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# A Delay Decomposition Approach to Stability Analysis of Discrete-Time Systems with Time-Varying Delay

Xun-Lin Zhu and Guang-Hong Yang

*Abstract*— **This paper studies the problem of stability analysis for discrete-time delay systems. By using a delay decomposition approach and the discrete Jensen inequality, a new stability criterion is presented in terms of linear matrix inequalities (LMIs) and proved to be less conservative than the existing ones. A numerical example is given to illustrate the effectiveness and advantages of the proposed method.**

## I. INTRODUCTION

During the last two decades, the stability problem of linear continuous-time systems with time-delay has received considerable attention [6]-[9]. The practical examples of time delay systems include engineering, communications and biological systems. The existence of delay in a practical system may induce instability, oscillation and poor performance.

Compared with continuous-time systems with time-delay, discrete-time systems with time-varying delay have strong background in engineering applications, among which network based control has been well recognized to be a typical example (see [3]-[5], [12]). One should notice that little effort has been made towards investigating the stability of discrete time-delay systems. The reason is that for linear discrete-time systems with constant time-delay, one can transform them into the delay-free systems via state augmentation approach. However, the augmentation approach cannot be applied to linear discrete-time systems with time-varying delay. Recently, there have been some works investigating the stability of discrete systems with time-varying delay via Lyapunov approaches [13], [14].

By employing the Moon's inequality [10] to estimate the cross products between two vectors, [14] proposed a stability condition which was dependent on the minimum and maximum delay bounds. By defining a new Lyapunov functional and circumventing the utilization of some bounding inequalities for cross products between two vectors, [13] improved the result in [14], and the free-weighting matrix method (see [6]) was adopted to reduce the conservatism of the results. However, the introduction of the free-weighting matrices may increase the number of decision variables, then it may lead to the increase of the computational complexity inevitably.

By defining a new Lyapunov functional and using the discrete Jensen inequality, [17] presented stability criteria for discrete-time delay systems. Since the discrete Jensen inequality was adopted and no any free-weighting matrices were introduced, the computational complexity of the stability criteria in [17] was reduced greatly compared with the existing results. Furthermore, it was shown that the stability conditions in [17] were also less conservative than the corresponding ones in [13] and [14].

In this paper, the range of delay $d_k$ is divided into $d_M - d_m + 1$ cases: $d_k = d_m,\ d_k = d_m + 1,\ \cdots,\ d_k = d_M$. For each case, we estimate the upper bounds of the term $-\sum_{i=k-d_M}^{k-d_m-1} \eta^T(i)U_2\eta(i)$, respectively. Thus, the upper bound of the term $-\sum_{i=k-d_M}^{k-d_m-1} \eta^T(i)U_2\eta(i)$ is estimated more exactly, and the presented stability condition is less conservative than the corresponding one in [17].

This paper is organized as follows. Section II gives the problem statement. The stability criterion of discrete-time delay systems is presented in Section III. Section IV gives an example to illustrate the effectiveness of the presented stability criteria. Section V concludes this paper.

## II. PROBLEM STATEMENT

Consider the following discrete-time system with a time-varying state delay [13]:

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-d_k) \\ x(k) = \phi(k) \quad k = -d_M,\ -d_M+1,\ \cdots,\ 0, \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $A$ and $A_d$ are constant matrices with appropriate dimensions, $d_k$ is a time-varying delay in the state, and it satisfies

$$d_m \le d_k \le d_M, \tag{2}$$

where $d_m$ and $d_M$ are constant positive integers representing the lower and upper delays, respectively.

The purpose of this paper is to find new stability criteria which are of less conservatism and less computational complexity than the existing results.

For the system (1)-(2), the Moon's inequality was used in [14] to bound the inner product between two vectors, and the obtained stability condition is listed as follows:

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Xun-Lin Zhu is with the School of Computer Science and Communication Engineering, Zhengzhou University of Light Industry, Zhengzhou, Henan, 450002, China. hntjxx@163.com

Guang-Hong Yang is with the College of Information Science and Engineering, Northeastern University, Shenyang, Liaoning, 110004, China. Corresponding author. yangguanghong@ise.neu.edu.cn, yang_guanghong@163.com

**Lemma 1.** [14] The system (1)-(2) is asymptotically stable if there exist matrices $P=P^T>0,\ Q=Q^T>0,\ X=X^T>0,\ Z=Z^T>0$, and $Y$ satisfying

$$\Upsilon=\begin{bmatrix} -P & 0 & PA & PA_d \\ * & -d_M^{-1}Z & Z(A-I) & ZA_d \\ * & * & \Upsilon_1 & -Y \\ * & * & * & -Q \end{bmatrix}<0, \tag{3}$$

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}\geq 0, \tag{4}$$

where

$$\Upsilon_1=-P+d_MX+Y+Y^T+(d_M-d_m+1)Q.$$

By using the free-weighting method, [13] presented an improved result on Lemma 1 as follows:

**Lemma 2.** [13] The system (1)-(2) is asymptotically stable if there exist matrices $P=P^T>0,\ Q=Q^T\geq 0,\ R=R^T\geq 0,\ Z_i=Z_i^T>0\ (i=1,\ 2),\ M,\ S,\ N$ satisfying

$$\Xi=\begin{bmatrix} \Xi_1+\Xi_2+\Xi_2^T+\Xi_3 & \Xi_4 \\ * & \Xi_5 \end{bmatrix}<0 \tag{5}$$

where

$$\Xi_1=\begin{bmatrix} \Xi_{11} & A^TPA_d & 0 \\ * & A_d^TPA_d-Q & 0 \\ * & * & -R \end{bmatrix},$$
$$\Xi_{11}=A^TPA-P+(d_M-d_m+1)Q+R,$$
$$\Xi_2=\begin{bmatrix} M+N & S-M & -S-N \end{bmatrix},$$
$$\Xi_3=d_M\begin{bmatrix} A-I & A_d & 0 \end{bmatrix}^T(Z_1+Z_2)\begin{bmatrix} A-I & A_d & 0 \end{bmatrix},$$
$$\Xi_4=\begin{bmatrix} \sqrt{d_M}M & \sqrt{d_M-d_m}S & \sqrt{d_M}N \end{bmatrix},$$
$$\Xi_5=diag\{-Z_1,\ -Z_1,\ -Z_2\}.$$

By using the Jensen inequality method, [17] presented an improved result on Lemma 2 as follows:

**Lemma 3.** [17] The system (1)-(2) is asymptotically stable if there exist matrices $P=P^T>0,\ Q_i=Q_i^T\geq 0,\ U_i=U_i^T>0\ (i=1,\ 2,\ 3)$ satisfying

$$\Lambda=\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & U_3 & U_1 \\ * & \Lambda_{22} & U_2 & U_2 \\ * & * & -Q_3-U_2-U_3 & 0 \\ * & * & * & -Q_2-U_1-U_2 \end{bmatrix}<0, \tag{6}$$

where

$$\Lambda_{11}=A^TPA-P+(d_M-d_m+1)Q_1+Q_2+Q_3-(U_1+U_3)+(A-I)^TU(A-I),$$
$$\Lambda_{12}=A^TPA_d+(A-I)^TUA_d,$$
$$\Lambda_{22}=A_d^TPA_d-Q_1-2U_2+A_d^TUA_d,$$
$$U=d_M^2U_1+(d_M-d_m)^2U_2+d_m^2U_3.$$

Corresponding to the Jensen integral inequality [2], we can get the following discrete Jensen inequality which will be exploited for the stability analysis of the system (1)-(2):

**Lemma 4.** [17] For any constant matrix $M\in R^{n\times n},\ M=M^T>0$, integers $\gamma_2\geq\gamma_1$, vector function $\omega:\ \{\gamma_1,\ \gamma_1+1,\ \cdots,\ \gamma_2\}\rightarrow R^n$ such that the sums in the following are well defined, then

$$\begin{aligned} &-(\gamma_2-\gamma_1+1)\sum_{i=\gamma_1}^{\gamma_2}\omega^T(i)M\omega(i) \\ &\leq-\Big(\sum_{i=\gamma_1}^{\gamma_2}\omega(i)\Big)^TM\Big(\sum_{i=\gamma_1}^{\gamma_2}\omega(i)\Big). \end{aligned} \tag{7}$$

## III. MAIN RESULT

In this section, a new stability criterion for system (1)-(2) will be presented by suing a delay decomposition method.

For convenience, we denote $\bar{d}=d_M-d_m$.

For the system (1)-(2), we give the following stability condition using the discrete Jenson inequality.

**Theorem 1.** System (1)-(2) is asymptotically stable if there exist matrices $P=P^T>0,\ Q_j=Q_j^T\geq 0,\ U_j=U_j^T>0\ (j=1,\ 2,\ 3)$ satisfying

$$\Omega_i<0 \qquad (i=0,\ 1,\ \cdots,\ \bar{d}) \tag{8}$$

where

$$\Omega_0=\begin{bmatrix} \Omega_{11} & U_3 & U_1 & A^TP & (A-I)^TU \\ * & \Omega_{22}^{(0)} & U_2 & A_d^TP & A_d^TU \\ * & * & -Q_2-U_1-U_2 & 0 & 0 \\ * & * & * & -P & 0 \\ * & * & * & * & -U \end{bmatrix},$$

$$\Omega_j=\begin{bmatrix} \Omega_{11} & 0 & U_3 & U_1 & A^TP & (A-I)^TU \\ * & \Omega_{22}^{(j)} & \Omega_{23}^{(j)} & \Omega_{24}^{(j)} & A_d^TP & A_d^TU \\ * & * & \Omega_{33}^{(j)} & 0 & 0 & 0 \\ * & * & * & \Omega_{44}^{(j)} & 0 & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -U \end{bmatrix},$$

$$\Omega_{\bar{d}}=\begin{bmatrix} \Omega_{11} & U_3 & U_1 & A^TP & (A-I)^TU \\ * & \Omega_{22}^{(\bar{d})} & U_2 & 0 & 0 \\ * & * & \Omega_{33}^{(\bar{d})} & A_d^TP & A_d^TU \\ * & * & * & -P & 0 \\ * & * & * & * & -U \end{bmatrix},$$

$$\Omega_{11}=-P+(\bar{d}+1)Q_1+Q_2+Q_3-U_1-U_3,$$
$$\Omega_{22}^{(0)}=-Q_1-Q_3-U_3-U_2,$$
$$\Omega_{22}^{(j)}=-Q_1-\frac{\bar{d}}{j}U_2-\frac{\bar{d}}{\bar{d}-j}U_2,$$
$$\Omega_{23}^{(j)}=-\frac{\bar{d}}{j}U_2,$$
$$\Omega_{24}^{(j)}=-\frac{\bar{d}}{\bar{d}-j}U_2,$$
$$\Omega_{33}^{(j)}=-Q_3-\frac{\bar{d}}{j}U_2-U_3,$$
$$\Omega_{44}^{(j)}=-Q_2-U_1-\frac{\bar{d}}{\bar{d}-j}U_2,$$
$$\Omega_{22}^{(\bar{d})}=-Q_3-U_3-U_2,$$
$$\Omega_{33}^{(\bar{d})}=-Q_1-Q_2-U_1-U_2,$$
$$U=d_M^2U_1+\bar{d}^2U_2+d_m^2U_3.$$

*Proof:* Choose a Lyapunov functional candidate as:

$$V(k) = V_1(k)+V_2(k)+V_3(k)+V_4(k)+V_5(k) +V_6(k)+V_7(k)+V_8(k), \tag{9}$$

where

$$\begin{aligned}
V_1(k) &= x^T(k)Px(k),\\
V_2(k) &= \sum_{i=k-d_k}^{k-1} x^T(i)Q_1x(i),\\
V_3(k) &= \sum_{i=k-d_M}^{k-1} x^T(i)Q_2x(i),\\
V_4(k) &= \sum_{i=k-d_m}^{k-1} x^T(i)Q_3x(i),\\
V_5(k) &= \sum_{j=-d_M+1}^{-d_m}\sum_{i=k+j}^{k-1} x^T(i)Q_1x(i),\\
V_6(k) &= d_M\sum_{i=-d_M}^{-1}\sum_{m=k+i}^{k-1} \eta^T(m)U_1\eta(m),\\
V_7(k) &= (d_M-d_m)\sum_{i=-d_M}^{-d_m-1}\sum_{m=k+i}^{k-1} \eta^T(m)U_2\eta(m),\\
V_8(k) &= d_m\sum_{i=-d_m}^{-1}\sum_{m=k+i}^{k-1} \eta^T(m)U_3\eta(m),\\
\eta(k) &= x(k+1)-x(k),
\end{aligned}$$

and $P=P^T>0,\ Q_i=Q_i^T\geq 0,\ U_i=U_i^T>0\ (i=1,\ 2,\ 3)$ are matrices to be determined. From Lemma 3, it yields that

$$\begin{aligned}
&-d_M\sum_{l=k-d_M}^{k-1}\eta^T(l)U_1\eta(l)\\
&\leq -\Big(\sum_{l=k-d_M}^{k-1}\eta(l)\Big)^T U_1\Big(\sum_{l=k-d_M}^{k-1}\eta(l)\Big)\\
&= -[x(k)-x(k-d_M)]^T U_1[x(k)-x(k-d_M)],
\end{aligned}\tag{10}$$

and

$$\begin{aligned}
&-d_m\sum_{l=k-d_m}^{k-1}\eta^T(l)U_3\eta(l)\\
&\leq -\Big(\sum_{l=k-d_m}^{k-1}\eta(l)\Big)^T U_3\Big(\sum_{l=k-d_m}^{k-1}\eta(l)\Big)\\
&= -[x(k)-x(k-d_m)]^T U_3[x(k)-x(k-d_m)].
\end{aligned}\tag{11}$$

Define $\Delta V(k)=V(k+1)-V(k)$, then along the solution of (1) we have

$$\begin{aligned}
\Delta V_1(k) =& x^T(k+1)Px(k+1)-x^T(k)Px(k)\\
=&[Ax(k)+A_d(k-d_k)]^T P[Ax(k)+A_d(k-d_k)]\\
&-x^T(k)Px(k),
\end{aligned}\tag{12}$$

$$\Delta V_2(k) \leq x^T(k)Q_1x(k)-x^T(k-d_k)Q_1x(k-d_k) +\sum_{i=k-d_M+1}^{k-d_m} x^T(i)Q_1x(i), \tag{13}$$

$$\Delta V_3(k) = x^T(k)Q_2x(k)-x^T(k-d_M)Q_2x(k-d_M), \tag{14}$$

$$\Delta V_4(k) = x^T(k)Q_3x(k)-x^T(k-d_m)Q_3x(k-d_m), \tag{15}$$

$$\Delta V_5(k) = (d_M-d_m)x^T(k)Q_1x(k)-\sum_{i=k-d_M+1}^{k-d_m} x^T(i)Q_1x(i), \tag{16}$$

$$\begin{aligned}
\Delta V_6(k) =& d_M\sum_{i=-d_M}^{-1}[\eta^T(k)U_1\eta(k)-\eta^T(k+i)U_1\eta(k+i)]\\
=& d_M^2\eta^T(k)U_1\eta(k)-d_M\sum_{m=k-d_M}^{k-1}\eta^T(m)U_1\eta(m)\\
\leq& d_M^2[(A-I)x(k)+A_d(k-d_k)]^T U_1\\
&\times[(A-I)x(k)+A_d(k-d_k)]\\
&-[x(k)-x(k-d_M)]^T U_1[x(k)-x(k-d_M)],
\end{aligned}\tag{17}$$

$$\begin{aligned}
\Delta V_7(k) =& (d_M-d_m)\sum_{i=-d_M}^{-d_m-1}[\eta^T(k)U_2\eta(k)\\
&-\eta^T(k+i)U_2\eta(k+i)]\\
=& (d_M-d_m)^2\eta^T(k)U_2\eta(k)\\
&-(d_M-d_m)\sum_{m=k-d_M}^{k-d_m-1}\eta^T(m)U_2\eta(m),
\end{aligned}\tag{18}$$

$$\begin{aligned}
\Delta V_8(k) =& d_m\sum_{i=-d_m}^{-1}[\eta^T(k)U_3\eta(k)-\eta^T(k+i)U_3\eta(k+i)]\\
=& d_m^2\eta^T(k)U_3\eta(k)-d_m\sum_{m=k-d_m}^{k-1}\eta^T(m)U_3\eta(m)\\
\leq& d_m^2[(A-I)x(k)+A_d(k-d_k)]^T U_3\\
&\times[(A-I)x(k)+A_d(k-d_k)]\\
&-[x(k)-x(k-d_m)]^T U_3[x(k)-x(k-d_m)].
\end{aligned}\tag{19}$$

Now, we estimate the upper bound of $-\bar{d}\sum_{i=k-d_M}^{k-d_m-1}\eta^T(i)U_2\eta(i)$ in $\Delta V_7(k)$ as follows:

case 1: if $d_k=d_m$, then it yields that:

$$\begin{aligned}
&-\bar{d}\sum_{i=k-d_M}^{k-d_m-1}\eta^T(i)U_2\eta(i)\\
&\leq -[x(k-d_m)-x(k-d_M)]^T U_2[x(k-d_m)-x(k-d_M)].
\end{aligned}\tag{20}$$

Combining (12)-(19) with (20), and from the Schur complement, it gets that $\Delta V(k)<0$ if $\Omega_0<0$.

case 2: if $d_m<d_k<d_M$, denote $i=d_k-d_m$, then it gets that:

$$\begin{aligned}
&-\bar{d}\sum_{i=k-d_M}^{k-d_m-1}\eta^T(i)U_2\eta(i)\\
&= -\bar{d}\sum_{i=k-d_k}^{k-d_m-1}\eta^T(i)U_2\eta(i)-\bar{d}\sum_{i=k-d_M}^{k-d_k-1}\eta^T(i)U_2\eta(i)\\
&\leq -\frac{\bar{d}}{i}[x(k-d_m)-x(k-d_k)]^T U_2[x(k-d_m)-x(k-d_k)]\\
&\quad -\frac{\bar{d}}{\bar{d}-i}[x(k-d_k)-x(k-d_M)]^T U_2[x(k-d_k)-x(k-d_M)].
\end{aligned}\tag{21}$$

Combining (12)-(19) with (21), and from the Schur complement, it gets that $\Delta V(k)<0$ if $\Omega_i<0$ for $1\leq i\leq\bar{d}-1$.

TABLE I

ALLOWABLE UPPER BOUND OF $d_M$ FOR GIVEN $d_m$

| Methods | $d_m = 2$ | $d_m = 4$ | $d_m = 6$ | $d_m = 10$ | $d_m = 12$ |
|---|---|---|---|---|---|
| Lemma 1 | 7 | 8 | 9 | 12 | 13 |
| Lemma 2 | 13 | 13 | 14 | 15 | 16 |
| Lemma 3 | 13 | 13 | 14 | 17 | 18 |
| Theorem 1 | 17 | 17 | 18 | 20 | 21 |

case 3: if $d_k = d_M$, then similar to case 1, one can get that $\Delta V(k) < 0$ if $\Omega_{\bar{d}} < 0$.

The proof is completed. ∎

**Remark 1.** By combining a delay decomposition method with the discrete Jensen inequality, Theorem 1 presents a new LMI-based stability criterion for the discrete system (1)-(2). Different from [17], $\bar{d}+1$ cases $d_k = d_m,\ d_k = d_m+1,\ \cdots,\ d_k = d_M$ are discussed in the proof of Theorem 1, respectively, such that the upper bound of the term $-\bar{d}\sum_{i=k-d_M}^{k-d_m-1} \eta^T(i)U_2\eta(i)$ is estimated more exactly, so the stability condition in Theorem 1 is less conservative than Lemma 3.

**Remark 2.** Similar to [17], Theorem 1 can be extended to the case of uncertain systems, and it is omitted here.

## IV. ILLUSTRATIVE EXAMPLES

In this section, an example is provided to illustrate the advantage of the proposed stability result.

**Example 1.** [13] Consider the following system

$$x(k+1) = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix} x(k-d_k), \tag{22}$$

where $d_k$ represents a time-varying state delay. The upper bounds on the time delay, $d_M$, which guarantee the stability of the system (1) for given lower bounds, $d_m$, are shown in Table 1. It is clear that the results obtained by Theorem 1 are less conservative than the ones obtained in [13], [14] and in [17].

## V. CONCLUSION

This paper studies the problem of stability for discrete-time delay systems. By combining a delay decomposition method with the discrete Jensen inequality, an LMI-based stability condition is derived. The presented stability condition is less conservative than the existing ones. A numerical example has illustrated the merits and effectiveness of the proposed method. As a future work, we will consider the problem about how to extend the delay decomposition method to the continuous-time case.

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