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A Transformation for a Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem

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***Abstract*— Unmanned aerial vehicles (UAVs) are being increasingly used for surveillance missions in civil and military applications. These vehicles can be heterogeneous in the sense that they can differ either in their motion constraints or sensing/attack capabilities. Given a surveillance mission that require a group of heterogeneous UAVs to visit a set of targets, this paper addresses a resource allocation problem of finding the optimal sequence of targets for each vehicle such that 1) each target is visited at least once by some vehicle, and 2) the total cost travelled by all the vehicles is minimized. This problem can be posed as a Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem (HMDMTSP). This paper presents a transformation of a Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem (HMDMTSP) into a single, Asymmetric, Traveling Salesman Problem (ATSP). As a result, algorithms available for the single salesman problem can be used to solve the HMDMTSP. To show the effectiveness of the transformation, the well known Lin-Kernighan-Helsgaun heuristic was applied to the transformed ATSP. Computational results show that good quality solutions can be obtained for the HMDMTSP relatively fast.**

***Index Terms*— Unmanned Aerial Vehicle, Heterogeneous Multiple Traveling Salesman Problem, Vehicle Routing.**

I. Introduction

Multiple, autonomous, Unmanned Aerial Vehicles (UAVs) are being developed for several civil and military applications for surveillance purposes. In these applications, multiple, heterogeneous UAVs with different sensing capabilities can be used to accomplish a given mission. Apart from the sensing capabilities, the UAVs can also have different motion constraints. Therefore, the cost of traveling between two locations can depend on the type of the UAV. This paper addresses a fundamental routing problem that arises in surveillance applications involving a team of heterogeneous UAVs. Given a set of heterogeneous vehicles and targets, the objective of the routing problem is to assign a sequence of targets to each vehicle such that each target is visited by some vehicle, and the total cost of traveling for all the vehicles is minimized. The cost of traveling between any two targets is a function of the vehicle and the pair of targets involved. It may be also assumed that these costs are part of the given data.

The routing problem can be formally stated as follows: Let there be n targets and m vehicles. Each vehicle starts at a distinct starting point or "depot". Let V be the set of vertices that correspond to the initial locations of the vehicles (or depots), with the m vertices, $\{V_1, \ldots, V_m\} = V$, representing the vehicle starting position, or depot (i.e., the vertex V_i corresponds to the depot of the i^{th} vehicle). Let (T) be the set of vertices representing the targets with n vertices, $\{T_1, \ldots, T_n\} = T$ representing the targets. Let $\mathbb{V}^i = V_i \bigcup T$ be the set of all the vertices corresponding to the i^{th} vehicle. Let $E^i = \mathbb{V}^i \times \mathbb{V}^i$ denote the set of all edges (pairs of vertices) corresponding to the i^{th} vehicle and let $C^i : E^i \rightarrow \Re_+$ denote the cost function with $C^i(a,b)$ representing the cost of traveling from vertex a to vertex b for vehicle i. We consider all the cost functions to be asymmetric, $i.e.$, $C^i(T_j, T_k)$ may not be equal to $C^i(T_k, T_j)$ for all $j, k \in \mathbb{V}^i, i = 1, \ldots, m$. A vehicle either does not visit any target or visits a subset of targets in T. If the i^{th} vehicle does not visit any targets, then its tour, $TOUR_i = \emptyset$ and its corresponding cost, $C(TOUR_i) = 0$. If the i^{th} vehicle visits at least one target, then its tour may be represented by an ordered set, $\{V_i, T_{i_1}, \ldots, T_{i_{r_i}}, V_i\}$, where $T_{i_l},\ l = 1, \ldots, r_i$ corresponds to r_i distinct targets being visited in that sequence by the i^{th} vehicle. There is a cost, $C(TOUR_i)$, associated with a tour for the i^{th} vehicle visiting at least one target and is defined as $C(TOUR_i) = C^i(V_i, T_{i_1}) + \sum_{k=1}^{r_i - 1} C^i(T_{i_k}, T_{i_{k+1}}) + C^i(T_{i_{r_i}}, V_i)$. This paper addresses the following Heterogeneous, Multiple depot, Multiple Traveling Salesman Problem (**HMDMTSP**): find tours for the vehicles so that

- each target is visited exactly once by any one vehicle, and,
- the overall cost defined by $\sum_{i \in V} C(TOUR_i)$ is minimized.

Note that the cost functions for each vehicle in the HMDMTSP are general and need not satisfy the triangle inequality. If all the UAVs are identical, then the HMDMTSP becomes the standard Multiple Depot, Multiple TSP. HMDMTSP is a generalization of the standard single TSP and is NP-Hard [1]. There are several approaches available in the literature to address a multiple TSP. They include exact algorithms [2], approximation algorithms [3], [4] and heuristics [2]. A multiple TSP can also be transformed to a single TSP wherein the standard approaches available for the single TSP can put to use. This objective of this paper to present a transformation that can convert the HMDMTSP into a standard single, asymmetric TSP.

Please refer to our companion paper [5] for an extensive

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literature review of the transformations available for the single depot, Multiple TSP. For the homogenous MDMTSP, a transformation was recently presented [5] with the assumption that the costs satisfied the triangle inequality. For the MTSP with 2 depots where there are no assumptions on the costs satisfying the triangle inequality, Rao gives a transformation in [6]. For the variant of the multiple depot TSP where each vehicle need not return to its initial depot and must visit at least one target, GuoXing [7] provides a transformation to a single ATSP. The transformation given in this paper is for multiple, heterogeneous vehicles with no assumptions on the costs satisfying the triangle inequality and is based on the transformation by Noon [8] for a heterogeneous, single depot, MTSP.

II. Transformation to a single ATSP

The HMDMTSP can be transformed by first posing the problem as a multiple, one in a set, ATSP. Then, the Noon-Bean transformation [8] available for a single, one in a set TSP is extended to transform the multiple, one in a set, ATSP into a single ATSP. To pose the HMDMTSP as a multiple, one in a set, ATSP, we replicate a distinct set of target vertices for each vehicle. Let the new set of target vertices replicated for vehicle i be denoted by $T_1^i, \ldots, T_n^i$. For each $k \in \{1, \ldots, n\}$, T_k^i is the replicated-target of T_k for vehicle i. Vehicle i is allowed to *only* visit replicated-targets in $\{T_1^i, \ldots, T_n^i\}$. The cost of traveling from target, T_j^i, to target, T_k^i, for vehicle i is $C^i(T_j, T_k)$ for all $j, k \in \{1, \ldots, n\}$. If a target, T_k^i, is visited by vehicle i, then it is required that none of the targets in the set $\{T_k^j : j \in \{1, \ldots, m\} \setminus \{i\}\}$ be visited by any of its corresponding vehicles. Now, consider the following multiple, one in a set, ATSP:

Find tours for all the vehicles such that

- for each $k = 1, \ldots, n$, exactly one target in $\{T_k^i : i \in \{1, \ldots, m\}\}$ is visited by any one vehicle,
- for each $i = 1, \ldots, m$, the i^{th} vehicle visits targets only in the set $\{T_1^i, \ldots, T_n^i\}$, and,
- the total cost of all the tours is minimized.

Refer to Fig. 1 that illustrates the multiple, one in a set, ATSP for 3 vehicles and 2 targets. It is clear that a feasible solution for the HMDMTSP can be easily transformed to a feasible solution of the multiple, one in a set, ATSP and vice versa. Now, we adapt the Noon-Bean transformation given in [8] for a single vehicle to the multiple, one in a set, ATSP. To do this, it is necessary to add a terminal vertex for each vehicle. Let V_i^d be the terminal vertex corresponding to vertex V_i. Now, there are essentially $n+2$ vertices denoted by $V_i, V_i^d, T_1^i, \ldots, T_n^i$ corresponding to vehicle i. Since, there are m vehicles, the new transformed graph has $m(n+2)$ vertices. The edges in the new transformed graph and their corresponding costs are specified as follows:

$$
\begin{aligned}
\widehat{C}(V_i, T_k^i) &= C^i(V_i, T_k) + M, \text{ for all } i \in \{1, \ldots, m\}, \\
&\quad k \in \{1, \ldots, n\}, \\
\widehat{C}(V_i, V_i^d) &= M, \text{ for all } i \in \{1, \ldots, m\}, \\
\widehat{C}(V_i^d, V_{i+1}) &= 0, \text{ for all } i \in \{1, \ldots, m-1\}, \\
\widehat{C}(V_m^d, V_1) &= 0, \\
\widehat{C}(T_j^i, T_k^{i+1}) &= C^{i+1}(T_j, T_k) + M, \text{ for all } i \in \\
&\quad \{1, \ldots, m-1\}, j, k \in \{1, \ldots, n\}, j \neq k, \\
\widehat{C}(T_j^m, T_k^1) &= C^1(T_j, T_k) + M, \text{ for all} \\
&\quad j, k \in \{1, \ldots, n\}, j \neq k, \\
\widehat{C}(T_j^i, T_j^{i+1}) &= 0, \text{ for all } i \in \\
&\quad \{1, \ldots, m-1\}, j \in \{1, \ldots, n\}, \\
\widehat{C}(T_j^m, T_j^1) &= 0, \text{ for all } j \in \{1, \ldots, n\}, \\
\widehat{C}(T_j^i, V_{i+1}^d) &= C^{i+1}(T_j, V_{i+1}) + M, \text{ for all } i \in \\
&\quad \{1, \ldots, m-1\}, j \in \{1, \ldots, n\}, \\
\widehat{C}(T_j^m, V_1^d) &= C^1(T_j, V_1) + M, \text{ for all } j \in \{1, \ldots, n\}.
\end{aligned} \tag{1}
$$

In the above equations, M is a large positive constant chosen to be equal to $2(n+m) \max_{i=1}^m \max_{j,k=1}^n C^i(T_j, T_k)$. Since the HMDMTSP can have a maximum of $(n+m)$ edges, the constant M is greater than the optimal cost of the HMDMTSP. An edge does not exist in the transformed graph if it is not assigned a cost in the equations (1). Refer to Fig. 2 for an example illustrating the transformation for 3 vehicles and 2 targets. Now, the main result of this paper is in the following theorem:

Theorem 2.1: Given an optimal tour, x_{atsp}^{opt}, for the ATSP on the transformed graph, one can construct a set of tours $TOUR_1, \ldots, TOUR_m$ that are optimal for the HMDMTSP in $n+m$ steps.

Without loss of generality, we assume that the optimal tour x_{atsp}^{opt} starts from the vertex corresponding to the first vehicle, V_1. To prove theorem 2.1, we first state a list of facts about the optimal tour, x_{atsp}^{opt}, in the following lemma:

Lemma 2.1: The optimal tour, x_{atsp}^{opt}, for the single ATSP on the transformed graph satisfies the following conditions:

- **I** Consider the set of all the replicated-targets corresponding to target T_j, *i.e.*, $R_j = \{T_j^i : i = 1, \ldots, m\}$. Then, both the in-degree and the out-degree of the set R_j in x_{atsp}^{opt} is equal to 1. Essentially, there is exactly one edge entering and leaving the set R_j.
- **II** Let T_j^i be the first replicated-target in the set R_j visited by the directed path from V_1 in the optimal tour. After reaching T_j^i, the directed path visits all the remaining vertices in R_j before leaving R_j.
- **III** The directed path, $PATH_i$, from a depot vertex, V_i, to its corresponding terminal vertex, V_i^d, in the optimal tour, x_{atsp}^{opt}, will not visit any other depots or terminal vertices.
- **IV** The cost of the optimal tour, $\widehat{C}(x_{atsp}^{opt})$, is equal to the sum of the cost of all the directed paths, $\sum_{i=1}^m \widehat{C}(PATH_i)$.

Proof: The cost of any incoming or outgoing edge

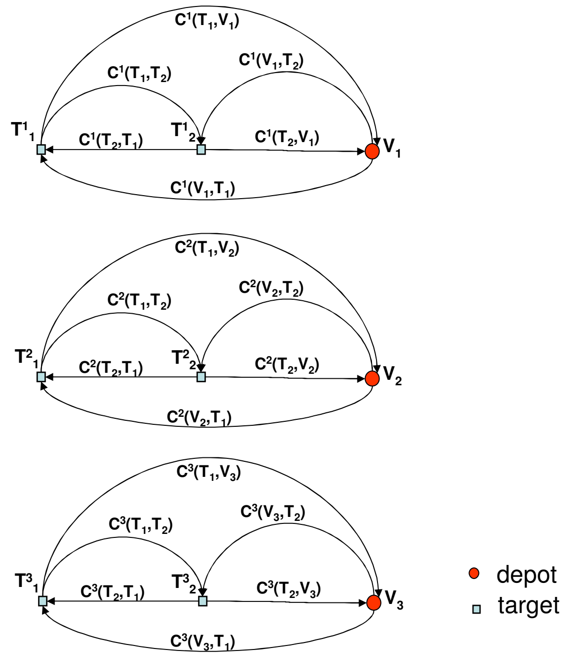

Fig. 1. Example illustrating the formulated multiple, one in a set, ATSP for 3 vehicles and 2 targets.

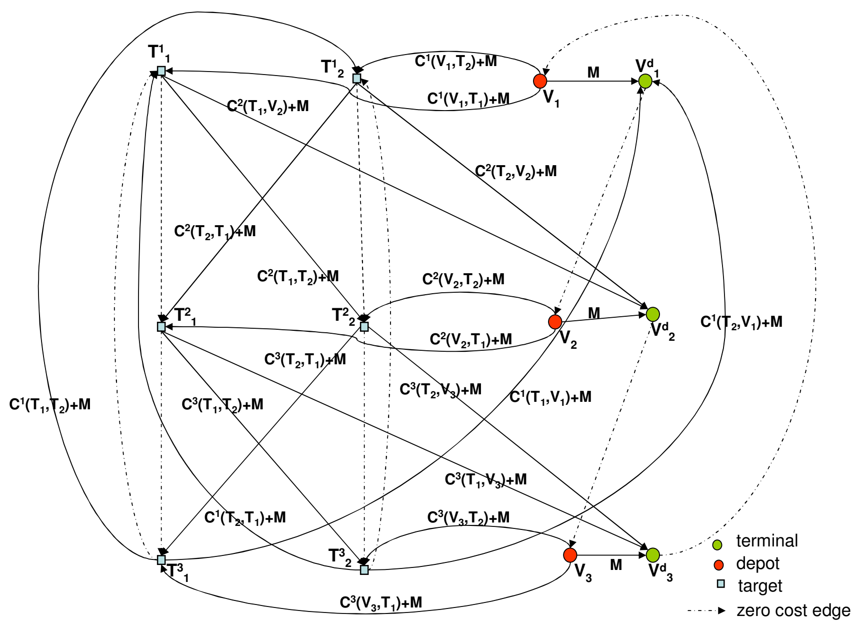

Fig. 2. Example illustrating the transformed single ATSP for 3 vehicles and 2 targets.

of the set R_j has an additional cost M associated with it. Without loss of generality, let an incoming edge be incident on target T_j^1. The transformed graph is such that a directed path from T_j^1 can use just the zero cost edges and visit each of the targets in $\{T_j^2, \ldots, T_j^m\}$. That is the directed path can visit T_j^2 after T_j^1, T_j^3 after T_j^2 and so on. After reaching T_j^m the directed path can leave the set R_j. If the directed path from T_j^1 leaves T_j^i for any $i < m$, then the remaining vertices in the set R_j can be only visited by any of the incoming edges of R_j whose cost is at least greater than M. Since M is a large constant (greater than the optimal cost of the HMDMTSP), the optimal tour, would have a least number of edges whose cost has a value M associated with it. For this reason, the number of edges incoming or outgoing of the set R_j in x_{atsp}^{opt} will be as minimum as possible. Therefore, the in-degree and the out-degree of the set R_j in the optimal tour must be equal to 1. Hence claim **I** of the lemma is true.

Claim **II** just follows from the proof of claim **I**. The directed path from V_i after visiting a subset of replicated-targets must either visit V_i^d or some other terminal vertex. The transformation is such that this directed path can visit any other terminal vertex only if claim **II** is violated. Therefore, the directed path from V_i after visiting a subset of replicated-targets must visit V_i^d. Hence, claim **III** must be true.

There is only one outgoing edge from each of the terminal vertices in the transformed graph. Also, all these outgoing edges are zero cost edges. Therefore, all the zero cost edges in the set $\mathcal{Z} = \{(V_1^d, V_2), (V_2^d, V_3), \ldots, (V_m^d, V_1)\}$ must be chosen in the optimal solution x_{atsp}^{opt}. From claim **III**, removing all the zero cost edges in $\mathcal{Z}$ from x_{atsp}^{opt} would essentially leave a set of m disconnected directed paths, $PATH_1, PATH_2, \ldots, PATH_m$, with each path starting from a depot and reaching its corresponding terminal vertex. Therefore, $\widehat{C}(x_{atsp}^{opt}) = \sum_{i=1}^m \widehat{C}(PATH_i)$. Hence proved. ∎

Lemma 2.2: Given an optimal solution x_{atsp}^{opt} for the transformed graph, there exists a set o f tours, $TOUR_1, TOUR_2, \ldots, TOUR_m$, for the HMDMTSP such that the total cost of the tours, $\sum_{i=1}^m C(TOUR_i)$, is equal to $\widehat{C}(x_{atsp}^{opt}) - (n+m)M$. Also, these tours can be constructed in $n+m$ steps.

Proof:
From lemma 2.1, the directed path corresponding to the i^{th} vehicle, $PATH_i$, does not visit any other depot or terminal. Also, each replicated-target set R_j in the optimal solution, x_{atsp}^{opt}, has an in-degree and out-degree equal to 1. Let α_i denote the number of distinct replica-target sets visited by the directed path, $PATH_i$, from depot V_i to V_i^d in the optimal tour.

Case $\alpha_i > 0$:
If $\alpha_i > 0$, let the distinct sets visited by $PATH_i$ be represented by $R_{i1}, R_{i2}, \ldots, R_{i\alpha_i}$. The directed path first visits the set R_{i1}, then R_{i2}, and so on. Also, let the tour of the i^{th} vehicle for the HMDMTSP constructed from $PATH_i$ be $TOUR_i = \{V_i, T_{i1}, T_{i2}, \ldots, T_{i\alpha_i}, V_i\}$. In the following argument, we first show that $\widehat{C}(PATH_i) = C(TOUR_i) + (\alpha_i + 1)M$ if i is equal to 1.

$$\begin{aligned}\sum_{(u,v)\in Path_1} \widehat{C}(u,v) &= \widehat{C}(V_1, T_{11}^1) + \sum_{k=1}^{\alpha_1 - 1} \widehat{C}(T_{1k}^m, T_{1(k+1)}^1) \\ &\quad + \widehat{C}(T_{1\alpha_1}^m, V_1^d).\end{aligned}$$

Now, substituting for the transformed costs from equation (1), we get,

$$\begin{aligned}\sum_{(u,v)\in Path_1} \widehat{C}(u,v) &= \\ & C^1(V_1, T_{11}) + M + \sum_{k=1}^{\alpha_1-1}(C^1(T_{1k}, T_{1(k+1)}) + M) \\ & + C^1(T_{1\alpha_1}, V_1) + M \\ &= C^1(V_1, T_{11}) + \sum_{k=1}^{\alpha_1-1} C^1(T_{1k}, T_{1(k+1)}) + C^1(T_{1\alpha_1}, V_1) \\ & + (\alpha_1 + 1)M \\ &= C(TOUR_1) + (\alpha_1 + 1)M.\end{aligned}$$

One can use a similar argument to also show that $\widehat{C}(PATH_i) = C(TOUR_i) + (\alpha_i + 1)M$ for any $i > 1$.

Case $\alpha_i = 0$:
In this case, $PATH_i$ consists of only one directed edge (V_i, V_i^d) whose edge cost is M. Also, let $TOUR_i = \emptyset$ and $C(TOUR_i) = 0$ when $\alpha_i = 0$.

As the optimal solution x_{atsp}^{opt} visits each of the replicated-targets exactly once, any target $j \in T$ must be present in the tours of exactly one of the vehicles. Therefore, $TOUR_1, \ldots, TOUR_m$, is a feasible solution to the HMDMTSP. An example in Fig. 3 shows an optimal solution of the ATSP and its corresponding tours for the HMDMTSP. Now, we show the relation about the costs.

$$\begin{aligned}\widehat{C}(x_{atsp}^{opt}) &= \sum_{i=1}^m \widehat{C}(PATH_i) \quad \text{(from lemma 2.1)} \\ &= \sum_{i=1,\alpha_i>0}^m \widehat{C}(PATH_i) + \sum_{i=1,\alpha_i=0}^m \widehat{C}(PATH_i) \\ &= \sum_{i=1,\alpha_i>0}^m (C(TOUR_i) + (\alpha_i+1)M) + \sum_{i=1,\alpha_i=0}^m M \\ &= \sum_{i=1}^m C(TOUR_i) + mM + M\sum_{i=1,\alpha_i>0}^m \alpha_i \\ &= \sum_{i=1}^m C(TOUR_i) + (m+n)M.\end{aligned}$$

Note that for any i, $TOUR_i$ can be constructed from $PATH_i$ in $\alpha_i + 1$ steps. Therefore, all the tours can be constructed from x_{atsp}^{opt} in $n+m$ steps. ∎

Lemma 2.3: Given a set of optimal tours, $TOUR_1^*$, $\ldots, TOUR_m^*$, for the HMDMTSP, one can construct a feasible solution, $\widehat{x}_{atsp}$, for the transformed graph such that $\sum_{i=1}^m C(TOUR_i^*) = \widehat{C}(\widehat{x}_{atsp}) - (n+m)M$.

Proof: If $TOUR_i^*$ is an empty set for any vehicle i, let $\widehat{PATH}_i$ consist of exactly one edge (V_i, V_i^d). If $TOUR_i^*$ is not an empty set for any vehicle i, let $TOUR_i^*$ be denoted by $\{V_i, T_{i1}, T_{i2}, \ldots, T_{i\alpha_i}, V_i\}$. Construct a directed path $\widehat{PATH}_i$ that starts from V_i, visits all the replicated-target

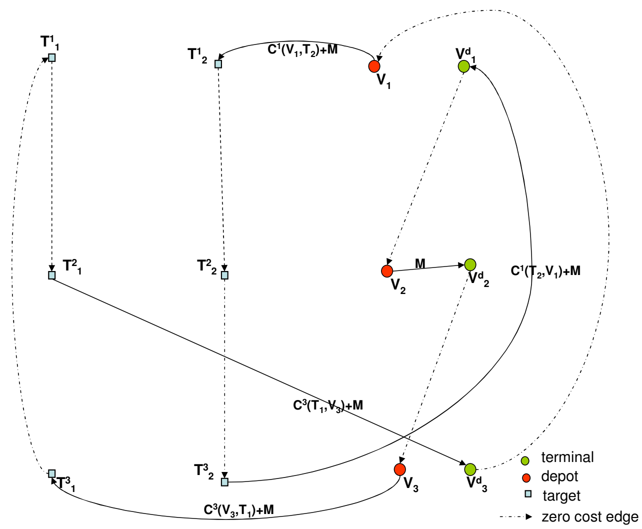

Fig. 3. Example illustrating an optimal ATSP tour on the transformed graph. Note that the optimal tour has 5 ($i.e.,\ m+n = 3+2 = 5$) edges with additional costs equal to M. The corresponding tours for the HMDMTSP in this example are: $TOUR_1 = \{V_1, T_2, V_1\}$, $TOUR_2 = \emptyset$, $TOUR_3 = \{V_3, T_1, V_3\}$.

sets in the order of $R_{i1}, R_{i2}, \dots, R_{i\alpha_i}$ and reaches the terminal V_i^d. By adding all the outgoing, zero cost, edges from the terminals (i.e., $\mathcal{Z}= \{(V_1^d, V_2), (V_2^d, V_3), \dots, (V_m^d, V_1)\}$) to all the directed paths $\widehat{PATH}_1,\dots,\ \widehat{PATH}_m$, one can construct a feasible solution $\widehat{x}_{atsp}$ for the single ATSP on the transformed graph. By using the same arguments of lemma 2.2 in the reverse direction, we can also show that $\sum_{i=1}^{m} C(TOUR_i^*) = \widehat{C}(\widehat{x}_{atsp}) - (n+m)M$. Hence proved. ∎

A. Proof of theorem 2.1

Let the tours be constructed for the HMDMTSP as in lemma 2.2. Now,

$$\begin{aligned}\sum_{i=1}^{m} C(TOUR_i) &= \widehat{C}(x_{atsp}^{opt}) - (n+m)M \text{ (from lemma 2.2)} \\ &\leq \widehat{C}(\widehat{x}_{atsp}) - (n+m)M \\ &= \sum_{i=1}^{m} C(TOUR_i^*) \text{ (from lemma 2.3).}\end{aligned}$$

Therefore, the tours, $TOUR_1, \dots, TOUR_m$, must be optimal for the HMDMTSP.

III. Computational Results

The objective of this section was to find the quality of the solutions produced for the HMDMTSP by applying a well known algorithm to the transformed problem. In particular, we applied the modified LKH heuristic [9] which is one the best heuristics [1] available to solve the single ATSP.

In all the simulations, the number of UAVs were fixed to be equal to 6. The minimum turning radius (r) of all the UAVs in the simulations was chosen to vary uniformly from 100 to 200 meters. Dubins' [10] result was used to calculate the minimum distance to travel for an UAV between any two targets. Dubin's [10] result states that the path joining the two targets (x_1, y_1, θ_1) and (x_2, y_2, θ_2) that has minimal length subject to the minimum turning radius constraints, is one of RSR, RSL, LSR, LSL, RLR and LRL. Here, any curved segment of radius r along which the vehicle executes a clockwise (counterclockwise) rotational motion is denoted by $R(L)$, and the segment along which the vehicle travels straight is denoted by S.

All the targets were uniformly generated in a square of area 5 × 5 km^2. For each generated target, an approach angle was selected uniformly in the interval $[0, 2\pi]$. Each vehicle was assigned a subset of targets (generated randomly) that the vehicle cannot visit. In essence, heterogeneity among vehicles was introduced in two ways: the Dubins' distance between any two targets will depend on the minimum turning radius of a vehicle; the initial vehicle-target assignment will also determine whether a vehicle can visit a target or not.

The number of targets were allowed to vary from 4 to 40. For a given number of vehicles (m) and targets (n), 20 instances were randomly generated. The solution quality of an instance I is defined as

$$100\left(\frac{C_{LKH}^I - C_{LB}^I}{C_{LB}^I - (n+m)M}\right),$$

where C_{LKH}^I is the cost of the solution obtained by applying the LKH heuristic on the transformed graph and C_{LB}^I is the lower bound for the single ATSP on the transformed graph. The LKH program by Helsgaun available at http://www.akira.ruc.dk/ keld/research/LKH/ was used to solve the ATSP. The program was run on a Pentium 4 CPU with 3GHz processing power and 1.24 GB RAM.

The results regarding the mean solution quality and their computation times are shown in Fig.4 and Fig.5 respectively. The results show that the quality of the solutions were approximately 15% on an average. As expected, the mean

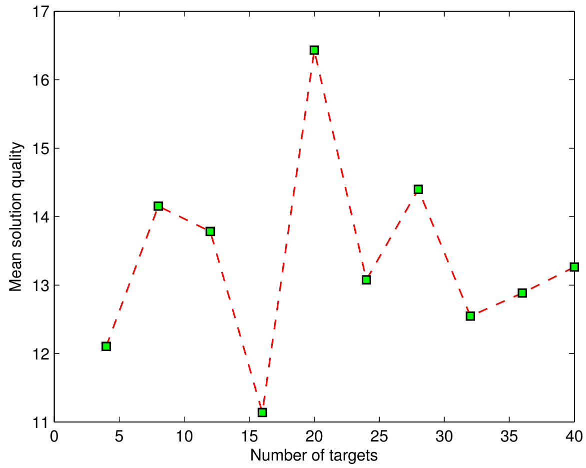

Fig. 4. Mean solution quality as a function of the number of targets.

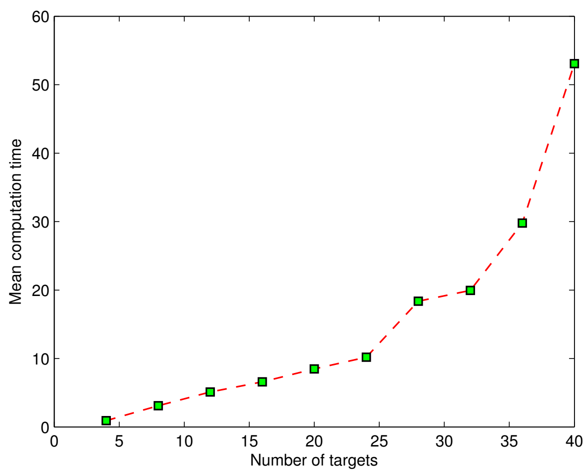

Fig. 5. Mean computation time in seconds as a function of the number of targets.

computation times increased with the number of targets. Fig. 6 shows the output of the tours found by applying the LKH heuristic for a 5 vehicle, 20 target problem. In this example, there were five UAVs used with their turning radius uniformly spaced from 100 m to 200 m. Each vehicle was assigned a distinct subset of targets that it is not supposed to visit. The solution found by the LKH heuristic used only two UAVs with a turning radius of 125 m and 175 m. The solution quality for this example was approximately 16%. Considering that a heterogeneous, multiple vehicle routing problem is a difficult problem to solve, these results indicate that the approach given in the paper is promising.

IV. Conclusions

A transformation was provided in this paper that converts a Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem (HMDMTSP) to a standard, Asymmetric Traveling Salesman Problem (ATSP). The limitation of the approach proposed in this paper is that the transformed graph

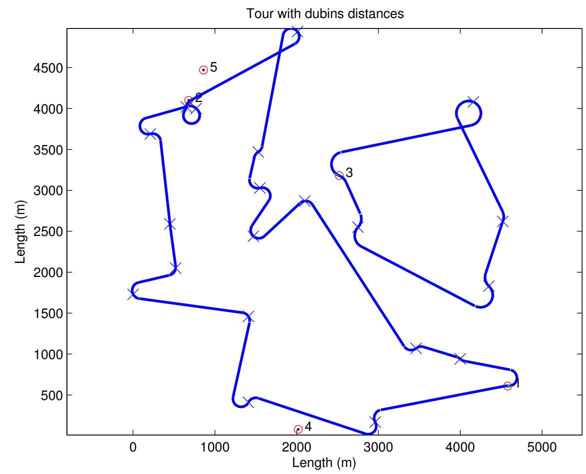

Fig. 6. An example showing the dubins tours found by applying the LKH heuristic on the transformed graph.

contains a large number of vertices (i.e., $(n+2)m$ vertices) where n (m) is the number of targets (vehicles). If there are large number of vehicles present in an instance, the transformed graph could still be a computationally challenging problem.

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