2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Nonlinear Crossover Model of Vehicle Directional Control

Timothy J. Gordon

Abstract— **This paper develops a simple nonlinear control model to represent the dynamics of a coupled vehicle-driver system. It is applied to an analysis of delay and preview time in closed-loop control. The model is a reduced-order non-linear model based on a separation between kinetic and kinematic driver control components. In kinetic control, vehicle dynamics and compensatory actions are represented in a fashion similar to the well-known linear "crossover" model; kinematic control takes the form of a reference velocity derived from the local roadway. Analytic stability results are developed, providing specific relationships between minimum preview, speed, time-delay and disturbance amplitude. The analysis is developed quite generally, but is applied in detail to straight-line path following on a flat horizontal road. Predictions are validated using results from one specific experimental study. The model is suitable for developing high-level control methods relating to active safety, as in automated collision avoidance systems.**

I. Introduction

THIS paper develops a simple nonlinear control model to represent the dynamics of a coupled vehicle-driver system. The driver sub-model may be applied to any level of vehicle dynamics model; however the aim of the paper is to derive fundamental relationships between various operating and performance parameters. A particular aim is to improve understanding of the steering control aspects of handling control under conditions of limited road friction.

The classical literature on driver modeling focuses on linear dynamics, with application of classical control [3, 6, 8, 10] or optimization [5, 7 9]. In these studies it is commonly assumed that a desired path is pre-defined; in fact this is not necessary, provided some visually-based reference is available [1, 2] – for example this could be for remaining with a road boundary, just as easily as from the tracking of a lane center or optimal path. In reference [4] an artificial neural network is used to model the recorded behavior of human drivers in a "black-box" form; here we take the opposite approach, developing a new type of model from the perspective of a simple nonlinear control formulation of the driving task.

Consider a simple concept model for controlling the motion of a ground vehicle - Figure 1. This vehicle motion is considered planar, the road surface is assumed horizontal, and other degrees of freedom such as roll, pitch and sideslip are considered secondary. Thus the mass center acceleration $\mathbf{a}_G$, velocity $\mathbf{v}_G$ and position $\mathbf{r}_G$ are described by 2-D vectors.

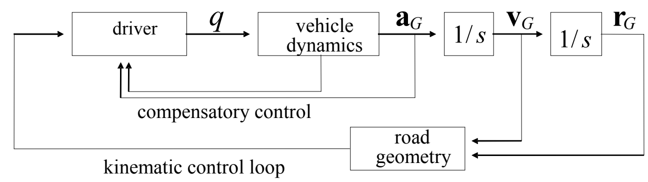

Fig. 1. *Concept Model for the Overall Driving Task*

Driver inputs $q(t)$ comprise steering, throttle and brake application. Control is separated into two nested loops. In the outer "kinematic control loop", the feedback is related to visual preview of the road geometry and other relevant objects (vehicles, obstructions etc.), and includes motion cues derived from the motion of the subject vehicle; position and velocity of the subject vehicle are essentially filtered by the road scene to provide the necessary visual information to control the vehicle. Localized effects of vehicle pitch and roll on the visual road scene are naturally regarded as being filtered out by the driver. In other words, we assume the driver executes an ongoing transformation between vehicle and world coordinates; the model can then be defined in global coordinates, even though in practice the driver only directly perceives relative position and motion.

The net result is a visually derived reference, passed to the inner "compensatory" control loop; this in turn activates the control (steering, braking and throttle) while making use of feedback from vehicle accelerations, steering torque, pedal force, plus any other relevant vehicle dynamic responses such as roll and pitch [9]. Note that vehicle yaw response, while relevant to both control loops, can be excluded from a simplified model of the process; assuming vehicle lateral (slip) velocities are small, yaw angle is directly related to $\mathbf{v}_G$, while yaw velocity is proportional to $\mathbf{a}_G$. For this reason we need not explicitly refer to yaw response, when a reduced-order model of the overall closed-loop control is developed in the next section.

II. Model Reduction

In Figure 1, the driver model integrates information from the road scene and vehicle response in some general way; we now make some more specific assumptions to develop the model. The driver is assumed to convert visual information into a kinematic reference for inner-loop control of the vehicle dynamics – Figure 2.

Manuscript received September 22, 2008. T.J.Gordon is Research Professor at the University of Michigan Transportation Research Institute, and Professor of Mechanical Engineering, University of Michigan, Ann Arbor, USA (e-mail: tjgordon@umich.edu).

978-1-4244-4524-0/09/$25.00 ©2009 AACC

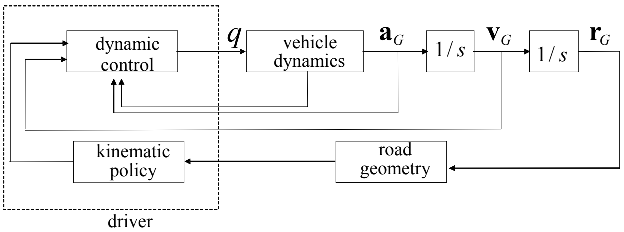

Fig. 2. *Kinematic Policy as a Pre-Filter for Steering Control*

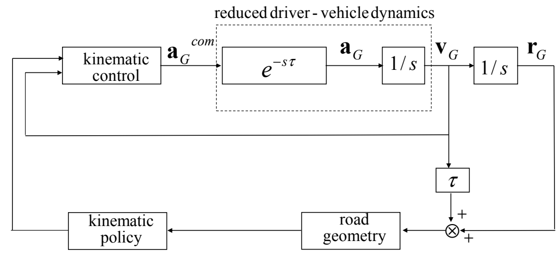

Fig. 3. *Nonlinear Crossover Model of the Vehicle-Driver System Dynamics*

The *kinematic policy* is a key part of the reduced-order model: it takes the previewed road scene as input and provides a vehicle velocity reference as output. In the case where road geometry and motion constraints are static (e.g. there is collision avoidance goal with respect to other vehicles) we presume the desired velocity vector is a function of position only; in this case the combination of given road geometry and kinematic policy can be represented as a stationary vector field $\mathbf{w}(\mathbf{x})$ which is the desired vehicle mass center velocity vector at any point on the roadway. Here $\mathbf{x} = (x, y)$ is represented as an independent spatial variable, while in application in the driving control loop we set $\mathbf{x} = \mathbf{r}_G(t)$.

To allow effective path following, the driver must be aware of the path curvature. This is conveniently encoded within the model by the kinematic policy to provide a velocity gradient reference, $\nabla\mathbf{w}$, in addition to the reference velocity $\mathbf{w}$. This is then sufficient to define a reference acceleration

$$\mathbf{a}^{ref} = (\mathbf{w}\cdot\nabla)\mathbf{w}\,. \tag{1}$$

We assume that the driver directly commands the vehicle acceleration with a constant time-delay – Figure 3. The driver command $\mathbf{a}_G^{com}$ is limited according to available road friction:

$$\left|\mathbf{a}^{com}(t)\right| \le a \tag{2}$$

Suppose, as a special case, the intention is to follow a straight-line reference using a fixed preview distance L – Figure 4. Using the (x, y) coordinates shown, the reference velocity is given by

$$\mathbf{w}(y) = \frac{U}{D}\begin{pmatrix} L \\ -y \end{pmatrix} \tag{3}$$

where $D = \sqrt{L^2 + y^2}$ and the reference speed U is assumed constant. In this case we can easily derive the reference acceleration (1):

$$\mathbf{a}^{ref} = (\mathbf{w}.\nabla)\mathbf{w} = \frac{U^2 L y}{D^4}\begin{pmatrix} y \\ L \end{pmatrix}. \tag{4}$$

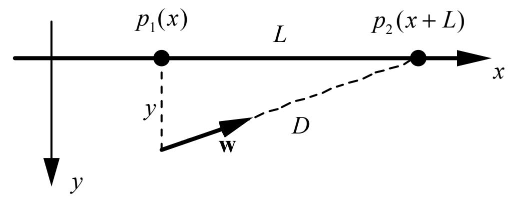

Fig. 4. *Fixed Preview Distance for Straight-line Driving*

The kinematic control block in Fig. 3 is now a sub-process providing target mass-center accelerations to the reduced driver-vehicle model. We develop some generic control laws in the next section using only local information – all roadway data is distilled into the reference field and its spatial derivatives, $\mathbf{w}(\mathbf{x})$ and $\nabla\mathbf{w}(\mathbf{x})$.

One adaptation assumed in Figure 3 is the scalar gain (τ) which represents the only explicit forward predictive element of the model. Because of the known time delay in the vehicle-driver block, the desired vehicle control is based on a predicted vehicle position:

$$\tilde{\mathbf{r}}_G = \mathbf{r}_G + \tau\,\mathbf{v}_G\,. \tag{5}$$

An extended predictor might be considered, making use of the acceleration vector $\tilde{\mathbf{r}}_G = \mathbf{r}_G + \tau\mathbf{v}_G + \frac{1}{2}\tau^2\mathbf{a}_G$, but this would only be relevant if the mass center accelerations were large. Some values typical of normal highway driving are $\tau = 0.2$, $|\mathbf{v}_G| = 20ms^{-1}$ $|\mathbf{a}_G| = 4ms^{-2}$, and in this case the magnitude of the acceleration term is only 2% that of the velocity term; not only would compensation for the acceleration term be difficult for a human driver to execute, under normal conditions its effect would be negligible.

III. Control Law

The control aim is to minimize the instantaneous error between the vehicle mass-center velocity and the reference velocity derived from kinematic road preview: $\mathbf{e}(t) = \mathbf{v}_G(t) - \mathbf{w}(\mathbf{r}_G(t))$. In the absence of disturbances the driver commands vehicle acceleration

$$\dot{\mathbf{v}}_G = \mathbf{u}(t) \tag{6}$$

and the control input $\mathbf{u}$ is to be a function of the mass center velocity and reference vector field terms:

$$\mathbf{u} = f(\mathbf{v}_G, \mathbf{w}, \nabla\mathbf{w}). \tag{7}$$

When $\mathbf{v}_G(t) = \mathbf{w}(\mathbf{r}_G(t))$ the vehicle accurately follows the reference field, and the control

$$\mathbf{u}_0(t) \equiv \mathbf{a}^{ref}(\mathbf{r}_G(t)) \tag{8}$$

is required to maintain tracking, e.g. to provide lateral acceleration in following a curve. To compensate for errors we introduce a residual control $\tilde{\mathbf{u}}(t)$ in the form

$$\tilde{\mathbf{u}}(t) = \mathbf{u}(t) - \mathbf{u}_0(t) = -k\,\mathbf{e}(t) \tag{9}$$

yielding the simple (nonlinear) control law

$$\mathbf{u} = \mathbf{u}_0 + \tilde{\mathbf{u}} = \mathbf{u}_0 - k\mathbf{e}\,. \tag{10}$$

IV. Linear Stability Analysis

Linear analysis of tracking a straight-line path is now considered, assuming path deviations are small: $|y| << L$, $|\dot{y}| << U$. Combining equations (3), (4) and (9) provides the following control law:

$$\mathbf{u}(t) = \frac{U^2 L y}{D^4}\begin{pmatrix} y \\ L \end{pmatrix} - k\left\{\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} - \frac{U}{D}\begin{pmatrix} L \\ y \end{pmatrix}\right\} \tag{11}$$

which reduces to the following equations for longitudinal and lateral control

$$u_x(t) = -k(\dot{x} - U) \tag{12}$$

$$u_y(t) = T^{-2} y - k(\dot{y} + T^{-1} y) \tag{13}$$

Longitudinal control simply maintains a constant vehicle speed, is decoupled from the lateral control, and is of little interest here. The lateral control law is to be applied within the Nonlinear Crossover Model: if $y(t)$ represents the instantaneous deviation, according to equation (5), the displacement 'presented' to the kinematic policy is actually

$$\hat{y} = y + \tau\dot{y} \tag{14}$$

So, replacing y in (13) by $\hat{y}$ while making no change to $\dot{y}$ - as above, accelerations terms are not used:

$$u_y(t) = T^{-2} y + T^{-2}\tau\dot{y} - k\dot{y} - kT^{-1} y - kT^{-1}\tau\dot{y}$$

where $T = LU^{-1}$ is the preview time; this can be re-written

$$u_y(t) = -K_P y - K_D \dot{y} \tag{15}$$

with

$$K_P = kT^{-1} - T^{-2}\,,\ K_D = k + k\tau T^{-1} - \tau T^{-2}\,. \tag{16}$$

The linearized control thus reduces to a simple proportional-derivative feedback of the lateral deviation. The simple form of the control is directly comparable with steering control models from the classical literature [3, 8, 10]. In particular, the separation into a feedforward 'pursuit control' component plus a 'compensatory feedback' control can be made explicit here by formally setting $T \to \infty$ in equations (14), (15) to reduce the feedforward component to zero:

$$u_y^{comp}(t) = -k\dot{y} \tag{17}$$

This is also the specific linear form of the residual control in equation (9). The open-loop transfer function for the plant-plus compensatory control for the linearized model is then

$$G_y^{comp}(s) = \lim_{T\to\infty}\frac{Y(s)}{U(s)} = \frac{ke^{-s\tau}}{s} \tag{18}$$

Thus the nonlinear control law presented in Section III, taken together with the simple preview point assumption in Figure 4, reduces to the well-known *crossover model* of human control [10,11]. Hence the general model presented will be referred to as the *nonlinear crossover model* (NCM).

Linearized stability of the NCM is now considered, with all terms in (16) fully included. Assuming the proportional gain is positive

$$K_P > 0 \tag{19}$$

the Nyquist or Bode condition for positive phase margin is

$$\arg(L(j\omega_c)) > -\pi \tag{20}$$

$$L(s) \equiv e^{-s\tau}\left(K_D s^{-1} + K_P s^{-2}\right). \tag{21}$$

Here L is the forward path ('loop') transfer function and ω_c is the crossover frequency (now including preview terms). From equations (16), (19) there is a lower limit on the compensatory gain:

$$k > T^{-1} \tag{22}$$

which increases as the preview time becomes short. Alternatively the driver control bandwidth, as represented by the crossover frequency of the open loop compensatory control, must exceed the lower bound derived from preview: this is standard [6] and intuitively obvious. In the next section a slightly stronger result is obtained using a nonlinear analysis

Condition (21) provides an upper limit on k; from (21), the crossover frequency is easily found to be

$$\omega_c^2 = \tfrac{1}{2}\left\{K_D^2 + \sqrt{K_D^4 + 4K_P^2}\right\} \tag{23}$$

and condition (20) reduces to

$$\tan^{-1}\left(\frac{\omega_c K_D}{K_P}\right) > \omega_c \tau \tag{24}$$

This condition constrains three independent control parameters (T, k, τ); to analyze the result it is convenient to scale relative to the preview time to provide corresponding dimensionless variables, and hence reduce the constraint to two variables:

$$\bar{\tau} = T^{-1}\tau,\ \bar{k} = Tk,\ \bar{K}_D = TK_D$$
$$\bar{K}_P = T^2 K_P,\ \bar{\omega}_c = T\omega_c \tag{25}$$

Equations (16), (23) and (24) are unchanged except for converting to dimensionless variables and formally setting T to unity. The phase margin condition (24) is then easily expressed in terms $(\bar{k}, \bar{\tau})$:

$$\tan^{-1}\left(\overline{\omega}_c\bar{\tau} + \frac{\overline{\omega}_c\bar{k}}{\bar{k}-1}\right) > \overline{\omega}_c\bar{\tau} \qquad (26)$$

where $\overline{\omega}_c(\bar{k}, \bar{\tau})$ is found from (23), i.e.

$$2\overline{\omega}_c^2 = (\bar{k} + \bar{k}\bar{\tau} - \bar{\tau})^2 + \sqrt{(\bar{k} + \bar{k}\bar{\tau} - \bar{\tau})^4 + 4(\bar{k}-1)^2} \qquad (27)$$

The stability region is plotted in Figure 5. The shaded region represents an upper bound on the normalized compensatory gain. From equation (25) the controller gain k is scaled linearly with respect to the preview time T, so as expected the upper limit becomes large for large previews.

The figure includes a superimposed curve at the stability boundary. Based on least-squares curve fitting, the normalized gain stability condition is given by

$$\bar{k} < \bar{k}_c(\bar{\tau}) \equiv \frac{-0.4808\bar{\tau} + 1.2941}{\bar{\tau} - 0.0094} \qquad (28)$$

In this linear analysis, closed-loop stability is guaranteed based on these limits on k, while there is no independent limit on the preview time T.

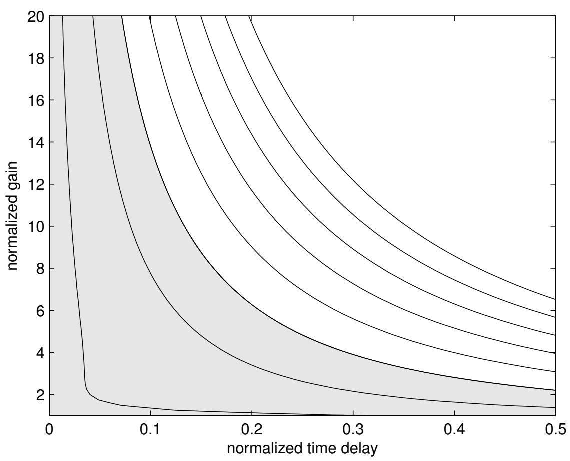

Fig. 5. *Phase Contours in the* $(\bar{k}, \bar{\tau})$ *plane- stability region shaded*

.

V. Nonlinear Stability Criterion

Returning to the general control model, a nonlinear stability analysis was presented in [1]. That analysis assumed zero time delay (Figure 3) but will form the basis of an analytical result that includes the time delay. In the following analysis we temporarily assume $\tau \to 0$. From the above definitions of $\mathbf{e}$ and $\tilde{\mathbf{u}}$

$$\mathbf{e}\cdot\dot{\mathbf{e}} = \mathbf{e}\cdot\tilde{\mathbf{u}} - \mathbf{e}\cdot[(\mathbf{e}\cdot\nabla)\mathbf{w}].$$

This can be re-written

$$\mathbf{e}\cdot\dot{\mathbf{e}} = \mathbf{e}\cdot\tilde{\mathbf{u}} - \mathbf{e}^T H\mathbf{e}$$

where H is the symmetric 2×2 matrix with components

$$h_{ij} = \tfrac{1}{2}\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right) \qquad (29)$$

measuring the point-wise divergence of the field [1]. Since H is symmetric it has real eigenvalues, which are assumed bounded below in the region Ω of interest

$$eig(H) \geq -c \ \text{ for } \ \mathbf{x} \in \Omega \qquad (30)$$

for some $c > 0$. From equation (9) the control law is

$$\mathbf{u}(t) = (\mathbf{w}(\mathbf{r}_G)\cdot\nabla)\mathbf{w}(\mathbf{r}_G) - k(\mathbf{v}_G(t) - \mathbf{w}(\mathbf{r}_G))$$

and hence the error equation takes the form

$$\mathbf{e}\cdot\dot{\mathbf{e}} = -ke^2 - \mathbf{e}^T H\mathbf{e}$$

where $e(t) = |\mathbf{e}(t)|$. Then, from (30) we obtain

$$\dot{e} \leq -(k-c)e \qquad (31)$$

which implies

$$\frac{d}{dt}\left(\exp\{(k-c)t\}e(t)\right) \leq 0$$

$$e(t) < e_0 \exp\{-(k-c)t\}$$

and hence the errors induced by an initial disturbance asymptotically tend to zero provided:

$$k > c \qquad (32)$$

In the absence of time delay, there is no upper gain limit, and the ability to satisfy this condition depends only on there being sufficient available surface friction and hence mass-center accelerations. We assume (c.f. equation (2))

$$|\mathbf{u}(t)| \leq a.$$

Since the direction of $\mathbf{e}$ is unknown, and some control authority is needed for making corrective control, we assume a conservative limit on the reference control

$$|\mathbf{u}_0(\mathbf{x})| \leq a - b \qquad (33)$$

with corrective accelerations bounded by b, $0 < b < a$:

$$|\tilde{\mathbf{u}}(t)| \leq b. \qquad (34)$$

Comparing equations (10) and (32), the lower limit on k is equivalent to placing an upper bound on the initial disturbance (velocity error):

$$e_0 < \frac{b}{c} \qquad (35)$$

Under the above control, $e(t)$ is monotonic decreasing and the gain can be maintained above the limit in (32) without violating (34).

VI. Application to Straight-line Tracking

Using the reference field (3), which assumes constant speed and preview distance L, it is straightforward to calculate matrix (29)

$$H = -\frac{UL}{2D^3}\begin{pmatrix} 0 & y \\ y & 2L \end{pmatrix} \qquad (36)$$

which has eigenvalues

$$\lambda = -\frac{UL(L \pm D)}{2D^3} \tag{37}$$

These are bounded below, as in (30) by $\lambda \geq -c$, where

$$c = \frac{UL(L+D)}{2D^3} = \frac{U}{2L}\left(\xi^3 + \xi^2\right) \tag{38}$$

where the dimensionless parameter $\xi = L/D$ is used to define the dependence on y. Since $0 < \xi \leq 1$, it is clear that c attains its greatest magnitude at $\xi = 1$, which occurs on the target path itself (y = 0), and hence the global minimum is

$$c = \frac{U}{L} = T^{-1} \tag{39}$$

In the following, (39) is used as a global bound for all y, and it is noticeable that equation (22) is "re-discovered" here in the nonlinear analysis by comparing equations (32) and (39). The linear analysis included a pre-compensated time delay, while here the time delay in vehicle control (from previewed aspects of the road scene) is assumed to be absent or fully compensated by the driver.

From equations (35) and (39) we determine a conservative upper bound on the initial disturbance:

$$e_0 < bT \tag{40}$$

We now define a specific form for the initial disturbance in straight-line tracking, by assuming a pure lateral deviation y with unchanged initial velocity

$$\mathbf{v}_G(0) = \begin{bmatrix} U \\ 0 \end{bmatrix};$$

then from (3)

$$\mathbf{e}(0) = \frac{U}{D}\begin{bmatrix} D-L \\ y \end{bmatrix}.$$

For simplicity, if we assume the lateral deviation is small compared to the preview distance, we obtain

$$e_0 = \frac{U|y|}{L} = T^{-1}|y|$$

and hence from (40) the minimum preview time is

$$T > T_{\min} = \sqrt{\frac{y_{\max}}{b}} \tag{41}$$

In general, for large deviations, the bound is more complex (and easily derived), but it is clear that the preview must increase with the level of disturbance, and in the real world case the driver must adapt preview to increase with path offset, or disturbance level. This general phenomenon is discussed by MacAdam in [5], where results from a simulator experiment are presented that indicate region of "preferred damping" just above a stability boundary - Figure 6. From the figure it can be seen that the boundary is estimated as a straight line with slope around 2.0 in the space of preview time vs. delay time.

To make a comparison with MacAdam's experimental stability margin, we continue to represent the disturbance as an initial velocity error, but allow that disturbance to grow during the period of the time delay. This corresponds to a situation where the driver cannot use preview to compensate for the disturbance, as would be the case for previewed path curvature. In the case of a pure lateral path offset, the disturbance does not grow during this initial delay interval, so the above result (41) is unaffected by $\tau > 0$. On the other hand an initial yaw deviation would certainly amplify the initial error, and this case is considered now.

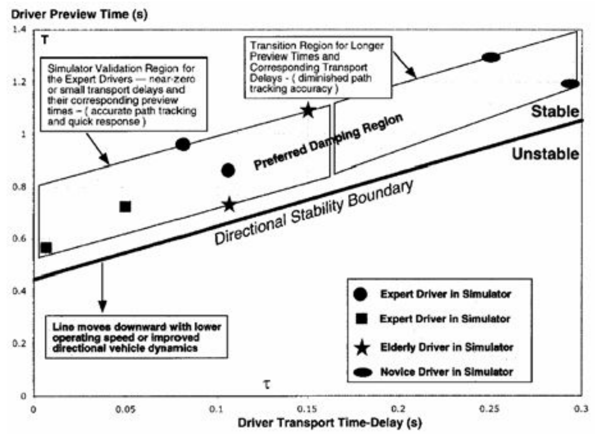

Fig. 6. *Empirical Stability Region - reference* [6]

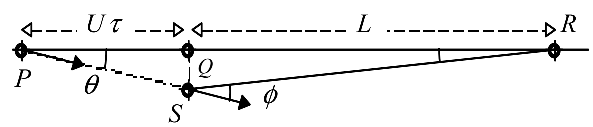

Fig. 7. *Initial Path Deviation Resulting from Impulsive Disturbance*

In Figure 7 we assume an initial error in the vehicle heading, $\theta << 1$. Points P, Q and R lie on the target path – P is the vehicle mass centre position at time $t = 0$, and the vehicle moves to S at time $t = \tau$. Since the disturbance is initially unknown to the driver, the intervention (10) is applied at S, and R represents the target point in the kinematic policy. The effect of the time delay is twofold: first the driver preview distance at which control initiates intervention is distance $|PR|$

$$L_p = L + U\tau \tag{42}$$

with corresponding preview time

$$T_p = T + \tau \tag{43}$$

However, the stability correction appropriate to the above analysis equates to the shorter preview time T. Secondly, the initial path deviation is amplified by the resulting path offset, so that at S the angular error is now $\theta + \phi$. Assuming small angles and zero speed deviation, the initial velocity error is given by

$$e_0 = U|\theta + \phi| \tag{44}$$

and from the figure

$$\phi = \frac{U\theta\tau}{L} = \frac{\theta\tau}{T} \qquad (45)$$

Comparing equation (40), the condition for stable tracking of the target line becomes

$$U|\theta + \phi| < bT \qquad (46)$$

Defining a characteristic time

$$T_0 \equiv \frac{U\theta}{b}, \qquad (47)$$

condition (46) reduces to

$$T_0 T_p < T^2. \qquad (48)$$

In the limit $\tau \to 0$ (when $T = T_p$), this implies $T > T_0$ places a lower limit on the preview time – in other words a longer minimum preview is required at high speed, not just a great preview distance. Also, the required preview increases with the amplitude of the disturbance and reduces when there available friction b increases. More generally, from (43), (48) is a quadratic inequality for T in terms of T_0:

$$T^2 - T_0 T - T_0\tau < 0 \qquad (49)$$

This is easily solved to give:

$$T_p > T_p^{\min} \equiv \tau + \tfrac{1}{2}T_0\left(1 + \sqrt{1 + \frac{4\tau}{T_0}}\right) \qquad (50)$$

The expression further simplifies in the case where the time delay is small compared to the minimum preview time $\tau << T_0$; ignoring terms in $\tau^2 T_0^{-2}$ and smaller we obtain

$$T_p^{\min} \approx T_0 + 2\tau \qquad (51)$$

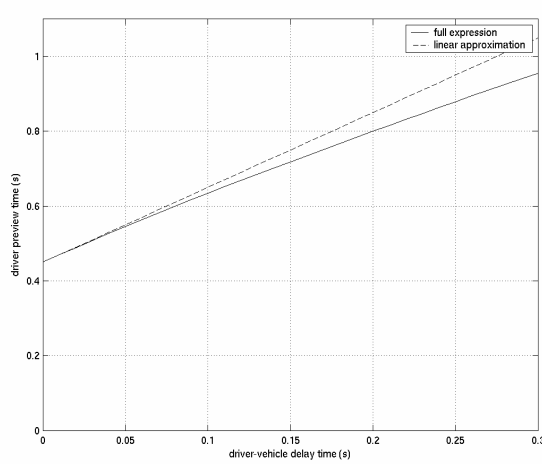

Fig. 8. *Theoretical Stability Boundary and Approximation*

In Figure 8, T_0 has been chosen to match the intercept of Figure 6, yielding an excellent match between theory and experiment. The model also predicts that for driver-vehicle delay τ larger than about 0.2 seconds, the required preview time is slightly less than the linear boundary shows. This is a testable hypothesis, applicable to a future driving simulator study. Note that the analysis here is specific to straight-line driving, while MacAdam's results are based on driver performance on a 'slalom-like course'; however the similarity in the results is striking. Further analysis of curved path tracking is feasible and currently ongoing.

VII. CONCLUSIONS

On the basis of a very simple "acceleration command with delay" model of the coupled dynamics of a highway vehicle plus driver, a number of analytical results have been obtained relating preview time, controller gain and preview distance. The linearized control law reduces to a specific form of a proportional derivative, and pure compensatory component in this case reduces to the standard crossover model of human control – hence the term "Nonlinear Crossover Model" is used for the control system.

Both linear and nonlinear analysis have been applied to the analysis of stable tracking, with appropriate agreement found at the low-gain limit. Predicted stability limits for the linearized model have been represented in a non-dimensionalized form. Nonlinear results relate time delay, preview time, acceleration limits and disturbance amplitudes. Predictions have been verified against the findings of an earlier study of friction-limited dynamic control in a driving simulator.

REFERENCES

[1] T. J.Gordon, M.C. Best and P. J. Dixon, "An automated driver based on convergent vector fields", *Proceedings of the Institution of Mechanical Engineers Part D Journal of Automobile Engineering*, vol. 216 pp. 329-347.

[2] T. J. Gordon and N. Magnuski, "Modeling normal driving as a collision avoidance process", *Proceedings of 8th International Symposium on Advanced Vehicle Control* (AVEC), Taipai, Taiwan, 2006.

[3] K. Guo and H. Guan, "Modelling of Driver/Vehicle Direction Control System", *Vehicle System Dynamics*, vol. 22, pp. 141-184, 1993

[4] C.C. MacAdam and G. E. Johnson, "Application of elementary neural networks and preview sensors for representing driver steering control behaviour", *Vehicle System Dynamics*, vol. 25, pp. 3–30, 1996.

[5] C. C. MacAdam, "Understanding and modeling the human driver", *Vehicle System Dynamics*, vol. 40, pp.101-134, 2003.

[6] A. Modjtahedzadeh and R. A. Hess, "A model of driver steering control behavior for use in assessing vehicle handling qualities: *Trans. ASME, J. Dynamic Systems, Measurement and Control*, vol. 115, pp. 456–464, 1993.

[7] G. Prokop, "Modeling human vehicle driving by model predictive online optimization", *Vehicle System Dynamics*, vol. 35, pp. 19–53, 2001.

[8] M. Tomizuka and D. E. Whitney, "The Human operator in manual preview tracking (an experiment and its modeling via optimal control)", *Trans. ASME, J. Dynamic Systems, Measurement and Control*, vol. 98, pp. 407–413, 1976.

[9] A. Y. Ungoren and H. Peng, "An adaptive lateral preview driver model", *Vehicle System Dynamics*, vol. 43, pp. 245-259, 2005.

[10] D. H. Weir and D. T. McRuer, "Dynamics of driver vehicle steering control", *Automatica*, , vol. 6, pp. 87–98. 1970.

[11] R. Jagacinski and J. Flach, "Control Theory for Humans", Lawrence Erlbaum Associates Inc., Mahwah, New Jersey, 2003.