

Sliding Mode Control of Uncertain Multivariable Nonlinear Systems Applied to Uncalibrated Robotics Visual Servoing

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Abstract—An output-feedback sliding mode controller using monitoring functions was recently introduced for linear uncertain single-input-single-output (SISO) systems with unknown control direction. Here, a generalization is developed for multivariable systems with strong nonlinearities. The monitoring scheme is extended to handle the uncertainty of the plant high frequency gain matrix K_p . Our strategy provides global stability properties and exact output tracking. Experimental results with a robotics visual servoing system, using a fixed but uncalibrated camera, illustrate the robustness and practical viability of the proposed scheme.

I. INTRODUCTION

The design of output-feedback control of uncertain systems without knowledge of the high frequency gain sign, *i.e.*, control direction, has been an instigating problem since the early 1980s [1]. In the adaptive control literature, the so called Nussbaum gain [2] has been used to design stable systems under this relaxed assumption, including the multivariable case [3]-[5]. However, this approach is of arguable practical interest, due to the large transients and inherent lack of robustness [1], [6]. More recently, an output-feedback tracking sliding mode control (SMC) for SISO uncertain linear plants with unknown control direction was introduced in [7]. In lieu of the Nussbaum gain [2], the control sign was adjusted based on monitoring functions [8].

In this paper, we extend the controller of [7] to nonlinear multiple-input-multiple-output (MIMO) plants with relative degree one and affine in the control using a unit vector model-reference sliding mode control approach. Here, the nonlinearities are allowed to be strong (*e.g.*, polynomial) and unmatched, *i.e.*, not in the span of the control input matrix.

One motivation to use a unit vector [9], [10] instead of a vector “sign(·)” switching function, is that a less restrictive prior knowledge of the plant *high frequency gain* (HFG) matrix K_p can be obtained as compared to using direct norm-bounds on the uncertainty [11] or by requiring the positive definiteness property of K_p [12], [13]. In fact, we only need to require $-K_p S_q$ to be a Hurwitz matrix where S_q is a control pre-compensator matrix [9], [10].

In this work, K_p is allowed to be uncertain so that S_q is not known *a priori*. Inspired by the recent developments in supervisory control [14] and the *spectrum-unmixing sets* introduced in [15], we propose a switching mechanism that selects a suitable static pre-compensator matrix S_q out of a

finite index set of matrices \mathcal{Q} through an appropriate monitoring function. Global asymptotic stability with respect to a compact set and exact output tracking are then demonstrated.

Application to a robotics visual servoing system using a fixed but uncalibrated camera illustrates the effectiveness of the proposed multivariable controller in real-world conditions. Remarkably, the usual restriction of the camera orientation angle $\psi \in (-\frac{\pi}{2}, \frac{\pi}{2})$, presented in [16]-[20], is effectively removed.

II. NOTATION AND TERMINOLOGY

The following notation and basic concepts are employed:

- ISS means Input-to-State-Stable [21] and classes \mathcal{K} , \mathcal{K}_∞ functions are defined as usual [22, pp. 144].
- The Euclidean norm of a vector x and the corresponding induced norm of a matrix A are denoted by $|x|$ and $|A|$.
- We adopt Filippov’s definition for the solution of discontinuous differential equations [23] and the concept of *extended equivalent control* [10], also applicable to the reaching phase of a sliding mode.

III. PROBLEM FORMULATION

This paper considers the global tracking problem of MIMO nonlinear systems transformable into the *normal form*

$$\dot{\eta} = \phi_0(\eta, y, t), \quad (1)$$

$$\dot{y} = K_p u + \phi_1(\eta, y, t), \quad (2)$$

where $u \in \mathbb{R}^l$ is the control input, $y \in \mathbb{R}^l$ is the measured output and $\eta \in \mathbb{R}^{n-l}$ is the unmeasured state of the η -subsystem, also referred to as an *inverse system*. The uncertain functions ϕ_0 and ϕ_1 are piecewise continuous in t and locally Lipschitz continuous in the other arguments. For each solution of (1)–(2) there exists a maximal time interval of definition given by $[0, t_M)$, where t_M may be finite or infinite. Thus, finite-time escape is not precluded *a priori*.

It is further considered that the plant control direction is unknown (and constant) in the sense that all uncertain parameters of the HFG matrix K_p belong to some compact set Ω_p . In Ω_p , it is only assumed that:

- (A1) (i) $\det(K_p) \neq 0$, (ii) there exists a *known* constant $c > 0$ such that $|K_p^{-1}| \leq c$ and (iii) there exists a finite index set \mathcal{Q} of *known* matrices $S_q \in \mathbb{R}^{l \times l}$ such that $-K_p S_q$ is Hurwitz for some $q \in \mathcal{Q}$.

According to (A1), we focus the simplest relative degree one case amenable to Lyapunov design. The case of general relative degree with unknown control direction will be left for a future work. The Hurwitz condition is necessary and

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sufficient for the attractiveness of the sliding surface in the case of unit vector sliding mode control [9], [10].

This assumption significantly relaxes the usual requirement of positive definiteness and symmetry of K_p [12], [13]. Symmetry is a non generic property. It can be easily destroyed by arbitrarily small uncertainties in K_p . Moreover, if K_p is positive definite, then this implies that $-K_p$ is Hurwitz but the converse is not true.

For the SISO case, (A1) can be interpreted as follows: the first two conditions indicate that the scalar $K_p \neq 0$ can be positive or negative, *i.e.*, the control direction is unknown. Moreover, in this case the index set is $\mathcal{Q} = \{0, 1\}$ and a scalar S_q ($S_0 = -1, S_1 = 1$) is required to make $-K_p S_q$ negative. Here, a switching scheme is provided for cycling through the elements of the finite index set \mathcal{Q} [15] so that stability and the tracking objective are achieved.

Global Tracking Problem

The problem consists in designing a control law u , via output-feedback and without the knowledge of the plant control direction, to drive the *output tracking error*

$$e(t) = y(t) - y_m(t) \quad (3)$$

asymptotically to zero (exact tracking), starting from any plant/controller initial conditions and maintaining uniform closed-loop signal boundedness. The *desired trajectory* $y_m(t)$ is assumed to be generated by the *reference model*:

$$\dot{y}_m = A_m y_m + r, \quad A_m = -\text{diag} \{\gamma_1, \dots, \gamma_l\}, \quad (4)$$

where $r, y_m \in \mathbb{R}^l$, $\gamma_i > 0$ ($i = 1, \dots, l$) and $r(t)$ is assumed piecewise continuous and uniformly bounded.

In order to achieve the control objective our strategy requires a norm observer for the state η of the inverse system (1), according to the following definition.

Definition 1: A first order norm observer for system (1) is a SISO dynamic system of the form (y is the plant output):

$$\dot{\bar{\eta}} = -\lambda_o \bar{\eta} + \varphi_o(y, t), \quad (5)$$

with input $\varphi_o(y, t)$ and output $\bar{\eta}$, such that: (i) $\lambda_o > 0$ is a constant; (ii) $\varphi_o(y, t)$ is a non-negative function continuous in y , piecewise continuous and upperbounded in t ; and (iii) for each initial states $\eta(0)$ and $\bar{\eta}(0)$

$$|\eta(t)| \leq |\bar{\eta}(t)| + \bar{k}_o (|\bar{\eta}(0)| + |\eta(0)|) e^{-\lambda_o t}, \quad (6)$$

$\forall t \in [0, t_M)$, with some constant $\bar{k}_o > 0$.

It is well known that, in the time-invariant case, if the inverse system (1) is ISS [21] then it admits such norm observer and the plant is minimum-phase. Here, we assume that:

(A2) The inverse system (1) admits a *known* norm observer (5) with φ_o and λ_o *known*.

In [www.coep.ufrj.br/~liu/ACC09], two classes of system with time-varying inverse system are given for which (5) can be implemented. The first case incorporates a class of nonlinearities ϕ_0 where a linear growth condition is required only *w.r.t.* the unmeasured state η . The other one, adapted from [24], illustrates a case where strong polynomial

nonlinearities in η are also allowed. In both cases, (1) has an ISS-like property *w.r.t.* an appropriate function of y and t .

In order to obtain a norm bound for ϕ_1 in (2), we additionally assume that:

(A3) There exists a *known* locally Lipschitz class \mathcal{K}_∞ function $\alpha_1(|\eta|)$ and a *known* non-negative function $\varphi_1(y, t)$ continuous in y , piecewise continuous and upperbounded in t such that $|\phi_1(\eta, y, t)| \leq \alpha_1(|\eta|) + \varphi_1(y, t)$.

Note that (A3) is not restrictive since ϕ_1 is assumed to be locally Lipschitz continuous in η and y . Furthermore, the bounding functions α_1 and φ_1 impose particular growth condition only *w.r.t.* the t -dependence.

From Tracking to Regulation Problem

From (2)–(4), the e -dynamics can be written as

$$\dot{e} = A_m e + K_p(u - u^*), \quad (7)$$

where $u^* := K_p^{-1}(-\phi_1 + A_m y + r)$.

Then, the global tracking problem can be reformulated as the regulation problem described as follows. Find an output-feedback sliding mode control law u in such a way that, for all initial conditions $(\eta(0), e(0), \bar{\eta}(0))$: (i) the solutions of (1)–(2), (5) and (7) are bounded and (ii) $e(t)$ tends asymptotically to zero as $t \rightarrow \infty$.

The *ideal control* u^* is considered as a matched input disturbance in (7). From (A1)–(A3), it can be norm bounded by available signals

$$|u^*| \leq c [\alpha_1(2|\bar{\eta}|) + \varphi_1(y, t) + |A_m y + r|] + \pi_1, \quad (8)$$

modulo the exponential decaying term $\pi_1 := k_1(|\bar{\eta}(0)| + |\eta(0)|)e^{-\lambda_o t}$, where $k_1 > 0$ is a constant, c is given in (A1) and π_1 comes from the exponential term in (6). To develop this inequality we have used the fact that α_1 is locally Lipschitz and $\psi(a+b) \leq \psi(2a) + \psi(2b)$, $\forall a, b \geq 0$ and $\forall \psi \in \mathcal{K}_\infty$.

IV. OUTPUT-FEEDBACK SLIDING MODE CONTROLLER

This section presents a first generalization of the proposed controller in [7] for a class of MIMO nonlinear plants with unknown control direction and relative degree one.

Let q^* be the *unknown* index of the index set \mathcal{Q} , given in (A1), for which the corresponding *unknown* matrix $\mathcal{S} = S_{q^*}$ assures that $-K_p \mathcal{S}$ is Hurwitz. Thus, the Lyapunov equation $(K_p \mathcal{S})^T P + P(K_p \mathcal{S}) = I$ has a solution $P = P^T > 0$.

Now, if the control direction is known (q^* is known) one can apply the *unit vector control* (UVC) law [10]

$$u = -\mathcal{S} \varrho(\bar{\eta}, y, t) \frac{e}{|e|}, \quad (9)$$

to (7) and verify that, if the modulation function ϱ satisfies

$$\varrho \geq c_d |u^*(t)| + \delta, \quad \delta \geq 0, \quad (10)$$

modulo the exponential decaying term $c_d \pi_1$, then the time derivative of $V = e^T P e$ along the solutions of (7) satisfies:

$$\dot{V} \leq -2\lambda_m V + c_d \pi_1, \quad \forall t \in [t_i, t_M),$$

for any $t_i \in [0, t_M)$, where $0 < \lambda_m < \min_i \{|\gamma_i|\}$, c_d is an appropriate positive constant and π_1 comes from (8). Hence, by using the comparison lemma [23], one has:

$$|e(t)| \leq \zeta(t), \quad \forall t \in [t_i, t_M), \quad (11)$$

where

$$\zeta(t) := |e(t_i)|e^{-\lambda_m(t-t_i)} + \pi_2,$$

with $\pi_2 := \Psi_2(|\bar{\eta}(0)| + |\eta(0)|)e^{-\lambda_c t}$, $0 < \lambda_c < \min\{\lambda_o, \lambda_m\}$ and $\Psi_2 \in \mathcal{K}$ (see [10, Lemma 1] for details).

The major problem is that q^* is unknown, thus we cannot implement the UVC law (9). In [7], a switching scheme based on monitoring function was developed to cope with the lack of knowledge of the control direction. Only linear SISO plants with relative degree one were considered. In that case, K_p was a scalar and after a finite number of changes in the control sign ($S_q = \pm 1$), the correct control direction could be detected. For MIMO nonlinear plants, the UVC law is redefined as

$$u = -S_q \varrho(\bar{\eta}, y, t) \frac{e}{|e|}, \quad \forall t \in [0, t_M), \quad (12)$$

where ϱ satisfies (10) and a switching mechanism also based on a monitoring function is used to decide when the static pre-compensator matrix S_q [14] should be switched within the collection of matrices with $q \in \mathcal{Q}$.

V. SWITCHING SCHEME AND MONITORING FUNCTION

We now construct the monitoring function φ_m based on the norm bound for e given in (11) following the ideas of [7], [8]. Reminding that inequality (11) holds when the matrix S_q is correct ($S_q = \mathcal{S}$), it seems natural to use ζ as a benchmark to decide whether a switching of S_q is needed, *i.e.*, the switching occurs only when (11) is violated. However, since π_2 is not available for measurement we consider the following function, defined in the interval $[t_k, t_{k+1})$, to replace ζ :

$$\varphi_k(t) = |e(t_k)|e^{-\lambda_m(t-t_k)} + a(k)e^{-\lambda_c t}, \quad (13)$$

where the switching time t_k sets the change of index $q \in \mathcal{Q}$, thus *cycling* through the S_q matrices and $a(k)$ is any positive monotonically increasing unbounded sequence.

The *monitoring function* φ_m can thus be defined as

$$\varphi_m(t) := \varphi_k(t), \quad \forall t \in [t_k, t_{k+1}) \subset [0, t_M). \quad (14)$$

Note that from (13) and (14), one has $|e(t_k)| < \varphi_k(t_k)$ at $t = t_k$. Hence, the switching time t_k is defined by

$$t_{k+1} := \begin{cases} \min\{t > t_k : |e(t)| = \varphi_k(t)\}, & \text{if it exists,} \\ t_M, & \text{otherwise,} \end{cases} \quad (15)$$

where $k \in \{0, 1, \dots\}$ and $t_0 := 0$ (see Figure 1). The following inequality is directly obtained from (14)

$$|e(t)| \leq \varphi_m(t), \quad \forall t \in [0, t_M). \quad (16)$$

Figure 1 illustrates the tracking error norm $|e|$ as well as the monitoring function φ_m .

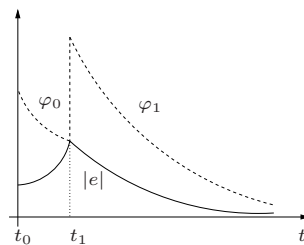


Fig. 1. The trajectories of φ_m and $|e|$.

VI. STABILITY RESULTS

In order to fully account for all initial conditions in the closed-loop system, let

$$z^T(t) := [z^0(t), e(t)], \quad z^0(t) := [\eta^T(0) \bar{\eta}^T(0)]e^{-\gamma t}, \quad (17)$$

where z^0 denotes the *transient state* [10] and $\gamma > 0$ is a generic constant. The main result is now stated.

Theorem 1: Assume that (A1)–(A3) hold. Consider the error equation (7) with UVC law (12) and monitoring function (13)–(14). If the modulation function satisfies (10), then the control direction switching stops. The complete error system, with state $z(t)$, is globally asymptotically stable *w.r.t.* a compact set, independent of the initial conditions, and ultimately exponentially convergent to zero. Moreover, all signals in the closed loop system remain uniformly bounded and if $\delta > 0$ in (10), then the sliding mode on the manifold $e=0$ is reached in finite time.

Proof: See [www.coep.ufrj.br/~liu/ACC09]. ■

Remark 1: [S_q Selection] We know that if $-K_p S_q$ is Hurwitz all trajectories of the system converge to the origin of the error state space [10, Lemma 1]. Moreover, if $-K_p S_q$ is not Hurwitz, then for *almost* every initial condition (*i.e.*, except for a set of measure zero) the system trajectories diverge unboundedly or do not converge to the origin. This is a contradiction, since if the switching stops, according to *Theorem 1*, the state must converge to the origin. Then, almost always, the ultimate matrix S_q selected is such that $-K_p S_q$ is Hurwitz.

Remark 2: [Initial Transients] Note that, due to the initial identification phase of a stabilizing S_q , it is not possible to state Lyapunov stability with respect to the state space origin. Indeed, even if the initial state norm is small, one cannot guarantee that the initial transient is correspondingly small during the initial phase. However, either the trajectories start in an invariant compact set or they tend to this set asymptotically. Ultimately, the trajectories converge exponentially to the origin, as stated in *Theorem 1*.

VII. APPLICATION TO ROBOTICS VISUAL SERVOING

To illustrate the applicability of the proposed switching scheme, we consider a simple case where the plant is a MIMO integrator without inverse subsystem. The proposed strategy is used to solve the visual servoing control problem for a robot manipulator performing planar movements in the cartesian space and using a fixed and uncalibrated camera with optical axis orthogonal to the robot workspace.

A. Visual Servoing Kinematic Control

Firstly, one considers the kinematic control problem for a robot manipulator. In this approach, the end-effector position $x \in \mathbb{R}^n$ is given by the forward kinematics map $x = f(\theta)$, where $\theta \in \mathbb{R}^l$ is the manipulator joint angle vector.

The differential kinematics equation can be obtained from the time derivative of the forward kinematics map given by $\dot{x} = J(\theta)\dot{\theta}$, where $J(\theta) = \frac{\partial f}{\partial \theta} \in \mathbb{R}^{n \times l}$ is the manipulator Jacobian. Then, considering $\dot{\theta}_i$ as the control input v_i ($i = 1, \dots, l$) one obtains the following control system

$$\dot{x} = J(\theta)v. \quad (18)$$

A cartesian control law u can be transformed to joint control signals by using

$$v = J(\theta)^{-1}u, \quad (19)$$

provided that u does not drive the robot manipulator to singular configurations.

Now, consider the visual servoing approach for closed-loop manipulator position control by through the camera image information. Let $y \in \mathbb{R}^2$ be the end-effector position in the image frame and $y_m \in \mathbb{R}^2$ be the desired time-varying trajectory for a target feature fixed on the arm tip. Then, the control goal can be described by

$$y \rightarrow y_m(t), \quad e = y - y_m \rightarrow 0, \quad (20)$$

where $e \in \mathbb{R}^2$ is the image error. We only consider planar movements in the cartesian space, thus $n = l = 2$ and $x \in \mathbb{R}^2$, without loss of generality. The visual servoing system consists of a monocular fixed CCD camera with optical axis *perpendicular* to the robot frame. Then, with some assumptions on the lens properties, the camera/workspace transformation can be represented by [26]

$$y = K_p x + y_0, \quad (21)$$

with

$$K_p = \frac{f_0}{f_0 + z_0} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix},$$

where y_0 is a constant term, which depends on the position of the camera frame with respect to the robot frame, K_p is the *camera/workspace transformation matrix* and considers the camera orientation angle ψ (or camera misalignment) with respect to the robot frame, f_0 is the camera focal length, z_0 is the depth from the camera frame to the robot workspace (in general $z_0 \gg f_0$), and $\alpha_1, \alpha_2 > 0$ are the scaling factors of the camera [pixel/mm].

Thus, the cartesian control problem in the image frame is described from (21) by

$$\dot{y} = K_p u. \quad (22)$$

Then, based on (22) and considering a feedforward and proportional control law given by

$$u = K_p^{-1}[\dot{y}_m + K(y_m - y)], \quad (23)$$

one has that the image error dynamics is governed by $\dot{e} + K e = 0$. Hence, by a proper choice of K as a positive definite matrix, $e \rightarrow 0$ exponentially as $t \rightarrow \infty$.

However, if the intrinsic and extrinsic parameters of camera model are *uncertain* (uncalibrated camera), matrix K_p is also uncertain. Therefore, the control law (23) cannot be implemented directly. As an approach to solve this problem, some adaptive schemes were proposed in order to cope with the uncertainty in the camera parameters [16]-[20]. As a rule, in the existing adaptive visual servoing methods, the camera orientation error ψ must be restricted to the range $(-\frac{\pi}{2}, \frac{\pi}{2})$.

In what follows, the combination of the proposed sliding mode approach and the switching scheme based on the monitoring function is applied to circumvent the limitations of adaptive control methods with respect to transient behavior, robustness and restricted range of camera miscalibration.

B. Visual Servoing based on Sliding Mode Control and Monitoring Function

The proposed control scheme can be applied to the visual servoing control problem in the presence of uncertainty. As a remarkable feature, we will show that an arbitrary camera orientation misalignment is allowed.

The monitoring function φ_m (13)-(14) is used to switch the matrix S_q in (12). The finite set of matrices S_q , $q \in \mathcal{Q} = \{0, 1, 2, 3\}$ can be chosen as

$$S_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, S_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

For arbitrary camera misalignment ψ , $-K_p S_q$ is Hurwitz for some S_q and therefore, the usual restriction $|\psi| < \frac{\pi}{2}$ can be removed. Note that the considered plant (22) is linear. Potentially, the proposed control system could also deal with nonlinear systems.

VIII. EXPERIMENTAL RESULTS

This section describes the experimental setup and discusses the test results.

A. Experimental setup

The proposed controller was implemented on a 6-DOF Zebra Zero robot manipulator (Integrated Motions, Inc.) performing end-effector motions on a vertical plane. Due to the robot large gear ratios and a high gain velocity control loop, a purely kinematic model is adopted since the robot dynamic effects are negligible.

A KP-D50 CCD camera (Hitachi, Ltd.) with a lens focal length $f_0 = 6$ [mm] and scaling factors $\alpha_1 = 119$ [pixel/mm] and $\alpha_2 = 102$ [pixel/mm] was mounted in front of the Zebra Zero. Figure 2 shows the camera point-of-view corresponding approximately to the orientation $\psi \approx 0$.

The average depth from the image plane to the robot vertical workspace was $z_0 = 1$ [m]. The extracted visual features are the centroid coordinates of the image of a red disc fixed on the robot wrist. The images with 640×480 [pixel] are acquired using a Meteor frame-grabber (Matrox, Ltd.) at 30 frames per second (FPS).

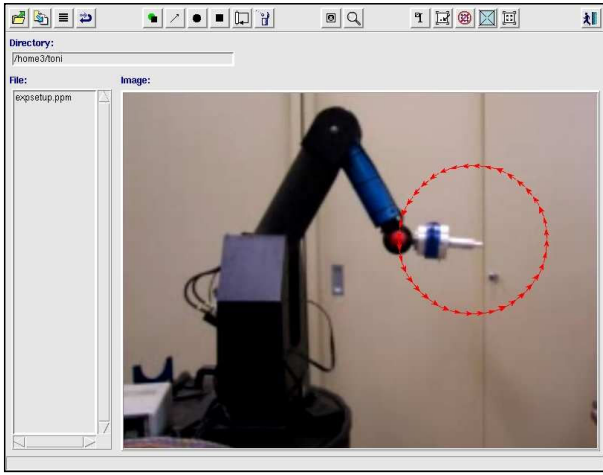


Fig. 2. Experimental station.

The visual servo controller was coded in C language and executed in 35 [ms] on a 200 MHz Pentium Pro processor with 64 Mbyte RAM using Linux OS. The joint velocity command generated by the visual servoing control law feeds the Zebra Zero ISA board which closes a velocity loop using an HCTL1100 microcontroller (HP Inc.) working in proportional velocity mode with 0.52 [ms] sampling time.

The image processing in RGB format is performed on a subwindow 100×100 [pixel] wide. The first estimation of the centroid coordinates is performed off-line using an *ad-hoc* Graphical User Interface developed in Tcl/Tk language [27] as shown in Figure 2. During the task execution, the feature is computed using the image moments algorithm [28]. Due to noise sensitivity, the proportional gain in the velocity loop cannot be made high enough to eliminate the steady state error due to gravity effects. The gravity disturbance was identified off-line using a least squares method and then effectively cancelled [29].

B. Test results

The experimental tests were performed without previous calibration of the camera position. The desired trajectory y_m is a circle generated by the model (4), with $\gamma_1 = \gamma_2 = 1$ and $r^T = [r_1 \ r_2]$ with

$$r_1 = y_1(0) + R[1 - \cos(\omega_r t)], \quad (24)$$

$$r_2 = y_2(0) + R[\sin(\omega_r t)], \quad (25)$$

where $y^T(0) = [y_1(0) \ y_2(0)]$ is the initial position of the centroid coordinates in the image frame, R and ω_r determine the radius and the angular velocity of the reference trajectory, respectively. The robot manipulator should then perform the tracking of a circular trajectory specified by $R = 40$ [pixel] and $\omega_r = \frac{\pi}{5}$ [rad/s]. The initial position of the centroid was $y_1(0) = 330$ [pixel], $y_2(0) = 275$ [pixel] and the camera orientation was intentionally changed to different values by software, without modifying the controller, in order to verify the effectiveness of the adaptation scheme.

The monitoring function φ_m is obtained from (13)-(14) with $a(k) = k + 1$, $\lambda_m = \lambda_c = 0.9$. In addition, a constant of 15 was added to φ_m to reduce spurious modifications in the control direction estimate due to the measurement noise.

It is well known that the measurement noise and the low sampling rate of the CCD camera can cause control chattering which can be alleviated by using a boundary layer in the UVC law (see, e.g., [11]).

The modulation function ϱ in (12) was implemented in order to satisfy (8) and (10), considering $\phi_1 = 0$ in u^* . All tests were designed to avoid manipulator Jacobian singularities in (19).

Figure 3 shows the time history of the monitoring function φ_m and the error norm $|e|$. The experimental test was performed with $\psi \approx \pi$ [rad], while the nominal value was assumed to be $\psi \approx \pi/2$ [rad]. Thus we had a very large mismatch from the “nominal” orientation. For the nominal orientation, the correct matrix pre-compensator would be S_0 so that this was the initial pre-compensator applied to the controller. Therefore, it would be necessary three switchings to reach the correct matrix S_3 (for $\psi = \pi$) and guarantee a stable model following. However, in order to test the cyclic switching and the robustness of the proposed scheme under time-varying control direction, the camera misalignment angle was set to $\psi \approx \pi/2$ in the last portion of the experiment, before the third switching. Note that, at the fourth switching (4th SW), the correct matrix S_0 (for $\psi = \pi/2$) is selected again ($-K_p S_0$ is Hurwitz) and thereafter $|e| \rightarrow 0$.

Figure 4 describes the time history of the image error e and the control signal u , respectively. Note that the asymptotic convergence of the error to a small residual set is evident. The target trajectory is illustrated in Figure 5.

IX. CONCLUSIONS

An output-feedback controller was developed for uncertain nonlinear multivariable systems with relative degree one and unknown high frequency gain matrix. Based on sliding mode and monitoring function, the controller leads to global asymptotic stability with respect to some compact set and ultimate exponential convergence of the tracking error state to zero. The proposed strategy was successfully tested with a robotics visual servoing experimental setup. The method was shown to be robust to large calibration uncertainties, thus removing the usual restriction on the camera misalignment to be less than $\pi/2$ [rad].

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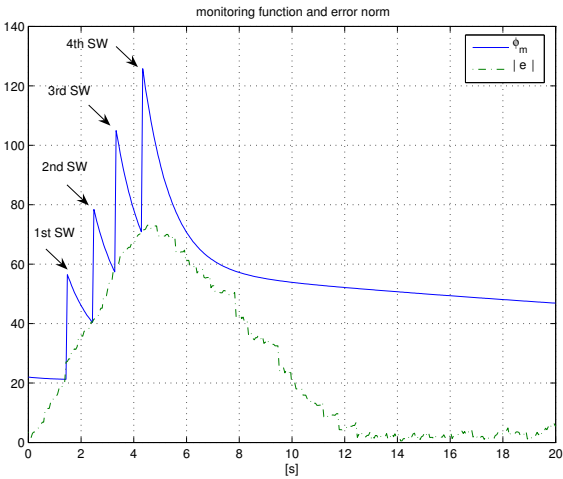


Fig. 3. Monitoring function (φ_m) and error norm ($|e|$).

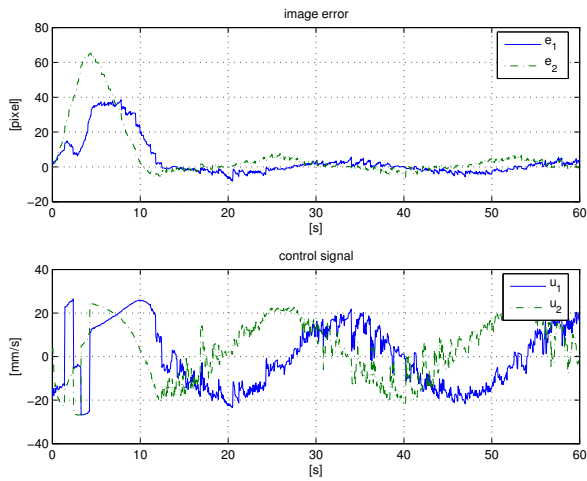


Fig. 4. Image error (e) and control signal (u).

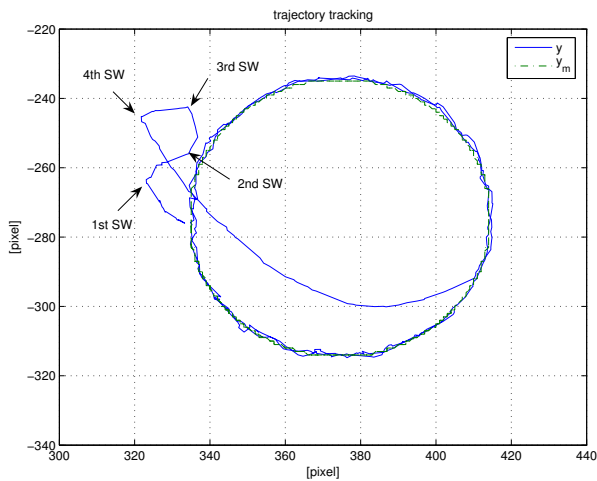


Fig. 5. Trajectory tracking in the image frame.

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