# Motion coordination through cooperative payload transport

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Abstract—In this paper we consider a group of agents collaboratively transporting a flexible payload. The reaction forces between the agents and the payload are modeled as the gradients of the nonlinear potentials that describe the deformation of the payload. We develop decentralized control laws without explicit communication such that the agents and the payload will eventually move with the same velocity meanwhile the contact forces are regulated.

### I. INTRODUCTION

Motion coordination and cooperative control have received a lot of attention during the past few years. The main challenge in cooperative control is to design a decentralized control law that depends on local information to guarantee a global behavior. A number of studies, to name a few, [1], [2], [3], [4], [5], [6], [7], have successfully proposed distributed control laws to achieve emergent group behaviors, such as, consensus, flocking and schooling. One of the main design methodologies is to employ potential function method to propose local feedback rules in the form of artificial attraction and repulsion forces between neighboring agents, for example, [1], [6].

The local information between agents, such as relative distances, is usually obtained through "explicit" information flows, including sensor measurements and direct communication. While this "explicit" information flow exists in many cooperative control applications, there are situations where no "explicit" communication is required to achieve cooperative tasks. For example, suppose that several people move a table and only one person knows where to go. Then, even without explicitly talking to or seeing each other, those people are able to adjust their velocities and forces, and finally succeed in moving the table towards the target without breaking the table. In this case, the communication is *implicit*, and people receive the information (e.g. where to go, how fast to go) by feeling the contact forces and the trend where the table is going. Through everybody adjusting their forces and velocities, the whole group eventually moves towards the goal.

Inspired by this example, we consider a group of agents handling a flexible payload. As the agents move, the payload may be squeezed or stretched, which generate contact forces to the agents. The contact forces between the agents and the payload are then modeled as the gradients of the nonlinear

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potentials that are induced by the deformation of the payload. As a starting case to investigate, we assume that the deformation of the payload is so small that the payload can be modeled as a rigid body. This assumption is reasonable when the payload is made by surrounding a large rigid load with bumpers or elastic materials. Another illustration of this setup is multiple grippers grasping a rigid load, where the grippers possess compliance from installed flexible mechanisms. Reference [8] studied a similar problem, where one of the grippers is rigid and the others are flexible with built-in linear springs, and developed stabilization control laws that achieve both position and force control. In this paper, our objective is to design decentralized control laws such that the contact forces are regulated at some setpoint and that the agents and the payload move with the same velocity in the limit. Because all the agents are acting to the payload, the reaction forces to the agents can be considered as *implicit* communication while the payload acts as the "medium". Indeed, in the graph representation, this implicit communication topology is a bidirectional star graph with the payload at the center (see Fig. 2).

When the desired velocity is available to all the agents, we propose a decentralized control law that consists of an internal velocity feedback and an external force feedback from the payload. Instead of designing *artificial* attractive/repulsive force feedback as in formation control, the passive contact forces serve as the *actual* potential force feedback. We also study the situation where no desired velocity is pre-designed. In this case, we augment the decentralized control with an integral control term so that through the interactions between the agents and the payload, they still achieve the same velocity in the limit with contact forces regulated.

The subsequent sections are organized as follows: we formulate our problem in Section II. In Section III, we study the situation where the desired velocity  $v^d$  information is available to each agent and propose decentralized control laws that guarantee velocity convergence to  $v^d$  and force regulation. A modified integral control law is developed in Section IV to ensure that the agents and payload move with a common velocity when no pre-designed desired velocity is available. Simulation results are presented in Section V. Conclusions and future work are discussed in Section VI.

### II. PROBLEM FORMULATION

Consider that N planar agents hold a common flexible load as shown in Fig. 1. Each agent is modeled as a point robot. Suppose that the load is initially not deformed and



Fig. 1. Multiple agents are attached to a common flexible load. Initially the payload is not deformed.

that each agent *i* is attached to the load at the point  $a_i$ , i.e.  $x_i(0) = a_i(0) = x_c(0) + r_i$ , where  $x_c \in \mathbb{R}^2$  and  $x_i \in \mathbb{R}^2$  are the inertial positions for the CM (Center of Mass) of the load and the *i*th agent, and  $r_i$  is a fixed vector in the inertial frame. Assuming that the initial orientation of the load  $\theta$  is zero, we define

$$a_i(t) := x_c(t) + R(\theta)r_i, \qquad (1)$$

where

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{pmatrix}.$$
 (2)

Note that  $a_i(t)$  represents the position where the *i*th agent is attached if the payload is not deformed at time *t*. As the agents move, however, the flexible payload may be deformed (squeezed or stretched) and therefore  $x_i(t) \neq a_i(t)$ . The deformation of the payload, described by

$$z_i = x_i - a_i, \quad i = 1, \cdots, N, \tag{3}$$

generates a reaction force  $f_i$  to agent *i*. We assume that the reaction force  $f_i$  is the gradient of a positive definite potential function  $P_i(z_i)$ , that is,

$$f_i = \nabla P_i(z_i). \tag{4}$$

Note that when  $z_i = 0$ , the payload is not squeezed or stretched by agent *i*. Therefore, no force would be generated to agent *i*. In other words,  $P_i(z_i)$  satisfies the following:

$$P_i(z_i) = 0 \quad \Longleftrightarrow \quad z_i = 0 \tag{5}$$

$$\nabla P_i(z_i) = 0 \quad \Longleftrightarrow \quad z_i = 0. \tag{6}$$

We further assume that the deformation is small enough so that the load can be approximated as a rigid payload. This can be considered as the agents attached to a rigid payload by nonlinear passive springs that represent  $f_i$ 's in (4). Therefore, the dynamics of the agents and the load, restricted to purely translational motion, are,

$$m_i \ddot{x}_i = F_i - f_i, \quad i = 1, \cdots, N \tag{7}$$

$$M_c \ddot{x}_c = \sum_{i=1}^N f_i \tag{8}$$

where  $m_i$  and  $M_c$  are the mass-inertias of the *i*th agent and the load,  $F_i$  is the applied force to the *i*th agent, and  $f_i$  is defined in (4).

Our control objective is to design  $F_i$  in a decentralized way such that all the agents and the payload converge to the same constant velocity, while regulating the spring forces on the load, i.e.  $f_i$  maintained at a setpoint  $f_i^d$ . Because the load is moving with a constant speed eventually,  $f_i^d$ 's are subject to the following constraint:

$$\sum_{i=1}^{N} f_i^d = 0.$$
 (9)

In the following sections, we analyze two situations: first, the desired velocity  $v^d$  is pre-designed and available to each agent; second,  $v^d$  is not predesigned.

## III. DECENTRALIZED CONTROL WITH $v^d$ PRE-DESIGNED

We note from (4) that the reaction force  $f_i$  depends on the deformation  $z_i$  and if  $z_i$  can be regulated to some desired state,  $f_i$  would also be maintained accordingly. To this end, we assume that for a given  $f_i^d$ , there exists a locally unique solution  $z_i^d$ , such that

$$f_i^d = \nabla P_i(z_i^d) \tag{10}$$

and

$$\nabla^2 P_i(z_i^d) > 0. \tag{11}$$

Therefore, achieving a desired constact force  $f_i^d$  is now equivalent to driving the deformation  $z_i$  in (3) to the desired one  $z_i^d$ .

**Proposition** 1: Consider the decentralized control law

$$F_i = -\Gamma_i(\dot{x}_i - v^d) + f_i^d \tag{12}$$

where  $\Gamma_i = \Gamma_i^T > 0$ . The equilibrium  $\mathscr{E}$ 

$$\mathscr{E} = \{ (\dot{x}_i, \dot{x}_c, f_i) \mid \dot{x}_i = v^d, \quad \dot{x}_c = v^d \quad and \quad f_i = f_i^d \}$$
(13)

is asymptotically stable.

The dynamics (7) with the proposed control (12) now take the form:

$$m_i \ddot{x}_i = -\Gamma_i (\dot{x}_i - v^d) + f_i^d - f_i,$$
(14)

which consists of an internal feedback that drives the agent's velocity to  $v^d$ , and an external feedback that regulates the contact force. In the formation control literature, the external feedback is usually derived by designing *artificial* potential reaction forces [1], [6] while in our problem, the reaction forces between the agents and the payload follow directly from the *actual* potentials.

The closed-loop system (7), (8) and (12) can be considered as a cooperative system of N + 1 agents, if the payload is treated as an additional agent. The interactions between the N + 1 agents then display a star topology with the payload at the center as seen in Fig. 2. Therefore, controlling the forces (and thus the deformations) between the agents and the payload simultaneously guarantees that the relative positions between agents are maintained tightly.



Fig. 2. The implicit communication between agents and payload exhibits a bidirectional star topology.

We also note that the payload has no direct control input and therefore is a *passive* node. Without the  $v^d$  information, this passive node still achieves  $v^d$  in the limit. This phenomenon has been studied in [9], where an estimation scheme was proposed for those agents without the  $v^d$  information to reconstruct  $v^d$  from the local interactions. In our case, the payload recovers  $v^d$  information from the contact forces  $f_i$ 's, which are indeed the local interactions with the agents.

*Proof of Proposition 1*: We take the following energymotivated Lyapunov function

$$V = \sum_{i} [P_{i}(z_{i}) - P_{i}(z_{i}^{d}) - f_{i}^{d}(z_{i} - z_{i}^{d})] + \frac{1}{2} \sum_{i} \xi_{i}^{T} m_{i} \xi_{i} + \frac{1}{2} \xi_{c}^{T} M_{c} \xi_{c},$$
(15)

where

$$\xi_i = \dot{x}_i - v^d \quad \text{and} \quad \xi_c = \dot{x}_c - v^d. \tag{16}$$

Under the assumption that (11) is satisfied, the first term in (15) is positive definite. Then, the time derivative of *V* yields

$$\dot{V} = \sum_{i} (f_{i} - f_{i}^{d})^{T} \dot{z}_{i} + \sum_{i} \xi_{i}^{T} m_{i} \ddot{x}_{i} + \xi_{c}^{T} M_{c} \ddot{x}_{c}.$$
 (17)

From (3), the kinematics of  $z_i$  is given by

$$\dot{z}_i = \dot{x}_i - \dot{a}_i = \dot{x}_i - \dot{x}_c.$$
 (18)

We next rewrite (17) from (7), (8), (18), (9) and (12) as

$$\dot{V} = \sum_{i} (f_{i} - f_{i}^{d})^{T} (\dot{x}_{i} - \dot{x}_{c}) + \sum_{i} \xi_{i}^{T} (F_{i} - f_{i}) + \xi_{c}^{T} \sum_{i} f_{i} \\
= \sum_{i} (f_{i} - f_{i}^{d})^{T} (\xi_{i} - \xi_{c}) + \sum_{i} \xi_{i}^{T} (-\Gamma_{i} \xi_{i} + f_{i}^{d} - f_{i}) \\
+ \xi_{c}^{T} \sum_{i} f_{i} \\
= -\sum_{i} \xi_{i}^{T} \Gamma_{i} \xi_{i} + \xi_{c} \sum_{i} f_{i}^{d} \\
= -\sum_{i} \xi_{i}^{T} \Gamma_{i} \xi_{i} \leq 0,$$
(19)

which implies the stability of the equilibrium  $\mathscr{E}$ .

To conclude asymptotic stability, we apply LaSalle Invariance Principle by investigating the largest invariant set  $\mathcal{M}$ where  $\dot{V} = 0$ , i.e.  $\xi_i = 0$ . From (16), we note that  $\xi_i = 0$ implies that  $\dot{x}_i = v^d$ . We further obtain from  $\dot{\xi}_i = 0$  that  $\ddot{x}_i = 0$ , which leads to  $F_i = f_i$  from (7). Thus, it is clear from (12) that  $f_i^d = f_i$ . We now show that in  $\mathcal{M}$ ,  $\dot{x}_c = v^d$ . To see this, we note that  $f_i^d = f_i$  implies that  $z_i = z_i^d$ . Since  $z_i^d$  is constant,  $\dot{z}_i = 0$  in  $\mathcal{M}$ , that is, from (3),  $\dot{x}_i = \dot{a}_i$ . Because we only consider the translational motion and because  $\dot{x}_i = v^d$  in  $\mathcal{M}$ , we conclude that  $\dot{a}_i = \dot{x}_c = v^d$ .  $\Box$ 

### IV. DECENTRALIZED CONTROL WITHOUT $v^d$ PREDESIGNED

When  $v^d$  is not preassigned to each agent, we follow the adaptive design in [9] and develop an integral control with which each agent adaptively estimates the group velocity and all the agents move with the same velocity in the limit. We now define  $\hat{v}_i^d$  as the velocity estimate for *i*th agent and propose the following update law for  $\hat{v}_i^d$ :

$$\dot{v}_i^d = \Lambda_i (f_i^d - f_i) \tag{20}$$

in which  $\Lambda_i = \Lambda_i^T > 0$ . Note that  $\hat{v}_i^d$  stops updating when  $f_i^d \equiv f_i$ , that is, the contact force is regulated at the desired setpoint and  $z_i$  remains constant, which further implies that agent *i* and the payload have the same velocity. If all the agents have the same velocity as the payload, then they move at the same velocity. Moreover, the relative distances between the agents are also maintained.

We next modify the design in (12) as

$$F_i = -\Gamma_i(\dot{x}_i - \hat{v}_i^d) + m_i \dot{v}_i^d + f_i^d, \qquad (21)$$

and present the following proposition:

**Proposition** 2: Consider the decentralized control laws in (20) and (21). The equilibrium  $\mathscr{E}^*$ 

$$\mathscr{E}^* = \{ (\dot{x}_i, \dot{x}_c, f_i) \mid \dot{x}_i = \bar{v}, \ \dot{x}_c = \bar{v} \ and \ f_i = f_i^d \}$$
(22)

is asymptotically stable, where  $\bar{v} \in \mathbb{R}^2$  is a constant. Furthermore,  $\bar{v}$  can be characterized as the weighted average of the initial payload velocity  $\dot{x}_c(0)$  and the initial velocity estimates  $\hat{v}_i^d(0)$ ,  $i = 1, \dots, N$ :

$$\bar{v} = (\sum_{i=1}^{N} \Lambda_i^{-1} + M_c)^{-1} (M_c \dot{x}_c(0) + \sum_{i=1}^{N} \Lambda_i^{-1} \hat{v}_i^d(0)).$$
(23)

Expanding the dynamics (7) with the control (20) and (21), we obtain

$$m_{i}\ddot{x}_{i} = -\Gamma_{i}(\dot{x}_{i} - \hat{v}_{d}^{i}(0)) + m_{i}\dot{v}_{i}^{d} + f_{i}^{d} - f_{i} + \underbrace{\Lambda_{i}\Gamma_{i}\int(f_{i}^{d} - f_{i})}_{\text{intergral force control}}$$

$$(24)$$

which is of the integral force feedback form [10], [11]. Such an integral force control has been shown in [10] to be robust with respect to small time delay in force measurements.

*Proof of Proposition 2*: We first rewrite (21) as

$$F_i = -\Gamma_i \xi_i + m_i \dot{v}_i^d + f_i^d \tag{25}$$

where

$$\xi_i = \dot{x}_i - \hat{v}_i^d \tag{26}$$

and  $\hat{v}_i^d$  is updated as in (20). It follows from (24) that

$$m_i \dot{\xi}_i = -\Gamma_i \xi_i + f_i^d - f_i. \tag{27}$$

We then choose the following Lyapunov function

$$V_{1} = \sum_{i} [P_{i}(z_{i}) - P_{i}(z_{i}^{d}) - f_{i}^{d}(z_{i} - z_{i}^{d})] + \frac{1}{2} \sum_{i} \xi_{i}^{T} m_{i} \xi_{i} + \frac{1}{2} \dot{x}_{c}^{T} M_{c} \dot{x}_{c} + \frac{1}{2} \sum_{i} (\hat{v}_{i}^{d})^{T} \Lambda_{i}^{-1} \hat{v}_{i}^{d}, \qquad (28)$$

whose time derivative is given by

$$\dot{V}_{1} = -\sum_{i} (f_{i}^{d} - f_{i})\dot{z}_{i} + \sum_{i} \xi_{i}^{T} m_{i} \dot{\xi}_{i} + \dot{x}_{c}^{T} M_{c} \ddot{x}_{c} + \sum_{i} (f_{i}^{d} - f_{i})^{T} \hat{v}_{i}^{d}.$$
(29)

Noting that

$$\dot{z}_i = \xi_i + \hat{v}_i^d - \dot{x}_c, \qquad (30)$$

we rewrite (29) from (9), (27) and (25) as

$$\dot{V}_{1} = -\sum_{i} (f_{i}^{d} - f_{i})^{T} (\xi_{i} + \hat{v}_{i}^{d} - \dot{x}_{c}) + \sum_{i} (f_{i}^{d} - f_{i})^{T} \hat{v}_{i}^{d} + \dot{x}_{c} \sum_{i} f_{i} + \sum_{i} \xi_{i}^{T} (-\Gamma_{i} \xi_{i} + f_{i}^{d} - f_{i}) = -\sum_{i} \xi_{i}^{T} \Gamma_{i} \xi_{i} \leq 0$$
(31)

which implies the stability of the equilibrium  $\mathscr{E}^*$ . We again apply LaSalle Invariance Principle to investigate the largest invariant set  $\mathscr{M}^*$ . On  $\mathscr{M}^*$ ,  $\dot{V}_1 = 0$  means  $\xi_i = 0$  and thus  $\dot{\xi}_i = 0$ , which further implies from (27) that  $f_i = f_i^d$ . We obtain from (20) that  $\dot{v}_i^d = 0$ . It then follows from  $\ddot{x}_i = \dot{v}_i^d = 0$ that  $\dot{x}_i$  is constant on  $\mathscr{M}^*$ . Since  $\dot{f}_i = \dot{f}_i^d = 0$  and  $\dot{z}_i = \dot{x}_i - \dot{x}_c$ , we conclude  $\dot{x}_i = \dot{x}_c$ ,  $\forall i$ , which means that all the agents and the payload share the same constant velocity  $\bar{v}$ . Noting from (26) and  $\xi_i = 0$ , we further obtain  $\dot{x}_c = \hat{v}_i^d$ ,  $i = 1, \dots, N$ . On the other hand, from (8) and (20), we compute

$$M_c \dot{x}_c(t) = \int_0^t \sum_{i=1}^N f_i(s) ds + M_c \dot{x}_c(0)$$
(32)

and

$$\hat{v}_{i}^{d}(t) = \int_{0}^{t} \Lambda_{i}(f_{i}^{d} - f_{i}(s)) ds + \hat{v}_{i}^{d}(0).$$
(33)

We rewrite (33) as

$$\Lambda_i^{-1}(\hat{v}_i^d(t) - \hat{v}_i^d(0)) = \int_0^t f_i^d - f_i(s)ds$$
(34)

and note from (9), (32) and (34) that

$$\sum_{i} \Lambda_{i}^{-1}(\hat{v}_{i}^{d}(t) - \hat{v}_{i}^{d}(0)) = \int_{0}^{t} \sum_{i=1}^{N} (f_{i}^{d} - f_{i}(s)) ds \quad (35)$$

$$= -M_c(\dot{x}_c(t) - \dot{x}_c(0)). \quad (36)$$

Because on  $\mathcal{M}^*$ ,  $\dot{x}_c = \dot{x}_i = \hat{v}_i^d := \bar{v}$  and  $\bar{v}$  is constant, we conclude (23).  $\Box$ 

The situation where only one agent, say agent 1, has the  $v^d$  information becomes a special example of the design (20)-(21). In fact, agent 1 can choose to shut off the estimation (20) by selecting  $\Lambda_1 = 0$  and letting  $\hat{v}_1^d(0) = v^d$ . A simple computation from (23) shows that  $\lim_{\Lambda_1 \to 0} \bar{v} = v^d$ , which means that the group will eventually move with the velocity  $v^d$ . We then present the following corollary without proof:

**Corollary** 1: Suppose that agent 1 has the  $v^d$  information and implements (20) and (21) with  $\hat{v}_1^d(0) = v^d$  and  $\Lambda_1 = 0$  while the other agents apply the control (20) and (21),  $i = 2, \dots, N$ . Then the equilibrium  $\mathcal{E}$  in (13) is asymptotically stable.

#### V. SIMULATION

In this simulation, we consider three agents moving a spherical payload, which is initially centered at the origin with radius 1 in the inertial frame. The agents are located at  $[1 \ 0]^T$ ,  $[\cos(2\pi/3) \sin(2\pi/3)]^T$  and  $[\cos(-2\pi/3) \sin(-2\pi/3)]^T$  in the inertial frame, which means that initially the payload is not deformed. We then select  $f_1^d = [-0.05 \ 0]^T$ ,  $f_2^d = -0.05[\cos(2\pi/3) \sin(2\pi/3)]^T$ and  $f_3^d = -0.05[\cos(-\pi/3) \sin(-\pi/3)]^T$  so that (9) is satisfied. The masses of the payload  $M_c$  and the agents  $m_i$ are 5Kg and 1Kg, and the desired velocity  $v^d$  is chosen as  $[0.1 \ 0.1]^T$ .

We assume that the nonlinear reaction force is of the following form:

$$f_i = kz_i + k_1 ||z_i||^2 z_i \tag{37}$$

where k and  $k_1$  are positive constants and  $z_i$  is defined in (3). Note that when the agent barely touches the payload, i.e.  $z_i = 0$ ,  $f_i$  is zero. We then select k = 1,  $k_1 = 0.1$  and the feedback gain  $\Gamma_i = I$ . Fig. 3 illustrates the snapshots of the formation of the agents and the payload moving in the direction of  $[0.1 \ 0.1]^T$ . At  $t_0$ , since the payload is not deformed, the three circles in Fig. 3, which denote the three agents, are centered on the edge of the payload, shown by the big circle. At  $t_2$ , those three circles are pushed into the big circle, which means that the payload is squeezed by the three agents and will be moving with those deformations as seen at  $t_3$ . The simulation results in Fig. 4 show that the contact forces are regulated at the desired values.



Fig. 3. Snapshots of the agents and the payload: at  $t_0$ , the three circles, denoting the three agents, are centered at the edge of the big circle, denoting the payload. This means that the agents barely touch the payload and there is no deformation. At  $t_2$  and  $t_3$ , the three circles are pushed into the big circle, which illustrates that the payload is now squeezed by the three agents.



Fig. 4. The norms of  $||f_i - f_i^d||$ , i = 1, 2, 3, converge to zero.

We also simulate the situation where no agents has the desired velocity information. In this case, we employ the adaptive design (20)-(21) in Section IV to achieve the desired contact forces and the convergence to a common velocity. We select the initial estimates  $\hat{v}_i^d(0)$ , i = 1, 2, 3, as  $[0.02 \ 0.02]^T$ ,  $[-0.04 - 0.04]^T$ , and  $[0.1 \ 0.1]^T$ , respectively. The adaptation gain  $\Lambda_i$  is chosen to be  $0.2I_2$ . Simulation in Fig. 5 shows that the contact forces are well regulated at the desired setpoint  $f_i^d$ . Furthermore, as computed from (23), the velocities of the three agents should converge to  $[0.02 \ 0.02]^T$ , which matches the results in Fig. 6.



Fig. 5. With the adaptive design (20)-(21), the contact forces are regulated at their desired values.

#### VI. CONCLUSIONS AND FUTURE WORK

We study a motion coordination problem where a group of agents move a flexible payload. The contact forces, which describe the relative information between the agents and the payload, build up an *implicit* information topology flow. When the desired constant velocity is available to each



Fig. 6. With the adaptive design (20)-(21), the velocities of the agents converge to  $[0.02 \ 0.02]^T$ , as computed from (23).

agent, we develop a decentralized controller that achieves the desired velocity convergence and the force regulation. We also consider the situation where the desired velocity is not available. In this case, we propose an integral force control that guarantees that the agents and the payload move with the same velocity while the contact forces are regulated. Future work will pursue time-varying desired velocity tracking and experimental results of the proposed controllers.

### ACKNOWLEDGEMENT

This work is supported in part by the Center for Automation Technologies and Systems (CATS) at Rensselaer Polytechnic Institute under a block grant from the New York State Office of Science, Technology, and Academic Research (NYSTAR). The work of the second author is also supported in part by the Outstanding Overseas Chinese Scholars Fund of Chinese Academy of Sciences (No. 2005-1-11).

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