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An Augmented Multiple Model Strategy for Disturbance Estimation and Control

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Abstract — Classical model-based control strategies assume a single disturbance model. In practice, the type of disturbance is often unknown, can change with time, or multiple different disturbance types can occur simultaneously. In this paper a multiple model predictive control strategy is developed to handle different disturbances, including multiple disturbances occurring simultaneously. A detailed discussion of disturbance model bank generation, state estimation and disturbance model weighting is provided, and an unconstrained multiple model predictive control solution is formulated. Simulation results demonstrate successful estimation and control of single and multiple simultaneous disturbances.

1. Introduction

Model predictive control (MPC) techniques use models to predict plant responses to input signals propagated into the future, allowing for optimal calculation of control actions. One early development and implementation of MPC was based on step response models and known as dynamic matrix control (DMC; Cutler and Ramaker, 1980). A common criticism of DMC was the poor disturbance rejection to step input disturbances (Shinskey, 2001). This limitation was considered by Lundstrom et al. (1995), who developed an observer-based formulation for improved disturbance rejection. Muske and Badgwell (2002) present a general formulation that handles step disturbances entering the system at the output or input. Conditions that guarantee detectability and offset-free control for the augmented system are derived. Pannocchia and Rawlings (2003) derive conditions that guarantee offset-free control of non-square systems with more measured outputs than manipulated inputs.

While the importance of disturbance modeling and estimation in model predictive control is well-known, existing strategies are designed around a single disturbance model. For systems where more than one type of disturbance is likely to affect the system, a single fixed disturbance model is not sufficient to adequately handle disturbances that can range from steps to ramps to periodic behavior.

The contribution of this paper is the development and formulation of a multiple model disturbance estimation and control strategy designed to identify, estimate and reject active disturbances present in a system operating at steady state, thereby controlling the system in the presence of a range of possible disturbances. Previous multiple model-based approaches have generally focused on handling multiple operating conditions. Athans et al. (1977) develop a multiple model adaptive control (MMAC) strategy to control aircraft under different flight conditions. Kothare et al. (2000) use a linear parameter varying (LPV) strategy to regulate a nuclear steam generator over a wide range of loads. The monograph edited by Murray-Smith and Johanson (1997) has chapters on a number of different multiple model methods for control. Our previous work has focused on the control of nonlinear processes using a multiple model predictive control (MMPC) approach; for example, Aufderheide and Bequette (2003) develop a DMC-based MMPC algorithm. Kuure-Kinsey and Bequette (2007) develop a state space and state-estimation based MMPC strategy to avoid the limitations imposed by a DMC-based approach. In this paper we extend the state estimation-based approach to handle multiple disturbances, rather than focusing on handling nonlinear systems.

2. Control Structure

The multiple model predictive control structure can be represented by the control block diagram in figure 1

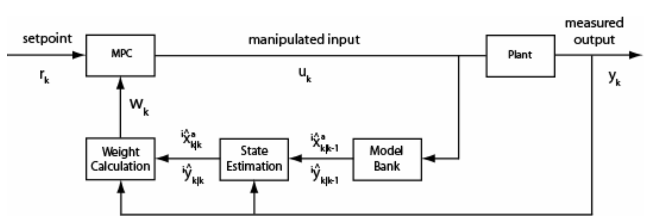

Figure 1: Control block diagram (Kuure-Kinsey and Bequette, 2007)

The strategy has four primary elements: a disturbance model bank, state estimation, a disturbance model weight calculation and a predictive control calculation.

3. Disturbance Model Bank

The multiple model disturbance estimation and control strategy is based on the use of *n* different disturbance models in the model bank for a given underlying nominal model. The linear state space model structure is used

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$$x_k = Ax_{k-1} + Bu_{k-1} + B^d d_{k-1}$$
$$y_k = Cx_k \quad (1)$$

It is important to note, however, that there is no inherent restriction to the use of a linear nominal model, and extension of the method to a nonlinear model follows a similar approach.

In practice, there is process and measurement noise as given in the model

$$x_k = Ax_{k-1} + Bu_{k-1} + B^d d_{k-1} + \omega_{k-1}$$
$$y_k = Cx_k + \nu_k \quad (2)$$

The ν_k and ω_k terms represent measurement and state noise, respectively. There are four types of disturbances that typically affect process systems, and it is important that the strategy is able to detect all of them. To accomplish this, the nominal linear model in (2) is augmented with a disturbance model representing each of the four common disturbances being estimated.

The first type of disturbance to estimate is an additive output disturbance. The additive output disturbance enters the model through the output term

$$x_k = Ax_{k-1} + Bu_{k-1}$$
$$d_k = d_{k-1} + \omega_{k-1} \quad (3)$$
$$y_k = Cx_k + d_k + \nu_k$$

The second type of disturbance is a step input disturbance. Here, the step input disturbance enters the model through the input

$$x_k = Ax_{k-1} + Bu_{k-1} + B^d d_{k-1}$$
$$d_k = d_{k-1} + \omega_{k-1} \quad (4)$$
$$y_k = Cx_k + \nu_k$$

The third type of disturbance to estimate is a ramp input disturbance

$$x_k = Ax_{k-1} + Bu_{k-1} + B^d d_{k-1}$$
$$d_k = d_{k-1} + \Delta d_{k-1}$$
$$\Delta d_k = \Delta d_{k-1} + \omega_{k-1} \quad (5)$$
$$y_k = Cx_k + \nu_k$$

Note the presence of two disturbance terms in (5). The d_k term is the disturbance term being estimated, and Δd_k is an estimate of the rate of change of the disturbance, emphasizing the fact that the disturbance enters the model at a constant rate and not as a discrete step. The fourth type of disturbance to estimate is a periodic disturbance. Here, the disturbance enters through a disturbance input.

$$x_k = Ax_{k-1} + Bu_{k-1} + B^d d_{k-1}$$
$$h_k = \Delta h_{k-1}$$
$$\Delta h_k = -\theta^2 \Delta t\, h_{k-1} + d_{k-1} \quad (6)$$
$$d_k = d_{k-1} + \omega_{k-1}$$
$$y_k = Cx_k + \nu_k$$

The expected period of the disturbance is θ, and the θ^2 term places the poles on the imaginary axis. The input and change in input where the disturbance occurs, h_k and Δh_k respectively, are new states being estimated.

The four disturbance models have a common structure

$$\underbrace{\begin{bmatrix} {}^i x_k \\ {}^i d_k^a \end{bmatrix}}_{{}^i x_k^a} = \underbrace{\begin{bmatrix} {}^i A_1 & {}^i A_2 \\ {}^i A_3 & {}^i A_4 \end{bmatrix}}_{{}^i A^a} \underbrace{\begin{bmatrix} {}^i x_{k-1} \\ {}^i d_{k-1} \end{bmatrix}}_{{}^i x_{k-1}^a} + \underbrace{\begin{bmatrix} {}^i B_1 \\ {}^i B_2 \end{bmatrix}}_{{}^i B^a} u_{k-1} + {}^i\Omega^a \omega_{k-1}$$
$${}^i y_k = \underbrace{\begin{bmatrix} {}^i C_1 & {}^i C_2 \end{bmatrix}}_{{}^i C^a} \underbrace{\begin{bmatrix} {}^i x_k \\ {}^i d_k \end{bmatrix}}_{{}^i x_k^a} + \nu_k \quad (7)$$

The augmented state vector is divided into two sub-vectors: ${}^i x_k$ represents the original states in the nominal linear model, and ${}^i d_k^a$ represents all the disturbance states being estimated: disturbance, rate of change of disturbance, and periodic states.

4. State Estimation

The model predicted outputs, ${}^i y_k$, from the disturbance model are corrected by estimating disturbance states. This disturbance term, ${}^i d_k^a$, in the four disturbance models, is used to account for all uncertainties between the model and plant under control. To estimate this disturbance term, it is important to first recognize that the augmented structure in (7) is written more compactly as

$${}^i x_k^a = {}^i A^a\, {}^i x_{k-1}^a + {}^i B^a u_{k-1} + {}^i\Omega^a \omega_{k-1}$$
$${}^i y_k = {}^i C^a\, {}^i x_k^a + \nu_k \quad (8)$$

The state estimation procedure uses the standard predictor/corrector equations, defined in (9) – (11).

$${}^i \hat{x}_{k|k-1}^a = {}^i A^a\, {}^i \hat{x}_{k-1|k-1}^a + {}^i B^a u_{k-1} \quad (9)$$
$${}^i \hat{x}_{k|k}^a = {}^i \hat{x}_{k-1|k-1}^a + {}^i L_k \left(y_k - {}^i C^a\, {}^i \hat{x}_{k|k-1}^a \right) \quad (10)$$
$${}^i \hat{y}_{k|k} = {}^i C^a\, {}^i \hat{x}_{k|k}^a \quad (11)$$

The predictor/corrector equations predict the augmented states without the measurement, then update the augmented states based on the difference between the plant measurement and the uncorrected model prediction. The augmented state update is dependent on ${}^{i}L_k$, which is an appropriate observer gain based on the disturbance model. For the first model, the additive output disturbance model, ${}^{1}L_k$ is equivalent to the use of a deadbeat observer (Muske and Badgwell, 2002). For the remaining disturbance models, ${}^{i}L_k$ is defined by the solution to the Riccati equation. The ${}^{i}Q$ and ${}^{i}R$ terms in the Riccati equation are stochastic terms representing the variance on the input disturbance and output measurement. In practice, the variances unknown, and ${}^{i}Q$ and ${}^{i}R$ become tuning parameters for the Kalman filter, conventionally expressed as the ratio ${}^{i}Q/{}^{i}R$, for scalar noise terms.

Each of the four disturbance models in the disturbance model bank has a Q and R matrix. Since Q/R is a tuning parameter and the disturbance models are unique, there is no restriction on the magnitude and values of ${}^{i}Q$ and ${}^{i}R$, and are unique for each disturbance model.

5. Model Weighting Calculation

Once the disturbance models are updated with information from the most recent measurement, the predicted outputs are passed to the model weighting calculation. For each disturbance model and associated predicted output, a corresponding weight is calculated. The weights are normalized so that the sum of all four weights is unity, and the closer a disturbance model's weight is to unity, the better that disturbance is at representing the current disturbance state of the plant. The weights are calculated based on residuals of each model in the model bank.

$$ {}^{i}\varepsilon_k = y_k - {}^{i}\hat{y}_{k|k-1} \tag{12} $$

The model predicted outputs are calculated based on state estimation updates in (9-11). The model weights are based on Bayesian probability (Athans et al., 1977)

$$ {}^{i}\rho_k = \frac{\exp\left(-\tfrac{1}{2}\,{}^{i}\varepsilon_k^T\;{}^{i}\Lambda\;{}^{i}\varepsilon_k\right){}^{i}\rho_{k-1}}{\sum_{j=1}^{4}\exp\left(-\tfrac{1}{2}\,{}^{j}\varepsilon_k^T\;{}^{i}\Lambda\;{}^{j}\varepsilon_k\right){}^{j}\rho_{k-1}} \tag{13} $$

The ${}^{i}\Lambda$ in (13) is a diagonal scaling matrix for the residuals, and is based on the covariances of each disturbance model. As the covariances are not known in practice, the ${}^{i}\Lambda$ matrix is a tuning parameter that is adjusted to achieve desired control behavior. Similar to Q/R in Kalman filtering, each disturbance model in the disturbance model bank has a unique ${}^{i}\Lambda$ matrix. For the work in this paper, the same ${}^{i}\Lambda$ matrix is used for each disturbance model in a given disturbance model bank.

The term ${}^{i}\rho_k$ represents the probability of the i^{th} disturbance model representing the plant at the k^{th} time step. The probability calculation is recursive, as it relies on information from the previous time step ${}^{i}\rho_{k-1}$. Due to this recursion, if the probability of any disturbance model reaches zero, there is no way for that probability to become non-zero at a future time step. To account for this and allow every disturbance model to remain active in the calculation, an artificial lower limit on the probability is enforced. Any probability that drops below this limit, represented by δ, is set equal to δ. The disturbance model weights are then calculated by normalizing the probabilities, according to the formula

$$ {}^{i}w_k = \begin{cases} \dfrac{{}^{i}\rho_k}{\sum_{j=1}^{4}{}^{j}\rho_k} & {}^{i}\rho_k > \delta \\ 0 & {}^{i}\rho_k < \delta \end{cases} \tag{14} $$

The probability calculation in (13) is recursive, so an initial value for the probability of each disturbance model is required prior to the first control calculation. Without *a priori* knowledge of the system, each disturbance model starts with the same initial weight.

6. Model Predictive Control

While the multiple model disturbance estimation and control strategy is designed to estimate the current disturbance state in the system, it is also a control strategy. The underlying control strategy used is based on linear model predictive control. At each time step, an optimization problem is formulated and solved. The objective function is to minimize control action over a prediction horizon of *p* time steps. The decision variables are *m* control moves, where *m* is the control horizon. Only the first control move is applied to the system, the model is updated, and the entire process is repeated at the next time step. The model used in the control calculation is the average linear model defined as

$$ \bar{y}_{k+j|k} = \sum_{i=1}^{4}{}^{i}w_k\;{}^{i}\hat{y}_{k+j|k} \tag{15} $$

The vector ${}^{i}w_k$ is the weight vector defined in (14), and the ${}^{i}\hat{y}_{k+j|k}$ terms are the individual disturbance model predicted outputs generated from the disturbance models defined in section 4. From the average system model in (15), the next step is to derive a model predictive control solution. In the objective function that follows, the first term represents the error over the prediction horizon and the second term is a penalty on control actions.

$$ \min\,\Phi = \left(Y_{sp} - \bar{Y}\right)^T W_y \left(Y_{sp} - \bar{Y}\right) + \Delta U^T W_u \Delta U \tag{16} $$

Y_{sp} is a vector of setpoints, ΔU is a vector of optimal control moves, and $\bar{Y}$ is the vector of predicted outputs from $\overline{y}_{k+1|k}$ to $\overline{y}_{k+p|k}$ .For the average linear model defined in (15), the solution to the unconstrained problem is given by

$$\Delta U = \left(S_e^T S_c^{a^T} S_c^a S_e + W_u\right)^{-1} S_e^T S_c^{a^T}\left(Y_{sp} - S_x^a - S_c^a U_0\right) \quad (17)$$

7. Simulation Results

The unconstrained multiple model disturbance estimation strategy developed in this paper is applicable to both linear and nonlinear systems. Given the absence of research on the subject of estimating disturbances in a multiple model framework, it is important to start by analyzing the performance of the strategy applied to linear systems. The advantage to studying linear systems is the elimination of nonlinearities between the plant and models in the disturbance model bank. This isolates the source of error between plant and model down to the disturbances entering the system, allowing for an accurate study of the efficacy of the disturbance estimation strategy.

7.1 Van de Vusse Reactor

The van de Vusse reactor is chosen for its challenging nonlinear behavior that includes input multiplicity and nonminimum phase behavior (Sistu and Bequette, 1995).

The nature of the four disturbance models results in different disturbances being estimated. The additive output disturbance model assumes the disturbance is to the measured output, the concentration of species B, and estimates accordingly. The step input and ramp input disturbance models estimate an input disturbance, the dilution rate for the van de Vusse reactor. The periodic disturbance model estimates the most likely disturbance to be periodic in nature, the feed concentration.

To populate the disturbance model bank, a nominal linear model is required. The model is derived by linearizing around the nominal operating conditions given in Bequette (2003). The resulting linear model serves as both the plant and nominal linear model for the disturbance model bank. The disturbance models are generated using the procedure and equations outlined in section 3.

7.2 Tuning Q/R

Each of the four disturbance models uses an observer gain, ${}^iL_k$, to update and correct the model states and predicted outputs based on plant measurement information. For the additive output disturbance model, the observer gain is static. For the remaining three disturbance models, the observer gain is based on the solution to the Riccati equation. The solution, and resulting observer gain, is a function of the covariance terms ${}^iQ$ and ${}^iR$, which are tuning parameters for the disturbance models. Each of the disturbance models is estimating a different set of disturbances, so it makes sense that the ${}^iQ$ / ${}^iR$ ratio for each disturbance model has a different magnitude.

To provide a guideline for expected relative magnitudes of the Q/R ratio, each of the disturbance models is studied independently. The Q/R ratio is varied and performance results for linear model predictive control are analyzed to determine an optimal Q/R ratio. The step input disturbance model is analyzed first. The performance of Q/R is investigated for rejection of a step disturbance in the input, the dilution rate. For Q/R varying between 0.01 and 100 by factors of 10, the disturbance rejection results for a step input disturbance of 0.15 min^{-1} at time 6 minutes are shown in figure 2.

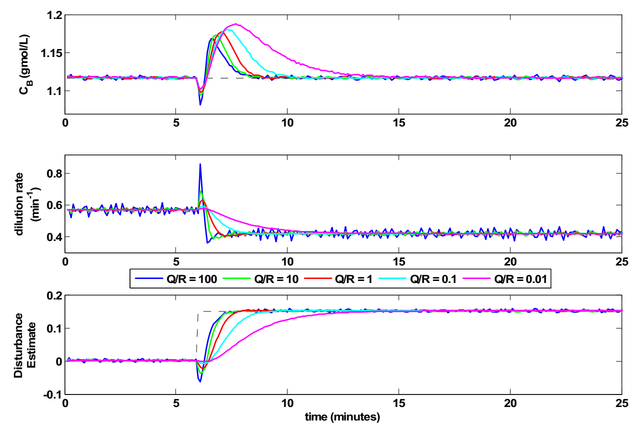

Figure 2: Tuning the Q/R ratio for rejection of a 0.15 min^{-1} disturbance at t = 6 minutes using the step input disturbance model; p = 50, m = 3, ${}^i\Lambda$ = 5000, δ = 0.01, Δt = 0.1 minutes

The disturbance rejection results in figure 2 show that there is a strong dependence in performance on the Q/R ratio. There is a trade-off between speed of disturbance rejection and manipulated input action. As the Q/R ratio increases, the step disturbance is rejected faster at the expense of increased manipulated input action. A Q/R ratio of 10 is selected as the optimal trade-off for the step input disturbance model.

Similar analysis is carried out for the ramp input and periodic disturbance models, resulting in a Q/R of 0.1 for the ramp input disturbance model and a Q/R of 100 for the periodic disturbance model. From the disturbance rejection results for the three disturbance models at varying Q/R ratios, it is possible to draw a general conclusion about the relative magnitudes of Q/R for the three disturbance models. This relationship is illustrated by

$$\tfrac{Q}{R}(periodic) > \tfrac{Q}{R}(step) > \tfrac{Q}{R}(ramp) \quad (18)$$

The relationship in (18) makes sense in terms of the magnitude of disturbances each model rejects. Ramp disturbances are the result of small continuous perturbations, requiring a smaller Q/R ratio than periodic disturbances that result from larger perturbations. Magnitudes of step disturbances are typically moderate in nature, resulting in a Q/R between that of ramp and periodic disturbance models. The Q/R ratios identified through this trade-off analysis are used throughout the remainder of the paper.

7.3 Setpoint Regulation

While the multiple model disturbance estimation and control strategy is designed to estimate a current disturbance state and control the system in its presence, small setpoint changes around the steady state operating condition are also reasonably expected and therefore an important scenario to study. This also provides a chance to determine how the strategy performs in a situation where there are no active disturbances to the system. For setpoint changes, the control and disturbance estimation results are shown in figure 3.

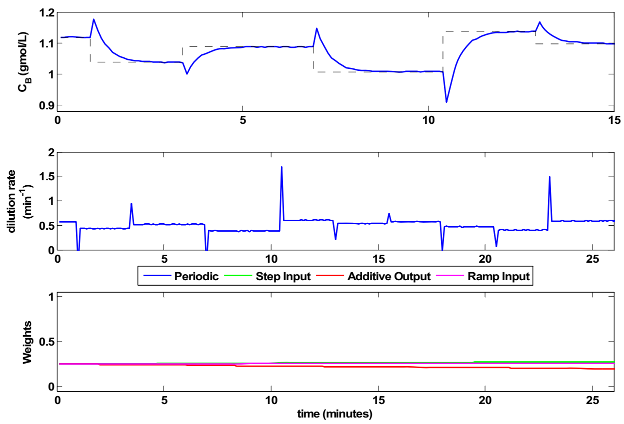

Figure 3: Setpoint regulation with no active disturbances; p = 50, m = 3, $^{i}\Lambda$ = 5000, δ = 0.01, Δt = 0.1 minutes

The results in figure 3 illustrate the performance of the multiple model disturbance estimation and control strategy in the absence of active disturbances. Each of the models in the disturbance model bank is designed to estimate a disturbance. With no active disturbances, there is only measurement noise to account for and the result is that no disturbance model evolves to dominance. This makes intuitive sense as there is no unique disturbance to distinguish one disturbance model from another.

7.4 Single Disturbance

The setpoint regulation results in section 7.3 show that the multiple model disturbance estimation and control strategy is able to handle setpoint changes if necessary. The true purpose of the strategy, however, is to detect and estimate disturbances. To test this, the van de Vusse reactor is maintained at its steady-state operating condition as a step input disturbance is introduced to the system. For the van de Vusse reactor, this corresponds to a discrete step change in the manipulated input, the dilution rate. For such a step input change, with a disturbance of -0.2 min^{-1} entering the system at t = 9 minutes, control and disturbance estimation results are shown in figure 4.

The results in figure 4 show that the multiple model disturbance estimation strategy is able to control the van de Vusse reactor in the presence of a step input disturbance. The disturbance model weights evolve to properly select and identify the step input disturbance model in the disturbance model bank. Note also that the step input model accurately estimates the magnitude of the active disturbance while the other disturbance models do not.

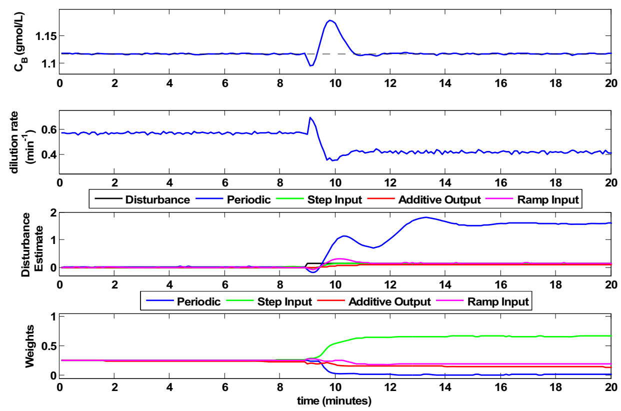

Figure 4: Estimation and rejection of a step input disturbance at t = 9 minutes; p = 50, m = 3, $^{i}\Lambda$ = 5000, δ = 0.01, Δt = 0.1 minutes

7.5 Multiple Disturbance

The result in section 7.4 demonstrates the ability of the multiple model disturbance estimation and control strategy to detect and subsequently reject and control the reactor in the presence of a single disturbance entering the system. Though less likely to occur from a physical point of view, it is nonetheless instructive to evaluate how the multiple model disturbance estimation and control strategy performs in the presence of a multiple disturbances.

The multiple disturbance scenario is a step input disturbance occurring with an underlying periodic disturbance. For the van de Vusse reactor, this corresponds to a continual and unmeasured sinusoidal disturbance in the feed concentration, due perhaps to a poorly tuned upstream controller, with a sudden discrete and unmeasured step in the dilution rate. With the disturbances occurring simultaneously at t = 9 minutes, the control and disturbance estimation results are shown in figure 5.

The results in figure 5 demonstrate that the strategy is able to successfully control the reactor in the presence of multiple disturbances, one periodic and one discrete. Note also the disturbance model weight evolution. Initially, prior to the introduction of the disturbances, the weights are roughly equal, as the setpoint regulation results suggest. Upon the introduction of the disturbances at t = 9 minutes, the step input disturbance model dominates initially. This makes sense as the step input disturbance is discrete with all its impact felt immediately. Over the next 100 minutes, the periodic disturbance model steadily begins to dominate. This also makes sense as the periodic nature of the disturbance has a continual effect while the step input disturbance is static after the initial step.

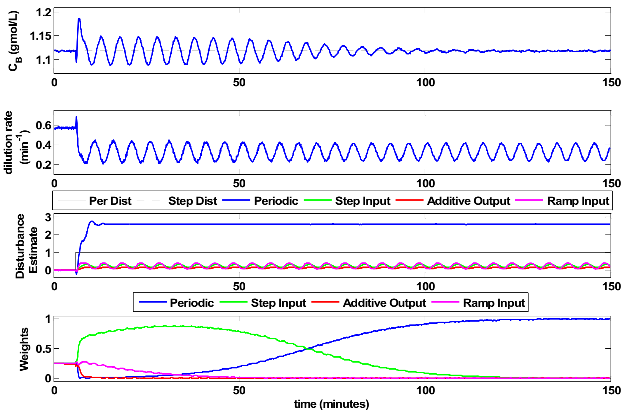

Figure 5: Estimation and rejection of a periodic input disturbance and step input disturbance at t = 9 minutes; p = 50, m = 3, ${}^{i}\Lambda = 5000$, $\delta = 0.01$, $\Delta t = 0.1$ minutes

8. Summary

This paper develops a multiple model approach to controlling nonlinear systems in the presence of disturbances and estimating the type of disturbance active in the system. The strategy builds on classical multiple model control by incorporating four different disturbance models to a nominal linear model representing the expected system operating condition. The disturbance models represent the common disturbances expected in a nonlinear system: additive output, step and ramp input and periodic.

Acknowledgements

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