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Consensus in Leaderless Networks of High-Order-Integrator Agents

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***Abstract*— This paper investigates the consensus problem for a group of high-order-integrator agents with fixed topology. A linear distributed consensus protocol is proposed, which only depends on the agent's own information and its neighbors' partial information. A necessary and sufficient condition for convergence to consensus is established. It is proved that the topology having a spanning tree is a necessary condition for convergence to consensus. Based on the consensus protocol for networks of high-order-integrator agents, a consensus controller is provided for a group of identical agents with dynamics described by a completely controllable single-input linear time-invariant (LTI) system. It is shown that the consensus of this kind of networks is equivalent to that of networks of high-order-integrator agents. Finally, the parameter design of the protocol is discussed.**

I. INTRODUCTION

Consensus problems for networks of dynamic agents have been extensively studied by researchers from distinct points of view. As to the mathematical models of agents, there are discrete-time forms ([1]-[5]), single-integrator dynamics ([6]-[11]), double-integrator dynamics ([12]-[13]) and so on. The assumptions on network topology, which is adopted to describe the complex interconnections among agents, include bidirectional network, unidirectional network, fixed topology, switching topology, random topology, small-world network, leader-follower framework, model-reference framework, network with communication time-delays etc. Applications of this research pertain to cooperative control of unmanned air vehicles, autonomous formation flight, control of communication networks, distributed sensor fusion in sensor networks, swarm-based computing, rendezvous in space (see [17]-[21] and the references therein).

This paper mainly investigates the consensus problem for networks of high-order-integrator agents. The idea of employing high-order integrator to describe the dynamics of agents is inspired by the following facts. First, any completely controllable continuous-time LTI system with state-space equation $\dot{x} = Ax + Bu$ can be equivalently brought into a collection of decoupled and independently controlled chains of integrators, under an appropriate nonsingular linear transformation and a suitable state feedback (see [23]). Second, denoting system $\dot{x} = Ax+Bu$ as matrix pair (A, B), the set of all completely controllable pairs (A, B) is open and dense in the space composed of all matrix pairs (A, B) (see [22] and the references therein). Third, the high-order-integrator model of agents is a generalization of the single-integrator and the double-integrator models studied in the literature (see, e.g., [6], [7] and [12]). Finally, take single-input LTI system $\dot{x} = Ax + bu$ for example, where $(A,\ b)$ is completely controllable and can be transformed into a high-order integrator. (Note that any completely controllable multi-input LTI system can be transformed into a completely controllable single-input LTI system, see [24].) If there exists a protocol solving the consensus problem for networks of agents with dynamics expressed by the high-order integrator, then a consensus controller can be designed for networks of identical agents with dynamics $\dot{x} = Ax + bu$. (The consensus controller is given later on.) Hence it is of physical interest and of theoretical interest to investigate the consensus problem for networks of high-order-integrator agents. Some related work on the consensus problem for networks of high-order-integrator agents can be found in [14]/[15].

The objective of this paper is to steer a group of high-order-integrator agents to a constant state. To do this, we propose a linear distributed protocol which only depends on the agent's information and its neighbors' partial information. We employ weighted graph to model the interactions among agents. For networks of high-order-integrator agents under the protocol, a necessary and sufficient condition for convergence to consensus is established. It is proved that the underlying graph having a spanning tree is a necessary condition for convergence to consensus. The consensus state for such networks is found out as well. It is shown that only the agents, which can act as a root of a spanning tree in the graph, contribute to the consensus state. Based on the consensus protocol for networks of high-order-integrator agents, a consensus controller is provided for networks of identical agents with dynamics described by a completely controllable single-input LTI system (LTI networks for short). The convergence to consensus of LTI networks is equivalent to that of networks of high-order-integrator agents under the same topology. At last, the parameter design of the protocol for networks of high-order-integrator agents is discussed. Some necessary/sufficient conditions for the protocol solving consensus are established.

The remainder of the paper is organized as follows. Section II presents some mathematical preliminaries and notations. Section III states the agent model and the definition of consensus. Section IV establishes the main results. The last section makes some conclusions.

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II. MATHEMATICAL PRELIMINARIES

In this section, we present some notations and some preliminary results on algebraic graph theory, which will be useful for the subsequent sections.

A. Notations

Throughout this paper, we let $\mathbb{R}$ and $\mathbb{C}$ be the set of real numbers and the set of complex numbers, respectively. $\mathbb{R}^n$ is the $n-$dimensional real vector space and $\| \cdot \|$ denotes the Euclidian norm. $\mathbb{M}_n$ ($\mathbb{M} = \mathbb{R}$ or $\mathbb{C}$) is the set of $n-$by-n matrices. I_n is an identity matrix with order $n \times n$. For a given matrix $M \in \mathbb{M}_n$, $\sigma(M)$, $\mathbf{R}(M)$ and $\mathrm{rank}(M)$ are the spectrum (set of eigenvalues), the range and the rank of M, respectively. $\mathrm{diag}\{a_1, \cdots, a_n\}$ defines a diagonal matrix with diagonal elements being $a_1, \cdots, a_n$. $e_1 = [1\ 0\ \cdots\ 0]^T \in \mathbb{R}^m$. $\mathbf{1}_N = [1, \cdots, 1]^T \in \mathbb{R}^N$. $\underline{N} = \{1, \ldots, N\}$ is a index set. $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $\overline{z}$ and $|z|$ are the real part, the imaginary part, the conjugate complex number and the module of $z \in \mathbb{C}$, respectively. $\otimes$ denotes the Kronecker product.

B. Algebraic Graph Theory Preliminaries

A digraph (undirected graph) $\mathcal{G}$ consists of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, and an arc (edge) set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. An arc (edge) of $\mathcal{G}$, denoted by $e_{ij} = (v_i, v_j)$, is an ordered (unordered) pair of distinct vertices of $\mathcal{V}$; v_i and v_j are called the tail and the head of e_{ij} respectively. If $e_{ij} = (v_i, v_j)$ is an arc, then we say v_i is a neighbor of v_j. In this paper, we assume $e_{ii} \notin E$ and the elements of $\mathcal{E}$ are unique. Denote the collection of neighbors of v_i by $\mathcal{N}_i = \{v_j :\ e_{ji} = (v_j, v_i) \in \mathcal{E}\}$. A directed path from v_i to v_j means that there is a sequence of distinct arcs in $\mathcal{E}$, (v_i, v_1), (v_1, v_2), $\ldots$, (v_r, v_j). A directed tree is a directed graph, where every vertex has exactly one tail except for one special vertex without any tail. The special vertex is called the root of the tree. We say a digraph has a spanning tree if there exists a subset of the arcs $\mathcal{E}' \subset \mathcal{E}$ such that the digraph $\mathcal{G}' = (\mathcal{V}, \mathcal{E}')$ is a directed tree. A digraph is called strongly connected, if there exists a path between any pair of distinct vertices; for undirected graph it is called connected.

Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}_N$ be a matrix with rows and columns indexed by the vertices of $\mathcal{G}$, all entries of which are nonnegative. A weighted graph is a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a nonnegative matrix $\mathcal{A}$, denoted by $\mathcal{G}(\mathcal{A})$, such that $(v_i, v_j) \in \mathcal{E}$ if and only if $a_{ji} > 0$. Here $\mathcal{A}$ is called the adjacency matrix of $\mathcal{G}$, and a_{ji} is said to be the weight of link (v_i, v_j). If $\mathcal{G}(\mathcal{A})$ is undirected, then $\mathcal{A}^T = \mathcal{A}$.

The Laplacian matrix $\mathcal{L}_{\mathcal{G}(\mathcal{A})} = [l_{ij}] \in \mathbb{R}_N$ of $\mathcal{G}(\mathcal{A})$, abbreviated as $\mathcal{L}$, is defined as

$$l_{ij} = \begin{cases} \sum_{v_j \in \mathcal{N}_i} a_{ij}, & i = j \\ -a_{ij}, & i \neq j \end{cases} .$$

We refer to the diagonal matrix $\mathcal{D} = \mathrm{diag}\{d_1, \cdots, d_N\}$ with $d_i = \sum_{v_j \in \mathcal{N}_i} a_{ij},\ i \in \underline{N}$ as the in-degree matrix of $\mathcal{G}(\mathcal{A})$, where d_i is called the in-degree of vertex v_i. Then $\mathcal{L} = \mathcal{D} - \mathcal{A}$. We next present some basic results on the spectral properties of $\mathcal{L}$, which are useful for the development of this paper (see [6], [7], [26]):

(1) $\mathcal{L}$ has at least one zero eigenvalue and all the nonzero eigenvalues have positive real parts. The zero eigenvalue is simple if and only if the associated graph $\mathcal{G}$ has a spanning tree. A right eigenvector of $\mathcal{L}$ associated to the zero eigenvalue is $\mathbf{1}_N$, i.e., $\mathcal{L}\mathbf{1}_N = 0$.

(2) For undirected graph, the associated Laplacian $\mathcal{L}$ is positive semi-definite; if the undirected graph is connected, then $\mathrm{rank}(\mathcal{L}) = N - 1$.

III. AGENT MODEL AND CONSENSUS PROBLEM

Consider a network of N dynamic agents. The dynamics of each agent is described by the following mth-order integrator

$$\begin{aligned} &\xi_i^{(m)} = u_i, \\ &\xi_i(0) = \xi_{i0},\ \cdots,\ \xi_i^{(m-1)}(0) = \xi_{i0}^{(m-1)},\ t \geq 0, \end{aligned} \tag{1}$$

where $m \geq 1$ is a positive integer and denotes the order of the differential equations; for convenience, $\xi_i \in \mathbb{R}$ is called the information variable of agent i; $\xi_i^{(k)},\ k \in \underline{m-1}$ is the kth-order derivative of ξ_i; $u_i \in \mathbb{R}$ is the control input; $\bar{\xi}_{i0} := [\xi_{i0}\ \cdots\ \xi_{i0}^{(m-1)}]^T$ is the initial state of agent i. Note that the control inputs u_i are usually called consensus protocol in the literature. The interactions among agents are realized in their control inputs. We employ weighted graph $\mathcal{G}(\mathcal{A})$ to model the interaction topology of the network. Each vertex in the vertex set represents an agent of the network. Each arc/edge e_{ji} in the arc/edge set indicates that there is a communication link from agent j to agent i. $a_{ij} > 0$ is the weight of the communication link e_{ji}. Let $\bar{\xi}_i(t) = [\xi_i(t)\ \cdots\ \xi_i^{(m-1)}(t)]^T$ be the state of agent i.

Denote $\bar{\xi}(t) = [\bar{\xi}_1^T(t)\ \cdots\ \bar{\xi}_N^T(t)]^T$ and $\bar{\xi}_0 = [\bar{\xi}_{10}^T\ \cdots\ \bar{\xi}_{N0}^T]^T$ as the stacked vector of the agents' states and the stacked vector of the agents' initial states, respectively. In this paper, we devote to solving the following consensus problem for network in (1).

Definition 1: Consider network in (1). If for any initial state $\bar{\xi}_0$, the states of agents satisfy

$$\lim_{t \to \infty} \| \bar{\xi}_i(t) - \bar{\xi}_j(t) \| = 0$$

for all $i, j \in \underline{N}$, then we say the network solves a consensus problem asymptotically. Furthermore, if there exists $\bar{\xi}^* \in \mathbb{R}^m$ such that for any initial state $\bar{\xi}_0$

$$\lim_{t \to \infty} \| \bar{\xi}_i(t) - \bar{\xi}^* \| = 0$$

for all $i \in \underline{N}$, then we call $\bar{\xi}^*$ to be the consensu state of the network.

IV. MAIN RESULTS

In this section, we consider the consensus problem defined in Section III for network in (1) with fixed topology. To do this, we propose the following distributed consensus protocol

$$u_i = -\sum_{k=1}^{m-1} c_k \xi_i^{(k)} - \sum_{j \in \mathcal{N}_i} \kappa_i a_{ij}(\xi_i - \xi_j), \tag{2}$$

where $c_k > 0,\ k \in \underline{m-1}$ are the feedback gains of absolute information, $a_{ij} > 0$ are the weights of communication links, $\mathcal{N}_i$ is the collection of neighbors of agent i, and $\kappa_i > 0$ is the feedback gain of relative information. Given protocol (2), the dynamics of agent i can be expressed as

$$\dot{\bar{\xi}}_i = E_m \bar{\xi}_i - \sum_{j \in \mathcal{N}_i} \kappa_i a_{ij} F_m(\bar{\xi}_i - \bar{\xi}_j), \tag{3}$$

where

$$E_m = \begin{bmatrix} \mathbf{0} & I_{m-1} \\ 0 & \theta^T \end{bmatrix},\ F_m = \begin{bmatrix} \mathbf{0} & 0 \\ 1 & \mathbf{0}^T \end{bmatrix},$$
$$\theta = [-c_1\ \cdots\ -c_{m-1}]^T,\ \mathbf{0} = [0\ \cdots\ 0]^T \in \mathbb{R}^{m-1}.$$

Furthermore, the closed-loop network has the following concise form of dynamics

$$\dot{\bar{\xi}} = [I_N \otimes E_m - (\Upsilon \mathcal{L}) \otimes F_m]\bar{\xi} \triangleq \Omega \bar{\xi}, \tag{4}$$

where $\mathcal{L}$ is the associated Laplacian of $\mathcal{G}(\mathcal{A})$ and $\Upsilon = \mathrm{diag}\{\kappa_1,\ \cdots,\ \kappa_N\}$.

Remark 1: The property $\Upsilon \mathcal{L} \mathbf{1}_N = 0$ indicates that $\Upsilon \mathcal{L}$ is a Laplacian matrix of $\mathcal{G}(\hat{\mathcal{A}})$ with $\hat{\mathcal{A}} = \Upsilon \mathcal{A}$. The gain matrix Υ can be used to modify the eigenvalues of $\Upsilon \mathcal{L}$ and has effect on the convergence speed.

A. Necessary and Sufficient Condition for Convergence

Before proceeding, we establish the following two lemmas which are needed for the main result.

Lemma 1: Suppose Ω is given in (4) and $\mu_1 = 0,\ \mu_2,\ \cdots,\ \mu_N \in \sigma(\Upsilon \mathcal{L})$, then the characteristic polynomial of Ω is

$$\prod_{i=1}^{N} (s^m + c_{m-1}s^{m-1} + \cdots + c_1 s + \mu_i), \tag{5}$$

and the algebraic multiplicity of zero eigenvalue of Ω is equal to that of $\mathcal{L}$. Moreover, $\mathbf{1}_N \otimes e_1$ with $e_1 \in \mathbb{R}^m$ ($\frac{1}{c_1} w_l^T \otimes [c_1\ \cdots\ c_{m-1}\ 1]$) is a right (left) eigenvector of Ω associated to the zero eigenvalue, where w_l^T is the left eigenvector of $\mathcal{L}$ associated to the zero eigenvalue and satisfies $w_l^T \mathbf{1}_N = 1$.

Lemma 2: Consider network in (4), if the network solves a consensus problem, then all the equilibria of system (4) are of the form $a_0 \mathbf{1}_N \otimes e_1$ with $a_0 \in \mathbb{R}$ and $e_1 \in \mathbb{R}^m$.

Here is the main result.

Theorem 1: Consider network in (4) with fixed topology $\mathcal{G}(\mathcal{A})$, then the network asymptotically solves a consensus problem if and only if $\mathcal{G}(\mathcal{A})$ has a spanning tree and all the nonzero eigenvalues of Ω have negative real parts. Moreover, the consensus state is $\chi(\bar{\xi}_0)e_1$, where $e_1 \in \mathbb{R}^m$, $\chi(\bar{\xi}_0) = \left(\frac{1}{c_1}\sum_{i=1}^{N} w_i \tilde{c}^T \bar{\xi}_{i0}\right)$, $\tilde{c} = [c_1\ \cdots\ c_{m-1}\ 1]^T$, and $w_l = [w_1\ \cdots\ w_N]^T$ is given in Lemma 1.

Proof: (Sufficiency.) If $\mathcal{G}(\mathcal{A})$ has a spanning tree, then $\mathcal{L}$ has a simple zero eigenvalue. According to Lemma 1, Ω has only one zero eigenvalue. Hence there exists a nonsingular matrix S, such that $S^{-1}\Omega S = \mathrm{diag}\{0,\quad J'\}$, where $J' \in \mathbb{R}_{Nm-1}$ is an upper-triangular Jordan matrix with diagonal entries being the eigenvalues of Ω. From the assumption that all the nonzero eigenvalues of Ω have negative real parts, it follows $\lim_{t\to\infty} \exp(J't) = 0$. Denote the first column of S as v_r and the first row of S^{-1} as v_l^T. Then $\Omega v_r = 0$, $v_l^T \Omega = 0$ and $v_l^T v_r = 1$. Since the zero eigenvalue of Ω is simple, Lemma 1 indicates that $v_r \in \mathrm{span}\{\mathbf{1}_N \otimes e_1\}$ with $e_1 \in \mathbb{R}^m$ and $v_l \in \mathrm{span}\{\frac{1}{c_1} w_l \otimes [c_1\ \cdots\ c_{m-1}\ 1]^T\}$. Thus for any initial state $\bar{\xi}_0$

$$\begin{aligned} & \lim_{t\to\infty} \bar{\xi}(t) = \lim_{t\to\infty} \exp(\Omega t)\bar{\xi}_0 \\ = \ & S\,\mathrm{diag}\{1,\ \lim_{t\to\infty} \exp(J't)\}S^{-1}\bar{\xi}_0 \\ = \ & S\,\mathrm{diag}\{1,\ \underbrace{0,\ \ldots,\ 0}_{Nm-1}\}S^{-1}\bar{\xi}_0 \\ = \ & v_r v_l^T \bar{\xi}_0 = \chi(\bar{\xi}_0)\mathbf{1}_N \otimes e_1, \end{aligned}$$

where $\chi(\bar{\xi}_0)$ and e_1 are given in the theorem. This implies that the network solves the consensus problem asymptotically with the consensus state being $\chi(\bar{\xi}_0)e_1$.

(Necessity.) Suppose the network solves a consensus problem, then Lemma 2 indicates that the equilibrium set of system (4) is $\mathrm{span}\{\mathbf{1}_N \otimes e_1\}$ with $e_1 \in \mathbb{R}^m$. Thus for any initial state $\bar{\xi}_0$, we have $\lim_{t\to\infty} \exp(\Omega t)\bar{\xi}_0 = (\lim_{t\to\infty} \exp(\Omega t))\bar{\xi}_0 \in \mathrm{span}\{\mathbf{1}_N \otimes e_1\}$. This implies that $\mathbf{R}(\lim_{t\to\infty} \exp(\Omega t)) \subseteq \mathrm{span}\{\mathbf{1}_N \otimes e_1\}$. Hence $\mathrm{rank}(\lim_{t\to\infty} \exp(\Omega t)) \le 1$. In addition, it is obvious that Ω has at least one zero eigenvalue due to $\mathcal{L}$ always having one. Define the Jordan matrix of Ω as J. If the sufficient condition does not hold, then $\mathcal{G}(\mathcal{A})$ hasn't a spanning tree, or $\mathcal{G}(\mathcal{A})$ has a spanning tree but Ω has a nonzero eigenvalue with nonnegative real part. For the first case, the Laplacian $\mathcal{L}$ of $\mathcal{G}(\mathcal{A})$ has at least two zero eigenvalues, and so does Ω. Suppose the algebraic multiplicity and the geometric multiplicity of the zero eigenvalue of Ω are k and l, respectively. Then $N > k \ge 2$ due to the distributed protocol. If $l = k$, then the associated Jordan matrix of Ω has the form $J = \mathrm{diag}\{0_k,\ \bar{J}\}$, $\bar{J} \in \mathbb{R}_{(Nm-k)}$. It follows that $\lim_{t\to\infty} \exp(Jt) = \mathrm{diag}\{I_k,\ 0_{Nm-k}\}$, which results in a contradiction to $\mathrm{rank}(\lim_{t\to\infty} \exp(\Omega t)) \le 1$. If $l < k$, then there is at least one Jordan block with order no less than 2. Thus $\mathrm{rank}\,(\lim_{t\to\infty} \exp(\Omega t)) \ge 2$. This leads to a contradiction as well. It is obvious that the other case contradicts the definition of consensus. The proof is completed. ■

Remark 2: Theorem 1 implies that $\mathcal{G}(\mathcal{A})$ having a spanning tree is a necessary condition for network in (4) solving a consensus problem. Note that when $m = 1$, (1) becomes the single-integrator model studied in [6], [7]. However, due to the effect of high-order dynamics of agents, the aforementioned property of networks of high-order-integrator agents is different from that of networks of single-integrator agents established in [7], where the underlying graph having a spanning tree is a necessary and sufficient condition for networks of single-integrator agents solving consensus problem. In addition, when $\mathcal{G}(\mathcal{A})$ has a spanning tree, the left eigenvector w_l^T of $\mathcal{L}$ satisfies that $w_i > 0$ if and only if the ith vertex of $\mathcal{G}(\mathcal{A})$ is the root of a spanning tree in $\mathcal{G}(\mathcal{A})$ (see Lemma 3.3 in [8] or Theorem 9 in [16]). This fact plus the expression of $\chi(\bar{\xi}_0)e_1$ given in Theorem 1 indicates that only the agents which act as roots in $\mathcal{G}(\mathcal{A})$, contribute to the consensus state.

Remark 3: In contrast to the consensus protocol proposed in [14]/[15], protocol (2) only depends on the agent's own state information and the state variables ξ_j of its neighbors rather than the kth derivatives of ξ_j, $j \in \mathcal{N}_i$. To a great extent, this property reduces the communication or sensing cost and/or the computing cost for a network of high-order-integrator agents reaching the consensus state given in Theorem 1, although the protocol in [14]/[15] can also steer the network to the consensus state by selecting appropriate initial states of agents.

Remark 4: To clarify the importance and the necessity of high-order-integrator model (1), we introduce briefly the transformation from a completely controllable single-input LTI system $\dot{x} = Ax + bv$ into a controlled high-order integrator, and give a consensus controller for networks of N identical agents with dynamics modeled by the LTI system. Herein, $x \in \mathbb{R}^m$ is the state and $v \in \mathbb{R}$ is the control input. First, let $x = T\tilde{x}$, where $T \in \mathbb{R}_m$ is a nonsingular matrix such that $T^{-1}AT = A_c, T^{-1}b = b_c = [0 \ \cdots \ 0 \ 1]^T \in \mathbb{R}^m$ and $(A_c,\ b_c)$ is the associated controllable canonical form to $(A,\ b)$. Then $\dot{\tilde{x}} = A_c\tilde{x} + b_c v$. Suppose the characteristic polynomial of A is $s^m - a_m s^{m-1} - \cdots - a_2 s - a_1$, and denote $\tilde{x} = [\ \tilde{x}_1 \quad \tilde{x}_2 \quad \cdots \quad \tilde{x}_m\ ]^T$. Next, take the state feedback $v = -[\ a_1 \quad a_2 \quad \cdots \quad a_m\ ]\tilde{x} + u$, where $u \in \mathbb{R}$ is an external input. It follows that $\tilde{x}_1^{(m)} = u$, and $\tilde{x}_{k+1},\ k \in \underline{m-1}$ is the kth order derivative of $\tilde{x}_1$. Consequently, the LTI system is transformed into the controlled high-order integrator with respect to $\tilde{x}_1$. Furthermore, based on protocol (2), we can give a consensus controller for a group of N agents with dynamics

$$\dot{x}_i = Ax_i + bv_i,\ i \in \underline{N} \tag{6}$$

as follows:

$$v_i = f^T T^{-1} x_i - \sum_{j \in \mathcal{N}_i} \kappa_i a_{ij} e_1^T T^{-1}(x_i - x_j), \tag{7}$$

where $f = [-a_1 \ -(c_1 + a_2) \ \cdots \ -(c_{m-1} + a_m)]^T \in \mathbb{R}^m$, $c_1,\ \cdots,\ c_{m-1}$ are given in (2) and $e_1 \in \mathbb{R}^m$. Through direct computation, we can obtain that there is a nonsingular linear transformation between the closed-loop system of (6)(7) and that of (1)(2). This indicates that the network (6) with protocol (7) reaches consensus if and only if the network (1) with protocol (2) reaches consensus. Furthermore, the consensus state of network (6) with protocol (7) is $\left(\frac{1}{c_1}\sum_{i=1}^N w_i \tilde{c}^T T^{-1} x_i(0)\right) T e_1$, where $e_1 \in \mathbb{R}^m$, $x_i(0)$ is the initial state of agent i, and $w_i,\ \tilde{c}$ are given in Theorem 1. We omit the details for saving space.

B. Design of Protocol (2)

Denote $h_{m-1}(s) = s^{m-1} + c_{m-1}s^{m-2} + \cdots + c_1$ and $g_{mi}(s) = s^m + c_{m-1}s^{m-1} + \cdots + c_1 s + \mu_i$ with $0 \neq \mu_i \in \sigma(\Upsilon\mathcal{L})$. Theorem 1 shows that if $\mathcal{G}(\mathcal{A})$ has a spanning tree, then network in (4) solves a consensus problem iff the real polynomial $f(s) := h_{m-1}(s)\prod_{i=2}^N g_{mi}(s)$ is Hurwitz stable. In this subsection, we investigate the design of parameters $c_k,\ k \in \underline{m-1}$ and gains $\kappa_i,\ i \in \underline{N}$ for the convergence to consensus of network (4).

We first consider the case of $m = 2$, that is, the double-integrator system $\ddot{\xi}_i = u_i$ with $u_i = -c_1\dot{\xi}_i + \sum_{j \in \mathcal{N}_i} \kappa_i a_{ij}(\xi_j - \xi_i)$, which was studied in [12].

Theorem 2: Consider network of double-integrator agents with fixed topology $\mathcal{G}(\mathcal{A})$. Then the network solves consensus if and only if $\mathcal{G}(\mathcal{A})$ has a spanning tree and

$$c_1 > \max_{0 \neq \mu_i \in \sigma(\Upsilon\mathcal{L})} \frac{|\operatorname{Im}(\mu_i)|}{\sqrt{\operatorname{Re}(\mu_i)}}. \tag{8}$$

Proof: According to Theorem 1, we only need to prove $h_1(s)\prod_{i=2}^N g_{2i}(s)$ is Hurwitz stable iff c_1 satisfies (8). It is obvious that $h_1(s)$ is Hurwitz stable iff $c_1 > 0$. Next, if $\operatorname{Im}(\mu_i) = 0$ then $g_{2i}(s)$ is Hurwitz stable iff $c_1 > 0$ and $\mu_i > 0$; if $\operatorname{Im}(\mu_i) \neq 0$ then $s^2 + c_1 + \mu_i$ is Hurwitz stable iff $g_{2i}(s)\overline{g}_{2i}(s) = s^4 + 2c_1 s^3 + (c_1^2 + 2\operatorname{Re}(\mu_i))s^2 + 2c_1\operatorname{Re}(\mu_i)s + \mu_i\overline{\mu}_i$ is Hurwitz stable, and iff $c_1 > 0,\ c_1^2 + \operatorname{Re}\mu_i > 0,\ (c_1^2 + 2\operatorname{Re}(\mu_i))\operatorname{Re}(\mu_i) > \mu_i\overline{\mu}_i + \operatorname{Re}^2(\mu_i)$, which is equivalent to (8). This completes the proof. ■

Remark 5: Theorem 2 indicates that when $\mathcal{G}(\mathcal{A})$ is connected, the network reaches consensus if and only if $c_1 > 0$. In this scenario, let $\gamma = \min\{|\operatorname{Re}(\lambda)| :\ \lambda$ is the root of $h_1(s)\prod_{i=2}^N g_{2i}(s)\}$. By some computation, we can obtain

$$\gamma = \begin{cases} \frac{c_1}{2}, & 0 < c_1 < 2\sqrt{\mu_2} \\ \frac{c_1 - \sqrt{c_1^2 - 4\mu_2}}{2}, & c_1 \geq 2\sqrt{\mu_2} \end{cases}.$$

According to linear system theory, we know γ reflects the convergence speed. From the expression of γ, it is easy to see that γ is a continuous function of c_1, denoted by $\gamma(c_1)$. Then $\gamma(c_1)$ is non-increasing with respect to c_1 over $[2\sqrt{\mu_2},\ \infty)$ and $\max_{0<c_1<\infty}\gamma(c_1) = \sqrt{\mu_2}$ with the maximum point being $c_1 = 2\sqrt{\mu_2}$. This means that if $c_1 = 2\sqrt{\mu_2}$, then the convergence speed reaches the maximum value $\sqrt{\mu_2}$.

For the general case, we need the following lemma, which is concerning to Hurwitz stability of a family of disk polynomials. We start by presenting some notations.

Let $\mathcal{F}_D = \{\delta(s) = \delta_m s^m + \cdots + \delta_1 s + \delta_0 :\ \delta_k \in D_k,\ k \in \{0\} \cup \underline{m}\}$ be a family of complex polynomials, where $D_k = \{z \in \mathbb{C} :\ |z - \beta_k| \leq r_k,\ r_k \geq 0, \beta_k \in \mathbb{C}\}$ and $0 \notin D_m$. Denote $\beta(s) = \beta_m s^m + \cdots + \beta_1 s + \beta_0$, $\gamma_1(s) = r_0 - \sqrt{-1}r_1 s - r_2 s^2 + \sqrt{-1}r_3 s^3 + r_4 s^4 - \cdots$ and $\gamma_2(s) = r_0 + \sqrt{-1}r_1 s - r_2 s^2 - \sqrt{-1}r_3 s^3 + r_4 s^4 + \cdots$. Let $q_1(s) = \frac{\gamma_1(s)}{\beta(s)}$ and $q_2(s) = \frac{\gamma_2(s)}{\beta(s)}$ be two proper rational functions. Define $\|q_k\|_\infty = \sup_{\omega \in \mathbb{R}}\left|\frac{\gamma_k(\sqrt{-1}\omega)}{\beta(\sqrt{-1}\omega)}\right|,\ k = 1,\ 2$. When $\beta(s)$ is a real polynomial, $\|q_1\|_\infty = \|q_2\|_\infty$ (see [25]).

Lemma 3: ([25]) All the polynomials of $\mathcal{F}_D$ are Hurwitz stable if and only if $\beta(s)$ is Hurwitz stable and $\|q_k\|_\infty < 1,\ k = 1,\ 2$.

Given a graph $\mathcal{G}(\mathcal{A})$ which has a spanning tree, let $d = \max_{i \in \underline{N}}\{d_i\}$ (d_i are given in Section II) be the maximum in-degree of all the vertices and $\mu_2 \in \sigma(\mathcal{L})$ be the nonzero eigenvalue with minimum positive real part. The Geršgorin disk theorem (see [27]) proves that for any $\mu_i \in \sigma(\mathcal{L}),\ i \in \underline{N},\ \mu_i \in \mathcal{B}(d) := \{z \in \mathbb{C} :\ |z - d| \leq d\}$. Suppose $\varepsilon > 0$ is an appropriate small number. We next find a disk in the complex plane such that all the nonzero eigenvalues of $\mathcal{L}$ are located in it. Moreover, the disk satisfies that the origin of

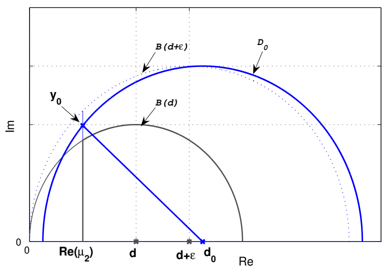

Fig. 1. Disk D_0 which satisfies $\sigma(\mathcal{L})/\{0\} \subset D_0$, $0 \notin D_0$ and has center $(d_0,\ 0)$, $d_0 > 0$. .

the complex plane is not in it and its center is located at the positive half-real axis. Set $\mathcal{B}(d+\varepsilon) = \{z \in \mathbb{C}:\ |z-d-\varepsilon| \leq d+\varepsilon\}$, $\partial(\mathcal{B}(d+\varepsilon)) = \{z \in \mathbb{C}:\ |z-d-\varepsilon| = d+\varepsilon\}$ and $\partial(\mathcal{B}(d)) = \{z \in \mathbb{C}:\ |z-d| = d\}$. Then $\mathcal{B}(d) \subset \mathcal{B}(d+\varepsilon)$. Denote the intersection points between the line $z = \mathrm{Re}(\mu_2)$ and the upper half circles of $\partial(\mathcal{B}(d))$ and $\partial(\mathcal{B}(d+\varepsilon))$ as z_1 and z_2, respectively. Let $y_0 = \frac{\mathrm{Im}(z_1+z_2)}{2}$. Solve the equation of s: $(\mathrm{Re}(\mu_2)-s)^2 + y_0^2 = (d+\varepsilon)^2$ and denote the positive root as d_0. Then $d_0 > d+\varepsilon$ and $D_0 = \{z \in \mathbb{C}:\ |z-d_0| \leq d+\varepsilon\}$ is the just disk which we want to find. In other words, $\sigma(\mathcal{L})/\{0\} \subset D_0$, $0 \notin D_0$ and the center $(d_0,\ 0)$ with $d_0 > 0$ is located at the positive half-real axis. Fig. 1 shows the upper part of the disk D_0.

Taking $\beta_0 = d_0$, $r_0 = d+\varepsilon$, $\beta_k = c_k, r_k \geq 0, k \in \underline{m-1}$ and $\beta_m = 1, r_m \geq 0$, we can obtain the following result based on Lemma 3 .

Theorem 3: Consider network in (4) with fixed topology $\mathcal{G}(\mathcal{A})$. Suppose $\mathcal{G}(\mathcal{A})$ has a spanning tree and $\kappa_i = 1,\ i \in \underline{N}$. If there exist some parameters $c_k,\ k \in \underline{m-1}$ and some numbers $r_i \geq 0,\ i \in \underline{m}$ such that $h_{m-1}(s)$ and $\hat{\beta}(s) = s^m + c_{m-1}s^{m-1} + \cdots + c_1 s + d_0$ are Hurwitz stable, and $\|\frac{\gamma_1}{\hat{\beta}}\|_\infty < 1$, where $\gamma_1(s)$ is given in Lemma 3, then the network solves a consensus problem.

Theorem 3 is a direct result of Theorem 1 and Lemma 3, hence the proof is omitted.

For $m = 3$, model (1) is a triple integrator and protocol (2) is $u_i = -c_2\ddot{\xi}_i - c_1\dot{\xi}_i + \sum_{j\in\mathcal{N}_i}\kappa_i a_{ij}(\xi_j - \xi_i)$. In this scenario, we have the following result.

Corollary 1: Consider network of triple-integrator agents with topology graph $\mathcal{G}(\mathcal{A})$. Suppose $\mathcal{G}(\mathcal{A})$ has a spanning tree and $\kappa_i = 0,\ i \in \underline{N}$. If $c_1,\ c_2$ satisfy $c_1 > 0,\ c_2 > 0,\ c_1c_2 > d_0$ and

$$c_2^4 - 4c_1c_2^2 + c_1^2 + 24d_0c_1 \leq 0, \tag{9}$$

then the network solves a consensus problem.

Proof: From Theorem 1, we only need to prove $h_2(s)\prod_{i=2}^{N} g_{3i}(s)$ is Hurwitz stable. Denote the family $\mathcal{F}_D$ in Lemma 3 as $\mathcal{F}_D = \{\delta(s) = s^3 + c_2s^2 + c_1s + \mu:\ \mu \in D_0 = \{z \in \mathbb{C}:\ |z-d_0| \leq d+\varepsilon\}\}$ with $\varepsilon > 0$. Then $g_{3i}(s) \in \mathcal{F}_D,\ i = 2,\ \cdots,\ N$. From $c_1 > 0,\ c_2 > 0$ and $c_1c_2 > d_0$, it follows that $h_2(s)$ and $\widetilde{\beta}(s) = s^3 + c_2s^2 + c_1s + d_0$ is Hurwitz stable. We next prove $\|\frac{d+\varepsilon}{\widetilde{\beta}}\|_\infty < 1$. Note that $|\widetilde{\beta}(\sqrt{-1}\omega)|^2 = \omega^6 + (c_2^2 - 2c_1)\omega^4 + (c_1^2 - 2d_0c_2)\omega^2 + d_0^2$. Then $|\widetilde{\beta}(\sqrt{-1}\omega)|^2$ is a continuous function of ω and $|\widetilde{\beta}(\sqrt{-1}\omega)|^2 = |\widetilde{\beta}(-\sqrt{-1}\omega)|^2$. Therefore, we only consider the case of $\omega \in [0,\ \infty)$. Let $n(s) = s^3 + (c_2^2 - 2c_1)s^2 + (c_1^2 - 2d_0c_2)s + d_0^2,\ s \geq 0$. Then $|\widetilde{\beta}(\sqrt{-1}\omega)|^2 = n(\omega^2)$. In addition, $\dot{\gamma}(s) = 3s^2 + 2(c_2^2 - 2c_1)s + (c_1^2 - 2ddc_2)$, which is a quadratic function of s. Hence if the discriminant $\Delta := 4(c_2^2 - 2c_1)^2 - 12(c_1^2 - 2d_0c_2) \leq 0$, that is, inequality (9) holds, then $\dot{\gamma}(s) \geq 0$ for $s \geq 0$. This implies that $\gamma(s)$ is increasing when $s \geq 0$. Consequently, $|\widetilde{\beta}(\sqrt{-1}\omega)|^2 = \gamma(\omega^2)$ is increasing over $[0,\ \infty)$ (this is derived from the monotonicity of composite functions). Thus inequality (9) results in $|\widetilde{\beta}(\sqrt{-1}\omega)|^2 \leq |\widetilde{\beta}(0)|^2 = d_0^2 > (d+\varepsilon)^2$. It follows that $\|\frac{d+\varepsilon}{\widetilde{\beta}}\|_\infty < 1$. According to Lemma 3, all the polynomials in the family $\mathcal{F}_D$ are Hurwitz stable. This completes the proof. ■

Remark 6: The inequalities about c_1 and c_2 in Corollary 1 are solvable. For example, if $c_1 = c_2^2$ and $c_2^3 \geq 12d_0$, then the inequalities hold.

Observe that when the topology of network in (4) is undirected, the eigenvalues of its Laplacian matrix are non-negative real numbers. In this case, we give the following result which is obtained from Nyquist criterion.

Theorem 4: Consider network in (4) with fixed undirected topology $\mathcal{G}(\mathcal{A})$. Suppose $\mathcal{G}(\mathcal{A})$ is connected. Then the network solves a consensus problem if and only if there exist some parameters $c_k,\ k \in \underline{m-1}$ and gains $\kappa_i > 0,\ i \in \underline{N}$ such that $h_{m-1}(s)$ is Hurwitz stable and the net encirclement of $(-\frac{1}{\mu_i},\ 0)$ by the Nyquist plot $\frac{1}{sh_{m-1}(s)}$ is zero for all $\mu_i \in \sigma(\Upsilon\mathcal{L})/\{0\},\ i = 2,\ \cdots,\ N$.

In Theorem 4, $\frac{1}{sh_{m-1}(s)}$ has a first-order integrating link when $h_{m-1}(s)$ is Hurwitz stable. This implies that there is no intersection point at minus infinity between the Nyquist plot of $\frac{1}{sh_{m-1}(s)}$ and the real axis, and there are at most $[\frac{m-1}{2}]$ (the maximum integer no larger than $\frac{m-1}{2}$) intersection points between the Nyquist plot and the negative half-real axis. Moreover, the real parts of these points are finite. After selecting parameters $c_k,\ k \in \underline{m-1}$ which make $h_{m-1}(s)$ Hurwitz stable, we can adjust gains $\kappa_i,\ i \in \underline{N}$ such that the nonzero eigenvalues of $\Upsilon\mathcal{L}$ are located at the exterior of the Nyquist plot of $\frac{1}{sh_{m-1}(s)}$.

V. Conclusions

This paper has considered the consensus problem for networks of high-order-integrator agents with fixed topology. A linear distributed protocol has been proposed, which steers all the agents to a constant state. It has been proved that the underlying graph having a spanning tree is a necessary condition for convergence to consensus. The parameter design for the protocol has been discussed. In addition, a consensus controller has been provided for networks of identical agents with dynamics modeled by a completely controllable single-input LTI system, based on the protocol of networks of high-order-integrator agents,.

The work of this paper will motivate other research topics. By taking into account the constraints caused by the impossibility of measuring all state variables, designing an observer-based protocol may be a necessary research direction. Studying the consensus problem for the case of switching topology and communication time-delay will be an interesting and significant work.

APPENDIX

Proof of Lemma 1. Let P be a nonsingular matrix such that $P^{-1}\mathcal{L}P = J$, where J is the Jordan matrix associated to $\mathcal{L}$. Then

$$\begin{aligned} & det(sI_{Nm} - \Omega) \\ = \;& det\big(sI_{Nm} - (P^{-1} \otimes I_m)\Omega(P \otimes I_m)\big) \\ = \;& \textstyle\prod_{i=1}^{N} det(sI_m + \mu_i F_m - E_m) \\ = \;& \textstyle\prod_{i=1}^{N} (s^m - c_{m-1}s^{m-1} - \cdots - c_1 s + \mu_i), \end{aligned}$$

which implies that the number of zero eigenvalues of Ω is equal to that of $\mathcal{L}$. Since $\mathcal{L}\mathbf{1}_N = 0$, $E_m e_1 = 0$ with $e_1 \in \mathbb{R}^m$ and $[-c_1 \ \cdots \ -c_{m-1} \ 1]E_m = 0$, we can obtain $\frac{1}{\sqrt{N}}\mathbf{1}_N \otimes e_1$ with $e_1 \in \mathbb{R}^m$ ($-\frac{1}{c_1}w_l^T \otimes [-c_1 \ \cdots \ -c_{m-1} \ 1]$) is a right (left) eigenvector of Ω associated to the zero eigenvalue. ■

Proof of Lemma 2. Let $\bar{\xi} = S\zeta$, where S is a permutation matrix such that $\zeta = [\zeta_1^T \ \cdots \ \zeta_m^T]^T$ with $\zeta_k = [\xi_1^{(k-1)} \ \cdots \ \xi_N^{(k-1)}]^T$, $k \in \underline{m}$ and $\xi_i^{(0)} = \xi_i$, $i \in \underline{N}$. Then

$$\dot{\zeta} = [E_m \otimes I_N - F_m \otimes (\Upsilon\mathcal{L})]\zeta \triangleq \Xi\zeta, \qquad (10)$$

where E_m, F_m are given in (3). Let $\beta = [\beta_1^T \ \cdots \ \beta_m^T]^T$ with $\beta_k \in \mathbb{R}^N$, $k \in \underline{m}$ be a right eigenvector of Ξ associated to the eigenvalue s. It follows that $\beta_2 = s\beta_1, \ \cdots, \ -\mathcal{L}\beta_1 + c_1\beta_2 + \cdots + c_{m-1}\beta_m = s\beta_m$. Thus $-\mathcal{L}\beta_1 = (s^m - c_{m-1}s^{m-1} - \cdots - c_1 s)\beta_1$. As a result, $\beta = [1 \ s \ \cdots \ s^{m-1}]^T \otimes \beta_1^T$, and $\mu := -(s^m - c_{m-1}s^{m-1} - \cdots - c_1 s)$ is an eigenvalue of $\mathcal{L}$ with a corresponding right eigenvector β_1. To prove the lemma, we only need to prove the equilibria of system (10) are of the form $a_0 e_1 \otimes \mathbf{1}_N$ with $a_0 \in \mathbb{R}$ and $e_1 \in \mathbb{R}^m$. First, it is obvious that $e_1 \otimes \mathbf{1}_N$ is an equilibrium of system (10). Next, if there exists an equilibrium of system (10), denoted by ζ_0, such that $\zeta_0 \notin \mathrm{span}\{e_1 \otimes \mathbf{1}_N\}$. Then $\Xi\zeta_0 = 0$. Hence there is a vector $\zeta_{01} \in \mathbb{R}^N$ satisfying $\mathcal{L}\zeta_{01} = 0$, such that $\zeta_0 = e_1 \otimes \zeta_{01}$. Thus $\zeta_0 \notin \mathrm{span}\{e_1 \otimes \mathbf{1}_N\}$ if and only if $\zeta_{01} \notin \mathrm{span}\{\mathbf{1}_N\}$. If we take the initial state of system (10) to be ζ_0, then the corresponding solution is $\zeta(t) = \zeta_0 = e_1 \otimes \zeta_{01}$. The fact $\zeta_{01} \notin \mathrm{span}\{\mathbf{1}_N\}$ contradict $\lim_{t\to\infty}(\xi_i(t) - \xi_j(t)) = 0$ for all $i, \ j \in \underline{N}$. Therefore all the equilibria of system (10) belong to $\mathrm{span}\{e_1 \otimes \mathbf{1}_N\}$. This completes the proof. ■

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