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Analysis and Synthesis of Self-Powered Linear Structural Control with Imperfect Energy Storage

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***Abstract*— Self-powered vibration control systems are characterized by a distributed network of regenerative force actuators, which are interfaced with a common power bus. Also connected to the power bus is an energy-storing subsystem, such as a supercapacitor, flywheel, or battery. The entire system is controlled using switch-mode power electronics, and the only power required for system operation is that necessary to perform these switching operations. The resultant energy conservation constraint restricts the set of feedback laws that are feasible. This paper reports on an LMI feasibility constraint for linear self-powered feedback laws, in terms of actuator and storage hardware parameters. Two design applications of this constraint are illustrated. The first is the determination of the least-efficient energy storage parameters necessary to realize a given passive control law. It is shown that this problem is quasiconvex, and may be posed as a generalized eigenvalue problem. The second example uses an extension of positive-real-constrained $\mathcal{H}_2$ optimal control, to optimize a control law subject to the feasibility constraint. Both examples are illustrated in the context of base-excited vibrating structures, subjected to stationary stochastic excitation.**

***Index Terms*— Vibration, Regeneration, Mechatronics**

I. Introduction

In many structural control applications, restrictions on power availability have generated considerable interest in self-powered vibration suppression systems, capable of operating entirely on the energy they absorb. For example, supplemental passive electrical and mechanical impedances accomplish this task. Over the last decade a new class of actuation system, first formally defined in [1], has been proposed that actively controls the storage, transmission, and reuse of absorbed energy. Called *Regenerative Force Actuation (RFA)*, such systems have similarities with semiactive (i.e., adaptive viscous damping) systems in that they have external power supply demands that are orders of magnitude below their power flow capabilities. Unlike semiactive systems, they are capable of electrically storing and reusing the energy they remove from a mechanical system. Additionally, when multiple actuators are used to control a structure, their electronics are connected together, enabling them to "share" power. Such a system of actuators is called an *RFA Network* [2].

To illustrate an example of an RFA network, consider first the electromechanical actuator depicted in Fig. 1a. This diagram shows a linear brushless motor being used as a force actuator. The drive circuitry, shown here simply as a box with a transistor symbol, consists of passive components (i.e. inductors, capacitors, and diodes) and transistors. By controlling the transistors in this circuit, the system converts mechanical power fv to electrical power $V_B i_B$. The transistors are operated as power-electronic switches, which open and close different circuit paths. This mode of operation requires very little external power, even if the electric power flowing through the network is quite high. As these transistor switches (together with the sensors and control intelligence) constitute the only power requirements for operation, the system is highly efficient.

Using a standard "H-bridge" drive topology, the actuator in Fig. 1a is approximately equivalent to Fig. 1b. Thus, velocity v produces a back-EMF $V = K_f v$, which induces stator current i. The resultant electromechanical force is $f_e = K_f i$.

An RFA network then consists of m forcing devices situated throughout a mechanical system, and interfaced through a DC-link power bus $B - B'$, as shown in Fig. 2. Also interfaced to $B - B'$ is an energy-storage subsystem, depicted as capacitor C_S. (Other devices, such as flywheels or batteries, could also be used.) This energy-storage subsystem is non-ideal, in the sense that it dissipates energy whenever it delivers or accepts power from bus $B - B'$, and because it exhibits leakage. These effects are represented by resistances R_L and R_S, respectively.

Decay of stored energy is characteristic of all storage devices. However, the inclusion of R_S in the model also serves another purpose. Suppose one of the purposes of the RFA network is to harvest energy for other applications, or for its own control intelligence, sensor, and switching systems. Then R_S can be used to (crudely) model the power demands of other electrical subsystems that are driven using this harvested energy.

Regenerative actuation has been examined in the context of automotive suspension systems using hydraulic as well as electromechanical devices [1]. It has also received attention in flexible aerospace structures with the use of a piezoelectric actuator with an inductor for energy storage [3], [4]. In civil engineering applications, regenerative actuation was first proposed in [5]. This work, along with similar approaches proposed in [6], focused on single-device implementations with energy storage. The concept of power-sharing between actuators was examined in [2], which presented an approach to the realization and electronic control of an arbitrary m-device electromechanical RFA network.

Fig. 3 shows three examples of the many potential uses of RFA networks in structural vibration control systems. Fig. 3a shows a civil structure application, for use in the reduction

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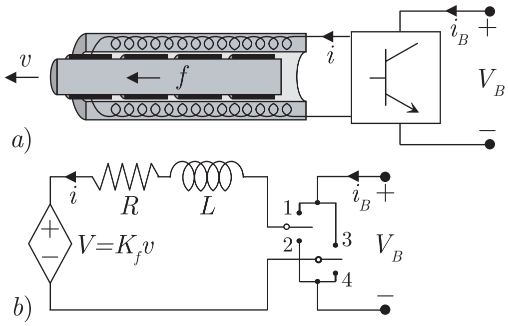

Fig. 1. Mechanical (a) and electrical (b) schematic of an electromechanical actuator

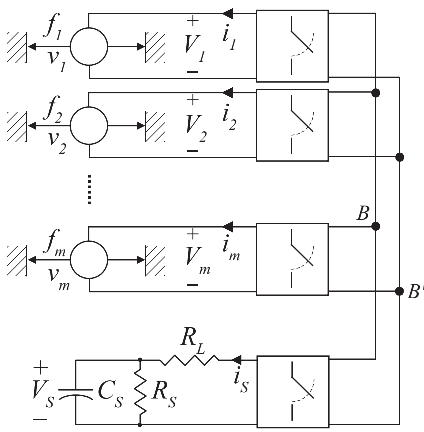

Fig. 2. Schematic for an m-device RFA network

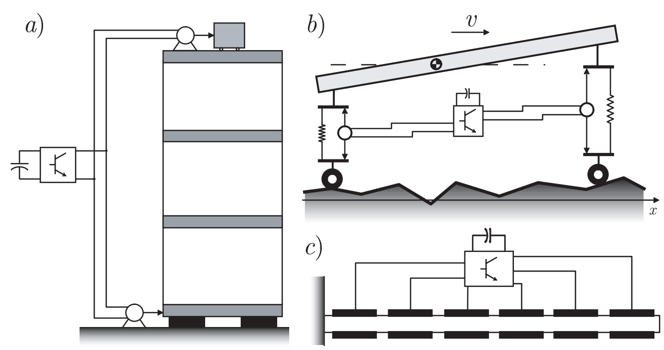

Fig. 3. Civil (a), automotive (b), and aerospace (c) applications of RFA networks

of earthquake response. Fig. 3b shows an RFA network, for use in automotive suspensions. Fig. 3c shows a potential aerospace application in which many piezoelectric actuators, located along the length of a flexible cantilever beam, are used to control vibrations and/or harvest energy.

Although a number of device realizations have been proposed for RFA networks, the area of control synthesis for these systems has been slower to develop. A development in [1] establishes a criterion for linear control laws for which in steady-state excitation, the average energy generated is positive. However this approach does not account for dissipation in the network, or losses in the energy storage system, and does not guard against the circumstance where the controller fully drains the energy supply in the course of the transient dynamic response. In [7], it was shown that RFA networks with no energy storage (i.e., with energy transmission capability only) may be viewed as imposing non-local and asymmetric supplemental damping matrices on structures. It was further illustrated that such damping concepts could be extended to RFA networks with energy-storage subsystems. However, such approaches fall short of exploiting the full capability of the energy storage subsystem.

This paper presents some new results, toward a generalized approach to the design of linear feedback controllers for energy-storing RFA networks such as the one in Fig. 2.

II. Constraints on Power Flow

Let $\mathbf{f}_e$ be the vector of electromechanical forces for each actuator in an RFA network, and let $\mathbf{v}$ be the corresponding vector of actuator velocities. The fact that bus $B - B'$ in Fig. 2 does not interface with an external power source imposes an "energy conservation" constraint on the system. This manifests itself as a dynamic constraint on $\mathbf{f}_e$, which depends on $\mathbf{v}$. It is reminiscent of the familiar passivity constraint from robust control theory [8]; i.e.,

$$\int_0^t \mathbf{f}_e^T(\tau)\mathbf{v}(\tau)d\tau \leqslant 0 \quad , \quad \forall t > 0 \tag{1}$$

Such a constraint would dictate that the cumulative energy injected into the structure, by the actuation system, must always be negative. In fact, this would exactly characterize the system forcing constraint if the electrical network (i.e., the actuators, energy storage subsystem, and switching network) were lossless. However, this circumstance is unrealistic. In reality, the RFA network is not capable of reusing all the electrical energy it generates.

The total resistive power dissipation for the network is

$$P_D(t) = \mathbf{f}_e^T(t)\mathbf{Z}_e^{-1}\mathbf{f}_e(t) + R_L i_S^2(t) + \frac{1}{R_S}V_S^2(t) \tag{2}$$

where

$$\mathbf{Z}_e = \text{diag}\left\{...K_{fk}^2/R_k...\right\} \tag{3}$$

Meanwhile, the total power delivered to storage is

$$P_A(t) = \mathbf{f}_e^T(t)\mathbf{v}(t) + V_S(t)\left(i_S(t) - V_S(t)/R_S\right) \tag{4}$$

Then the total power flowing out of bus $B - B'$ at time t is $P_A(t) + P_D(t)$. Assuming this bus has minimal energy storage capability, $P_A + P_D = 0$; i.e.,

$$\mathbf{f}_e^T(t)\mathbf{v}(t) + \mathbf{f}_e^T(t)\mathbf{Z}_e^{-1}\mathbf{f}_e(t) + V_S(t)i_S(t) + R_L i_S^2(t) = 0 \tag{5}$$

Technically, this assumption is untrue because bus $B - B'$ has some capacitance, which is necessary to smooth out the high-frequency oscillations in the power bus voltage resulting from the switching operations. However, the energy stored in this capacitance is assumed to be small.

From a basic power flow analysis, it is straight-forward to show that the differential equation for the stored energy $E_S(t)$ in capacitor C_S is

$$\dot{E}_S(t) = \left(-\frac{2}{\tau_S} - \frac{1}{\tau_L}\right)E_S(t) + \sqrt{\frac{E_S^2(t)}{\tau_L^2} - \frac{2E_S(t)}{\tau_L}\left(\mathbf{f}_e^T(t)\mathbf{v}(t) + \mathbf{f}_e^T(t)\mathbf{Z}_e^{-1}\mathbf{f}_e(t)\right)} \tag{6}$$

where

$$\tau_S = R_S C_S \qquad\qquad \tau_L = R_L C_S \tag{7}$$

This total stored energy must always be real and positive; i.e.,

$$E_S(t) \in \mathbb{R}^+ \quad \forall t > 0 \tag{8}$$

This expression is the most general form of the dynamic constraint imposed on an RFA network due to energy conservation. As one would expect, it is a more conservative constraint than the pure passivity constraint (1). Furthermore, one arrives at the passivity constraint from (6) and (8) by letting $\tau_S \to \infty$, $\tau_L \to 0$, and $\mathbf{Z}_e \to \infty$.

III. Feasible Linear Control Laws

We now ask the interesting question of what linear, time-invariant feedback laws adhere to (8) for an arbitrary external excitation. We restrict our attention to collocated (i.e., "velocity feedback") controllers $\mathbf{Z} : \mathbf{v} \to \mathbf{f}_e$ where $\mathbf{Z}$ is a LTI transfer function with a finite-dimensional state-space realization

$$\mathbf{Z} = \left[\begin{array}{c|c} \mathbf{A}_Z & \mathbf{B}_Z \\ \hline \mathbf{C}_Z & \mathbf{D}_Z \end{array}\right] \tag{9}$$

It will be convenient to refer to a particular state space realization for $\mathbf{Z}$ by the set $\Delta_Z = \{\mathbf{A}_Z, \mathbf{B}_Z, \mathbf{C}_Z, \mathbf{D}_Z\}$. The set of all $\mathbf{Z}$ of order n_c that satisfy (8) for all $\mathbf{v} \in \Re^m \times \mathcal{L}_2^+$ shall be denoted $\mathcal{Z}^{n_c}$. One may ask why $\mathcal{Z}^{n_c}$ has been restricted to velocity feedback rather than, say, the more general domain of n_c-order full-state feedback controllers. However, it is a straight-forward proof to show that for any linear feedback controller that does not belong to $\mathcal{Z}^{n_c}$, there exists a $\mathbf{v} \in \Re^m \times \mathcal{L}_2^+$ that violates (8).

If there were no dissipation in the electronic system, all controllers in $\mathcal{Z}^{n_c}$ would be required to satisfy (1); the pure passivity constraint. In this case, the Positive Real Lemma [9] provides the necessary and sufficient condition that there exist state space realization Δ_Z for $\mathbf{Z}$, and $\mathbf{P}_Z = \mathbf{P}_Z^T > 0$, such that

$$\begin{bmatrix} \mathbf{A}_Z^T\mathbf{P}_Z + \mathbf{P}_Z\mathbf{A}_Z & \mathbf{P}_Z\mathbf{B}_Z - \mathbf{C}_Z^T \\ \mathbf{B}_Z^T\mathbf{P}_Z - \mathbf{C}_Z & -\mathbf{D}_Z^T - \mathbf{D}_Z \end{bmatrix} < 0 \tag{10}$$

In the more complicated case where the RFA network contains dissipation, it is more difficult to exactly characterize the complete set of linear control laws satisfying the energy conservation constraint. However, it *is* possible to determine *sufficient* conditions for $\mathbf{Z} \in \mathcal{Z}^{n_c}$. In fact, such a condition can be found, which is reminiscent of the LMI above, albeit somewhat more complicated.

THEOREM 1: For arbitrary $\tau_L > 0$ and $\tau_S > 0$, a controller $\mathbf{Z} \in \mathcal{Z}^{n_c}$ if $\exists$ an n_c-dimensional state-space realization $\Delta_Z \in \mathcal{D}^{n_c}$, where $\mathcal{D}^{n_c}$ connotes all Δ_Z for which $\exists \mathbf{P}_Z = \mathbf{P}_Z^T > 0$ and $\mathbf{X}$ satisfying the following two LMIs:

$$\mathbf{A}_Z^T\mathbf{P}_Z + \mathbf{P}_Z\mathbf{A}_Z + \tfrac{2}{\tau_S}\mathbf{P}_Z + \mathbf{X} + \mathbf{X}^T \leqslant 0 \tag{11a}$$

$$\begin{bmatrix} -\mathbf{X}^T - \mathbf{X} & \mathbf{P}_Z\mathbf{B}_Z & \mathbf{C}_Z^T & -\mathbf{X}^T \\ \mathbf{B}_Z^T\mathbf{P}_Z & -\frac{1}{2}\mathbf{Z}_e & \mathbf{D}_Z^T + \frac{1}{2}\mathbf{Z}_e & \mathbf{B}_Z^T\mathbf{P}_Z \\ \mathbf{C}_Z & \mathbf{D}_Z + \frac{1}{2}\mathbf{Z}_e & -\frac{1}{2}\mathbf{Z}_e & \mathbf{0} \\ -\mathbf{X} & \mathbf{P}_Z\mathbf{B}_Z & \mathbf{0} & -\frac{1}{2\tau_L}\mathbf{P}_Z \end{bmatrix} \leqslant 0 \tag{11b}$$

The proof to this theorem is rather lengthy, and will be published in a forthcoming journal paper. Here, we simply note some of its implications.

First of all, we note that (11) is sufficient, but not necessary to ensure satisfaction of (8). However, it can be seen by inspection that the LMIs above distill to (10), the passivity constraint, in the case where $\tau_S \to \infty$, $\tau_L \to 0$, and $\mathbf{Z}_e \to \infty$. It is also straight-forward to show that, as expected, the constraints in Theorem 1 are always more conservative than those of pure passivity.

Further examination gives some insight into how the different resistances in the RFA network confine the set of feasible control laws. Consider, for example, the case where $\tau_S \to 0$, but where $\mathbf{Z}_e$ and τ_L are finite. This case corresponds to the case where the energy storage system is so lossy that it cannot retain energy for any significant time. In this case, the LMIs in Theorem 1 reduce to requirements $\mathbf{A}_Z = \mathbf{0}$, $\mathbf{B}_Z = \mathbf{0}$, $\mathbf{C}_Z = \mathbf{0}$, and that $\mathbf{D}_Z$ satisfy

$$\begin{bmatrix} -\mathbf{Z}_e & 2\mathbf{D}_Z^T + \mathbf{Z}_e \\ 2\mathbf{D}_Z + \mathbf{Z}_e & -\mathbf{Z}_e \end{bmatrix} \leqslant 0 \tag{12}$$

(In this case, it can be shown that the above constraint is also necessary.) As such, the capability of the RFA network reduces to static velocity feedback.

For any τ_S and τ_L, the LMIs in Theorem 1 require that

$$\mathbf{A}_Z^T\mathbf{P}_Z + \mathbf{P}_Z\mathbf{A}_Z + \tfrac{2}{\tau_S}\mathbf{P}_Z \leqslant 0 \tag{13}$$

which confines the poles of $\mathbf{Z}$ to have real parts less than $-1/\tau_S$. This constraint arises from leakage in the stored energy. Speaking qualitatively, the inequality above requires that the controller dynamics "decay faster than the stored energy does."

In the case where $\tau_L \to 0$, the LMIs in (11) reduce to

$$\begin{bmatrix} \mathbf{A}_Z^T\mathbf{P}_Z + \mathbf{P}_Z\mathbf{A}_Z + \frac{2}{\tau_S}\mathbf{P}_Z & \mathbf{P}_Z\mathbf{B}_Z & \mathbf{C}_Z^T \\ \mathbf{P}_Z\mathbf{B}_Z^T & -\frac{1}{2}\mathbf{Z}_e & \mathbf{D}_Z^T + \frac{1}{2}\mathbf{Z}_e \\ \mathbf{C}_Z & \mathbf{D}_Z + \frac{1}{2}\mathbf{Z}_e & -\frac{1}{2}\mathbf{Z}_e \end{bmatrix} \leqslant 0 \tag{14}$$

In this case, as well, the constraint above can be shown to be necessary as well as sufficient. This case corresponds to a system for which significant dissipation exists in the coils of the actuators, as well as in the leakage of the storage system, but for which energy can be transmitted between storage and bus $B - B'$ at very high efficiency. As this efficiency is made lower (i.e., as τ_L is increased), the domain of feasible controllers shrinks beyond those characterized by the LMI above, to exclude those controllers that require significant oscillatory or pulse power flow to and from storage.

At this point, it is reasonable to ask the following question: If RFA networks have a more restricted linear control domain than pure passivity, why not just implement $\mathbf{Z}$ using passive mechanical or electrical components? In truth, this may in many cases be a preferable option, especially for single-actuator control laws with n_c equal to 1 or 2. However, for single-actuator systems of higher order, such realizations can quickly become very impractical, involving the use of transformers (for electrical realizations [10]) or levers (for mechanical realizations [11]). For the multi-actuator case, they will require the use of gyrators, which although theoretically passive, generally require a power source for operation. As such, the theory here may be viewed as presenting an alternative approach to such passive realizations, in which all these components are replaced by a single storage system and a power electronic network. Also, because the RFA network stores energy rather than controllably dissipating it, this approach may be useful for applications where this harvested energy might be useful to power other systems.

IV. Pareto-Optimal Storage Parameters for Passive Feedback Laws

In this section and the next, we consider some uses of LMI constraints (11) in design of RFA networks. The issue addressed in this section concerns the design of hardware parameters τ_L, τ_S, and $\mathbf{Z}_e$, to realize a given passive control law $\mathbf{Z}$. In other words, we consider the case where $\mathbf{Z}$ is known to be passive, and the task at hand is to find the hardware to realize it. The key is to avoid *overdesign* of the system components. For example, the mass of energy storage is often inversely correlated with τ_L; i.e., storage systems with more efficient pulse power and charge/discharge capabilities often weigh more. Moreover, it is often the case that storage technologies are either very good at pulse power (i.e., low τ_L), very good at long-term storage (i.e., high τ_S), or some compromise of the two. For the actuator hardware, higher values in the diagonal of $\mathbf{Z}_e$ correspond to actuators with higher motor constants, which usually implies higher size and weight. Thus, there is a tangible benefit to the determination of the least efficient parameters (i.e., smallest combinations of τ_L, τ_S^{-1}, and $\mathbf{Z}_e$) that will accomplish a given control objective.

Consider again the feasibility LMIs in (11), and its use in the optimization problem

given:	Δ_Z, τ_S, $\mathbf{Z}_e$
minimize:	$\lambda = 1/\tau_L$
over:	λ, $\mathbf{P}_Z$, $\mathbf{X}$
subject to:	(11)

This is a generalized eigenvalue problem, and is therefore readily solvable for a global minimum through the use of any of several LMI-based optimization algorithms, such as interior-point or primal-dual methods [9].

We demonstrate the optimization through a simple example. Fig. 4a shows a nondimensionalized two-mass, one-actuator system. The controller for this system was designed through root-locus techniques, and is equal to

$$\mathbf{Z}(s) = -\frac{2s^2+4s+2}{s^2+1.2s+1.36} \tag{15}$$

which can readily be verified to be passive. State space parameters Δ_Z, in controllable canonical form, can be found by inspection form (15).

Fig. 4b shows the resultant optimization, obtained using the standard tools in the Matlab Robust Control toolbox. Each pareto front in the figure corresponds to a different value for the nondimensionalized $\mathbf{Z}_e$, which in this single-actuator case is a scalar quantity. The results are intuitive. As $\mathbf{Z}_e$ increases, the feasible domain of $\{\tau_L, \tau_S^{-1}\}$ combinations (indicated by the shaded regions in the plot) increases as well, implying that if an actuator is more efficient, the storage system can afford to be less so, and still produce $\mathbf{Z}$. Furthermore, each pareto front implies that as the dissipation due to transmission of stored energy (related to the size of τ_L) increases, the storage system must be more efficient at retaining energy (i.e., τ_S must be higher) in order to produce $\mathbf{Z}$. The plot also implies that as $\mathbf{Z}_e$ is made to increase, there is a point of diminishing returns, above which the losses in storage become the "weak link" in the power management system. Clearly, an increase in $\mathbf{Z}_e$ from 100 to 1000 is of only marginal benefit for storage redesign.

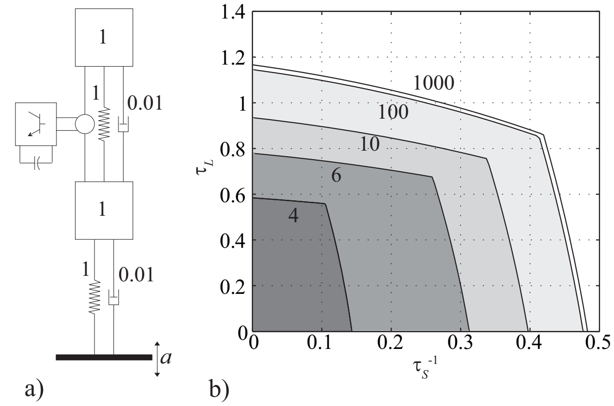

Fig. 4. Nondimensional 2-degree-of-freedom example structure (a), and successive $\{\tau_L, \tau_S\}$ pareto fronts for various values of $\mathbf{Z}_e$ (b)

V. Control Synthesis for Fixed Hardware Parameters

The previous section concerned the optimization of hardware parameters necessary to produce a given $\mathbf{Z}$ which is known to be passive. In this section, we consider the complimentary case, in which the hardware parameters are fixed, and $\mathbf{Z}$ is to be optimized. This is a considerably more difficult design problem because it is in general nonconvex.

Consider the dynamics of a passive, linear, vibrating structure, characterized by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_a\mathbf{a} + \mathbf{B}_f\mathbf{f}_e \tag{16}$$

$$\mathbf{y} = \mathbf{C}_y\mathbf{x} + \mathbf{D}_y\mathbf{f}_e \tag{17}$$

$$\mathbf{v} = \mathbf{C}_v\mathbf{x} \tag{18}$$

The structure is assumed to be passive. We will also assume, without loss of generality, that the particular realization for $\mathbf{x}$ is chosen such that $\mathbf{C}_v$ can be partitioned as

$$\mathbf{C}_v = \begin{bmatrix}\mathbf{C}_{v1} & \mathbf{0}\end{bmatrix} \tag{19}$$

where $\mathbf{C}_{v1}$ is square. We also assume $\mathbf{C}_{v1}$ is invertible, which is equivalent to stating that no combination of actuators can apply forces that do not affect the structural dynamics. In (16), $\mathbf{a}$ is a vector of exogenous stochastic inputs, taken to be uncorrelated white noise processes with spectral intensity $\mathbf{\Phi}_a = \mathbf{I}$. In (17), $\mathbf{y}(t)$ is a vector of response quantities by which performance is assessed. The standard $\mathcal{H}_2$ (i.e., LQG) performance measure is assumed:

$$J = \mathcal{E}\,\|\mathbf{y}\|_2^2 \tag{20}$$

where $\|\cdot\|_2$ is the Euclidean norm. However, note that strictly speaking, the above problem does not fall into the standard $\mathcal{H}_2$/LQG paradigm, because no noise injection is assumed on $\mathbf{v}$, which is the feedback signal. As such, the optimal $\mathbf{Z}$ is not required to be strictly proper.

For this system, the following theorem is a standard result.

THEOREM 2: Let $\mathbf{Z} \in \mathcal{Z}^{n_c}$. Then

$$J = J_0 + \mathcal{E}\,\|\mathbf{D}_y(\mathbf{Z}\mathbf{v} - \mathbf{K}\mathbf{x})\|_2^2 \tag{21}$$

where

$$\mathbf{K} = -\left[\mathbf{D}_y^T\mathbf{D}_y\right]^{-1}\left[\mathbf{B}_f^T\mathbf{P}_K + \mathbf{D}_y^T\mathbf{C}_y\right] \tag{22}$$

$$\mathbf{0} = \mathbf{A}^T\mathbf{P}_K + \mathbf{P}_K\mathbf{A} + \mathbf{C}_y^T\mathbf{C}_y - \mathbf{K}^T\mathbf{D}_y^T\mathbf{D}_y\mathbf{K} \tag{23}$$

and

$$J_0 = \mathrm{tr}\left[\mathbf{B}_a^T\mathbf{P}_K\mathbf{B}_a\right] \tag{24}$$

In this paper we consider a special form for $\mathbf{Z}$, which will facilitate its design for favorable J. Let $\mathbf{K}$ in Theorem 2 be partitioned with the same dimensions as $\mathbf{C}_{v1}$, as

$$\mathbf{K} = \begin{bmatrix}\mathbf{K}_1 & \mathbf{K}_2\end{bmatrix} \tag{25}$$

and define

$$\mathbf{K}_Z = \begin{bmatrix}\mathbf{0} & \mathbf{K}_2\end{bmatrix} \qquad \mathbf{G}_Z = \mathbf{K}_1\mathbf{C}_{v1}^{-1} \tag{26}$$

Then specifically, we restrict our attention to $\mathbf{Z}$ of the same order as the structure (i.e., $n_c = n_x$), and which permit the realization

$$\mathbf{Z} = \left[\begin{array}{c|c}\mathbf{A} + \mathbf{B}_f\mathbf{K}_Z + \mathbf{L}\mathbf{C}_v & \mathbf{B}_f\mathbf{G}_Z - \mathbf{L} \\ \hline \mathbf{K}_Z & \mathbf{G}_Z\end{array}\right] \tag{27}$$

where $\mathbf{L}$ is to be designed. This controller just implements feedback gain $\mathbf{K}$ on a standard Luenberger observer, except that because $\mathbf{v}$ is assumed to be measured precisely, the first m components of the system state are known as well (i.e. $\mathbf{C}_{v1}^{-1}\mathbf{v}$) and are used instead of the observed states to effect control. Our focus on this controller form is due to the following two corollaries, to Theorem 1 and 2, respectively.

COROLLARY 1: For $\mathbf{Z}$ as in (27), $\mathbf{Z} \in \mathcal{Z}^{n_c}$ if and only if $\exists\mathbf{P}_Z = \mathbf{P}_Z^T > 0$ and $\mathbf{X}$ such that

$$\hat{\mathbf{A}}^T\mathbf{P}_Z + \mathbf{C}_v^T\mathbf{F}^T + \mathbf{P}_Z\hat{\mathbf{A}} + \mathbf{F}\mathbf{C}_v + \tfrac{2}{\tau_S}\mathbf{P}_Z + \mathbf{X} + \mathbf{X}^T \leqslant 0 \tag{28}$$

$$\begin{bmatrix} -\mathbf{X}^T - \mathbf{X} & & & (sym) \\ \mathbf{G}_Z^T\mathbf{B}_f^T\mathbf{P}_Z - \mathbf{F}^T & -\frac{1}{2}\mathbf{Z}_e & & \\ \mathbf{K}_Z & \mathbf{G}_Z + \frac{1}{2}\mathbf{Z}_e & -\frac{1}{2}\mathbf{Z}_e & \\ -\mathbf{X} & \mathbf{P}_Z\mathbf{B}_f\mathbf{G}_Z - \mathbf{F} & \mathbf{0} & -\frac{1}{2\tau_L}\mathbf{P}_Z \end{bmatrix} \leqslant 0 \tag{29}$$

where

$$\hat{\mathbf{A}} = \mathbf{A} + \mathbf{B}_f\mathbf{K}_Z \tag{30}$$

and

$$\mathbf{F} = \mathbf{P}_Z\mathbf{L} \tag{31}$$

COROLLARY 2: For $\mathbf{Z}$ as in (27), $J < \gamma$ if and only if $\exists\mathbf{T} = \mathbf{T}^T$ and $\mathbf{Y} = \mathbf{Y}^T$ such that

$$J_0 + \mathrm{tr}\left[\mathbf{D}_y\mathbf{Y}\mathbf{D}_y^T\right] < \gamma \tag{32}$$

$$\begin{bmatrix}\mathbf{Y} & \mathbf{K}_Z \\ \mathbf{K}_Z^T & \mathbf{T}\end{bmatrix} > 0 \tag{33}$$

$$\begin{bmatrix}\mathbf{A}^T\mathbf{T} + \mathbf{T}\mathbf{A} + \mathbf{T}\mathbf{L}\mathbf{C}_v + \mathbf{C}_v^T\mathbf{L}^T\mathbf{T} & \mathbf{T}\mathbf{B}_a \\ \mathbf{B}_a^T\mathbf{T} & -\mathbf{I}\end{bmatrix} < 0 \tag{34}$$

Corollary 1 is directly evident, through substitution of (27) into (11). Corollary 2 follows as the standard LMI interpretation of Theorem 2, with appropriate substitutions of (27). (See, for example, [9] for a proof.)

The objective here is to take Corollaries 1 and 2, and unify them in some way to produce a convex optimization problem for minimization of γ, subject to LMI constraints. However this cannot be done without some kind of relaxation, because the union of the two sets of LMIs results in a nonconvex problem. Here, we make relaxation analogous to that made by Gapski and Geromel [12], who examined similar problems related to positive-real-constrained $\mathcal{H}_2$ optimal control. Specifically, for some scalar θ, we force the equality

$$\mathbf{P}_Z = \theta\mathbf{T} \tag{35}$$

This gives the following theorem

THEOREM 3: For $\mathbf{Z}$ as in (27), $J < \gamma$ and $\mathbf{Z} \in \mathcal{Z}^{n_c}$ if $\exists\mathbf{P}_Z = \mathbf{P}_Z^T$, $\mathbf{Y} = \mathbf{Y}^T$, $\mathbf{X}$, and θ such that LMIs (28) and (29) hold, together with

$$\theta J_0 + \mathrm{tr}\left[\mathbf{D}_y\mathbf{Y}\mathbf{D}_y^T\right] < \theta\gamma \tag{36}$$

$$\begin{bmatrix}\mathbf{Y} & \theta\mathbf{K}_Z \\ \theta\mathbf{K}_Z^T & \mathbf{P}_Z\end{bmatrix} > 0 \tag{37}$$

$$\begin{bmatrix}\mathbf{A}^T\mathbf{P}_Z + \mathbf{P}_Z\mathbf{A} + \mathbf{F}\mathbf{C}_v + \mathbf{C}_v^T\mathbf{F}^T & \mathbf{P}_Z\mathbf{B}_a \\ \mathbf{B}_a^T\mathbf{P}_Z & -\theta\mathbf{I}\end{bmatrix} < 0 \tag{38}$$

and where (31) relates $\mathbf{L}$ to $\mathbf{F}$.

It is emphasized that this theorem provides sufficient but not necessary conditions for $J < \gamma$, because of equation (35). As such, the above LMIs can be used to conservatively design $\mathbf{F}$ (and, through it, $\mathbf{L}$) to meet performance objectives subject to energy conservation constraints. The degree of conservativeness is compounded, by the particular mathematical structure of $\mathbf{Z}$ in (27). However, as with the hardware optimization in the previous section, the resultant problem is quasiconvex, and thus may be solved easily. Specifically, this optimization can be stated as

given:	$\tau_L, \tau_S, \mathbf{Z}_e, \mathbf{A}, \mathbf{B}_f, \mathbf{B}_a, \mathbf{C}_y, \mathbf{D}_y$
minimize:	γ
over:	$\mathbf{F}, \mathbf{P}_Z, \mathbf{Y}, \mathbf{X}, \theta, \gamma$
subject to:	(28), (29), (36), (37), (38)

As an example, consider the civil structure shown in Fig. 5. As shown, this five-story, base-isolated structure has control devices installed between the base and the ground, and between the roof and a mass damper. The structure was adapted from one considered by [13]. (The mass damper, tuned to the second natural frequency, has been added for this example.) The spectral content of the ground acceleration a is assumed to be a Kanai-Tajimi spectrum with a natural frequency of 17

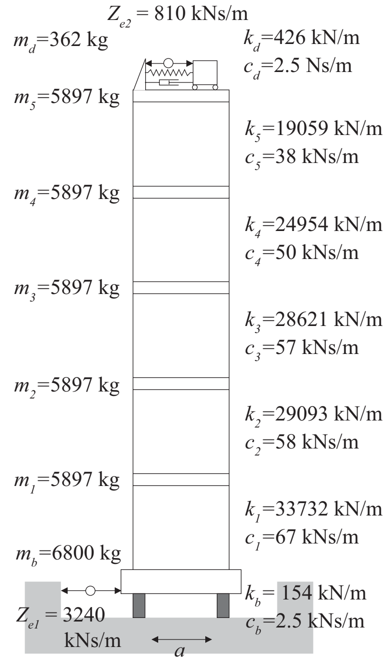

Fig. 5. Example structure

rad/s and a damping ratio of 0.3. Components of $\mathbf{y}(t) \in \Re^{14}$ are defined as

$$y_1 = d_b/4\text{cm} \qquad y_i = d_{i-1}/1\text{mm}\ ,\ i \in \{2..6\} \tag{39}$$

$$y_7 = a_b/0.1\text{g} \qquad y_i = a_{i-7}/0.1\text{g}\ ,\ i \in \{8..12\} \tag{40}$$

$$y_{13} = f_1/10\text{kN} \quad y_{14} = f_2/10\text{kN} \tag{41}$$

The Z_{ei} parameters shown in the figure are similar to those used in [14].

To model the energy storage system, we need only specify τ_S and τ_L. Here, we consider the design of $\mathbf{Z}$ for $\tau_S \in (1\text{s}, 100\text{s})$. Note that this is conservative, as many supercapacitors can store energy for days. Similarly, we consider $\tau_L \in (1\text{ms}, 100\text{ms})$. Fig. 6 shows a surface plot of the performance of the optimized $\mathbf{Z}$, normalized by J_0, the performance of the optimal full-state controller. This surface clearly shows the benefit of small τ_L and τ_S^{-1}, as well as their relevance to control design. With a very efficient storage system, the self-powered control design comes close to achieving the performance of a fully-active system.

VI. Conclusions

In this paper we have shown how imperfections in stored energy can affect the feasibility of linear control, for self-powered vibration suppression systems. The primary effort here has been to illustrate that it is possible to design a storage system with a given controller in mind, as well as to design a controller with knowledge about the storage system that will be used to implement it. The main result is the pair of feasibility LMIs (11), and their implications for these two design tasks. We showed that the problem of storage optimization for a given passive controller is quite tractable, as it can be posed as a generalized eigenvalue problem, and solved through convex optimization. Although the problem of control optimization, for a given storage system, is more

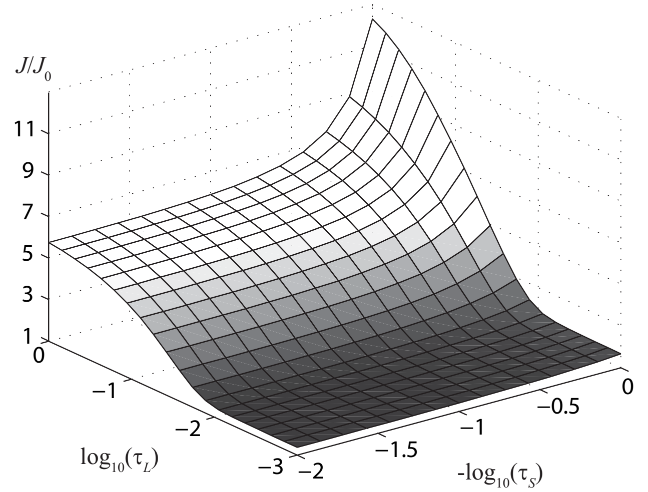

Fig. 6. Performance J/J_0, for optimized $\mathbf{Z}$, as a function of τ_L and τ_S

complex, we illustrated how some extant methods in the literature for positive-real-constrained $\mathcal{H}_2$ optimal control might be extended to this problem.

Still, the control design methods discussed in this paper are conservative, and therefore sub-optimal. One open question concerns the determination of the *true* optimal performance achievable with a given storage system. Another avenue for extension of these ideas concerns the use of more accurate models of the nonlinear dynamic behavior of energy storage and power electronic systems. These remain challenging problems requiring further work.

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