2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

FrA20.5

Design of a VSDD Brace Control System for Parameter Estimation of Shear Structures

Dongyu Zhang and Erik A. Johnson

Abstract—**In this paper, a new function of Variable Stiffness & Damping Device (VSDD) brace control systems is explored: facilitating structural parameter estimation and damage detection. In previous studies by the authors, a substructure identification method has been proposed to identify the structural parameters (story stiffness and damping coefficients) of a shear building. An error analysis shows that the accuracy of the estimated parameters can be greatly improved by amplifying the interstory acceleration near the substructure natural frequency. To achieve this structural response change, two kinds of control strategies are studied herein to design a VSDD system: an on-off passive strategy and a semiactive strategy. Since the accuracy of substructure identification is only dependent on the controlled substructure responses, it is shown that the proposed control-facilitated identification is robust to one common control system error: feedback measurement noise. Finally, a numerical example of a 5-story shear building structure is used to illustrate the efficacy of the proposed control method for improving the accuracy of the substructure identification.**

I. Introduction

Variable stiffness and damping devices (VSDDs), such as smart dampers and controllable stiffness elements, are controllable passive devices that potentially offer the reliability of the passive devices, yet maintain the versatility and adaptability of fully active systems [1]. Due to superior performance relative to passive devices and high reliability compared with active control, VSDDs for semiactive brace control systems have been recently implemented in building structures to mitigate large structural responses for a variety of dynamic loads, such as earthquakes and strong winds. However, such large natural hazards occur rather infrequently, so the VSDD control system remains unused the most of time. This paper presents a new technique that makes use of the capacity of the VSDD control system to facilitate structural parameter identification and improve damage detection accuracy by implementing alternative control algorithms when the control system is in idle status (*i.e.*, not immediately required for vibration mitigation). This new technique not only increases the cost-effectiveness of implementing a VSDD brace control system into the structure by adding new functions for the control system, but potentially improves the performance of the control system for vibration mitigation by providing a more accurate structural model for better control algorithm design.

Manuscript received September 22, 2008. This work was supported in part by the National Science Foundation through CAREER 00-94030, ANI 03-25875 and CMMI 08-26634. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Dongyu Zhang is with the Sonny Astani Department of Civil and Environmental Engineering, University of Southern California, Los Angeles, CA 90089 USA. (e-mail: dongzhan@usc.edu)

Erik A. Johnson is with the Sonny Astani Department of Civil and Environmental Engineering, University of Southern California, Los Angeles, CA 90089 USA. (phone: 213-740-0610; fax: 213-744-1426; e-mail: JohnsonE@usc.edu).

Many conventional parameter identification methods in Structural Health Monitoring (SHM) often confront the following difficulties in their application: (*i*) too many structural parameters must be identified from very limited measurements; (*ii*) the changes in measurements are usually insensitive to changes in the structural parameters. Both situations will generally lead to large errors in the estimated parameters. Several methods have been proposed to use structural control systems to overcome or alleviate these problems. Some researchers [2,3] propose to change structural modal features, like frequency, in multiple configurations by applying different control algorithms, using these multiple information sources together to solve the first problem of the rank-deficiency for the original identification and improve the accuracy of damage detection. To tackle the second difficulty of low sensitivity, other researchers [4,5,6] attempt to design structural control systems to enhance the sensitivity of the structural measurements to the change of structural parameters in order to reduce the identification error.

Although these studies have somewhat demonstrated improvement of identification accuracy by applying their control algorithms, there is still a big challenge for these techniques: how will the imperfections in the control system affect the identification results? Some control system error always exists, such as time delay for computation, unmodeled actuator dynamics, measurement noise in feedback and so forth. For all of the aforementioned control-facilitated identification methods, the structural control system is deeply involved in the whole identification procedure; thus, it is inevitable that these errors will affect the final result of the identification, possibly even eliminating the identification benefits of using control. Moreover, because of the complexity of the closed-loop control system, the effects of control system error on the identification accuracy become extremely difficult to analyze and predict. Thus, it would be very beneficial to develop some approaches where control improves the identification accuracy but the identification is robust to errors in the control forces.

In previous research by the authors [7], a substructure identification method was proposed to identify the structural parameters of a shear building, specifically the story stiffness and damping coefficients. By using the dynamic equilibrium of each floor, a series of identification problems can be formulated, from which all structural parameters can be estimated from top to bottom in an inductive manner. An error analysis shows that the accuracy of the identification is determined by the frequency response of the interstory acceleration in a frequency range around the story substructure natural frequency; strongly amplifying this response can greatly improve the identification accuracy.

Based on this error analysis result, two kinds of control strategies to design a VSDD system are studied herein to improve the accuracy of parameter identification: (*i*) an on-off passive strategy wherein the activated VSDD system functions as a passive system, adding fixed stiffness and damping to the structure, but adds nothing when inactive; (*ii*) a semiactive strategy wherein the VSDD device tries to mimic, as closely as possible, the control force trajectory of an optimally designed active control system. Since the identification accuracy of the substructure identification is not directly dependent on the control system itself, but on the control system's amplification of a certain interstory acceleration response, any control system errors (*e.g.*, feedback measurement noise and time delay) that do not significantly deteriorate the control system performance will not have a large side effect on the accuracy of controlled identification. Thus, the proposed controlled substructure identification should be quite robust to control system errors. Further, it is shown herein that certain control system uncertainties, specifically feedback measurement noise, may even have the potential to improve the control system performance, further enhancing identification accuracy.

This paper is organized as follows: a brief review is first given of the formulation and identification error analysis results of the substructure identification method. Then, two optimization problems are proposed to find the optimal parameters of two control algorithms to amplify the interstory acceleration responses and thereby increase the identification accuracy. Next, an analysis is performed to show that one of the common control system errors, measurement noise in feedback responses, will not deteriorate but rather improve the performance of the designed control system. Finally, a numerical example of a 5-story shear building illustrates the improvements of the controlled identification.

II. SUBSTRUCTURE IDENTIFICATION

A. Method Formulation

Fig. 1 shows an n-story shear structure with a VSDD semiactive control system. The equations of motion of this structure, without the effect of the control system, can be written in separate form as follows for top, middle and bottom floors, respectively:

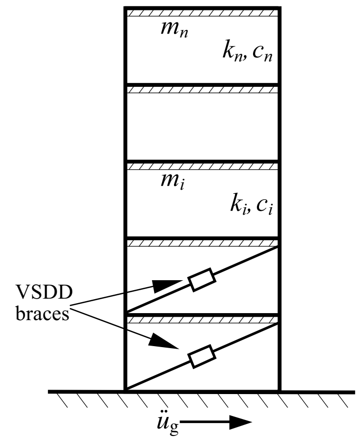

Fig. 1. A shear structure with a VSDD brace control system.

top floor ($i = n$):

$$m_n\ddot{x}_n + c_n(\dot{x}_n - \dot{x}_{n-1}) + k_n(x_n - x_{n-1}) = 0 \tag{1}$$

middle floor ($2 \le i \le n-1$):

$$\begin{aligned} m_i\ddot{x}_i + c_i(\dot{x}_i - \dot{x}_{i-1}) + k_i(x_i - x_{i-1}) \\ + c_{i+1}(\dot{x}_i - \dot{x}_{i+1}) + k_{i+1}(x_i - x_{i+1}) = 0 \end{aligned} \tag{2}$$

bottom floor ($i = 1$):

$$\begin{aligned} m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{u}_g) + k_1(x_1 - u_g) \\ + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = 0 \end{aligned} \tag{3}$$

where m_i is the mass of the i-th floor; c_i and k_i are the damping coefficient and stiffness of the i-th story, respectively; x_i is the displacement of the i-th floor relative to an inertial reference frame; u_g is the displacement of the ground, and an overdot represents derivative with respect to time. It is assumed here that the floor masses are known.

The motion of the top floor is affected only by the top story structural parameters and by the motion of the floor below. Thus, the identification will start as follows with the top floor. Adding $-m_n\ddot{x}_{n-1}$ to both side of (1), taking the Fourier transform and rearranging the terms (utilizing $\ddot{X} = (j\omega)^2 X$ for stationary initial conditions), gives

$$\frac{1}{1 - jc_n/(m_n\omega) - k_n/(m_n\omega^2)} = \frac{\ddot{X}_{n-1} - \ddot{X}_n}{\ddot{X}_{n-1}} \tag{4}$$

where $\ddot{X}_i = \ddot{X}_i(j\omega)$ is the Fourier transform of the i-th floor acceleration (herein, $j\omega$ is often omitted for notational simplicity). Since the right side of (4) only involves the structural acceleration, which can be easily computed from measured responses, the structural parameters $[k_n \ \ c_n]^T$ can be identified by solving the following optimization problem:

$$\underset{k_n, c_n}{\arg\min} \quad J(k_n, c_n) = \sum_{l=1}^{N} \left| f_l(k_n, c_n) - \hat{f}_l\left(\hat{\ddot{X}}_{n-1}, \hat{\ddot{X}}_n\right) \right|^2 \tag{5}$$

where $f_l(k_n, c_n) = \dfrac{1}{1 - jc_n/(m_n\omega_l) - k_n/(m_n\omega_l^2)}$,

$\hat{f}_l\left(\hat{\ddot{X}}_{n-1}, \hat{\ddot{X}}_n\right) = \dfrac{\hat{\ddot{X}}_{n-1}(j\omega_l) - \hat{\ddot{X}}_n(j\omega_l)}{\hat{\ddot{X}}_{n-1}(j\omega_l)} = \dfrac{\hat{\ddot{X}}_{n-1,l} - \hat{\ddot{X}}_{n,l}}{\hat{\ddot{X}}_{n-1,l}}$, and

$\hat{\ddot{X}}_{i,l} = \hat{\ddot{X}}_i(j\omega_l)$ denotes the Fourier transform (or frequency response) of the i-th measured floor acceleration at distinct frequencies $\omega_l = l \cdot \Delta\omega$ ($l = 1,2,\ldots,N$) at which the Fourier transforms are calculated with frequency interval $\Delta\omega$.

After the n-th story parameters $[k_n \quad c_n]^{\mathrm{T}}$ have been identified, the following induction method can be used to identify structural parameters of the other stories. Adding $-m_i\ddot{x}_{i-1}$ to both sides of (2) and following a similar procedure to transform to the frequency domain gives

$$\frac{1}{1 - jc_i/(m_i\omega) - k_i/(m_i\omega^2)} = \frac{\ddot{X}_{i-1} - \ddot{X}_i}{\ddot{X}_{i-1} + (\ddot{X}_{i+1} - \ddot{X}_i)\left[jc_{i+1}/(m_i\omega) + k_{i+1}/(m_i\omega^2)\right]} \tag{6}$$

So, if the structural parameters $[k_{i+1} \quad c_{i+1}]^{\mathrm{T}}$ are known, then the right side of (6) is known at distinct frequencies and a similar optimization problem can be formulated to identify structural parameters $[k_i \quad c_i]^{\mathrm{T}}$, written as:

$$\underset{k_i, c_i}{\arg\min} \quad J(k_i, c_i) = \sum_{l=1}^{N} \left| g_l(k_i, c_i) - \hat{g}_l\left(\hat{\ddot{X}}_{i-1}, \hat{\ddot{X}}_i, \hat{\ddot{X}}_{i+1}\right) \right|^2 \tag{7}$$

where $g_l(k_i, c_i) = \dfrac{1}{1 - jc_i/(m_i\omega) - k_i/(m_i\omega^2)}$ and

$$\hat{g}_l\left(\hat{\ddot{X}}_{i-1}, \hat{\ddot{X}}_i, \hat{\ddot{X}}_{i+1}\right) = \frac{\hat{\ddot{X}}_{i-1,l} - \hat{\ddot{X}}_{i,l}}{\hat{\ddot{X}}_{i-1,l} + \left(\hat{\ddot{X}}_{i+1,l} - \hat{\ddot{X}}_{i,l}\right)\left[\dfrac{jc_{i+1}}{m_i\omega} + \dfrac{k_{i+1}}{m_i\omega^2}\right]}.$$

Because structural parameters $[k_n \quad c_n]^{\mathrm{T}}$ have been identified from (5), and can serve as the known input parameters for (7), the parameters $[k_{n-1} \quad c_{n-1}]^{\mathrm{T}}$ can be identified. Following the same routine, all structural parameters $[k_i \quad c_i]^{\mathrm{T}}$ ($i = 1,\ldots,n$) can be identified in turn. When the parameters of the first story are to be identified, a simple replacement of $\ddot{X}_{i-1}$ with $\ddot{U}_{\mathrm{g}}$ is needed in (7).

The proposed substructure identification method has several advantages. (*i*) It is not necessary to simultaneously measure the acceleration of all floors; only two or three are needed for each identification step, potentially reducing the cost of an SHM system (particularly for wireless sensor networks where moving sensors around becomes convenient or battery usage limits sensor life [9]). (*ii*) In each step of the optimization procedure, there are only two optimization variables, making the optimization procedure much easier to execute and more likely to converge. (*iii*) Since the identification problem in each step of the substructure identification is simple and similar in its formulation, it becomes possible to perform *analytical* identification error analysis that reveals insight into how uncertainties in the identification process, like measurement noise, affect the final identification accuracy. Further, the error analysis paves a way for combining the substructure identification with structural control systems to further improve the accuracy of the substructure identification.

B. Identification Error Analysis

Understanding how the uncertainty in the measurement affects the identification accuracy plays an important role in better evaluating the accuracy of the identification method, as well as developing ways to improve it. In a previous study [8], based on the linearization of the original identification problem, the authors derived an approximate identification error analysis algorithm for least-square-error identification problems. This algorithm is applied to the substructure method, obtaining a simple *analytical* result for the relative identification error of the structural parameters. For the top story, the relative error, $\Delta\boldsymbol{\theta}_n$, in $\boldsymbol{\theta}_n = [k_n \quad c_n]^{\mathrm{T}}$ is

$$\Delta\boldsymbol{\theta}_n \approx \sum_{l=1}^{N} \mathrm{Re}\left\{ \begin{bmatrix} W_{11,l} & W_{12,l} \\ W_{21,l} & W_{22,l} \end{bmatrix} \cdot \begin{bmatrix} \hat{N}_{n-1,l}^{*} \big/ \left(\ddot{X}_{n,l} - \ddot{X}_{n-1,l}\right)^{*} \\ \hat{N}_{n,l}^{*} \big/ \left(\ddot{X}_{n,l} - \ddot{X}_{n-1,l}\right)^{*} \end{bmatrix} \right\} \tag{8}$$

(8) shows that the estimation error is proportional to the Fourier transforms, $\hat{N}_{i,l}$, of the measurement noise (obvious) but (interestingly) inversely proportional to the Fourier transform of the interstory acceleration. Because the frequency-dependent weighting functions $W_{jk,l}(j\omega)$ (which are themselves dependent on the parameters of the top story substructure [8]) are large only in the vicinity of the substructure natural frequency $\omega_{n0} = \sqrt{k_n/m_n}$, the estimation errors in the top story are strongly influenced by the interstory acceleration only near that frequency. For non-top stories, the error is a little more complicated with additional terms related to the error in the estimation of the story above

$$\begin{aligned} \Delta\boldsymbol{\theta}_i \approx{} & \sum_{l=1}^{N} \mathrm{Re}\left\{ \begin{bmatrix} U_{11,l} & U_{12,l} & U_{13,l} \\ U_{21,l} & U_{22,l} & U_{23,l} \end{bmatrix} \cdot \begin{bmatrix} \hat{N}_{i-1,l}{}^{*} \big/ \left(\ddot{X}_{i,l} - \ddot{X}_{i-1,l}\right)^{*} \\ \hat{N}_{i,l}{}^{*} \big/ \left(\ddot{X}_{i,l} - \ddot{X}_{i-1,l}\right)^{*} \\ \hat{N}_{i+1,l}{}^{*} \big/ \left(\ddot{X}_{i,l} - \ddot{X}_{i-1,l}\right)^{*} \end{bmatrix} \right\} \\ & + \sum_{l=1}^{N} \mathrm{Re}\left\{ \begin{bmatrix} U_{14,l} & U_{15,l} \\ U_{24,l} & U_{25,l} \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\left(\ddot{X}_{i+1,l} - \ddot{X}_{i,l}\right)^{*}}{\left(\ddot{X}_{i,l} - \ddot{X}_{i-1,l}\right)^{*}} \dfrac{\Delta k_{i+1}}{k_{i+1}} \\ \dfrac{\left(\ddot{X}_{i+1,l} - \ddot{X}_{i,l}\right)^{*}}{\left(\ddot{X}_{i,l} - \ddot{X}_{i-1,l}\right)^{*}} \dfrac{\Delta c_{i+1}}{c_{i+1}} \end{bmatrix} \right\} \end{aligned} \tag{9}$$

Weighting functions $U_{jk,l}(j\omega)$ are also large only near ω_{i0}.

Thus, the relative errors in estimates of both k_i and c_i can be expressed in terms of two kinds of errors: error related to the noise of measured floor acceleration, and error due to the uncertainty of the parameter estimates in the story above (which, of course, does not appear in the error terms for the top story parameter estimation). Moreover, amplifying the interstory acceleration of the story to be identified near its substructure natural frequency $\omega_{i0} = \sqrt{k_i/m_i}$ can significantly reduce the identification errors.

III. CONTROL ALGORITHM DESIGN FOR SUBSTRUCTURE IDENTIFICATION

Based on the identification error analysis in the previous section, the goal of new identification-focused control algorithms for a VSDD control system is to amplify the interstory acceleration near the substructure natural frequency. Two kinds of control strategies to design VSDD system are studied herein. First, an on-off passive strategy uses the VSDD in active and inactive modes, the former acting as a passive element that adds fixed stiffness and damping to the structure, and the latter exerting no forces. Second, in a semiactive strategy, the VSDD device tries to mimic, as closely as possible, the control force trajectory of an optimally designed active control system. It is worth emphasizing that the designed identification-focused control algorithms will be implemented with a fail-safe mechanism: if excessive excitation is detected, the control system will immediately switch back to the original control algorithm that is designed to mitigate structural motion and damage. Therefore, the new algorithm will not weaken the main function of the control system, response mitigation, but add extra value to the installed control system.

For simplicity in designing the new control algorithms, two assumptions are made:

1. Beside control forces, the structure is only excited by the ground motion, which is modeled by a filtered band limited Gaussian white noise process.
2. The control system is ideal and no control system errors, such as feedback measurement noise, actuator time delay and so forth, exist.

Based on the above assumptions, it can be shown that the frequency responses of the interstory accelerations of the close-loop controlled structure are zero-mean Gaussian random variables. Hence, instead of trying to directly amplify the frequency responses of the interstory acceleration, which are random in nature due to the stochastic excitation, the control system is designed to maximize the variances of these responses with frequency weighting as shown below in (10) and (12).

A. Passive Control Algorithm Design

Let $\boldsymbol{\kappa} = [\kappa_1 \ldots \kappa_p]^T$ be a vector composed of the stiffness and damping that the VSDD system will add to the structure. The following optimization problem is posed to maximize the frequency weighted i-th interstory acceleration

$$\arg\max_{\boldsymbol{\kappa}} J(\boldsymbol{\kappa}) = \arg\max_{\boldsymbol{\kappa}} \int_0^\infty \mathrm{E}\left[\left|W(j\omega)\left(\ddot{X}_i - \ddot{X}_{i-1}\right)\right|^2\right] d\omega \qquad (10)$$

$$\text{subject to } \kappa_k^{\max} \geq \kappa_k \geq 0,\ k = 1,2,\cdots,p$$

where $\ddot{X}_i - \ddot{X}_{i-1}$ is the Fourier transform of the i-th closed-loop interstory acceleration; the $\kappa_k^{\max}$ are the upper limit of the corresponding stiffness or damping parameter of the brace; all design variables κ_k should be non-negative due to the passive nature of the devices; $W(j\omega)$ is the frequency weighting function

$$W(j\omega) = \frac{k_i}{(m_i\omega)^2\left[1 - jc_i/(m_i\omega) - k_i/\left(m_i\omega^2\right)\right]^2} \qquad (11)$$

As shown in Fig. 2, the magnitude of $W(j\omega)$ peaks around frequency ω_{i0} and quickly vanishes further away. The role of this weighting function is to implicitly force the control system to focus on maximizing the target interstory acceleration only around the frequency ω_{i0}, so that the identification error can be greatly reduced.

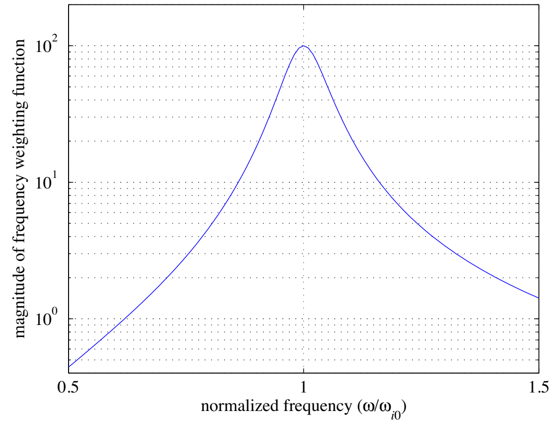

Fig. 2. Magnitude of frequency weighting function.

B. Semiactive Control Algorithm Design

A clipped optimal control strategy [1,10] is used to design the new semiactive algorithm to enhance the identification accuracy. The clipped optimal control is composed of two controllers in series: the primary controller is designed by a linear state feedback control algorithm assuming that the actuators are fully active, and a clipping algorithm is used as a secondary controller to make the VSDD mimic the control force close to that computed by the primary controller.

Due to the dissipative nature of VSDDs, a VSDD cannot always provide the exact control force as calculated by the primary controller. The performance of the clipped optimal control system, compared with the corresponding fully active system, is largely dependent on the dissipativity of the control forces from the primary controller [11]. Therefore, a dissipativity constraint for the primary controller is integrated into the optimization procedure of the algorithm design (12) to assure that the control forces applied to the structure are dissipative during most of the time history, so that the semiactive system effectively tracks the active system.

Let $\mathbf{L}$ be the state feedback gain matrix of the primary controller in a clipped optimal semiactive control system. An approximate optimal semiactive strategy can be found by solving for an active primary controller state feedback gain subject to a constraint that it be dissipative much of the time

$$\arg\max_{\mathbf{L}} J(\mathbf{L}) = \int_0^\infty \left|W(j\omega)\cdot\left(\ddot{X}_i - \ddot{X}_{i-1}\right)\right|^2 d\omega \qquad (12)$$

$$\text{subject to } \rho_{u_l v_l} \leq \varepsilon < 0,\ l = 1,2,\cdots,p$$

where $\rho_{u_l v_l}$ is the correlation coefficient between the control force u_l and the velocity v_l across the l-th VSDD brace. ε is a negative number between 0 and –1, with smaller ε requiring the control force be more dissipative; ε is chosen to be –0.5 in the numerical examples herein. The weighting function $W(j\omega)$ is the same as given in (11).

After the primary controller is designed, a secondary clipped optimal controller is concatenated afterward to form the full controller, where desired control force $u_l(t)$ is exerted at time t if $u_l(t)\cdot v_l(t) \leq 0$ (*i.e.*, if it is dissipative), and zero force otherwise.

IV. The Effect of Feedback Measurement Noise on the Controlled Identification Performance

The control algorithms in the previous section are designed assuming an ideal control system. However, some control system error always exists and will inevitably affect, more or less, the performance of the designed control system for improving identification accuracy. In this section, an analysis is made to examine how one common control system error, feedback measurement noise, will affect the performance of the control system. Since the identification accuracy of substructure identification is directly dependent, not on the control force, but on the closed-loop substructure responses (the interstory acceleration in a critical frequency range), the effect of feedback measurement noise on the accuracy of the substructure identification can be analyzed by examining how the noise will change the interstory acceleration response from that originally designed.

Both passive and semiactive control systems can be represented by the flowchart in the Fig. 3, where u_g is the ground excitation; u_c is the control system force applied to the structure; $\mathbf{z}$ represents the structural state response vector, containing the displacement and velocity responses of all floors; $\mathbf{n_z}$ is the measurement noise vector of the structural state-space response; $\mathbf{H}_1$ and $\mathbf{H}_2$ are the structure transfer functions from ground excitation and control forces to the structural state responses, respectively; $\mathbf{C}$ is the identification-oriented controller; and $\mathbf{T}$ is a linear gain that transforms the state-space response into the i-th interstory acceleration.

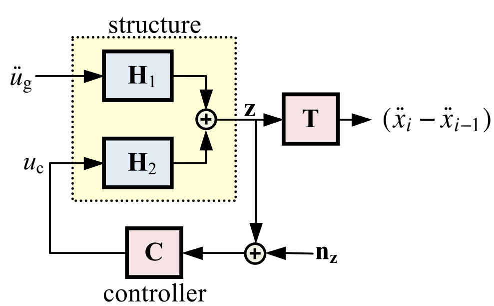

Fig. 3. Control system with state feedback measurement noise.

To analyze the effects of the measurement noise $\mathbf{n_z}$, two assumptions must be made:

1. Feedback measurement noises $\mathbf{n_z}$ can be modeled as a band limited white Gaussian process and are independent of the ground excitation.
2. The controller $\mathbf{C}$ is a linear controller.

Clearly the clipped optimal algorithm in the semiactive control is a non-linear controller and does not satisfy the second assumption above. However, it is known that if the primary controller in a clipped optimal controller is designed such that the control forces calculated from the primary controller are dissipative during most of the time, the clipped optimal controller can be approximated by an equivalent linear controller that contains the original primary controller only. Therefore, in this analysis, this equivalent linear controller is adopted to replace the original clipped optimal controller in order to analytically calculate the structural responses of clipped optimal control system.

Since both the controllers and the structures are linear time invariant (LTI), the whole closed-loop structure is a LTI system. By applying the principal of superposition, the output of this system in the frequency domain, the Fourier transform of the i-th interstory acceleration $\left(\ddot{X}_i - \ddot{X}_{i-1}\right)$, can be calculated as

$$\ddot{X}_i - \ddot{X}_{i-1} = \mathbf{T}(\mathbf{I}-\mathbf{H}_2\mathbf{C})^{-1}\mathbf{H}_1\ddot{U}_g + \mathbf{T}(\mathbf{I}-\mathbf{H}_2\mathbf{C})^{-1}\mathbf{H}_2\mathbf{C}\mathbf{N}_z \quad (13)$$

where $\ddot{X}_i(j\omega)$, $\ddot{X}_{i-1}(j\omega)$, $\ddot{U}_g(j\omega)$ and $\mathbf{N_z}(j\omega)$ are the Fourier transforms of the corresponding time domain responses of $\ddot{x}_i(t)$, $\ddot{x}_{i-1}(t)$, $\ddot{u}_g(t)$ and $\mathbf{n_z}(t)$, respectively. Applying the independence condition in the first assumption, the variance of the Fourier transform of the i-th closed-loop controlled interstory acceleration can be calculated as

$$\begin{aligned} \mathrm{E}\left[\left|\ddot{X}_i - \ddot{X}_{i-1}\right|^2\right] = \mathrm{E}\left[\left|\mathbf{T}(\mathbf{I}-\mathbf{H}_2\mathbf{C})^{-1}\mathbf{H}_1\ddot{U}_g\right|^2\right] + \\ \mathrm{E}\left[\left|\mathbf{T}(\mathbf{I}-\mathbf{H}_2\mathbf{C})^{-1}\mathbf{H}_2\mathbf{C}\mathbf{N}_z\right|^2\right] \end{aligned} \quad (14)$$

The variance in (14) contains two parts: the first part, due to ground excitation, is just equal to the variance of the responses from the ideally controlled system without feedback noise; the second part is contributed by the feedback noise variance. Since the second part is always greater than zero, the variance of the responses from the non-ideally controlled system (with feedback noise) will be larger than that from the ideally controlled system; this indicates that the control system with feedback noise should outperform the control system without noise in terms of improving the parameter identification accuracy. Thus, the proposed control-identification method should be robust to feedback measurement noise.

V. Numerical Example

A 5-story uniform shear structure, with two VSDD braces installed, one in each of the first two stories, is used to illustrate the effectiveness of the proposed control methods to improve identification accuracy. The structure parameters are $m_i = 1\times10^5$ kg, $c_i = 8\times10^5$ N·sec/m and $k_i = 16\times10^7$ N/m ($i = 1, \ldots, 5$). The ground excitation $\ddot{u}_g$ is generated by a Gaussian random pulse process passed through a 4-th order lowpass Butterworth filter with a 12 Hz cut-off frequency. 300 second ground and floor acceleration responses, with sampling rate 200 Hz, are calculated to carry out the identification. It is assumed that the magnitudes of the measurement noises of all acceleration responses $\ddot{x}_i$ are the same, with root-mean-square (RMS) equal to 5% of the RMS of the ground excitation.

100 substructure identification tests are performed for the structure without control, with passive control and with semiactive control, respectively; while there is measurement noise in the acceleration measurements, it is assumed first

that the control systems are ideal with no noise in the feedback term. The relative RMS errors (RMSEs) in the identified parameters (in percentage) are listed in Table 1. From the result, it is clearly seen that both control algorithms do greatly improve the parameter identification accuracy: taking the third story parameter for example, the RMSEs of stiffness and damping parameter estimates are reduced by a factor of 4.2 and 8.4, respectively, for the passive control method, and by a factor of 3.9 and 6.1, respectively, for semiactive control method.

TABLE 1. RELATIVE (PERCENT) RMSE OF IDENTIFIED PARAMETERS WITHOUT CONTROL AND WITH IDEAL PASSIVE AND SEMIACTIVE CONTROL.

story #	no control		Passive		semiactive	
	k_i	c_i	k_i	c_i	k_i	c_i
1	1.64	5.25	0.50	2.40	0.29	0.99
2	2.76	7.50	0.47	1.90	0.47	1.41
3	2.81	32.4	0.66	3.85	0.71	5.33
4	0.71	10.4	0.22	1.15	0.21	1.17
5	0.40	4.63	0.19	0.78	0.15	0.63

In order to verify the analysis conclusion that the proposed control-identification methods are robust to the feedback measurement noise, 20% Gaussian white noise is added into the structural state feedback; that is, the RMS of noise $\mathbf{n}_\mathbf{z}$ is equal to 20% of the RMS of the corresponding state response. Similarly, 100 identification tests are performed with the noise-contaminated passive and semiactive control systems; the results of these tests are shown in Table 2.

TABLE 2. RELATIVE (PERCENT) RMSE OF IDENTIFIED PARAMETERS WITH PASSIVE AND SEMIACTIVE CONTROL WITH 20% FEEDBACK NOISE.

story #	Passive		semiactive	
	k_i	c_i	k_i	c_i
1	0.51	1.66	0.31	1.05
2	0.45	1.43	0.42	1.23
3	0.65	3.00	0.63	5.46
4	0.21	1.11	0.21	1.46
5	0.17	0.86	0.14	0.70

By comparing the corresponding results between Table 1 and Table 2, it can be observed that, for the passive control method, most identification results with feedback noise are indeed more accurate than those without noise, as predicted in the analysis herein. For the semiactive method, however, the effect of feedback noise is not uncertain: some results improve but others deteriorate. This result may be due to the fact that only the primary linear controller is used in the analysis for the effects of feedback noise whereas, in reality, the true control system is nonlinear. As a result, although the estimates of some parameters do not improve with feedback noise, both control methods can still provide quite accurate identification result under fairly large feedback noise. This verifies the assertion that the proposed control-identification methods are quite robust to feedback measurement noise.

VI. CONCLUSIONS

In this paper, a method is introduced for designing an SHM-focused control algorithm for passive and semiactive strategies to enhance structural parameter identification accuracy for the substructure identification method. A previous study [7,8] showed that increasing the interstory acceleration response, in a frequency range around the substructure natural frequency, can reduce estimation error. Thus, an optimization problem is formulated for each control strategy to find the optimal control parameters that effect this kind of change in structural response. Since the proposed controlled substructure identification has identification error dependent on the controlled responses, not on the actual feedback forces themselves, the proposed method is quite robust to control system errors. An analysis shows that the proposed control-identification method is quite robust to the inclusion of feedback noise. A numerical example of a 5-story shear building is used to demonstrate that both passive and semiactive strategies can significantly improve the identification accuracy for structural stiffness parameters, and both methods can still provide accurate identification results in the presence of fairly large feedback measurement noise.

REFERENCES

[1] S. J. Dyke, B. F. Spencer Jr., M. K. Sain, and J. D. Carlson, “Modeling and control of magnetorheological damper for seismic response reduction,” *Smart Materials and Structures*, vol.5, no. 5, pp. 565–575, Oct. 1996.

[2] J. S. Lew and J. N. Juang, “Structural damage detection using virtual passive controllers,” *Journal of Guidance, Control and Dynamics*, vol. 25, no. 3, pp. 419–424, May–June 2002.

[3] M. I. S. Elmasry and E. A. Johnson, “Parametric frequency domain identification in multi-configuration structures,” *15th ASCE Engineering Mechanics Conference (EM2002)*, New York, 2002, pp. 444–450.

[4] B. H. Koh and L. R. Ray, “Feedback Controller Design for Sensitivity Based Damage Localization,” *Journal of Sound and Vibration*, vol. 27, no. 1–2, pp. 317–335, May 2004.

[5] J. A. Solbeck and L. R. Ray, “Damage identification using sensitivity-enhancing control and identified models,” *Journal of Vibration and Acoustics*, vol. 128, no. 2, pp. 210–220, Apr. 2006.

[6] L. J. Jiang, J. Tang, and K. W. Wang, “An optimal sensitivity-enhancing feedback control approach via eigenstructure assignment for structural damage identification,” *Journal of Vibration and Acoustics*, vol. 129, no. 6, pp. 771–783, Dec. 2007.

[7] D. Zhang and E. A. Johnson, “Substructure parameter identification method for shear type structure,” *4th World Conference on Structural Control and Monitoring*, San Diego, 2006, paper 4WCSCM-371.

[8] D. Zhang and E. A. Johnson, “Substructure identification for shear type structure I: identification method,” *Journal of Structural Control and Health Monitoring*, submitted.

[9] T. S. Fu, K. Chintalapudi, J. Paek, E. A. Johnson, and R. Govindan, “Distributed Damage Localization Scheme for Structural Health Monitoring System using Wireless Sensor Networks,” *4th World Conference on Structural Control and Monitoring*, San Diego, 2006, paper 4WCSCM-336.

[10] J. C. Ramallo, E. A. Johnson, and B. F. Spencer Jr., “Smart base isolation systems,” *Journal of Engineering Mechanics*, vol. 128, no. 10, pp. 1088–1099, Oct. 2002.

[11] E. A. Johnson and B. Erkus, “Dissipativity and performance analysis of smart dampers via LMI synthesis,” *Structural Control and Heath Monitoring*, vol. 14, no. 3, pp. 471–496, Apr. 2007.