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Networked Predictive Control of Constrained Nonlinear Systems: Recursive Feasibility and Input-to-State Stability Analysis

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***Abstract*— The present paper is concerned with the robust state feedback stabilization of uncertain discrete-time constrained nonlinear systems in which the loop is closed through a packet-based communication network. In order to cope with model uncertainty, time-varying transmission delays and packet dropouts which typically affect networked control systems, a robust control policy, which combines model predictive control with a network delay compensation strategy, is proposed. The contribution of the paper is twofold. First, the issue of guaranteeing the recursive feasibility of the optimization problem associated to the receding horizon control law has been addressed, such that the invariance of the feasible region under the networked closed-loop dynamics can be guaranteed. Secondly, the Input-to-Stability property of the networked closed-loop system with respect to bounded perturbations has been analyzed.**

***Index Terms*— Networked Control Systems, Nonlinear Control, Model Predictive Control.**

I. INTRODUCTION

In the past few years, control applications in which sensor data and actuator commands are sent through a shared communication network have attracted increasing attention in control engineering, since network technologies provide a convenient way to remotely control large distributed plants [1]. These systems, usually referenced as Networked Control Systems (NCS's), are affected by the dynamics introduced by both the physical link and the communication protocol, that, in general, need to be taken in account in the design of the NCS. Various control schemes have been presented in the current literature to design effective NCS's for linear time-invariant systems [5], [7], [12], [20], [21]. Moreover, if the system to be controlled is subjected to constraints and nonlinearities, the formulation of an effective networked control strategy becomes a really hard task [19]. In this framework, the present paper provides theoretical results that motivate, under suitable assumptions, the combined use of nonlinear Model Predictive Control (MPC) with a Network Delay Compensation (NDC) strategy [2], [18], in order to cope with the simultaneous presence of model uncertainties, time-varying transmission delays and data-packet losses. In the current literature, for the specific class of MPC schemes which impose a fixed terminal constraint set, X_f, as a stabilizing condition, the robust stability properties of the overall c-l system, in absence of transmission delays, has been shown to depend on the invariance properties of X_f, [11], [17]. In this regard, by resorting to invariant set theoretic arguments [3], [9], this paper aims to show that the devised NCS can robustly stabilize a nonlinear constrained system even in presence of data transmission delays and model uncertainty. In particular, the issue of recursive feasibility in constrained networked nonlinear MPC, first addressed in [16], in this paper is shown to be key point to prove the Input-to-State Stability (ISS) of the scheme w.r.t. additive perturbations. Indeed, by exploiting a novel characterization of regional ISS in terms of time-varying Lyapunov functions, the closed-loop system is shown to be ISS with respect to the aforementioned class of disturbances, even in presence of unreliable networked communication links.

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II. MAIN NOTATION AND BASIC DEFINITIONS

Let $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{Z}$, and $\mathbb{Z}_{\geq 0}$ denote the real, the non-negative real, the integer, and the non-negative integer sets of numbers, respectively. The Euclidean norm is denoted as $|\cdot|$. For any discrete-time sequence $\boldsymbol{\phi} : \mathbb{Z}_{\geq 0} \to \mathbb{R}^m$, $\|\phi\| \triangleq \sup_{k\geq 0}\{|\phi_k|\}$ and $\|\phi_{[\tau]}\| \triangleq \sup_{0\leq k\leq \tau}\{|\phi_k|\}$, where ϕ_k denotes the value that the sequence $\boldsymbol{\phi}$ takes on in correspondence with the index k. The set of discrete-time sequences of $\boldsymbol{\upsilon}$ taking values in some subset $\Upsilon \subset \mathbb{R}^m$ is denoted by $\mathcal{M}_\Upsilon$. Given a compact set $A \subseteq \mathbb{R}^n$, let ∂A denote the boundary of A. Given a vector $x \in \mathbb{R}^n$, $\mathrm{d}(x, A) \triangleq \inf\{|\xi - x|, \xi \in A\}$ is the point-to-set distance from $x \in \mathbb{R}^n$ to A. Given two sets $A \subseteq \mathbb{R}^n$, $B \subseteq \mathbb{R}^n$, $\mathrm{dist}(A, B) \triangleq \inf\{\mathrm{d}(\zeta, A), \zeta \in B\}$ is the minimal set-to-set distance. The difference between two given sets $A \subseteq \mathbb{R}^n$ and $B \subseteq \mathbb{R}^n$, with $B \subseteq A$, is denoted as $A \backslash B \triangleq \{x : x \in A, x \notin B\}$. Given two sets $A \in \mathbb{R}^n$, $B \in \mathbb{R}^n$, the Pontryagin difference set C is defined as $C = A \backsim B \triangleq \{x \in \mathbb{R}^n : x + \xi \in A, \forall \xi \in B\}$. Given a vector $\eta \in \mathbb{R}^n$ and a scalar $\rho \in \mathbb{R}_{>0}$, the closed ball centered in η of radius ρ is denoted as $\mathcal{B}^n(\eta, \rho) \triangleq \{\xi \in \mathbb{R}^n : |\xi - \eta| \leq \rho\}$. The shorthand $\mathcal{B}^n(\rho)$ is used when $\eta = 0$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing.

Let us consider the time-varying discrete-time dynamic system

$$x_{t+1} = g(t, x_t, \upsilon_t),\ t \in \mathbb{Z}_{\geq 0},\ x_0 = \overline{x}\,, \tag{1}$$

with $g(t, 0, 0) = 0$, $\forall t \geq \overline{T}$ with $\overline{T} \in \mathbb{Z}_{\geq 0}$, and where $x_t \in \mathbb{R}^n$ and $\upsilon_t \in \Upsilon \subset \mathbb{R}^r$ denote the state and the bounded input of the system, respectively. The discrete-time state trajectory of the system (1), with initial state $x_0 = \overline{x}$ and input sequence $\boldsymbol{\upsilon} \in \mathcal{M}_\Upsilon$, is denoted by $x(t, \overline{x}, \boldsymbol{\upsilon}_{0,t})$, $t \in \mathbb{Z}_{\geq 0}$.

Definition 2.1 (RPI set): A set $\Xi \subset \mathbb{R}^n$ is a Robust Positively Invariant (RPI) set for system (1) if, for all $t \in \mathbb{Z}_{\geq 0}$, it holds that $g(t, x_0, \upsilon) \in \Xi$, $\forall x \in \Xi$ and $\forall \upsilon \in \Upsilon$. □

In the following, the Regional Input-to-State Stability property, recently introduced in [13] (see also [6]), is recalled. It is worth noting that regional results are needed in the framework of nonlinear MPC due to the impossibility to obtain, in general, global bounds on the finite horizon costs used as Lyapunov function in the stability analysis. Nonetheless, in the framework of NCS's, due to the variability of transmission delays, a time invariant formulation is not suited, therefore it is necessary to extend the regional ISS analysis in order to cope with time-varying Lyapunov functions (see [4] and [14])

The following definition of regional ISS is provided for time-varying discrete-time nonlinear systems of the form (1).

Definition 2.2 (Regional ISS in Ξ): Given a compact set $\Xi \subset \mathbb{R}^n$, if Ξ is RPI for (1) and if there exist a $\mathcal{KL}$-function β and a $\mathcal{K}$-function γ such that

$$|x(t,\overline{x},\boldsymbol{\upsilon}_{0,t-1})| \leq \max\{\beta(|\overline{x}|,t),\gamma(\|\boldsymbol{\upsilon}_{[t-1]}\|)\}, \forall t \in \mathbb{Z}_{\geq 0}, \forall \overline{x} \in \Xi, \quad (2)$$

then the system (1), with $\boldsymbol{\upsilon} \in \mathcal{M}_\Upsilon$, is said to be regional Input-to-State Stable (ISS) for initial conditions in Ξ. □

In literature there exist some recent results concerning the characterization of the ISS property in terms of time-varying Lyapunov functions for perturbed (uncertain) discrete-time system [8], [10]; on the other hand those results guarantee the Input-to-State Stability property in a semi-global sense, and cannot be trivially used in the MPC setup due to the impossibility to obtain global bounds for the candidate ISS Lyapunov function. Indeed, for systems controlled by predictive control schemes the stability analysis needs to be carried out by using non smooth ISS-Lyapunov functions with an upper bound guaranteed only in a sub-region of the domain of attraction [13]. Therefore, a regional ISS result for a family of time-varying Lyapunov functions is needed to assess the stability properties of MPC-based NCS's.

To this end, let us first consider the following definition.

Definition 2.3 (Time-varying ISS-Lyapunov Function): Given a pair of compact sets $\Xi \subset \mathbb{R}^n$ and $\Omega \subseteq \Xi$, with Ξ RPI for system (1) and $\{0\} \subset \Omega$, a function $V(\cdot,\cdot): \mathbb{Z}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is called a (Regional) ISS-Lyapunov function in Ξ, if there exist $\mathcal{K}_\infty$-functions α_1, α_2, α_3, and $\mathcal{K}$-function σ_1 and σ_2, such that

1) the following inequalities hold $\forall \upsilon \in \Upsilon$ and $\forall t \in \mathbb{Z}_{\geq 0}$

$$V(t,x) \geq \alpha_1(|x|), \qquad \forall x \in \Xi, \quad (3)$$

$$V(t,x) \leq \alpha_2(|x|) + \sigma_1(|\upsilon|), \qquad \forall x \in \Omega, \quad (4)$$

$$V(t+1, g(t,x,\upsilon)) - V(t,x) \leq -\alpha_3(|x|) + \sigma_2(|\upsilon|), \forall x \in \Xi, \quad (5)$$

2) there exist some suitable $\mathcal{K}_\infty$-functions ϵ and ρ (with ρ such that $(id - \rho)$ is a $\mathcal{K}_\infty$-function, too) and a positive scalar $c \in \mathbb{R}_{>0}$ such that the set

$$\Theta \triangleq \{x : V(t,x) \leq b(\overline{\upsilon}), \forall t \in \mathbb{Z}_{\geq 0}\}, \quad (6)$$

verifies the inclusion

$$\Theta \subseteq \Omega \backsim \mathcal{B}^n(c), \quad (7)$$

with $\{0\} \in \Theta$ and where $b(s) \triangleq \alpha_4^{-1} \circ \rho^{-1} \circ \sigma_4(s)$, $\alpha_4 \triangleq \underline{\alpha}_3 \circ \overline{\alpha}_2^{-1}$, $\underline{\alpha}_3(s) \triangleq \min(\alpha_3(s/2), \epsilon(s/2))$, $\overline{\alpha}(s) \triangleq \alpha_2(s) + \sigma_1(s)$, $\sigma_4 = \epsilon(s) + \sigma_2(s)$ and $\overline{\upsilon} \triangleq \max_{\upsilon \in \Upsilon}\{|\upsilon|\}$.

□

Now, under the following assumption, the characterization of the regional ISS property in terms of Lyapunov functions can be stated.

Assumption 1: For every $t \in \mathbb{R}_{>0}$, the state trajectories $x(t,\overline{x}_0,\boldsymbol{\upsilon}_{0,t-1})$ of the system (1) are continuous in $\overline{x}_0 = 0$ and $\upsilon = 0$ with respect to the initial condition $\overline{x}_0$ and the disturbance sequence $\boldsymbol{\upsilon}_{0,t-1}$. □

Theorem 2.1 (Lyapunov characterization of regional ISS): Suppose that Assumption 1 holds. If the system (1) admits a (time-varying) ISS-Lyapunov function in Ξ, then it is regional ISS in Ξ with respect to υ and

$$\lim_{t\to\infty} \mathrm{d}(x(t,\overline{x},\boldsymbol{\upsilon}_{0,t-1}),\Theta) = 0, \quad \forall \overline{x} \in \Xi. \qquad □$$

Theorem 2.1 can be proven following the same lines of the main regional-ISS result in [13]. For a detailed proof see [15].

Notably, the ISS-Lyapunov inequalities (3),(4) and (5) differ from those posed in the original regional ISS formulation [13], since an input-dependent upper bound is admitted in (4) (thus allowing for a more general characterization). Moreover, with regard to the regional ISS result presented in [6], the ISS-Lyapunov function $V(t,x)$ is allowed to belong a family of time-varying functions. Remarkably, the possibility to incorporate an input-dependent upper bound in (4) and to admit a time-varying characterization will be instrumental for characterizing the ISS property for NCS's, as it will clearly emerge in Section IV.

III. Problem Formulation

Consider the nonlinear discrete-time dynamic system

$$x_{t+1} = f(x_t, u_t, \upsilon_t), \ t \in \mathbb{Z}_{\geq 0}, \ x_0 = \overline{x}, \quad (8)$$

where $x_t \in \mathbb{R}^n$ denotes the state vector, $u_t \in \mathbb{R}^m$ the control vector and $\upsilon_t \in \Upsilon$ is an uncertain exogenous input vector, with $\Upsilon \subset \mathbb{R}^r$ compact and $\{0\} \subset \Upsilon$. Assume that state and control variables are subject to the following constraints

$$x_t \in X, \ t \in \mathbb{Z}_{\geq 0}, \quad (9)$$

$$u_t \in U, \ t \in \mathbb{Z}_{\geq 0}, \quad (10)$$

where X and U are compact subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively, containing the origin as an interior point. Given the system (8), let $\hat{f}(x_t, u_t)$, with $\hat{f}(0,0) = 0$, denote the *nominal* model used for control design purposes. Moreover, let $\hat{x}_{t+j|t}$, $j \in \mathbb{Z}_{>0}$ denote the state prediction generated by the nominal model on the basis of the state informations at time t with the sequence $\mathbf{u}_{t,t+j-1} = \mathrm{col}\{u_t, \ldots, u_{t+j-1}\}$

$$\hat{x}_{t+j|t} = \hat{f}(\hat{x}_{t+j-1|t}, u_{t+j-1}), \ \hat{x}_{t|t} = x_t, t \in \mathbb{Z}_{\geq 0}, j \in \mathbb{Z}_{>0}. \quad (11)$$

Assumption 2 (Lipschitz): The nominal map $\hat{f}(x,u)$ is Lipschitz with respect to x in X, with Lipschitz constant $L_{f_x} \in \mathbb{R}_{>0}$. □

Introducing the *additive transition uncertainty* vector $d_t \triangleq f(x_t, u_t, \upsilon_t) - \hat{f}(x_t, u_t)$, the true state dynamics writes

$$x_{t+1} = \hat{f}(x_t, u_t) + d_t, \ t \in \mathbb{Z}_{\geq 0}, \ x_0 = \overline{x}. \quad (12)$$

Assumption 3 (Uncertainty): The transition uncertainty vector d_t belongs to the compact ball $D \triangleq \mathcal{B}^n(\overline{d})$, where $\overline{d} \triangleq \sup_{\upsilon \in \Upsilon}\{\mu(|\upsilon|)\}$, and μ is a $\mathcal{K}$-function. □

Under the posed assumptions, the control objective consists in guaranteeing the ISS property for the c-l system w.r.t. the prescribed class of uncertainties, while enforcing the fulfillment of constraints in presence of packet dropouts, bounded transmission delays and bounded disturbances.

With regard to the network dynamics and communication protocol, it is assumed that a set of data (packet) can be sent, at a given time instant, through the network by a node, while both the sensor-to-controller and the controller-to-actuator links are supposed to be affected by delays and dropouts due to the unreliable nature of networked communications. In order to cope with network delays, the data packets are Time-Stamped (TS) [20], that is, they contain the information on when they were delivered by the transmission node. In this work, we will consider first the case of networks with acknowledged communication protocols, also known as *TCP-like* [7], in which the destination node sends an acknowledgment packet (ACK) of successful packet reception to the source node, and then the results will be extended to non-acknowledged protocols, which are usually referred to as *UDP–like* [7]. In a TCP-like scenario, the acknowledgment messages are assumed to have the highest priority among all the routed packets, such that, after each successful packet reception, the source node receives a deterministic notification within a single time-interval.

In this regard, the presence of ACKs in TCP-like networks can be exploited by the controller (which is acknowledged of successful packet reception by the actuator) to internally reconstruct the true sequence of controls which have been applied to the plant [18] from time instat $t-\tau_c(t)$ to $t-1$, in order to get an estimation of the current state $\hat{x}_{t|t-\tau_c(t)}$, on the basis of the most recent available plant measurement $x_{t-\tau_c(t)}$. A graphical representation of the overall NCS layout is depicted in Figure 1, while the NDC and the controller will be described in the next sections.

A. Network delay compensation

In the sequel, $\tau_{ca}(t)$ and $\tau_{sc}(t)$ will denote the delays occurred respectively in the controller-to-actuator and in the sensor-to-controller links, while $\tau_a(t)$ will represent age (in discrete time instant) of the control sequence used by the smart actuator to compute the current input and $\tau_c(t)$ the age of the state measurement which had been used by the controller to compute the control actions at time t. Finally, $\tau_{rt}(t) \triangleq \tau_a(t)+\tau_c(t-\tau_a(t))$ is the so called *round trip time*, i.e., the age of the state measurement used to compute the input applied at time t.

The NDC strategy adopted in the present work, which relies on the one devised in [18] (originally developed for unconstrained systems nominally stabilized by a generic nonlinear controller), is based on exploiting the time stamps of the data packets in order to retain only the most recent informations at each destination node: when a novel packet is received, if it carries a more recent time-stamp than the one already in the buffer, then it takes the place of the older one and, in the TCP-like case, an acknowledgment of successful packet reception is sent to the node which transmitted the packet. The TS-based packet arrival management implies $\tau_a(t) \le \tau_{ca}(t)$ and $\tau_c(t) \le \tau_{sc}(t)$. Moreover, the NDC strategy comprises a Future Input Buffering (FIB) mechanism, which requires that the controller node send a packeted sequence of N_c control actions (with N_c to satisfy Assumption 4) to the actuator node (relying on a model-based prediction performed, in this case, by the MPC). In turn the smart actuator, at the arrival of each new packet, first stores the entire sequence in its internal buffer, then, at each time instant t, selects a time-consistent control action to apply to the plant, by setting $u_t = u_t^b$, where u_t^b is the $\tau_a(t)$-th element of the buffered sequence $\mathbf{u}^b_{t-\tau_a(t),t-\tau_a(t)+N_c-1}$, which is given by

$$\begin{aligned}\mathbf{u}^b_{t-\tau_a(t),t+N_c-1} &= \mathrm{col}[u^b_{t-\tau_a(t)},\ldots,u^b_t,\ldots,u^b_{t-\tau_a(t)+N_c-1}]\\ &= \mathbf{u}^c_{t-\tau_a(t),t+N_c-1|t-\tau_{rt}(t)}.\end{aligned}$$

where the sequence $\mathbf{u}^c_{t-\tau_a(t),t+N_c-1|t-\tau_{rt}(t)}$ had been computed at time $t-\tau_a(t)$ by the controller on the basis of the state measurement collected at time $t-\tau_{rt}(t) = t-\tau_a(t)-\tau_c(t-\tau_a(t))$. In most situations, it is natural to assume that the age of the data-packets available at the controller and actuator nodes subsume an upper bound [18], as specified by the following assumption.

Assumption 4 (Network reliability): The quantities $\tau_c(t)$ and $\tau_a(t)$ verify $\tau_c(t) \le \overline{\tau}_c$ and $\tau_a(t) \le \overline{\tau}_a$, $\forall t \in \mathbb{Z}_{>0}$, with $\overline{\tau}_c + \overline{\tau}_a + 1 < N_c$. Notably, we don't impose bounds on $\tau_{sc}(t)$ and $\tau_{ca}(t)$, allowing the presence of packet losses (infinite delay). Under these conditions, the round trip time verifies $\tau_{rt}(t) \le \overline{\tau}_{rt} = \overline{\tau}_c + \overline{\tau}_a \le N_c - 1$, $\forall t \in \mathbb{Z}_{>0}$. □

B. Current state reconstruction in TCP-like networks and shortening of the optimization horizon

At time t, the computation of the control sequence to be sent to the actuator must rely on a state measurement performed at time $t-\tau_c(t)$, $x_{t-\tau_c(t)}$. In order to recover the standard MPC formulation, the current (possibly unavailable) state x_t has to be reconstructed by means of the nominal model (11) and of the true input sequence $\mathbf{u}_{t-\tau_c(t),t}$ applied by the smart actuator to the plant, $\mathbf{u}_{t-\tau_c(t),t-1} \triangleq \mathrm{col}[u_{t-\tau_c(t)},\ldots,u_{t-1}]$ from time $t-\tau_c(t)$ to $t-1$. In this regard, the benefits due to the use of a state predictor in NCS's are deeply discussed in [18] and [20], [21]. The sequence $\mathbf{u}_{t-\tau_c(t),t-1}$ can be internally reconstructed by the controller thanks to the acknowledgment-based protocol. Moreover, in presence of delays in the controller-to-actuator path, we must consider that the computed control sequence may not be applied entirely to the plant. In order to ensure that the sequence used for prediction would coincide with the one that will be applied to the plant, we can retain, at time t, some of the elements of the control sequence computed at time $t-1$ (i.e., the subsequence $\mathbf{u}^b_{t,t+\overline{\tau}_a-1|t-1-\tau_c(t-1)}$), and optimize only over the remaining elements (i.e. the sequence $\mathbf{u}_{t+\overline{\tau}_a,t+N_c-1}$), initiating the finite horizon optimization with the state prediction $\hat{x}_{t+\overline{\tau}_a}$. We will show that the recursive feasibility of such a scheme can be guaranteed w.r.t. (suitably small) model uncertainty.

C. Finite horizon predictive controller

In the following, we will describe the mechanism used by the controller to compute the sequence of control actions to be forwarded to the smart actuator. It relies on the solution, at each time instant t, of a Finite Horizon Optimal Control Problem (FHOCP), which uses a constraint tightening technique [11] to robustly enforce the constraints.

First, let us introduce the following sets, obtained by restricting the nominal constraint X.

Definition 3.1 ($X_i(\overline{d})$): Under Assumptions 2 and 3, suppose[1], without loss of generality, $L_{f_x} \neq 1$. The tightened sets $X_i(\overline{d})$, are defined as

$$X_i(\overline{d}) \triangleq X \backsim \mathcal{B}^n\left(\frac{L^i_{f_x}-1}{L_{f_x}-1}\overline{d}\right), \forall i \in \mathbb{Z}_{>0} \ . \quad (13)$$

□

Problem 3.1 (FHOCP): Given a positive integer $N_c \in \mathbb{Z}_{\ge 0}$, at any time $t \in \mathbb{Z}_{\ge 0}$, let $\hat{x}_{t|t-\tau_c(t)}$ be the estimate of the current state obtained from the last available plant measurement $x_{t-\tau_c(t)}$ with the controls $\mathbf{u}_{t-\tau_c(t),t-1}$ already applied to the plant; moreover let $\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}$ be the state computed from $\hat{x}_{t|t-\tau_c(t)}$ by extending the prediction using the input sequence computed at time $t-1$, $\mathbf{u}^c_{t,t+\overline{\tau}_a-1}$. Then, given a stage-cost function h, the constraint sets $X_i(\overline{d}) \subseteq X$, $i \in \{\overline{\tau}_a(t)+1,\ldots,N_c\}$, a terminal cost function h_f and a terminal set X_f, the *Finite Horizon Optimal Control Problem* (FHOCP) consists in minimizing, with respect to a $N_c - \overline{\tau}_a$ steps input sequence, $\mathbf{u}_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)} \triangleq \mathrm{col}[u_{t+\overline{\tau}_a|t-\tau_c(t)},\ldots,u_{t+N_c-1|t-\tau_c(t)}]$, the cost function

$$\begin{aligned}&J^\circ_{FH}(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}, \mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}, N_c-\overline{\tau}_a)\\ &\triangleq \min_{\mathbf{u}_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}} \left\{ \sum_{l=t+\overline{\tau}_a}^{t+N_c-1} h(\hat{x}_{l|t-\tau_c(t)}, u_{l|t-\tau_c(t)}) + h_f(\hat{x}_{t+N_c|t-\tau_c(t)}) \right\}\end{aligned}$$

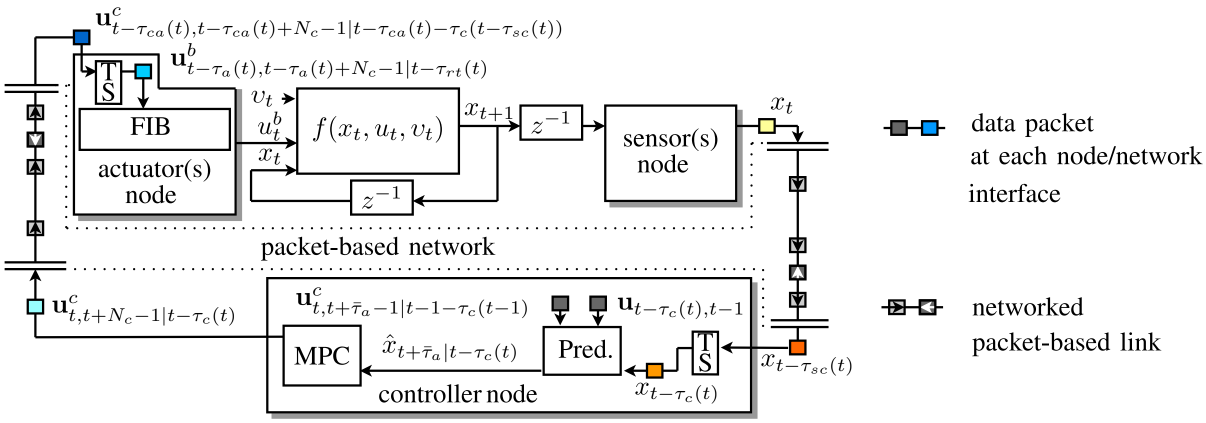
subject to the

i) nominal dynamics (11);

ii) input constraints $u_{t-\tau_c(t)+i|t-\tau_c(t)} \in U$, with $i \in \{\tau_c(t)+\overline{\tau}_a,\ldots,\tau_c(t)+N_c-1\}$;

[1] The very special case $L_{f_x}=1$ can be trivially addressed by a few suitable modifications to the Definition 3.1

Fig. 1 Scheme of the combined NCS-NDC strategy. In evidence the predictive controller (MPC), the Time-Stamping packet arrival management (TS) and the Future Input Buffering (FIB) mechanism at the actuator node.



iii) state constraints $\hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)} \in X_i(\overline{d})$, with $i \in \{\tau_c(t)+\overline{\tau}_a+1,\dots,\tau_c(t)+N_c\}$;

iv) terminal state constraint $\hat{x}_{t+N_c|t-\tau_c(t)} \in X_f$.

Finally, the sequence of controls forwarded by the controller to the actuator is constructed as $\mathbf{u}^c_{t,t+N_c-1|t-\tau_c(t)} \triangleq \mathrm{col}[\mathbf{u}^c_{t,t+\overline{\tau}_a-1|t-1-\tau_c(t-1)}, \mathbf{u}^{\circ}_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}]$ (i.e., it is obtained by appending the solution of the FHOCP a subsequence computed at time $t-1$). In the following, we will say that a sequence $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)} \triangleq \mathrm{col}[\mathbf{u}^c_{t,t+\overline{\tau}_a-1|t-1-\tau_c(t-1)}, \overline{\mathbf{u}}_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}]$ is *feasible* if the first subsequence yields to $\hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)} \in X_i(\overline{d})$, $\forall i \in \{\tau_c(t)+1,\dots,\tau_c(t)+\overline{\tau}_a$ and if the second subsequence (possibly suboptimal) satisfies all the constraints of the FHOCP set up at time t. □

The following definitions will be used in the sequel.

Definition 3.2 ($X_{MPC}(\tau)$): Given a non-negative integer $\tau \in \mathbb{Z}_{\geq 0}$, the set containing all the vectors $\overline{x}_0 \in \mathbb{R}^n$ for which there exists a sequence of N_c control moves which satisfies all the constraints specified below is said *feasible set with τ-delay restriction*, and it is denoted with $X_{MPC}(\tau)$.

$$X_{MPC}(\tau) \triangleq \left\{ \overline{x}_0 \in \mathbb{R}^n \left| \begin{array}{l} \exists\, \overline{\mathbf{u}}_{0,N_c-1} \in U^{N_c}: \\ \hat{x}(i,\overline{x}_0,\overline{\mathbf{u}}_{0,i-1}) \in X_{\tau+i}(\overline{d}), \\ \forall i \in \{1,\dots,N_c\} \quad \text{and} \\ \hat{x}(N_c,\overline{x}_0,\overline{\mathbf{u}}_{0,N_c-1}) \in X_f \end{array} \right. \right\} \tag{14}$$

□

For the sake of brevity, the set $X_{MPC}(0)$ will be denoted as X_{MPC}.

Definition 3.3 (Feasible sequence at time t): Given a delayed state measurement $x_{t-\tau_c(t)}$, available at at time t to the controller, let us consider the prediction $\hat{x}_{t|t-\tau_c(t)}$ of the actual state x_t obtained with the nominal model and with the actual control sequence applied from time $t-\tau_c(t)$ to t–1, $\overline{\mathbf{u}}_{t-\tau_c(t),t-1}$, which is known to the controller. Moreover consider a sequence of N_c control moves $\overline{\mathbf{u}}^c_{t,t+N_c-1}$ and its two subsequences $\overline{\mathbf{u}}^c_{t,t+\overline{\tau}_a-1}$ and $\overline{\mathbf{u}}^c_{t+\overline{\tau}_a,t+N_c-1}$ such that $\overline{\mathbf{u}}^c_{t,t+N_c-1} = \mathrm{col}[\overline{\mathbf{u}}^c_{t,t+\overline{\tau}_a-1}, \overline{\mathbf{u}}^c_{t+\overline{\tau}_a,t+N_c-1}]$.
The input sequence $\overline{\mathbf{u}}^c = \overline{\mathbf{u}}^c_{t,t+N_c-1}$ is said *feasible at time t* if the subsequence $\overline{\mathbf{u}}^c_{t,t+\overline{\tau}_a-1}$ yields to $\hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)} \in X_i(\overline{d})$, $\forall i \in \{\tau_c(t)+1,\dots,\tau_c(t)+\overline{\tau}_a\}$ and if the second subsequence satisfies all the constraints of the RHOCP initiated with $\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)} = \hat{x}(\overline{\tau}_a, x_{t-\tau_c(t)}, \overline{\mathbf{u}}^*_{t-\tau_c(t),t+\overline{\tau}_a-1})$. □

Remark 3.1: Note that what we call *feasible sequence in t* is not just an input sequence which satisfies the constraints of the RHOCP (specified in the horizon $[t+\overline{\tau}_a+1,\dots,t+N_c]$), but it is required to keep the nominal trajectories inside the restricted constraints for an horizon of N_c steps from $t+1$ to $t+N_c$, that is larger than the one considered by the optimization.

By accurately choosing the stage cost h, the constraints $X_i(\overline{d})$, the terminal cost function h_f, and by imposing a terminal constraint X_f at the end of the control horizon, it is possible to show that the recursive feasibility of the scheme can be guaranteed for $t \in \mathbb{Z}_{>0}$, also in presence of norm-bounded additive transition uncertainties and network delays. Moreover, in absence of transmission delays, this class of controllers has been proven to achieve the ISS property if the following assumptions are verified (see [17]).

Assumption 5: The transition cost function h is such that $\underline{h}(|x|) \leq h(x,u), \forall x \in X, , \forall u \in U$, where $\underline{h}$ is a $\mathcal{K}_\infty$-function. Moreover, h is Lipschitz w.r.t. x, uniformly in u, with L. constant $L_h \in \mathbb{R}_{>0}$. □

Assumption 6 (κ_f, h_f, X_f): There exist an auxiliary control law $\kappa_f(x): X \to U$, a function $h_f(x): \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, a positive constant $L_{h_f} \in \mathbb{R}_{>0}$, a level set of h_f, $X_f \subset X$ and a positive constant $\delta \in \mathbb{R}_{>0}$ such that the following properties hold:

i) $X_f \subset X$, X_f closed, $\{0\} \in X_f$;
ii) $\kappa_f(x) \in U$, $\forall x \in X_f \oplus \mathcal{B}^n(\delta)$;
iii) $\hat{f}(x,\kappa_f(x)) \in X_f$, $\forall x \in X_f \oplus \mathcal{B}^n(\delta)$;
iv) the closed-loop map $\hat{f}(x,\kappa_f(x))$, is Lipschitz in X_f, with L. constant $L_{f_c} \in \mathbb{R}_{>0}$;
v) $h_f(x)$ Lipschitz in X_f, with L. constant $L_{h_f} \in \mathbb{R}_{>0}$;
vi) $h_f(\hat{f}(x,\kappa_f(x))) - h_f(x) \leq -h(x,\kappa_f(x))$, $\forall x \in X_f \oplus \mathcal{B}^n(\delta)$. □

In addition, in order to establish the ISS property for the c-l system, we require the following assumptions to be verified together with 5 and 6.

Assumption 7: Let X_f be a sub-level set of h_f (i.e. $X_f = \{x \in \mathbb{R}^n: h_f(x) \leq \overline{h}_f\}$), then we assume that the transition cost function h and the terminal cost h_f satisfy the condition

$$\min_{u \in U} \left\{ \inf_{x \in \mathcal{C}_1(X_f) \setminus (X_f \oplus \mathcal{B}^n(\delta))} \{h_f(x) - h(x,u)\} \right\} > \overline{h}_f. \tag{15}$$

where $\delta \in \mathbb{R}_{>0}$ is a positive scalar for which Points iii) and vi) of Assumption 6 hold. □

Now, the following Lemma ensures that the original state constraints can be satisfied by imposing to the nominal trajectories in the RHOCP the restricted constraints introduced in Definition 3.1.

Lemma 3.1 (Robust Constraint Satisfaction): Any feasible control sequence $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}$, applied in open-loop to the perturbed system from time t to $t+N_c-1$, guarantees that the true (networked/perturbed) state will satisfy $x_{t+j}\in X$, $\forall j\in\{1,\ldots,N_c\}$. □

Proof: Given the state measurement $x_{t-\tau_c(t)}$, available at time t at the controller node, let us consider the combined sequence of controls $\mathbf{u}^*$ formed by: $i)$ the subsequence used for estimating $\hat{x}_{t|t-\tau_c(t)}$ (i.e., the true control sequence $\mathbf{u}_{t-\tau_c(t),t-1}$applied by the NDC to the plant from $t-\tau_c(t)$ to $t-1$) and $ii)$ a feasible control sequence $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}$, that is

$$\mathbf{u}^*_{t-\tau_c(t),t+N_c-1|t-\tau_c(t)} \triangleq \mathrm{col}[\mathbf{u}_{t-\tau_c(t),t-1},\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}]. \tag{16}$$

Then, the prediction error $\hat{e}_{t-\tau_c(t)+i|t-\tau_c(t)} \triangleq x_{t-\tau_c(t)+i} - \hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)}$, with $i\in\{1,\ldots,N_c+\tau_c(t)\}$ and $x_{t-\tau_c(t)+i}$ obtained by applying $\mathbf{u}^*_{t-\tau_c(t),t+N_c-1|t-\tau_c(t)}$ in open loop to the uncertain system (8), is upper bounded by

$$|\hat{e}_{t-\tau_c(t)+i|t-\tau_c(t)}| \le \frac{L^i_{f_x}-1}{L_{f_x}-1}\overline{d},\ \ \forall i\in\{1,\ldots,N_c+\tau_c(t)\}$$

where $\overline{d}$ is defined as in Assumption 3. Being $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}$ feasible, it holds that $\hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)} \in X_i(\overline{d}), \forall i \in \{\tau_c(t)+1,\ldots,N_c+\tau_c(t)\}$, then it follows immediately that $x_{t-\tau_c(t)+i} = \hat{x}_{t-\tau_c(t)+i|t-\tau_c(t)} + \hat{e}_{t-\tau_c(t)+i|t-\tau_c(t)}\in X$. ■

In the next section, the robust stability properties of the described control policy will be analyzed in presence of transmission delays and model uncertainty.

IV. Recursive Feasibility and Regional Input-to-State Stability

In the following, the set invariance theory [3] will be used to prove the robust stability of the devised NCS. The following definition will be used .

Definition 4.1 ($\mathcal{C}_i(X,\Xi)$): Given a set $\Xi\subseteq X$, the *i-step Controllability Set* to Ξ, $\mathcal{C}_i(X,\Xi)$, is the set of states which can be steered to Ξ by an admissible control sequence of length i, $\mathbf{u}_{0,i-1}$, under the nominal map $\hat{f}(x,u)$, subject to constraints (9) and (10), i.e.

$$\mathcal{C}_i(X,\Xi)\triangleq\left\{\begin{matrix} x_0\in X: \exists\mathbf{u}_{0,i-1}\in U\times\ldots\times U \text{ such that} \\ \hat{x}(x_0,\mathbf{u}_{0,i-1},t)\in X,\ \forall t\in\{1,\ldots,i-1\}, \\ \text{and } \hat{x}(x_0,\mathbf{u}_{0,i-1},i)\in\Xi. \end{matrix}\right\}$$ □

In the sequel, the shorthand $\mathcal{C}_1(\Xi)$ will be used in place of $\mathcal{C}_1(\mathbb{R}^n,\Xi)$ to denote the one-step controllability set to Ξ.

Resorting to recursive feasibility arguments, the following Theorem states that the set X_{MPC}, is RPI under the c-l networked dynamics, w.r.t. bounded uncertainties.

Theorem 4.1 (Invariance of the feasible set): Assume that at time instant t the control sequence computed by the controller, $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}$, is feasible. Then, in view of Assumptions 2-6, if the norm bound on the uncertainty satisfies

$$\overline{d}\le\min_{k\in\{0,\overline{\tau}_c\}}\left\{\min\left(\frac{L_{f_x}-1}{L^{N_c+k}_{f_x}-L^{N_c-1}_{f_x}}\mathrm{dist}\,(\mathbb{R}^n\backslash\mathcal{C}_1(X_f),X_f),\ \frac{L_{f_x}-1}{L^{N_c+k}_{f_x}-1}\mathrm{dist}\left(\mathbb{R}^n\backslash\hat{X}_{k+N_c}(\overline{d}),X_f\right)\right)\right\}, \tag{17}$$

then, the recursive feasibility of the scheme in ensured for every time instant $t+i, \forall i\in\mathbb{Z}_{>0}$, while the closed-loop trajectories are confined into X. Hence, the feasible set X_{MPC} is RPI under the c-l networked dynamics w.r.t. bounded uncertainties. □

Proof: [Theorem 4.1] The proof consists in showing that if, at time t, the input sequence computed by the controller $\overline{\mathbf{u}}^c_{t,t+N_c-1|t-\tau_c(t)}$ is feasible, then, under the perturbed c-l dynamics, there exists a feasible control sequence at time instant $t+1$ (i.e., the VHOCP is solvable and the overall sequence verifies the prescribed constraints).Finally, the recursive feasibility follows by induction. First, notice that Points ii) and iii) of Assumption 6 together imply that $\mathrm{dist}(\mathbb{R}^n\backslash\mathcal{C}_1(X_f),X_f) \ge \delta > 0$. Now, the proof will be carried out in three steps.

i) $\hat{x}_{t+N_c|t-\tau_c(t)} \in X_f \Rightarrow \hat{x}_{t+N_c+1|t+1-\tau_c(t+1)} \in X_f$: Let us consider the sequence $\mathbf{u}^*_{t-\tau_c(t),t+N_c-1|t-\tau_c(t)}$ defined in (16). It is straightforward to prove that the norm difference between the predictions $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)}$ and $\hat{x}_{t-\tau_c(t)+j|t+1-\tau_c(t+1)}$ (initiated respectively by $x_{t-\tau_c(t)}$ and $x_{t+1-\tau_c(t+1)}$), respectively obtained by applying to the nominal model the sequence $\mathbf{u}^*_{t-\tau_c(t),t-\tau_c(t)+j-1|t-\tau_c(t)}$ and its subsequence $\mathbf{u}^*_{t+1-\tau_c(t+1),t-\tau_c(t)+j-1|t-\tau_c(t)}$, can be upper bounded by

$$|\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)+i}-\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)}| \le \frac{1}{L_{f_x}}\sum_{l=1}^{i}L^{j-l+1}_{f_x}\overline{d} = \frac{L^j_{f_x}-L^{j-i}_{f_x}}{L_{f_x}-1}\overline{d} \tag{18}$$

where we have posed $i=\tau_c(t)-\tau_c(t+1)+1$ and with $j\in\{i,\ldots,N_c+\tau_c(t)\}$. Considering now the case $j=N_c+\tau_c(t)$, then (18) yields to $|\hat{x}_{t+N_c|t-\tau_c(t)+i} - \hat{x}_{t+N_c|t-\tau_c(t)}| = |\hat{x}_{t+N_c|t+1-\tau_c(t+1)} - \hat{x}_{t+N_c|t-\tau_c(t)}| \le (L^{N_c+\tau_c(t)}_{f_x} - L^{N_c+\tau_c(t)-i}_{f_x})/(L_{f_x}-1)\overline{d}$. If the following inequality holds $\forall k\in\{1,\ldots,\overline{\tau}_c\}$

$$\overline{d} \le \frac{L_{f_x}-1}{L^{N_c+k}_{f_x}-L^{N_c-1}_{f_x}}\mathrm{dist}\,(\mathbb{R}^n\backslash\mathcal{C}_1(X_f),X_f)\,,$$

then $\hat{x}_{t+N_c|t+1-\tau_c(t+1)}\in\mathcal{C}_1(X_f)$, whatever be the values of $\tau_c(t)$ and $\tau_c(t+1)$. Hence, there exists a control move $\overline{u}_{t+N_c|t+1-\tau_c(t+1)} = \overline{u}_f(\hat{x}_{t+N_c|t+1-\tau_c(t+1)}) \in U$, with $\overline{u}_f:\mathcal{C}_1(X_f)\to U$ defined as

$$\overline{u}_f(x)\triangleq\arg\min_{u\in U:\hat{f}(x,u)\in X_f}\{|u-\kappa_f(x)|\}\,, \tag{19}$$

which can steer the state vector from $\hat{x}_{t+N_c|t+1-\tau_c(t+1)}$ to $\hat{x}_{t+N_c+1|t+1-\tau_c(t+1)} \in X_f$.

ii) $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)} \in X_j(\overline{d}) \Rightarrow \hat{x}_{t-\tau_c(t)+j|t+1-\tau_c(t+1)} \in X_{j-i}(\overline{d})$, with $i=\tau_c(t)-\tau_c(t+1)+1$ and $\forall j\in\{\tau_c(t)+1,\ldots,N_c+\tau_c(t)\}$: Consider the predictions $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)}$ and $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)+i}$(initiated respectively by $x_{t-\tau_c(t)}$ and $x_{t-\tau_c(t)+i}$), respectively obtained with the sequence $\mathbf{u}^*_{t-\tau_c(t),t-\tau_c(t)+j-1|t-\tau_c(t)}$ and its subsequence $\mathbf{u}^*_{t-\tau_c(t)+i,t-\tau_c(t)+j-1|t-\tau_c(t)}$. Assuming that $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)} \in X\backsim\mathcal{B}^n((L^j_{f_x}-1)/(L_{f_x}-1)\overline{d})$, let us introduce $\eta\in\mathcal{B}^n((L^{j-i}_{f_x}-1)/(L_{f_x}-1)\overline{d})$. Let $\xi\triangleq\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)+i}-\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)}+\eta$, then, in view of Assumption 2 and thanks to (18), it follows that

$$|\xi|\leq|\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)+i}-\hat{x}_{t-\tau_{rt}(t)+j|t-\tau_c(t)}|+|\eta| \leq\frac{L_{f_x}^{j}-1}{L_{f_x}-1}\overline{d}, \quad (20)$$

and hence, $\xi \in \mathcal{B}^n\big((L_{f_x}^{j}-1)/(L_{f_x}-1)\overline{d}\big)$. Since $\hat{x}_{t-\tau_c(t)+j|t}\in X_j(\overline{d})$, it follows that $\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)}+\xi=\hat{x}_{t-\tau_c(t)+j|t-\tau_c(t)+i}+\eta\in X$, $\forall\eta\in\mathcal{B}^n((L_{f_x}^{j-i}-1)/(L_{f_x}-1)\overline{d})$, yielding to $\hat{x}_{t-\tau_c(t)+j|t+1-\tau_c(t+1)}\in X_{j-\tau_c(t)+\tau_c(t+1)-1}(\overline{d})$.

iii) $\hat{x}_{t+N_c|t-\tau_c(t)}\in X_f \Rightarrow \hat{x}_{t+1+N_c|t+1-\tau_c(t+1)}\in X_{N_c+\tau_c(t+1)}(\overline{d})$; Thanks to Point i), there exists a feasible control sequence at time $t+1$ which yields to $\hat{x}_{t+1+N_c|t+1-\tau_c(t+1)}\in X_f$. If $\overline{d}$ satisfies

$$\overline{d}\leq\min_{j\in\{N_c,\ldots,N_c+\overline{\tau}_c\}}\left\{\frac{L_{f_x}-1}{L_{f_x}^{j}-1}\mathrm{dist}(\mathbb{R}^n\backslash X_j(\overline{d}),X_f)\right\},$$

it follows that $\hat{x}_{t+1+N_c|t+1-\tau_c(t+1)}\in X_{N_c+\tau_c(t+1)}$, whatever be the value of $\tau_c(t+1)$.

Then, under the assumptions posed in the statement of Theorem 4.1, given $x_0\in X_{MPC}$, and being $\tau_c(0)=0$ (i.e. at the first time instant the actuator buffer is initiated with a feasible sequence)in view of Points i)–iii) it holds that at any time $t\in\mathbb{Z}_{>0}$ a feasible control sequence does exist and can be chosen as $\overline{\mathbf{u}}^c_{t+1,t+N_c+1|t+1-\tau_c(t+1)}=\mathrm{col}[\overline{\mathbf{u}}^c_{t+1,t+N_c-1|t-\tau_c(t)},\overline{u}_{t+N_c|t+1-\tau_c(t+1)}]$. Therefore the recursive feasibility of the scheme is ensured. ■

Next, we will show that the devised NCS is Regional ISS w.r.t. bounded uncertainties.

Theorem 4.2 (Regional Input-to-State Stability): Under Assumptions 2-6, if the bound on uncertainties verifies (17), then system (12), controlled by the proposed MPC–NDC strategy, is regional ISS in X_{MPC} with respect to additive perturbations $d_t\in\mathcal{B}^n(\overline{d})$. □

Proof: [Theorem 4.2] Recalling that we have posed the assumption that, at time $t=0$, the FIB contains a feasible control sequence, then, in a worst case situation, the system will be driven in open-loop for $\overline{\tau}_a$ time instants. With regard to the ISS property, this observation implies that the bound on the trajectories after $\overline{\tau}_a$ should depend on $x_{\overline{\tau}_a}$ and the regional ISS inequality (2) has to be modified as follows

$$|x(t+\overline{\tau}_a,\overline{x}_{\overline{\tau}_a},\boldsymbol{\upsilon})| \leq\max\big\{\beta(|\overline{x}_{\overline{\tau}_a}|,t),\gamma(\|\boldsymbol{\upsilon}_{[t+\overline{\tau}_a-1]}\|)\big\},\forall t\in\mathbb{Z}_{\geq0},\forall\overline{x}_{\overline{\tau}_a}\in\Xi,$$

where $\overline{x}_{\overline{\tau}_a}$ is the state at time $\overline{\tau}_a$ after the system has been driven for $\overline{\tau}_a$ steps by the open-loop policy stored in the buffer at time $t=0$. In view of previous consideration, the proof consists in showing that there exist a ISS-Lyapunov function $V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})$ for the closed-loop system.

To this end, let us define the following positive-definite function $V^\circ:\mathbb{R}^n\to\mathbb{R}_{\geq0}$

$$V^\circ(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}) \triangleq J_{FH}\Big(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)},\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)},N_c-\overline{\tau}_a\Big) \quad (21)$$

where $\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}=\hat{x}(t+\tau_a,x_{t-\tau_c(t)},\mathbf{u}_{t-\tau_c(t),t+\overline{\tau}_a-1})$ is a prediction obtained with the nominal model initiated with $x_{t-\tau_c(t)}$. Notice that V° corresponds to the optimal cost subsequent to the reduced horizon optimization. Now, consider the following candidate ISS-Lyapunov function $V:\mathbb{Z}_{\geq0}\times\mathbb{R}^n\to\mathbb{R}_{\geq0}$

$$\begin{aligned}&V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})\\&\triangleq J_{FH}\Big(x_{t+\overline{\tau}_a},\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)},N_c-\overline{\tau}_a\Big)\\&=\sum_{l=t+\overline{\tau}_a}^{t+N_c-1}h\Big(\hat{x}_{l|t+\overline{\tau}_a},u^\circ_{l|t-\tau_c(t)}\Big)+h_f(\hat{x}_{t+N_c|t+\overline{\tau}_a})\end{aligned} \quad (22)$$

where $\hat{x}_{t+\overline{\tau}_a+j|t+\overline{\tau}_a}$, $j\in\{1,\ldots,N_c-\overline{\tau}_a\}$ are obtained using the nominal model initialized with $\hat{x}_{t+\overline{\tau}_a|t+\overline{\tau}_a}=x_{t+\overline{\tau}_a}$ and the sequence $\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}$ (which is optimal for $\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}$ and not for $x_{t+\overline{\tau}_a}$). Notice that, since $\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}$ is not computed in correspondence of $x_{t+\tau_a}$, but exploiting a past state information $x_{t-\tau_c(t)}$, V becomes a time-varying function of the state. We will show in the following that $V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})$ verifies the ISS inequalities with time-invariant bounds.

Suppose, without loss of generality[2], that $L_{f_x}\neq1$. Now, let us point out that, in view of (18), the inclusion $x_{t+\overline{\tau}_a}\in\Omega\triangleq X_f\backsim\mathcal{B}^n((L_{f_x}^{\overline{\tau}_c+\overline{\tau}_a}-1)/(L_{f_x}-1)\overline{d})$ implies $\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}\in X_f$ whatever be the value of $\tau_c(t)$. Then, by Assumption 6, the control sequence $\tilde{\mathbf{u}}_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}\triangleq\mathrm{col}[\kappa_f(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}),\kappa_f(\hat{x}_{t+\overline{\tau}_a+1|t-\tau_c(t)}),\ldots,\kappa_f(\hat{x}_{t+N_c-1|t-\tau_c(t)})]$ is feasible for the RHOCP, hence the set X_{MPC} is not empty.

Now, our objective consists in finding a suitable comparison function to upper bound the candidate ISS-Lyapunov function $V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})$. By adding and subtracting $V^\circ(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)})$ to the right-hand side of (21), we obtain

$$\begin{aligned}&V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})\\&\leq\sum_{l=t+\overline{\tau}_a}^{t+N_c-1}h\Big(\hat{x}_{l|t+\overline{\tau}_a},u^\circ_{l|t-\tau_c(t)}\Big)-h\Big(\hat{x}_{l|t-\tau_c(t)},u^\circ_{l|t-\tau_c(t)}\Big)\\&+h_f(\hat{x}_{t+N_c|t+\overline{\tau}_a})-h_f(\hat{x}_{t+N_c|t-\tau_c(t)})\\&+J^\circ_{FH}\Big(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)},\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)},N_c-\overline{\tau}_a\Big),\end{aligned} \quad (23)$$

Considering that $\sum_{l=t+\overline{\tau}_a}^{t+N_c-1}h\left(\tilde{x}_{l|t-\tau_c(t)},\tilde{u}_{l|t-\tau_c(t)}\right)+h_f(\tilde{x}_{t+N_c|t-\tau_c(t)}\leq h_f(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)})$, then the following bound can be established

$$\begin{aligned}&J_{FH}\Big(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)},\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)},N_c-\overline{\tau}_a\Big)\\&\leq h_f(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)})-h_f(x_{t+\overline{\tau}_a})+h_f(x_{t+\overline{\tau}_a})\\&\leq L_{h_f}\frac{L_{f_x}^{\overline{\tau}_{rt}}-1}{L_{f_x}-1}\|\boldsymbol{d}_{[t+\overline{\tau}_a-1]}\|+h_f(x_{t+\overline{\tau}_a}).\end{aligned} \quad (24)$$

Finally, in view of Assumptions 2,5 and thanks to (18) and (24), the following inequality holds

$$V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})\leq\alpha_1(|x_{t+\overline{\tau}_a}|)+\sigma_1(\|\boldsymbol{d}_{[t+\overline{\tau}_a-1]}\|), \quad \forall x_{t+\overline{\tau}_a}\in X_f,\forall\mathbf{d}\in\mathcal{M}_{\mathcal{B}^n(\overline{d})} \quad (25)$$

where

$$\alpha_1(s)\triangleq L_{h_f}|s|$$

$$\sigma_1(s)\triangleq\frac{L_{f_x}^{\overline{\tau}_{rt}}-1}{L_{f_x}-1}\left[L_h\frac{L_{f_x}^{N_c-\overline{\tau}_a}-1}{L_{f_x}-1}+L_{h_f}L_{f_x}^{N_c-\overline{\tau}_a-1}+L_{h_f}\right]s$$

The lower bound on $V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})$ can be easily obtained using Assumption 5

$$V(t+\overline{\tau}_a,x_{t+\overline{\tau}_a})\geq\underline{h}(x_{t+\overline{\tau}_a}),\ \forall x_{t+\tau_a}\in X_{MPC} \quad (26)$$

Then, in view (25) of (26), the ISS inequalities (3) and (4) hold respectively with $\Xi=X_{MPC}$ and $\Omega=X_f\backsim\mathcal{B}^n((L_{f_x}^{\overline{\tau}_{rt}}-1)/(L_{f_x}-1)\overline{d})$. Moreover, in view

[2]The case $L_{f_x}=1$ can be trivially addressed with a few suitable modification to the proof of theorem 4.2

of Point i) in the proof of Theorem 4.1, given the (feasible) control sequence computed at time t, $\mathbf{u}^c_{t,t+N_c-1|t-\tau_c(t)} = \text{col}[\mathbf{u}^c_{t,t+\overline{\tau}_a-1|t-1-\tau_c(t-1)}, \mathrm{u}^\circ_{t+\overline{\tau}_a,t+N_c-1}]$, the sequence $\bar{\mathbf{u}}^c_{t+1,t+N_c|t+1-\tau_c(t+1)} = \text{col}[\mathbf{u}^c_{t+1,t+N_c-1|t-\tau_c(t)}, \bar{u}_f(\hat{x}_{t+N_c|t+1-\tau_c(t+1)})]$, with $\bar{u}_f(\cdot)$ defined as in (19), is a feasible sequence (in general, suboptimal) at time $t+1$. The subsequence $\mathbf{u}^c_{t+\overline{\tau}_a+1,t+N_c|t-\tau_c(t)}$ along the reduced horizon gives rise to a cost which verifies the inequality

$$\begin{aligned}
&J_{FH}\Big(\hat{x}_{t+\overline{\tau}_a+1|t+1-\tau_c(t+1)},\mathbf{u}^c_{t+\overline{\tau}_a+1,t+N_c|t-\tau_c(t)}, N_c-\overline{\tau}_a\Big)\\
&\le J_{FH}\Big(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)},\mathbf{u}^\circ_{t+\overline{\tau}_a,t+N_c-1|t-\tau_c(t)}, N_c-\overline{\tau}_a\Big)\\
&-h\Big(\hat{x}_{t+\overline{\tau}_a|t-\tau_c(t)}, u^\circ_{t|t-\tau_c(t)}\Big)\\
&+\sum_{l=t+\overline{\tau}_a+1}^{t+N_c-1} h\Big(\hat{x}_{l|t+1-\tau_c(t+1)},u^\circ_{l|t-\tau_c(t)}\Big)-h\Big(\hat{x}_{l|t-\tau_c(t)},u^\circ_{l|t-\tau_c(t)}\Big)\\
&+h\big(\hat{x}_{t+N_c|t+1-\tau_c(t+1)}, \bar{u}_f(\hat{x}_{t+N_c|t+1-\tau_c(t+1)})\big)\\
&+h_f\big(\hat{x}_{t+N_c+1|t+1-\tau_c(t+1)}\big)-h_f\big(\hat{x}_{t+N_c|t-\tau_c(t)}\big)
\end{aligned} \tag{27}$$

Now, by (23), in view of Assumptions 2,5, we have that

$$\begin{aligned}
&\Big|J_{FH}\Big(\hat{x}_{t+\overline{\tau}_a+1|t+1-\tau_c(t+1)},\mathbf{u}^\circ_{t+\overline{\tau}_a+1,t+N_c|t-\tau_c(t)},N_c-\overline{\tau}_a\Big)\\
&\quad -V(t+\overline{\tau}_a+1, x_{t+\overline{\tau}_a+1})\Big|\\
&\le \frac{L_{f_x}^{\overline{\tau}_{rt}}-1}{L_{f_x}-1}\left[L_h\frac{L_{f_x}^{N_c-\overline{\tau}_a}-1}{L_{f_x}-1}+L_{h_f}L_{f_x}^{N_c-\overline{\tau}_a-1}\right]||\boldsymbol{d}_{[t+\overline{\tau}_a]}||.
\end{aligned} \tag{28}$$

Moreover, in view of Point vi) of Assumption 6 and thanks to Assumption 7, it follows that

$$\begin{aligned}
&h\big(\hat{x}_{t+N_c|t+\overline{\tau}_a+1}, \bar{u}_f(\hat{x}_{t+N_c|t+\overline{\tau}_a+1})\big)\\
&\quad +h_f\big(\hat{x}_{t+N_c+1|t+\overline{\tau}_a+1}\big)-h_f\big(\hat{x}_{t+N_c|t+\overline{\tau}_a}\big)\\
&\qquad \le h_f\big(\hat{x}_{t+N_c|t+\overline{\tau}_a+1}\big)-h_f\big(\hat{x}_{t+N_c|t+\overline{\tau}_a}\big)\\
&\qquad \le L_{h_f}L_{f_x}^{N_c-\overline{\tau}_a-1}||\boldsymbol{d}_{[t+\overline{\tau}_a]}||
\end{aligned} \tag{29}$$

Finally, considering (27), (28) and (29) and by using Point v) of Assumption 6, the third ISS inequality can be obtained

$$\begin{aligned}
&V(t+\overline{\tau}_a+1, x_{t+\overline{\tau}_a+1})-V(t+\overline{\tau}_a, x_{t+\overline{\tau}_a})\\
&\quad\le -\alpha_2(|x_{t+\overline{\tau}_a}|)+\sigma_2(||\boldsymbol{d}_{[t+\overline{\tau}_a]}||),
\end{aligned} \tag{30}$$

$\forall x_{t+\overline{\tau}_a}\in X_{MPC}, \forall \boldsymbol{d}\in\mathcal{M}_{\mathcal{B}^n(\bar{d})}$, with

$$\begin{aligned}
&\alpha_2(s)\triangleq\underline{h}(s)\\
&\sigma_2(s)\triangleq\Bigg[L_h\frac{L_{f_x}^{\overline{\tau}_{rt}}-1}{L_{f_x}-1}+L_h\frac{L_{f_x}^{\overline{\tau}_c}-1}{L_{f_x}-1}\frac{L_{f_x}^{N_c-\overline{\tau}_a}-1}{L_{f_x}-1}+L_{h_f}L_{f_x}^{N_c-\overline{\tau}_a-1}\\
&\qquad +2\frac{L_{f_x}^{\overline{\tau}_{rt}}-1}{L_{f_x}-1}\left(L_h\frac{L_{f_x}^{N_c-\overline{\tau}_a}-1}{L_{f_x}-1}+L_{h_f}L_{f_x}^{N_c-\overline{\tau}_a-1}\right)\Bigg].
\end{aligned}$$

Finally, in view of (25), (26) and (30), it is possible to conclude that the closed-loop system is regional ISS in X_{MPC} with respect to $d\in\mathcal{B}^n(\bar{d})$. ■

Remark 4.1 (UDP-like networks): In the case of UDP-like networks, no ACKs are sent by the actuator node to the controller. In this scenario, the problem of delayed arrival of packeted input sequences to the actuator (which may lead to wrong open-loop predictions at the controller side, due to the fact that the truly applied input sequence is not known), could represent a major source of uncertainty, if no proper provisions are adopted. Thus, with the aim to recast the formulation in a deterministic framework, such that the sequence used by the controller to obtain $\hat{x}_t$ would coincide with the true input sequence applied by the actuator to the plant from time $t-\tau_c(t)$ to $t-1$, a possible solution consists in further shortening the optimization horizon w.r.t. the TCP-like case. In this set up, being N_c the length of the sequence computed by the controller to be forwarded to the actuator, the optimization is performed over the $N_c-\overline{\tau}_{rt}$ subsequence $\mathbf{u}_{t+\overline{\tau}_{rt},t+N_c-1|t-\tau_c(t)}$, and initiated with the predicted state $\hat{x}_{t+\overline{\tau}_{rt}|t-\tau_c(t)}$. The input sequence used to obtain $\hat{x}_{t+\overline{\tau}_{rt}|t-\tau_c(t)}$ is

$$\begin{aligned}
&\mathbf{u}^*_{t-\tau_c(t),t+\overline{\tau}_{rt}-1}\\
&\quad=\text{col}[\mathbf{u}^*_{t-\tau_c(t),t-2}, \mathbf{u}^c_{t-1,t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)}]
\end{aligned} \tag{31}$$

where $\mathbf{u}^*_{t-\tau_c(t),t-2}$ and $\mathbf{u}^c_{t,t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)}$ are respectively a subsequence of $\mathbf{u}^*_{t-1-\tau_c(t-1),t+N_c-2}$ and a subsequence of the control sequence $\mathbf{u}^c_{t-1,t+N_c-2|t-1-\tau_c(t-1)}$ computed at time $t-1$. At this point, noting that the first $\overline{\tau}_{rt}$ elements of $\mathbf{u}^c_{t-1,t+N_c-2|t-1-\tau_c(t-1)}$ coincide with the subsequence $\mathbf{u}^*_{t-1,t+\overline{\tau}_{rt}-2}$, then (31) can be rearranged as

$$\begin{aligned}
&\mathbf{u}^*_{t-\tau_c(t),t+\overline{\tau}_{rt}-1}\\
&\quad=\text{col}[\mathbf{u}^*_{t-\tau_c(t),t+\overline{\tau}_{rt}-2}, u^c_{t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)}]
\end{aligned}$$

where $u^c_{t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)}$ is the first element of the optimized sequence $\mathbf{u}^\circ_{t+\overline{\tau}_{rt}-1,t+N_c-\overline{\tau}_{rt}-2|t-1-\tau_c(t-1)}$, obtained by solving the RHOCP at time $t-1$; i.e., $u^c_{t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)} = u^\circ_{t+\overline{\tau}_{rt}-1|t-1-\tau_c(t-1)}$. By this position, with suitable few modifications to the proof Theorem 4.1, we can show that the proposed scheme is robustly recursively feasible in the UDP-like framework.

Remarkably, the further shortening of the optimization horizon may reduce the degree of optimality of the control action w.r.t. the TCP-like formulation. □

V. Example

In order to show the effectiveness of the devised control scheme, the closed-loop behavior of the following nonlinear system (forward-Euler discretized version of an undamped single-link flexible-joint pendulum) is simulated first in nominal conditions and then under the simultaneous presence of model uncertainty and unreliable communications between sensors, controller, and actuators

$$\begin{cases}
x_{(1)_{t+1}}=x_{(1)_t}+T_s x_{(2)_t}\\
x_{(2)_{t+1}}=x_{(2)_t}-\dfrac{T_s}{I}\big[MgL\sin(x_{(1)_t})+k\big(x_{(1)_t}-x_{(3)_t}\big)\big]\\
x_{(3)_{t+1}}=x_{(3)_t}+T_s x_{(4)_t}\\
x_{(4)_{t+1}}=x_{(4)_t}+\dfrac{T_s}{J}\left[k\left(x_{(1)_t}-x_{(3)_t}\right)+u\right]\\
x_0=\overline{x},\ t\in\mathbb{Z}_{\ge 0}
\end{cases} \tag{32}$$

where $x_{(i)_t}, i\in\{1,\ldots,4\}$ denotes the i-th component of the vector x_t, $T_s=0.05\,s$ is the sampling interval, $I=0.25\,kg\cdot m^2$ the inertia of the arm, $J=2\,kg\cdot m^2$ the rotor inertia, $g=9.8\,ms^2$ the gravitational acceleration, $M=1\,kg$ the mass of the link, $L=0.5\,m$ the distance between the rotational axis and the center of gravity of the pendulum-arm, $k=20\,N\cdot m/rad$ the stiffness coefficient of the link. The control objective consists in stabilizing the system toward the (open-loop unstable) 0-state equilibrium, while keeping in the trajectories within prescribed bounds depicted in Figure 2 (green).

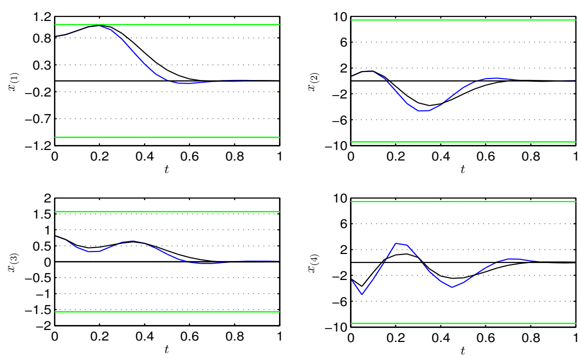
The following auxiliary linear controller is used $\kappa_f(x) = [-55.92\ \ -7.46\ 124.01\ 19.22]\cdot x$, with $X_f=\{x\in\mathbb{R}^4 : x^T\cdot P_f\cdot x\le 1\}$, $h_f(x)=10^3(x^T\cdot P_f\cdot x)$ and

$$P_f = 10^3\begin{bmatrix} 1.3789 & -0.0629 & -1.7904 & -0.1508\\ -0.0629 & 0.0186 & 0.1404 & 0.0074\\ -1.7904 & 0.1404 & 3.1580 & 0.2216\\ -0.1508 & 0.0074 & 0.2216 & 0.0292\end{bmatrix}$$

The predictive controller has been set up with control sequence length $N_c = 12$, and quadratic stage cost $h(x) = x^T \cdot Q \cdot x + Ru^2$, where $Q = \text{diag}(10, 0.1, 0.1, 0.1)$ and $R = 10^{-3}$.

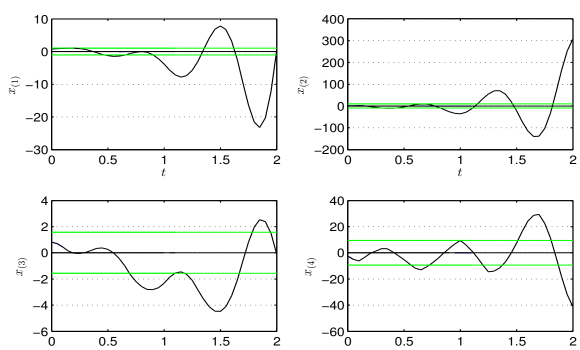
In the uncertain/unreliable networked scenario, a TCP-like protocol has been simulated, with delay bounds $\overline{\tau}_c = \overline{\tau}_a = 5$, while the nominal model is subjected to the following parametric uncertainty $M_{nom} = 1.05M$. Figure 2 shows the trajectories of the state variables in the nominal case (black) and in the uncertain/delayed conditions (blue). Notably, the constraints are fulfilled and the recursive feasibility of the scheme is guaranteed even in the networked case.

Fig. 2 Trajectories of the state variables of system (32) controlled by the combined strategy MPC+NDC over an unreliable network with uncertainty (blue : $\overline{\tau}_c = \overline{\tau}_a = 5$) and in nominal conditions (black : $\overline{\tau}_c = \overline{\tau}_a = 0$).



At the opposite, if a network delay compensation strategy is not used, then system (32), controlled by a nominal MPC, becomes unstable even for small delays $\overline{\tau}_c = \overline{\tau}_a = 2$, Figure 3.

Fig. 3 Trajectories of the state variables for system (32) controlled by a nominal NMPC, without delay compensation ($\overline{\tau}_c = \overline{\tau}_a = 2$). Feasibility gets lost and instability occurs.



Conclusion

In this paper a networked control system, based on the combined use of a model predictive controller with a network delay compensation strategy, has been designed with the aim to stabilize toward an equilibrium a constrained nonlinear discrete-time system, affected by unknown perturbations and subject to delayed packet-based communications in both sensor-to-controller and controller-to-actuator links. The characterization of the robust stability properties of the devised scheme represents a significant contribution in the context of nonlinear networked control systems, since it establishes the possibility to enforce the robust satisfaction of constraints under unreliable networked communications in the feedback and command channels, also presence of model uncertainty. In particular, sufficient condition to ensure the recursive feasibility of the scheme have been determined. Finally, by exploiting a novel characterization of the regional Input-to-State Stability in terms of time-varying Lyapunov functions, the closed-loop system has been shown to be regional Input-to-State stable with respect to bounded perturbations.

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