i\bar{A}_{di}\bar{A}_{dj(s)} \\ * & \bar{S}_1 - \begin{bmatrix} 0 & 0 \\ 0 & h\bar{b}_{j(s)}^T\bar{M}_{2i}\bar{b}_{j(s)} \end{bmatrix} & \bar{S}_3 \\ * & * & \bar{S}_2 \end{bmatrix} \tag{25c}$$

Then by adding inequality (24) to the right hand side of (23) and considering (25a-c), one can write the following for all $x(t) \in X_i,\ i \in I_1$

$$\begin{aligned}\dot{\bar{V}}_i \leq & \bar{\xi}^T(t,\tau(t))\bar{Z}_i\bar{\xi}(t,\tau(t)) \\ & + h^{-1}\int_{t-\tau(t)}^{t} \bar{\eta}^T(t,s,\tau(s))\bar{Y}_{j(s)}\bar{\eta}(t,s,\tau(s))ds\end{aligned} \tag{26}$$

where $\bar{\xi}(t,\tau(t)) = \left[\bar{x}^T(t),\ \bar{x}^T(t-\tau(t))\right]^T$ and $\bar{\eta}(t,s,\tau(s)) = \left[\bar{x}^T(t),\ \bar{x}^T(s),\ \bar{x}^T(s-\tau(s))\right]^T$. Note that (14) and (16) imply

$$\bar{\xi}^T(\cdot)(\bar{Z}_i + \tilde{\bar{E}}_i^T\tilde{\bar{U}}_i\tilde{\bar{E}}_i)\bar{\xi}(\cdot) < 0 \tag{27}$$

using the Schur complement, where $\tilde{\bar{U}}_i = \mathrm{diag}[\bar{U}_i, 0]$, $\tilde{\bar{E}}_i = [\bar{E}_i, 0]$, and $\bar{U}_i$ has only non-negative entries. Note also that from (3), the inequality $\bar{E}_i\bar{x}(t) \geq 0$ holds for all $x(t) \in X_i$. This leads to

$$\tilde{\bar{E}}_i\bar{\xi} \geq 0,\ \forall x(t) \in X_i,\ i \in I_1$$

and consequently it follows that

$$\bar{\xi}^T(\cdot)\tilde{\bar{E}}_i^T\tilde{\bar{U}}_i\tilde{\bar{E}}_i\bar{\xi}(\cdot) \geq 0, \qquad x(t) \in X_i, \qquad i \in I_1 \tag{28}$$

Therefore, the relations (3), (16) and (14) imply $\bar{\xi}^T(\cdot)\bar{Z}_i\bar{\xi}(\cdot) < 0$, for all $x(t) \in X_i,\ i \in I_1$. Furthermore, (15) implies $\bar{Y}_{j(s)} \leq 0$ and from (26), $\dot{\bar{V}}_i < 0$, $x(t) \in X_i$, $i \in I_1$. A similar procedure can be repeated for the case when the switching index belongs to I_0, leading to (9)-(11) and $\dot{V}_i < 0$, for all $x(t) \in X_i,\ i \in I_0$. Thus the system is asymptotically stable. ■

Remark 1: Theorem 1 assumes the absence of sliding modes. To avoid sliding modes at the boundaries, the following conditions can be added. Let the set $\{x \in R^n | \sigma_{ik} = c_{ik}^T x - d_{ik} = 0\}$ denote the sliding surface between the cells X_i and X_k. According to [8], $\dot{\sigma}_{ik}$ must be continuous across the boundary described in (4), which yields

$$\begin{aligned}& c_{ik}^T\left[A_i(F_{ik}s + l_{ik}) + A_{di}x(t-\tau(t)) + a_i + b_i\ 1(t-\tau(t))\right] \\ & \quad = c_{ik}^T\left[A_k(F_{ik}s + l_{ik}) + A_{dk}x(t-\tau(t)) + a_k + b_k\ 1(t-\tau(t))\right]\end{aligned}$$

for all $s \in R^{n-1},\ k \in \mathcal{N}_i,\ i \in I$. The above equation can be rewritten as follows

$$c_{ik}^T(A_i - A_k)F_{ik} = 0 \tag{29a}$$

$$c_{ik}^T(A_{di} - A_{dk}) = 0 \tag{29b}$$

$$c_{ik}^T\left[(A_i - A_k)l_{ik} + (a_i - a_k)\right] = 0 \tag{29c}$$

$$c_{ik}^T(b_i - b_k) = 0 \tag{29d}$$

Remark 2: Using a procedure similar to the one presented here, one can apply the results of [29] and define the following Lyapunov-Krasovsky functional

$$\begin{aligned}\bar{V}_i' = & \bar{x}^T(t)\bar{P}_i\bar{x}(t) + \int_{t-\tau(t)}^{t}\bar{x}^T(s)e^{\beta(s-t)}\bar{R}\bar{x}(s)ds \\ & + h^{-1}\int_{-h}^{0}\int_{t+s}^{t}\begin{bmatrix}\bar{x}(\theta) \\ \bar{x}(\theta-\tau(\theta))\end{bmatrix}^T e^{\beta(\theta-t)}\begin{bmatrix}\bar{S}_1 & \bar{S}_3 \\ \bar{S}_3^T & \bar{S}_2\end{bmatrix} \\ & \qquad \begin{bmatrix}\bar{x}(\theta) \\ \bar{x}(\theta-\tau(\theta))\end{bmatrix}d\theta ds\end{aligned}$$

to obtain the LMIs that determine the exponential stability of the system (5). It is to be noted that exponential stability is stronger than asymptotic stability, at the cost of more conservative LMIs.

IV. Robustness Analysis

Consider now the system (1) and define the matrices $\tilde{A}_i = A_i + \Delta A_i$, $\tilde{A}_{di} = A_{di} + \Delta A_{di}$, $\tilde{a}_i = a_i + \Delta a_i$, $\tilde{b}_i = b_i + \Delta b_i$ $(i \in I)$ and

$$\tilde{\bar{A}}_i = \bar{A}_i + \Delta\bar{A}_i, \quad \Delta\bar{A}_i = \begin{bmatrix}\Delta A_i & \Delta a_i \\ 0 & 0\end{bmatrix}$$

$$\tilde{\bar{A}}_{di} = \bar{A}_{di} + \Delta\bar{A}_{di}, \quad \Delta\bar{A}_{di} = \begin{bmatrix}\Delta A_{di} & 0 \\ 0 & 0\end{bmatrix}$$

$$\tilde{\bar{b}}_i = \bar{b}_i + \Delta\bar{b}_i, \qquad \Delta\bar{b}_i = \begin{bmatrix}\Delta b_i \\ 0\end{bmatrix}$$

$$\Delta\bar{\mathscr{A}} = \left\{\begin{bmatrix}\Delta A_j & \Delta a_j \\ 0 & 0\end{bmatrix},\ \forall j \in I\right\}, \quad \Delta\mathscr{A} = \left\{\Delta A_j,\ \forall j \in I\right\}$$

$$\Delta\mathscr{E} = \left\{\Delta a_j,\ \forall j \in I_1\right\}$$

$$\Delta\bar{\mathscr{B}} = \left\{\begin{bmatrix}\Delta b_j \\ 0\end{bmatrix},\ \forall j \in I_1\right\}, \qquad \Delta\mathscr{B} = \left\{\Delta b_j,\ \forall j \in I_1\right\}$$

$$\Delta\bar{\mathscr{A}}_d = \left\{\begin{bmatrix}\Delta A_{dj} & 0 \\ 0 & 0\end{bmatrix},\ \forall j \in I\right\}, \qquad \Delta\mathscr{A}_d = \left\{\Delta A_{dj},\ \forall j \in I\right\}$$

Let $||\cdot||$ denote the 2-norm. The following bounds are assumed to be given for the norm of respective matrices

$$\|\Delta A_i\| \leq \alpha_i, \qquad \|\Delta A_{di}\| \leq \beta_i$$

$$||\Delta\bar{A}_i|| \leq \bar{\alpha}_i, \qquad ||\Delta\bar{A}_{di}|| \leq \bar{\beta}_i, \qquad ||\Delta\bar{b}_i|| \leq \bar{\delta}_i$$

$$\max_{X \in \Delta\bar{\mathscr{A}}}||X|| \leq \bar{\alpha}^*, \ \max_{X \in \Delta\bar{\mathscr{A}}_d}||X|| = \max_{X \in \Delta\mathscr{A}_d}||X|| \leq \beta^*$$

$$\max_{X \in \Delta\mathscr{A}}||X|| \leq \alpha^*, \quad \max_{X \in \Delta\mathscr{E}}||X|| \leq \gamma^*$$

$$\max_{X \in \Delta\bar{\mathscr{B}}}||X|| = \max_{X \in \Delta\mathscr{B}}||X|| \leq \delta^*$$

The following theorem presents sufficient conditions for the stability of uncertain PWA systems described by (1).

Theorem 2: Consider the symmetric matrices $\bar{U}_i, U_i$ and $\bar{W}_i, W_i$, composed of non-negative entries. Then, the uncertain PWA time-delay system (1) is asymptotically stable in the absence of sliding modes if (11), (16), (13) and (18a-c) hold, and there exist positive definite matrices L_{ki}, $k = 1,...,10$, $\bar{L}_{ki}$, $k = 1,...,9$, $M_{1i},\ i \in I_0$ and $\bar{M}_{pi},\ p = 1,2,\ i \in I_1$ such that

$$\left[\begin{array}{ccccccccc}
hQ_i & hP_i\beta_i\alpha^* & hP_i\beta_i & hP_i\alpha^*A_{di} & hP_i\beta_i\beta^* & hP_i\beta_i^2 & hP_i\beta^*A_{di} & hP_i\beta_i\delta^* & hP_iA_{di}\delta^* \\
* & h\rho_{L_{1i}}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & h\rho_{X_{1i}}I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & h\rho_{L_{3i}}I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & h\rho_{L_{4i}}I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & h\rho_{X_{2i}}I & 0 & 0 & 0 \\
* & * & * & * & * & * & h\rho_{L_{6i}}I & 0 & 0 \\
* & * & * & * & * & * & * & hl_{2i}I & 0 \\
* & * & * & * & * & * & * & * & hl_{3i}I \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
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* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & *
\end{array}\right.$$

$$\left.\begin{matrix} hP_i\beta_i^2 & hP_i\beta_i & hP_i\beta_i\gamma^* & hP_iA_{di}\gamma^* & hP_iA_{di}[A_j \ a_j] & hP_iA_{di}[A_{dj} \ 0] \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ h\rho_{L_{9i}}I & 0 & 0 & 0 & 0 & 0 \\ * & h\rho_{L_{10i}}I & 0 & 0 & 0 & 0 \\ * & * & hl_{1i}I & 0 & 0 & 0 \\ * & * & * & hl_{4i}I & 0 & 0 \\ * & * & * & * & S_1-\Pi_j^1 & S_3 \\ * & * & * & * & * & S_2-\Pi_i^2 \end{matrix}\right] \geq 0 \quad (30)$$

where

$$\Pi_j^1 = h(L_{1i}+L_{2i}+L_{3i}+\begin{bmatrix}0 & 0\\ 0 & b_j^T(M_{1i}+L_{9i})b_j\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & a_j^TL_{10i}a_j\end{bmatrix})$$

$$\Pi_i^2 = h(L_{4i}+L_{5i}+L_{6i})$$

$$\begin{bmatrix} H_i' & hP_iA_{di} & P_i\alpha_i & P_i\beta_i & S_{31}+[0_{n\times n} \quad R_3+S_3] \\ * & -hM_{1i} & 0 & 0 & 0 \\ * & * & -\rho_{L_{7i}}I & 0 & 0 \\ * & * & * & -\rho_{L_{8i}}I & 0 \\ * & * & * & * & \bar{S}_2-(1-d)\bar{R}+\Gamma \end{bmatrix} < 0 \quad (31)$$

$$\Gamma = \begin{bmatrix}0_{n\times n} & 0_{n\times 1}\\ 0_{1\times n} & S_2+R_2\end{bmatrix}+\begin{bmatrix}0_{n\times(n+1)}\\ S_{32}\end{bmatrix}+\begin{bmatrix}0_{(n+1)\times n} & S_{32}^T\end{bmatrix}$$

$$\begin{bmatrix} X_{1i} & X_{1i}A_j \\ * & L_{2i}\end{bmatrix} > 0, \quad \begin{bmatrix} X_{2i} & X_{2i}A_{dj} \\ * & L_{5i}\end{bmatrix} > 0 \quad (32)$$

for any fixed $i \in I_0$ and for all $A_j \in \mathscr{A}$, $b_j \in \mathscr{B}$, $a_j \in \mathscr{E}$, $A_{dj} \in \mathscr{A}_d$. In addition, let the following LMIs hold

$$\left[\begin{matrix} h\bar{Q}_i & h\bar{P}_i\bar{\beta}_i\bar{\alpha}^* & h\bar{P}_i\bar{\beta}_i & h\bar{P}_i\bar{\alpha}^*\bar{A}_{di} & h\bar{P}_i\bar{\beta}_i\bar{\beta}^* & h\bar{P}_i\bar{\beta}_i^2 & h\bar{P}_i\bar{\beta}^*\bar{A}_{di} \\ * & h\bar{\rho}_{\bar{L}_{1i}}I & 0 & 0 & 0 & 0 & 0 \\ * & * & h\bar{\rho}_{\bar{X}_{1i}}I & 0 & 0 & 0 & 0 \\ * & * & * & h\bar{\rho}_{\bar{L}_{3i}}I & 0 & 0 & 0 \\ * & * & * & * & h\bar{\rho}_{\bar{L}_{4i}}I & 0 & 0 \\ * & * & * & * & * & h\bar{\rho}_{\bar{X}_{2i}}I & 0 \\ * & * & * & * & * & * & h\bar{\rho}_{\bar{L}_{6i}}I \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{matrix}\right.$$

$$\left.\begin{matrix} h\bar{P}_i\bar{\beta}_i\delta^* & h\bar{P}_i\bar{A}_{di}\delta^* & h\bar{P}_i\bar{\beta}_i & h\bar{P}_i\bar{A}_{di}\bar{A}_j & h\bar{P}_i\bar{A}_{di}\bar{A}_{dj} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ h\bar{l}_{2i}I & 0 & 0 & 0 & 0 \\ * & h\bar{l}_{3i}I & 0 & 0 & 0 \\ * & * & h\bar{\rho}_{\bar{L}_{9i}}I & 0 & 0 \\ * & * & * & \bar{S}_1-\bar{\Pi}_j^1 & \bar{S}_3 \\ * & * & * & * & \bar{S}_2-\bar{\Pi}_i^2 \end{matrix}\right] \geq 0 \quad (33)$$

$$\bar{\Pi}_j^1 = h(\bar{L}_{1i}+\bar{L}_{2i}+\bar{L}_{3i}+\begin{bmatrix}0 & 0\\ 0 & \bar{b}_j^T(\bar{M}_{2i}+\bar{L}_{9i})\bar{b}_j\end{bmatrix})$$

$$\bar{\Pi}_i^2 = h(\bar{L}_{4i}+\bar{L}_{5i}+\bar{L}_{6i})$$

$$\begin{bmatrix} \hat{\bar{H}}_i & \bar{P}_i & h\bar{P}_i\bar{A}_{di} & \bar{P}_i\bar{\alpha}_i & \bar{P}_i\bar{\beta}_i & \bar{P}_i\bar{\delta}_i & \bar{S}_3 \\ * & -\bar{M}_{1i} & 0 & 0 & 0 & 0 & 0 \\ * & * & -h\bar{M}_{2i} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\rho}_{\bar{L}_{7i}}I & 0 & 0 & 0 \\ * & * & * & * & -\bar{\rho}_{\bar{L}_{8i}}I & 0 & 0 \\ * & * & * & * & * & -\bar{l}_{1i}I & 0 \\ * & * & * & * & * & * & \bar{S}_2-(1-d)\bar{R} \end{bmatrix} < 0 \quad (34)$$

$$\begin{bmatrix} \bar{X}_{1i} & \bar{X}_{1i}\bar{A}_j \\ * & \bar{L}_{2i}\end{bmatrix} > 0, \quad \begin{bmatrix} \bar{X}_{2i} & \bar{X}_{2i}\bar{A}_{dj} \\ * & \bar{L}_{5i}\end{bmatrix} > 0 \quad (35)$$

for any fixed $i \in I_1$ and for all $\bar{A}_j \in \bar{\mathscr{A}}$, $\bar{b}_j \in \bar{\mathscr{B}}$, $\bar{A}_{dj} \in \bar{\mathscr{A}}_d$, where the following inequalities are satisfied

$$\rho_{L_{ki}} > 0, \quad L_{ki}-\rho_{L_{ki}}I > 0, \quad k = 1,\ 3,\ 4,\ 6,\ 7, \ldots,\ 10 \quad \forall i \in I_0 \quad (36)$$

$$\bar{\rho}_{\bar{L}_{ki}} > 0, \quad \bar{L}_{ki}-\bar{\rho}_{\bar{L}_{ki}}I > 0, \quad k = 1,\ 3,\ 4,\ 6,\ 7,\ 8,\ 9 \quad \forall i \in I_1 \quad (37)$$

$$\rho_{X_{ki}} > 0, \quad \rho_{X_{ki}}I - X_{ki} < 0, \quad k = 1 \text{ and } 2 \quad \forall i \in I_0 \quad (38)$$

$$\bar{\rho}_{\bar{X}_{ki}} > 0, \quad \bar{\rho}_{\bar{X}_{ki}}I - \bar{X}_{ki} < 0, \quad k = 1 \text{ and } 2 \quad \forall i \in I_1 \quad (39)$$

where

$$\hat{H}_i = H_i' + L_{7i} + L_{8i} + \begin{bmatrix}0 & 0\\ 0 & hl_{1i}+hl_{4i}\end{bmatrix}$$

$$\hat{\bar{H}}_i = \bar{H}_i' + \bar{L}_{7i} + \bar{L}_{8i} + \begin{bmatrix}0 & 0\\ 0 & \bar{l}_{1i}+h\bar{l}_{2i}+h\bar{l}_{3i}\end{bmatrix}$$

(note that H_i' and $\bar{H}_i'$ are defined in (12) and (17), respectively).

Proof: The proof is similar to the proof of Theorem 1, and is omitted here due to space restrictions.

V. Numerical Examples

In this section, two examples are provided to show the effectiveness of the proposed approach.

Example 1: In this example, the stability of a time-delay system is investigated and it is shown that while the LMIs proposed in [2] are infeasible, the ones introduced in this paper are quite effective. Consider the piecewise linear time-delay system $\dot{x}(t) = A_ix(t) + A_{di}x(t-\tau)$ with the system matrices given by

$$A_1 = A_3 = -\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \quad A_2 = A_4 = -\begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$A_{d1} = A_{d3} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad A_{d2} = A_{d4} = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix}$$

and let the cell partition be given by

$$E_1 = -E_3 = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \quad E_2 = -E_4 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad (40)$$

One can verify that using the LMIs proposed in [2], stability of the system is guaranteed only for time-delays less than 0.005, which is a very small margin. However, the LMIs derived in Theorem 1 ensure the stability for the time-delays as large as $h = 10^5$.

Example 2: Consider the piecewise linear time-delay system $\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau)$, with the same cell partition as in (40), and the system matrices given by

$$A_1 = A_3 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = A_4 = -\begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}$$

$$A_{d1} = A_{d3} = \begin{bmatrix} 0.1 & 5.0 \\ -5.0 & 0.1 \end{bmatrix}, \quad A_{d2} = A_{d4} = \begin{bmatrix} 1.0 & 5.0 \\ -5.0 & -1.0 \end{bmatrix}$$

The LMIs derived in Theorem 1 are feasible for time-delays less than or equal to $h = 0.0264$ in this example. Using simulations, the system is unstable for $\tau_{max} = 0.031$. This suggests that the result obtained in this example using the approach proposed for systems with no uncertainty is not too conservative.

Assume now that the matrices A_i and A_{di} $(i = 1, \ldots, 4)$ in the above example are subject to uncertainty. It can be verified that for $||\Delta A_i|| \le 0.1$ and $||\Delta A_{di}|| \le 0.1$ $(i = 1, \ldots, 4)$ the LMIs given in Theorem 2 are feasible for the time-delays less than or equal to $h = 0.024$.

VI. Conclusions

In this paper, robust stability of a class of piecewise affine (PWA) systems with time-varying delay is considered. It is assumed that the system is subject to bounded uncertainty. It is also assumed that the time delay is unknown and time-varying, but upper bounds on the magnitude of the delay and its rate of variation exist. Sufficient conditions in the form of linear matrix inequalities (LMI) are derived for robust stability of the system. Numerical examples are provided to show the usefulness of the proposed approach. The following open problems can be considered for future work: (i) finding less conservative LMIs using different Lyapunov functionals; (ii) robust performance analysis of time-delay PWA systems; (iii) finding stability conditions for other types of uncertainties such as polytopic and polynomial, and (iv) stability analysis for neutral-type time-delay PWA systems.

References

[1] M. Johansson, *Piecewise Linear Control Systems*. Berlin, Heidelberg: Springer, 2003.

[2] V. Kulkarni, M. Jun, and J. Hespanha, "Piecewise quadratic lyapunov functions for piecewise affine time-delay systems," in *American Control Conference*, Boston, Massachusetts, 2004, pp. 3885–3889.

[3] L. Rodrigues, "Stability analysis of piecewise-affine systems using controlled invariant sets," *Systems & Control Letters*, vol. 53, no. 2, pp. 157–169, 2004.

[4] A. Hassibi and S. P. Boyd, "Quadratic stabilization and control of piecewise-linear systems," in *Proceedings of American Control Conference*, Philadelphia, Pennsylvani, 1998, pp. 3659–3664.

[5] L. Rodrigues and S. P. Boyd, "Piecewise-affine state feedback for piecewise-affine slab systems using convex optimization," *Systems & Control Letters*, vol. 54, no. 9, pp. 835–583, 2005.

[6] L. Rodrigues and J. How, "Toward unified analysis and controller synthesis for a class of hybrid systems," *Nonlinear Analysis*, vol. 65, no. 12, pp. 2216–2235, 2006.

[7] L. Rodrigues and E. Boukas, "Piecewise-linear $\mathscr{H}_\infty$ controller synthesis with applications to inventory control of switched production systems," *Automatica*, vol. 42, no. 8, pp. 1245–1254, 2006.

[8] L. Rodrigues and J. P. How, "Observer-based control of piecewise-affine systems," *International Journal of Control*, vol. 76, no. 5, pp. 459–477, 2003.

[9] S. Azuma and J. Imura, "Optimal control of sampled-data piecewise-affine systems and its application to cpu processing control," in *Proceedings of the 42nd IEEE Conference on Decision Control*, Maui, Hawaii, 2003, pp. 161–166.

[10] A. G. Beccuti, G. Papafotiou, and M. Morari, "Optimal control of the boost dc-dc converter," in *Proceedings of the 44th IEEE Conference on Decision Control*, Seville, Spain, 2005, pp. 4457–4462.

[11] Y. Wei, L. Rodrigues, and B. Gordon, "Piecewise-affine control of a three dof aerobatic helicopter," in *Proceedings of American Control Conference*, Minneapolis, Minnesota, 2006, pp. 3924–3929.

[12] A. Momeni and A. G. Aghdam, "An adaptive tracking problem for a family of retarded time-delay plants," *International Journal of Adaptive Control and Signal Processing*, vol. 21, pp. 885–910, 2007.

[13] A. G. Aghdam and E. J. Davison, "Pseudo-decentralized switching control," *Automatica*, vol. 39, no. 2, pp. 317–324, 2003.

[14] ——, "Decentralized switching control for hierarchical systems," *Automatica*, vol. 43, no. 6, pp. 1092–1100, 2007.

[15] M. M. Tousi, I. Karuei, S. H. Zad, and A. G. Aghdam, "Supervisory control of switching control systems," *Systems & Control Letters*, vol. 57, pp. 132–141, 2008.

[16] S. Bibian and H. Jin, "Time delay compensation of digital control for dc switch mode power supplies using prediction techniques," *IEEE Transactions on Power Electronics*, vol. 15, no. 5, pp. 835–842, 2000.

[17] W. Zhang, M. Branicky, , and S. M. Phillips, "Stability of networked control systems," *IEEE Control Systems Magazine*, vol. 21, no. 1, pp. 84–99, 2001.

[18] E. I. Verriest, M. K. H. Fan, and J. Kullstam, "Frequency domain robust stability criteria for linear delay systems," in *Proceedings of the 32nd IEEE Conference on Decision and Control*, San Antonio, Texas, 1993, pp. 3473–3478.

[19] S. S. Wang, B. S. Chen, and T. P. Lin, "Robust stability of uncertain time-delay systems," *International Journal of Control*, vol. 46, pp. 963–976, 1987.

[20] E. Fridman and U. Shaked, "Parameter dependent stability and stabilization of uncertain time-delay systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 5, pp. 861–866, 2003.

[21] Q. G. W. C. Lin and T. H. Lee, "A less conservative robust stability test for linear uncertain time-delay systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 87–91, 2006.

[22] J. H. Su, "Further results on the robust stability of linear systems with a single time delay," *Systems & Control Letters*, vol. 23, no. 5, pp. 375–379, 1994.

[23] Q. L. Han and K. Q. Gu, "On robust stability of time-delay systems with norm bounded uncertainty," *IEEE Transactions on Automatic Control*, vol. 46, no. 9, pp. 1426–1431, 2001.

[24] M. N. A. Parlakci, "Robust stability of uncertain time-varying state-delayed systems," *IEE Proceedings on Control Theory and Applications*, vol. 153, no. 4, pp. 469–477, 2006.

[25] Y. G. Sun, L. Wong, and G. Xie, "Stability of switched systems with time-varying delays: delay-dependent common lyapunov functional approach," in *Proceedings of American Control Conference*, Minneapolis, Minnesota, 2006, pp. 1544–1549.

[26] G. S. Zhai, Y. Sun, X. K. Chen, and N. M. Anthony, "Stability and l_2 gain analysis for switched symmetric systems with time delay," in *Proceedings of American Control Conference*, Denver, Colorado, 2003, pp. 2682–2687.

[27] X. M. Sun, J. Zhao, and D. Hill, "Stability and l_2-gain analysis for switched delay systems: a delay-dependent method," *Automatica*, vol. 42, no. 10, pp. 1769–1774, 2006.

[28] Y. Wang, L. Xie, and C. E. de Souza, "Robust control of a class of uncertain nonlinear systems," *Systems & Control Letters*, vol. 19, pp. 139–149, 1992.

[29] S. Mondie and V. L. Kharitonov, "Exponential estimates for retarded time-delay systems: an lmi approach," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 268–273, 2005.