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Duty Ratio Control of a Rotary PWM Valve with Periodic Measurement Error

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Abstract— **The focus of this paper is on estimating system states in the presence of repetitive measurement noise. This estimation technique is applied to a novel self-spinning high speed rotary on/off valve that is being developed for pulse width modulating (PWM) fluid flow. The PWM duty ratio is determined by the valve spool's axial position. Unfortunately, the optical position measurement is corrupted by a repetitive noise, induced by the spool's rotary motion. Two models are developed to represent the periodic noise: one discrete time time invariant model based on internal model principle and a continuous time model that uses a set of periodic basis functions. Kalman filters are designed to estimate the spool position and the periodic noise. This estimates are used with a PI with feed-forward controller for the spool position reference tracking. Simulation and experimental results indicate the usefulness of estimating the periodic noise.**

I. INTRODUCTION

A novel self-spinning rotary high speed on/off valve, shown in Fig.1, has been proposed in [1]. The valve was designed to increase the overall hydraulic system efficiency by eliminating throttling through partially open control valves. The valve is composed of a rotating spool driven by the fluid flow, and a stationary sleeve. The spool consists of a center section and two outlet turbines. The center section contains helical barriers, which act as turbine blades, to capture the fluid angular momentum. Two outlet turbines reverse the flow direction relative to the inlet, and generate a reaction torque on the spool to aid its rotary motion. When the spool rotates, flow is apportioned to application (on) or to reservoir (off) by the barrier. Instead of partially opening the valve, the on/off valve modulates flow by modulating the duty ratio of the on/off times. Since the valve is nearly either fully on or fully off, throttling losses are minimized. The duty ratio is determined by the axial position of the spool relative to the rhombus inlet nozzle on the sleeve. The axial position of the spool is actuated hydro-statically using a small electro-hydraulic gerotor pump that is hydraulically connected to both ends of the valve sleeve. By pumping fluid from one end of the sleeve to the other end, the spool axial position is varied. The differential pressure needed to actuate the spool is less than $30psi$, and the power needed for this actuation is small. The spool axial position is measured using an

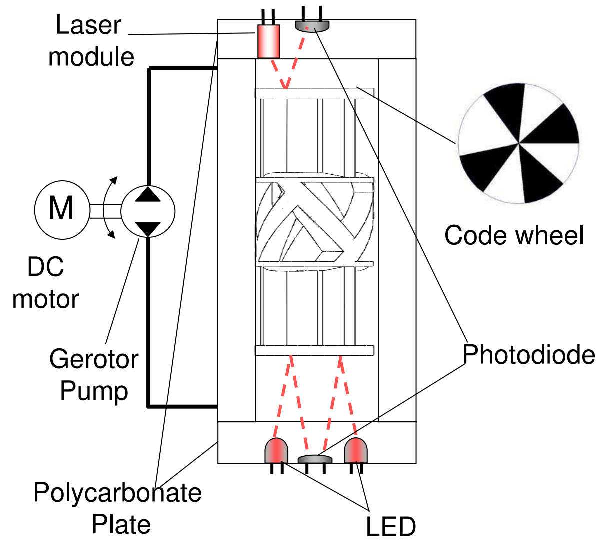

Fig. 1. System schematic

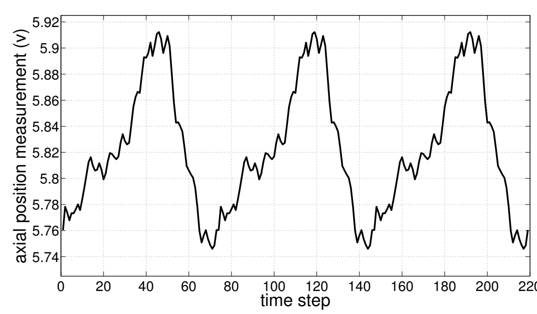

Fig. 2. Optical sensor output for a fixed spool axial position

optical sensor, which consists of two LEDs and a photodiode. Light emitted from the LEDs reflects off of the polished spool end, and is received by the photodiode. The LED light intensity varies monotonically with the light traveling distance. The spool axial position can be mapped to the photodiode response statically. Since the spool end can not be polished uniformly, rotary motion introduces a periodic measurement noise, as shown in Fig.2. The only information we have about this noise is that it has the same period as the

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spool rotary motion. In addition to this structured noise, other kind of noises exist as well, including electrical noise, effect of oil temperature on optical sensors, effect of air entrained in oil on light intensity, etc.

Distinguishing between the spool state and the periodic measurement noise is important. Otherwise, the actuator would respond to the corrupted tracking error, and correspondingly cause the spool to oscillate. This causees the DC motor to switch direction at a high frequency and wastes a lot of energy. In this paper, we develop approaches to estimating the periodic noise and canceling it out before feeding back the spool position state for control. Two approaches of modeling the periodic measurement noise were introduced in this paper: one is a discrete time time-invaraint periodic signal exo-generator, and the other one is a continuous time model that uses a set of weighted periodic basis functions.

In the next section, system model and the two periodic noise models will be presented. Procedure of designing a Kalman filter and a PI with feed-forward controller will be discussed in sections III and IV. Simulation and experimental results will be presented in section V. Concluding remarks and future research plans will be discussed in section VI.

II. SYSTEM MODELING

A. Spool Dynamics

A schematic of the system studied in this paper is shown in Fig.1. The fluid is treated as incompressible, and the spool dynamics is modeled as:

$$\dot{x} = \frac{Q(u)}{A_s} \tag{1}$$

x is the spool axial position, $Q(u)$ is the flow rate acting on the spool, and A_s is the spool end area.

The actuator is a small gerotor pump powered by a DC motor. The speed and the direction of the motor are controlled using an H-bridge. By pulse width modulating the enable signal to the H-bridge, we can control the motor speed, and therefore control the pump flow rate. The pump unit dynamics exhibits dead-band, saturation and asymmetry. The static relationship between input u and flow $Q(u)$ is calibrated experimentally.

The axial position, measured by the optical sensor, is corrupted by a periodic noise $d(t) = d(t+T)$ and an unstructured noise $n(t)$. $n(t)$ is modeled as a white noise with zero mean.

$$y(t) = x(t) + d(t) + n(t) \tag{2}$$

Since the actual spool position is taken to be the mean of the measurement, the noise d will be a repetitive signal with zero mean:

$$\int_t^{t+T} d(\tau)\cdot d\tau = 0$$

Modeling of d affects the structure of the estimator. The idea is to form the state space formulation of the discrete time noise model, so that it can be estimated from the measurement. If the spool rotates at a constant angular velocity, this noise can be represented as a function of time, and it has the same period as the rotary motion: Here we introduce two d models:

B. Repetitive Noise Dynamics

1) Model 1: In this model, the repetitive noise $d(t)$ is first discretized and then modeled using a time invariant exo-generator. Let $d(k) = d(k+T_s)$ be the discretized version of $d(t)$ with T_s being the sampling time. The discrete time period is $N = T/T_s$ (assuming that N is an integer). We define the zero mean noise exo-generator as:

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) \\ d(k) &= C_d x_d(k) \end{aligned} \tag{3}$$

with

$$A_d = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -1 & -1 & \cdots & -1 & -1 \end{pmatrix} \in \mathbb{R}^{N-1\times N-1}$$
$$C_d = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \in \mathbb{R}^{1\times N-1}$$

Notice that for any $x_d(0) \in \mathbb{R}^{N-1}$, $\sum_{k=1}^{N} d(k)$ is zero mean. The spool dynamics (1) are discretized as:

$$x(k+1) = x(k) + T_s \frac{Q(u(k))}{A_s} \tag{4}$$

Using (2) and the above, an augmented system combining the system state and the measurement noise states is:

$$\underbrace{\begin{pmatrix} x \\ x_d \end{pmatrix}}_{x_{aug1}}(k+1) = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & A_d \end{pmatrix}}_{A_{aug1}} \begin{pmatrix} x(k) \\ x_d(k) \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{T_s}{A_s} \\ 0 \end{pmatrix}}_{B_{aug1}} Q(u(k)) + w(k)$$
$$y(k) = \underbrace{\begin{pmatrix} 1 & C_d \end{pmatrix}}_{C_{aug1}} \begin{pmatrix} x(k) \\ x_d(k) \end{pmatrix} + n(k) \tag{5}$$

$x_{aug1} \in \mathbb{R}^{\mathrm{N}\times 1}$, $A_{aug1} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$, and $C_{aug1} \in \mathbb{R}^{1\times\mathrm{N}}$. The observability matrix of the current augmented system is $\mathrm{O} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$:

$$\mathrm{O} = \begin{pmatrix} C_{aug1} \\ C_{aug1}A_{aug1} \\ \vdots \\ C_{aug1}A_{aug1}^N \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 1 \\ 1 & -1 & -1 & \cdots & -1 \end{pmatrix}$$
$$det(\mathrm{O}) = (-1)^{(N+1)} N \neq 0$$

The observability matrix O is of full rank, and the pair (A_{aug1}, C_{aug1}) is observable. If d is not assumed to be zero mean, it will be impossible to distinguish the true spool state from the DC component of the disturbance, and the augmented system will be unobservable.

Using this approach, the dimension of x_d is typically very large. $N = \frac{T}{Ts}$, and T is the noise period. In our case $N \approx 100$. This leads to a high dimensional estimator which is computationally expensive and may not be very robust.

2) Model 2: In the second model, the periodic noise signal is represented by a linear combination of periodic basis functions $f_i(t) = f_i(t+T)$:

$$d(t) = \sum_i w_i f_i(t) \tag{6}$$

where w_i are the weights. A variety of basis functions can be implemented depending on the structure of the periodic noise. Compared with the first approach, this one is more flexible. Given a set of basis functions, the weights are defined as the states of the repetitive noise dynamics, while the basis functions will form the "C" matrix in $d(t) = C(t)x_d$. "C" is a periodic time varying matrix.

In this paper, we select Fourier series expansion, but instead of using an infinite number of harmonic frequencies, the first several terms are used to represent the periodical signal. Express it using the state space form:

$$d(t) = \underbrace{\begin{pmatrix}\cos(\omega t) & \sin(\omega t) & \cdots & \cos(n\omega t) & \sin(n\omega t)\end{pmatrix}}_{C_{d2}(t)} \underbrace{\begin{pmatrix} x_{d1} \\ \vdots \\ x_{d2n} \end{pmatrix}}_{x_d}$$

Notice that this model automatically ensures that $d(t)$ is zero mean.

An investigation was first conducted to determine the number of basis functions required.

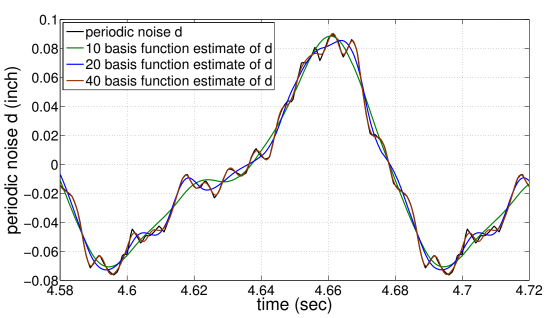

Fig. 3. Repetitive noise and its estimate using different number basis functions

We present the corrupted measurement of the spool axial position in Fig. 2 in section I. The DC component of the measurement represents the spool axial position. By extracting it from the noise, we get the repetitive noise as shown in Fig. 3. This repetitive noise is estimated using three groups of basis functions. They are designed using the same fundamental frequency, and they consist of 10, 20, and 40 basis functions respectively. The 40 basis functions model can capture more structure of the periodic noise compared with the other two groups, while 10 and 20 basis functions models do not exhibit a significant difference. The estimation error is not very sensitive to the number of basis function selected. To reduce the dimension of the repetitive noise dynamics while maintaining the estimation resolution, we use 10 basis functions in the simulations and the experiments below.

The augmented system combining the spool dynamics and the noise dynamics is a periodic system:

$$\frac{d}{dt}\underbrace{\begin{pmatrix} x \\ x_d \end{pmatrix}}_{x_{aug2}} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{A_{aug2}} \begin{pmatrix} x \\ x_d \end{pmatrix} + \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_{aug2}} \frac{Q}{A_s} + w$$

$$y = \underbrace{\begin{pmatrix} 1 & C_d(t) \end{pmatrix}}_{C_{aug2}(t)} \begin{pmatrix} x \\ x_d \end{pmatrix} + n \tag{7}$$

Checking the observability of the above augmented system by checking the observability grammian over one period:

$$\begin{aligned} M(0,T) &= \int_0^T \Phi^{\mathrm{T}}(t,0) C_{aug2}^{\mathrm{T}} C_{aug2}(t) \Phi(t,0) dt \\ &= \int_0^T C_{aug2}(t)^{\mathrm{T}} C_{aug2}(t) dt \end{aligned}$$

$$det(M(0,T)) = \left(\frac{T}{2}\right)^{2N} > 0 \tag{8}$$

Therefore, the states can be uniquely determined from the measurement through a proper state estimator.

III. STATE ESTIMATOR

In both augmented systems (5) and (7), we assume the process noise w and the un-modeled measurement noise n to be white noise with zero mean. A Kalman filter is designed as the state estimator.

For periodic noise model 1, since the augmented system is modeled in discrete time form, a discrete time Kalman filter is designed. Dimension of the estimator increases linearly as the sampling time decreases for a fixed spool rotating period. This may lead to a slow and un-robust Kalman filter dynamics.

$$\begin{aligned} \hat{x}_{aug1}^{-}(k) &= A_{aug1}\hat{x}_{aug1}(k-1) + B_{aug1}Q(u)(k-1) \\ \hat{x}_{aug1}^{+}(k) &= \hat{x}_{aug1}^{-}(k) + L(k)(y(k) - C_{aug1}(k)\hat{x}_{aug1}^{-}(k)) \end{aligned} \tag{9}$$

$L(k)$ is computed from solving a discrete time Riccati equation [2].

For periodic noise model 2, the dimension of the noise model is not a function of the sampling time. By properly selecting the type of basis functions, dimension of x_d can be optimized, and it allows us to choose a small sampling time. The same formulation can be easily adapted to other periods. A continuous time varying Kalman filter is needed:

$$\frac{d}{dt}\hat{x}_{aug2} = A_{aug2}\hat{x}_{aug2} + B_{aug2}Q(u) + L(t)(y - C_{aug2}\hat{x}_{aug2}) \tag{10}$$

where the injection gain $L(t)$ is computed based on a continuous time periodic Riccati equation.

IV. CONTROLLER DESIGN

A PI with feedforwad controller is designed using the spool state estimated from the Kalman filters. The control objective is for the duty ratio of the valve to track a reference trajectory. Equivalently, the spool axial position needs to track a desired trajectory $r(t)$. The tracking error and its integral are defined as:

$$e := r - x \tag{11}$$

$$\dot{e}_i = e \tag{12}$$

Define $E = \begin{pmatrix} e_i & e \end{pmatrix}^{\mathrm{T}}$, the dynamics of the position tracking error are:

$$\begin{aligned} \dot{e} &= \dot{r} - \frac{Q_d - Q_m}{As} \\ \dot{E} &= (A + BK)E - BKC_{aug}\tilde{x}_{aug} \end{aligned} \tag{13}$$

with $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ \frac{1}{A_s} \end{pmatrix}$ and K being the feedback PI gain. Q_d is the desired flow rate used as the feed-forward term: $Q_d = \dot{r}A_s$. We define Q_m as $Q_m = \frac{Q_d - Q(u)}{A_s}$.

Define $\tilde{x}_{aug} = x_{aug} - \hat{x}_{aug}$, we summarize the state estimate error dynamics as:

$$\dot{\tilde{x}}_{aug} = (A_{aug} - LC_{aug})\tilde{x}_{aug} \tag{14}$$

The closed loop dynamic of the combined error is:

$$\frac{d}{dt}\begin{pmatrix} E \\ \tilde{x}_{aug} \end{pmatrix} = \begin{pmatrix} A - BK & BKC_{aug} \\ 0 & A_{aug} - LC_{aug} \end{pmatrix} \begin{pmatrix} E \\ \tilde{x}_{aug} \end{pmatrix} \tag{15}$$

The closed-loop poles are the combination of the poles from the observer and the poles that would have resulted from using the same feedback on the true states. K and L can be designed separately, and the combined error dynamics can be stabilized.

The first augmented system is modeled in discrete time. Similar closed loop dynamics of the combined error can be derived in discrete time difference equation form. Therefore, $K(k)$ and $L(k)$ can be designed separately to stabilize the combined error dynamics as well.

For the first noise model, tracking error dynamic is converted into discrete time system by applying a zero-order-hold sampling to the system:

$$E(k+1) = \begin{pmatrix} 1 & T_s \\ 0 & 1 \end{pmatrix} E(k) + \begin{pmatrix} \frac{T^2 s}{2} \\ T_s \end{pmatrix} Q_m(k)$$

State feedback gain is determined by doing pole placement.

The control law is designed using flowrate, and it is converted to control input through an inverted mapping from $Q(u)$ to u, which is calibrated experimentally. In practice, the dynamics of the actuator can not be perfectly inverted, and the dead-band is not known precisely either.

If there is no stochastic noise n, poles of the estimator can guarantee that the estimates converge to the real states asymptotically. When considering the existence of bounded and stochastic noise, the output of the Kalman filter is guaranteed to be bounded; however, estimation error does not necessarily converge to zero.

V. SIMULATION AND EXPERIMENTAL RESULTS

A block diagram summarizing the state estimation and control strategy is shown in Fig.4.

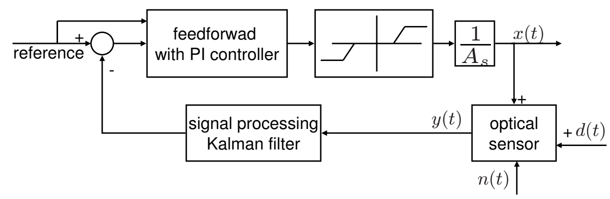

Fig. 4. System Block Diagram

As presented in section II-A, the gerotor pump unit is experimentally calibrated: Using the same experimental data,

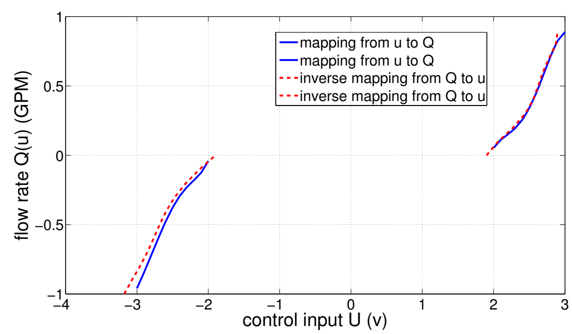

Fig. 5. Mapping between the control input and the gerotor pump flow rate

maps from u to Q and from Q to u were fitted separately. As shown in Fig. 5, the mapping is not exact.

We simulate the performance of the first repetitive noise model in Simulink. The measurement has a 25 Hz repetitive noise. Using a sampling time of $2ms$, we need 20 states to represent the disturbance dynamics. As shown in Fig. 6

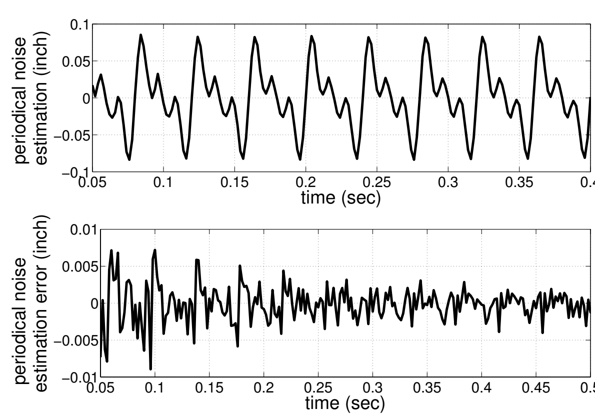

Fig. 6. Repetitive noise and its estimation error

and Fig. 7, spool position state can be distinguished from the repetitive noise, and the tracking performance is satisfactory.

If the sampling time is selected to be $1ms$, and the repetitive noise is of $10Hz$, we need 100 states to represent

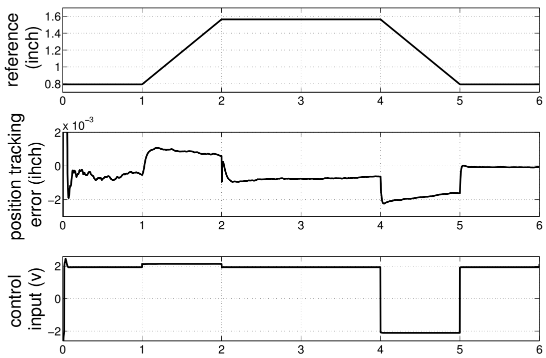

Fig. 7. Trapezoid reference signal, tracking error, and the control command

this noise. With the second model, faster sampling time does not require increase in the number of noise dynamic states.

Next, we consider the second noise model. A simplistic way of estimating the spool state from a noisy measurement is to design a continuous Kalman filter which lumps all the measurement noise into one term n_t without considering its temporal structure.

$$\dot{x} = \frac{Q(u)}{A_s}, \quad y = x + n_t$$
$$\dot{\hat{x}} = \frac{Q(u)}{A_s} + L(y - \hat{x}) \tag{16}$$

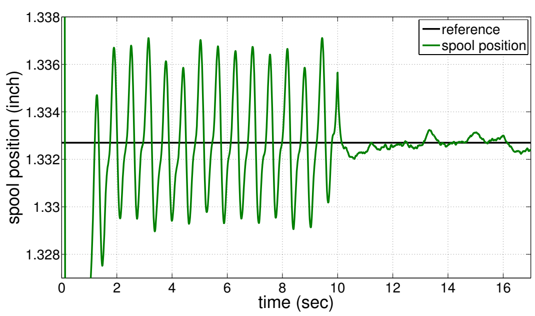

Fig. 8. Spool tracking error comparision: state estimated from lumped model Kalman filter is fed back in the first 10 sec, and after 10 sec, the state estimated form the basis function Kalman filter is fed back

Figure 8 shows the comparisons between the lumped estimation model and the periodic basis function model in simulation. The lumped model Kalman filter in (16) is used in the first 10 sec, and from 10sec, the feedback state is switched to the estimated state from the periodic basis function Kalman filter. With the lumped model Kalman filter, the repetitive noise corrupts the estimate of the true spool state so that the actuator responds to the repetitive noise (as shown in Fig. 9), making the spool oscillates. In contrast, with the periodic basis function Kalman filter, Fig. 8 shows that the spool position is regulated to the desired value in the presence of the periodic noise, and the control input does not respond to the periodic noise (Fig. 9).

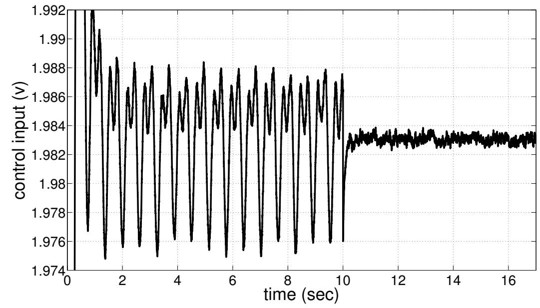

Fig. 9. Control input comparison between feeding back state from lumped model Kalman filter and feeding back state from basis function Kalman filter

We implemented the second approach experimentally. The control objective is to achieve disturbance rejection, so that the valve duty ratio remains constant. The spool was estimated to rotate at around $10.5Hz$, and the sampling time was $0.5ms$. 10 basis functions were used:

$$d(t) = \sum_{k=1}^{5} [x_{d2k-1} cos(k\omega t) + x_{d2k} sin(k\omega t)]$$

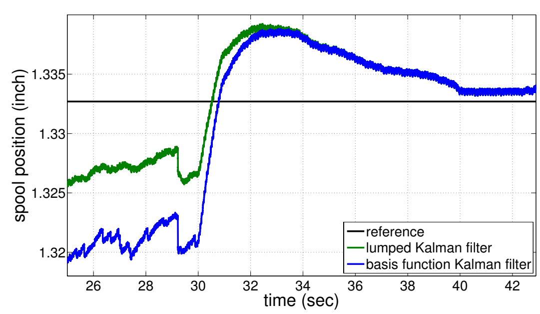

Fig. 10. Spool position tracking performance: before 30 sec, state estimated from the lumped model Kalman filter was fed back ,and at 30 sec, the feedback state was switched to that from the basis function Kalman

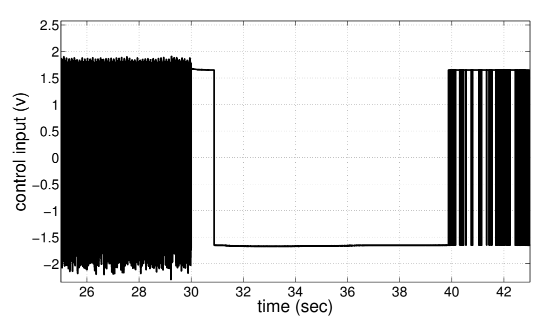

Fig. 11. Spool position tracking control input

Initially, the spool state estimated from a lumped Kalman filter is used as feedback state(green line in Fig. 10). The repetitive noise introduces a tracking error varying at a very high speed, which drives the actuator to compensate it. However, the actuator has a bandwidth limitation, and can not respond to the command as shown in Fig. 11. This causes the actuator to switch direction at a very fast speed. Since the tracking error amplitude is small, the effective control input is small, and the spool is not driven to oscillate. However, when performing an FFT analysis on the control

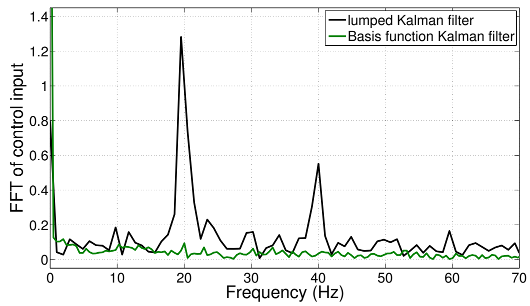

Fig. 12. FFT analysis on control input

input of both state feedback cases, as shown in Fig.12: we can see that when feeding back the state estimated from the lumped Kalman filter, the control input has a significant frequency component at a harmonic of the spool frequency ($\sim 19.53Hz$). When the feeding back state is switched to the estimated state from the basis function Kalman filter at 30 sec, the frequency component is removed from the input. In both cases , the spool is stabilized with a tracking error. By switching the feedback signal, the tracking error is reduced from $0.005inch$ to $0.0009inch$.

As shown in the experimental results, there is an discrepancy between the estimate of the spool frequency $10.5Hz$ and the true frequency analyzed using FFT $9.766Hz$. In practice, the angular velocity of the spool is not measured directly, and it can vary as the oil operating conditions varies. The sensitivity of the periodic noise estimate to the bias of the fundamental frequency is investigated in simulation.

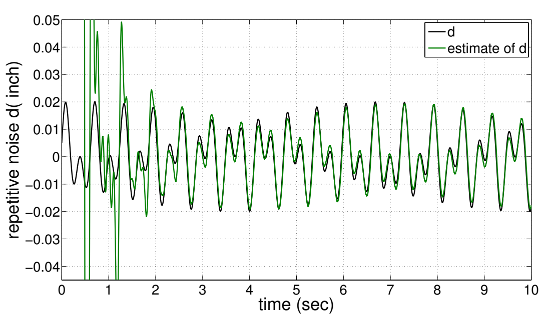

Fig. 13. Noise estimate with a fundamental frequency bias

A $0.5Hz$ bias on the fundamental frequency is imposed onto the basis function, while the repetitive noise has a frequency of $10Hz$. As shown in Fig. 13, this induces an estimation error on both the repetitive noise and the spool position. This is similar to the problem when using a lumped model Kalman filter. The difference is that estimation error on spool state is smaller compared with the error using a lumped model Kalman filter. In simulation, the actuator can

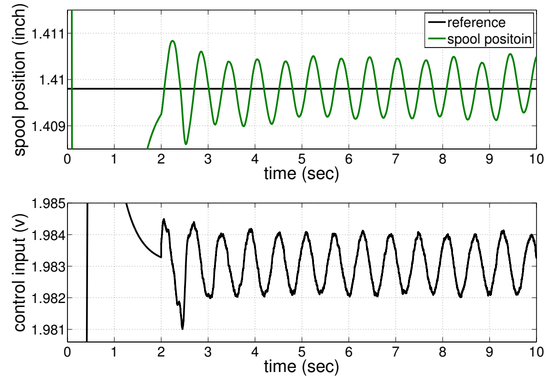

Fig. 14. Spool position tracking and control input with the fundamental frequency bias

respond to the state tracking error(shown in Fig. 14) fast enough, and the spool can oscillate. In practice, the error may be so small that the actuator input falls into dead-band, and can not cause oscillation. This simulation shows that states estimator error is sensitive to the fundamental frequency bias. Precisely knowing the repetitive noise period is crucial to accurate spool position control.

VI. CONCLUSION AND FUTURE WORK

Periodic measurement noise can be estimated using state estimators that incorporate models for the periodic noise. Simulation and experimental results show that the spool position can be distinguished from the periodic noise, and the spool achieved improved trajectory tracking performance. The time-invariant periodic noise model results in a high dimension system especially when small sampling times are used. On the other hand, the model that uses a set of periodic basis functions can be effective with a lower dimension.

Sensitivity of the noise model to the dimension of the noise dynamics and the accuracy of the fundamental frequency were investigated in simulation. It is found that knowledge of the period of the periodical signal is crucial. The noise period can potentially be obtained from the rotary position sensing using a coarse optical encoder. An event based Kalman filter has been developed to more accurately estimate the rotary states from the encoder [4]. Combining this with the work presented in this paper will be pursued next.

VII. ACKNOWLEDGMENTS

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REFERENCES

[1] H. Tu, M. Rannow, J. Van de ven, M. Wang, P. Li and T. Chase,2007, "'High Speed Rotary Pulse Width Modulated On/off Valve"' *Proceedings of the 2007 ASME-IMECE*, Paper No. IMECE2007-42559.

[2] D. Simon, *Optimal State Estimation,* Hoboken, NJ:John Wiley & Sons, Inc., 2006.

[3] K. Astrom, B. wittenmark, *Computer-Controlled Systems: Theory and Design (3rd Edition)* NJ: Prentice Hall, Inc., 1996

[4] M. Wang, P. Li, 2008, "'Event based Kalman filter observer for rotary high speed on/off valve"' *Proceedings of the 2008 ACC*, page 1546-51