2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Fault-Tolerant Controller Synthesis for Piecewise-Affine Systems

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Abstract— **In this paper, fault-tolerant, state feedback controllers are synthesized for piecewise-affine (PWA) systems while minimizing an upper bound of a quadratic cost function. The controllers are designed to deal with partial loss of control authority in the closed loop PWA system. The proposed controller design technique stabilizes and satisfies performance bounds for both the nominal and faulty systems. The new control technique is applied to a PWA model of a wheeled Mobile Robot (WMR) and the results are compared against a Linear Quadratic Regulator (LQR) in simulation.**

I. INTRODUCTION

Most dynamical systems exhibit nonlinear behavior [1], while many existing control methods address linear dynamics. Linear models of nonlinear systems are valid only within a small range around the equilibrium point about which the system is linearized. Controllers that are designed for linear models, may not even stabilize the system if the states go beyond the allowed range. This problem is more important especially if the system is prone to fault occurrence which may lead to jumps of states in the system. In [2] linear local controllers for performance are extended to PWA controllers that can guarantee global stability. PWA systems are a class of hybrid systems and are a good modeling framework for nonlinear phenomena . The theory of continuous-time PWA systems has been applied to several areas, such as, switched production and inventory control [3], aerospace systems [4], wheeled robots [5] and electric circuits [6]. PWA slab systems [7] are able to switch among several linear models based on variations of only one state variable. Each linear model can approximate some nonlinear phenomena in the system when the switching state is in a certain range. PWA systems pose challenging problems due to their switching nature. Switching among each closed loop model, either nominal or faulty, may destabilize the system even if each closed loop model is stable and has good performance in its allowed working region [8]. However, if controllers for the PWA nominal and faulty models in all regions are designed together in such a way that there exists a common Lyapunov function for all of them, it is guaranteed that any switching between closed loop PWA models for both nominal and faulty systems will be stable [8]. It is also possible to consider a performance criterion in the controller design problem. In this paper, an upper bound of a quadratic cost function is minimized for the PWA models of both nominal and faulty systems. The type of fault which is studied in this paper is partial loss of control authority, which is widely used to model the faults in actuators [9], [10]. The controller design is cast as a set of Linear Matrix Inequalities (LMIs) and solved with SeDuMi/YALMIP [11]. The resulting controller will not only handle large deviations from equilibrium points for systems with nonlinear phenomena, but it also has a robust behavior in the presence of faults without performance degradation. This controller design technique is applied to a WMR and the simulation results are compared favorably to those with an LQR controller.

The paper is organized as follows. Section II presents the PWA model. This is followed by the description of the controller structure and performance index in section III. Then, the controller synthesis method is presented, followed by simulations and conclusions.

II. PIECEWISE-AFFINE REPRESENTATION

Consider a piecewise-affine nominal system

$$\dot{x}(t) = A_i x(t) + B_i u(t) + b_i \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system and $u(t) \in \mathbb{R}^m$ is the input to the system. Associated with system (1) there is a corresponding polytopic partition of the state space representing working regions of each PWA model by $\mathcal{R}_i, i \in \{1, ..., M\}$ [7], [12]

$$\mathcal{R}_i = \{x \mid \bar{H}_i \bar{x} > 0\} \tag{2}$$

where $\bar{H}_i = \begin{bmatrix} H_i & h_i \\ 0 & 1 \end{bmatrix}$, $\bar{x} = \begin{bmatrix} x \\ 1 \end{bmatrix}$. H_i and h_i directly follow from the state space partitioning. For regions containing the equilibrium points, the regions are represented as

$$\mathcal{R}_i = \{x \mid H_i x > 0\} \tag{3}$$

Ellipsoidal Covering: We can build an exact approximation of the polytopic partitioning of the state space with ellipsoidal cell boundings for PWA slab systems [7]. This bounding enables a convex formulation of the quadratic stabilization problem for PWA slab systems [7]. The description of the ellipsoidal covering is

$$\epsilon_i = \{x \mid \| E_i x + f_i \| < 1\} \tag{4}$$

where E_i and f_i follow directly from the polytopic partitioning. More precisely, if $\mathcal{R}_i = \{x \mid d_1 < c_i^T x < d_2\}$, then the associated ellipsoidal covering is described by $E_i = 2c_i^T/(d_2 - d_1)$ and $f_i = -(d_2 + d_1)/(d_2 - d_1)$.

A PWA representation for a faulty system with partial loss of control authority is as follows:

$$\dot{x}(t) = A_i x(t) + B_{fi} u(t) + b_i \tag{5}$$

The state space partitioning for the faulty system is the same as the nominal system. The matrix B_{fi} encapsulates the fault in the system in each, valid for $\mathcal{R}_i$. Partial loss of control authority is a common type of fault that occurs in certain actuator channels [9],[10]. It can be modeled as a factor that multiplies the B matrix for the nominal system and reduces the amount of control authority. The faulty B_{fi} matrix modeling partial loss of control authority can be written as

$$B_{fi} = B(I - e_j e_j^T \delta_j) \tag{6}$$

where e_j is the j^{th} unit vector and δ_j is the amount of failure in the j^{th} actuator. The case of $\delta_j = 0$ corresponds to the nominal system and $\delta_j = 1$ corresponds to 100% loss of control authority [9].

III. Controller Structure and Performance

In this paper, it is assumed that for each of the nominal and faulty systems described by equations (1) and (5), respectively, there are M regions $\mathcal{R}_i, i \in \{1, ..., M\}$, in the partition of the state space. The state feedback control problem is parameterized by $K_i, i \in \{1, ..., M\}$ as

$$u = K_i x, \; x \in \mathcal{R}_i. \tag{7}$$

A performance criterion will be added to the design considerations by the quadratic cost function [13], [15]

$$J = \int_0^\infty (x^T \Upsilon x + u^T \Xi u) dt \tag{8}$$

where $x \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^m$ is the input to the system and $\Upsilon \geq 0$ and $\Xi > 0$ are weighting matrices.

Definition [13]: The controller in (7), provides guaranteed performance cost (8) if there exists a matrix $P = P^T > 0$ that satisfies

$$J \leq x^T(0) P x(0) \tag{9}$$

where P is a solution to

$$\dot{V}(x) + x^T \Upsilon x + u_i^T \Xi u_i < 0 \tag{10}$$

for $i \in \{1, ..., M\}$ and the quadratic candidate Lyapunov function is

$$V(x) = x^T P x \tag{11}$$

In order to avoid the dependency of the upper bound of the cost function J on initial conditions, it is assumed that the initial conditions are random variables with zero mean and covariance equal to identity [13]. Therefore

$$\mathbb{E}\{x(0)x^T(0)\} = I$$

$$\mathbb{E}\{x(0)\} = 0$$

where $\mathbb{E}$ is the expected value operator. Thus, the performance problem posed in the next section will be to minimize the maximum expected value of (8) which, from [13], obeys

$$\mathbb{E}\{x^T(0) P x(0)\} < \text{Trace(P)} \tag{12}$$

IV. Controller Synthesis

The candidate Lyapunov function (11) becomes a Lyapunov function and satisfies the guaranteed cost performance if $V > 0$ and the set of inequalities (10) hold. These conditions lead the following inequality for $x \in \epsilon_i$

$$\begin{aligned}[(A_i + B_i K_i)x + b_i]^T P x + x^T P((A_i + B_i K_i)x + b_i) \\ < -(x^T \Upsilon x + u_i^T \Xi u_i)\end{aligned} \tag{13}$$

Inequality (13) can be rewritten as

$$\begin{aligned}x^T[(A_i + B_i K_i)^T P + P(A_i + B_i K_i)]x + b_i^T P x + x^T P b_i \\ < -(x^T \Upsilon x + u_i^T \Xi u_i)\end{aligned} \tag{14}$$

Assuming $\bar{A}_i = (A_i + B_i K_i)$ (14) can be rewritten as

$$\begin{aligned}\begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} \bar{A}_i^T P + P\bar{A}_i & P b_i \\ b_i^T P & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \\ < - \begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} \Upsilon + K_i^T \Xi K_i & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix}\end{aligned} \tag{15}$$

Using (15) together with the S-procedure with multiplier $\lambda_i < 0$ [7], [14] yields

$$\begin{aligned}\begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} \bar{A}_i^T P + P\bar{A}_i + \Upsilon + K_i^T \Xi K_i & P b_i \\ b_i^T P & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} \\ < -\lambda_i \begin{bmatrix} x \\ 1 \end{bmatrix}^T \begin{bmatrix} E_i^T E_i & E_i^T f_i \\ f_i^T E_i & f_i^T f_i - 1 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix}\end{aligned} \tag{16}$$

Using new variables $Q = P^{-1}$ and $\mu_i = \lambda_i^{-1}$, sufficient conditions for quadratic stabilization are derived as

$$Q = Q^T > 0, \quad \mu_i < 0, \quad i = 1, ..., M$$

$$\begin{bmatrix} \Pi_i & Q^{-1} b_i + \mu_i^{-1} E_i^T f_i \\ (Q^{-1} b_i + \mu_i^{-1} E_i^T f_i)^T & -\mu_i^{-1}(1 - f_i^T f_i) \end{bmatrix} < 0 \tag{17}$$

where

$$\Pi_i = \bar{A}_i^T Q^{-1} + Q^{-1} \bar{A}_i + \mu_i^{-1} E_i^T E_i + \Upsilon + K_i^T \Xi K_i$$

Theorem 1: The fault-tolerant PWA controller stabilizes the nominal system if the following holds

$$Q = Q^T > 0, \quad \mu_i < 0, \quad i = 1, ..., M$$

$$\begin{bmatrix} \Gamma_i + \mu_i b_i b_i^T & Q\Upsilon^{1/2} & Y_i^T \Xi^{1/2} & \mu_i b_i f_i^T + QE_i^T \\ \Upsilon^{1/2} Q & -I_n & 0 & 0 \\ \Xi^{1/2} Y_i & 0 & -I_m & 0 \\ (\mu_i b_i f_i^T + QE_i^T)^T & 0 & 0 & -\mu_i (1 - f_i f_i^T) \end{bmatrix} < 0 \tag{18}$$

where $\Gamma_i = A_i Q + Q A_i^T + B_i Y_i + Y_i^T B_i^T$.

Proof: Applying Schur complement to the inequality (17) yields

$$\begin{aligned} & 1 - f_i^T f_i < 0 \\ & \Pi_i + (Q^{-1} b_i + \mu_i^{-1} E_i^T f_i) \\ & \times \mu_i (1 - f_i^T f_i)^{-1} (Q^{-1} b_i + \mu_i^{-1} E_i^T f_i)^T < 0 \end{aligned} \tag{19}$$

left multiplying the above inequality by Q and right multiplying it by $Q = Q^T$ and rearranging yields

$$\begin{aligned} & \bar{A}_i Q + Q \bar{A}_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q + \mu_i^{-1} Q E_i^T E_i Q \\ & \times (b_i + \mu_i^{-1} Q E_i^T f_i) \mu_i (1 - f_i^T f_i)^{-1} (b_i + \mu_i^{-1} Q E_i^T f_i)^T < 0 \end{aligned} \tag{20}$$

Using the Matrix Inversion Lemma as in [7] $(1 - f_i^T f_i)^{-1} = 1 + f_i^T (I - f_i^T f_i)^{-1} f_i$. Thus, inequality (20) can be rewritten as

$$\begin{aligned} & \bar{A}_i Q + Q \bar{A}_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q \\ & + \mu_i^{-1} Q E_i^T E_i Q + \mu_i b_i b_i^T + \mu_i^{-1} (Q E_i^T f_i)(Q E_i^T f_i)^T \\ & + b_i (Q E_i^T f_i)^T + Q E_i^T f_i b_i^T + (\mu_i b_i f_i^T + Q E_i^T f_i f_i^T) \mu_i^{-1} \\ & \times (1 - f_i f_i^T)^{-1} (\mu_i b_i f_i^T + Q E_i^T f_i f_i^T)^T < 0 \end{aligned} \tag{21}$$

Inequality (21) can be rewritten as

$$\begin{aligned} & \bar{A}_i Q + Q \bar{A}_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q + \mu_i b_i b_i^T \\ & + \mu_i^{-1} (E_i Q)^T (I + f_i f_i^T)(E_i Q) + b_i f_i^T (Q E_i^T)^T \\ & + (Q E_i^T)(b_i f_i^T)^T \\ & + (\mu_i b_i f_i^T + Q E_i^T - Q E_i^T (I - f_i f_i^T)) \mu_i^{-1} \\ & \times (I - f_i f_i^T)^{-1} (\mu_i b_i f_i^T + Q E_i^T - Q E_i^T (I - f_i f_i^T))^T < 0 \end{aligned} \tag{22}$$

Inequality (22) can be rearranged as

$$\begin{aligned} & \bar{A}_i Q + Q \bar{A}_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q + \mu_i b_i b_i^T \\ & + (\mu_i b_i f_i^T + Q E_i^T) \mu_i^{-1} (I - f_i f_i^T)^{-1} (\mu_i b_i f_i^T + Q E_i^T)^T \\ & + \mu_i^{-1} (E_i Q)^T (I + f_i f_i^T)(E_i Q) + b_i f_i^T (Q E_i^T)^T \\ & + (Q E_i^T)(b_i f_i^T)^T + \mu_i^{-1} (Q E_i^T)(I - f_i f_i^T)(Q E_i^T)^T < 0 \end{aligned} \tag{23}$$

which can then be rewritten as

$$\begin{aligned} & \bar{A}_i Q + Q \bar{A}_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q + \mu_i b_i b_i^T \\ & + (\mu_i b_i f_i^T + Q E_i^T) \mu_i^{-1} \\ & \times (I - f_i f_i^T)^{-1} (\mu_i b_i f_i^T + Q E_i^T)^T < 0 \end{aligned} \tag{24}$$

Using Schur complement and the fact that $1 - f_i^T f_i < 0$ is equivalent to $I - f_i^T f_i < 0$ since f_i is a scalar for PWA slab systems [7] yields

$$\begin{bmatrix} \Lambda_i & \mu_i b_i f_i^T + Q E_i^T \\ (\mu_i b_i f_i^T + Q E_i^T)^T & -\mu_i (1 - f_i f_i^T) \end{bmatrix} < 0 \tag{25}$$

where $\Lambda_i = \bar{A}_i Q + Q \bar{A}_i^T + \mu_i b_i b_i^T + Q\Upsilon Q + Q K_i^T \Xi K_i Q$. Replacing $\bar{A}_i$ by $(A_i + B_i K_i)$, introducing a new variable $Y_i = K_i Q$ and using Schur complement yields a convex representation of the sufficient conditions for quadratic stabilization as follows

$$Q = Q^T > 0, \quad \mu_i < 0, \quad i = 1, ..., M$$

$$\begin{bmatrix} \Gamma_i + \mu_i b_i b_i^T & Q\Upsilon^{1/2} & Y_i^T \Xi^{1/2} & \mu_i b_i f_i^T + QE_i^T \\ \Upsilon^{1/2} Q & -I_n & 0 & 0 \\ \Xi^{1/2} Y_i & 0 & -I_m & 0 \\ (\mu_i b_i f_i^T + QE_i^T)^T & 0 & 0 & -\mu_i (1 - f_i f_i^T) \end{bmatrix} < 0 \tag{26}$$

where $\Gamma_i = A_i Q + Q A_i^T + B_i Y_i + Y_i^T B_i^T$. □

Corollary: For the faulty system, the inequality (26) is equivalent to

$$Q = Q^T > 0, \quad \mu_i < 0, \quad i = 1, ..., M$$

$$\begin{bmatrix} \Gamma_i + \mu_i b_i b_i^T & Q\Upsilon^{1/2} & Y_i^T \Xi^{1/2} & \mu_i b_i f_i^T + QE_i^T \\ \Upsilon^{1/2} Q & -I_n & 0 & 0 \\ \Xi^{1/2} Y_i & 0 & -I_m & 0 \\ (\mu_i b_i f_i^T + QE_i^T)^T & 0 & 0 & -\mu_i (1 - f_i f_i^T) \end{bmatrix} < 0 \tag{27}$$

where $\Gamma_i = A_i Q + Q A_i^T + B_{fi} Y_i + Y_i^T B_{fi}^T$.

Proof: It follows trivially from (26), by replacing the faulty B_{fi} matrix (6) into that inequality. □

To design the controller gains for the guaranteed cost fault tolerant controller, the following convex problem will be solved.

Definition 4.1: (**Fault-Tolerant Controller Synthesis**)

$$\begin{aligned} \min \quad & \text{Trace(P)} \\ s.t. \quad & (26),\ (27) \end{aligned}$$

From the solution to this problem one gets the controller gains $K_i = Y_i Q^{-1}$.

V. Simulation Results for a WMR

In this section, the controller design technique that is introduced in this paper is applied to a path following problem of a WMR. The WMR is shown in Fig. 1 and is assumed to be rigid and to be driven by a torque T to control the heading angle ψ of the WMR. The forward velocity u_0

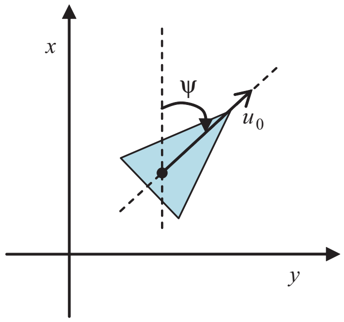

Fig. 1. Schematic of the Wheeled Mobile Robot (WMR)

is in the direction of the X-body axis and it is assumed to be already made constant by the proper design of a cruise controller. The heading angle of the WMR ψ is measured from the positive X-axis in the inertial frame. The kinematic equations of the WMR are

$$\begin{aligned} \dot{y} &= u_0 \ \sin\psi \\ \dot{\psi} &= R \end{aligned} \tag{28}$$

The dynamic equation of the WMR is

$$\dot{R} = \frac{1}{I}T \tag{29}$$

where T is the torque generated by the DC motors and is the input to the system. The moment of inertia of the WMR with respect to the center of mass is represented by $I = 1\ kg.m^2$ In this paper, it is desired that the WMR follows the path $y = 0$. The above differential equations are cast in matrix form as follows

$$\frac{d}{dt}\begin{bmatrix} y \\ \psi \\ R \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} u_0 \ \sin\psi \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \tag{30}$$

Piecewise-affine models of the system in equation (30) are derived for the following state-space partitioning

$$\begin{aligned} \mathcal{R}_1 &= \{x \in \mathbb{R}^3 \mid x_2 \in (-\frac{\pi}{15}, \frac{\pi}{15})\} \\ \mathcal{R}_2 &= \{x \in \mathbb{R}^3 \mid x_2 \in (-\frac{\pi}{5}, -\frac{\pi}{15})\} \\ \mathcal{R}_3 &= \{x \in \mathbb{R}^3 \mid x_2 \in (-\frac{3\pi}{5}, -\frac{\pi}{5})\} \end{aligned} \tag{31}$$

with $\mathcal{R}_4$ symmetric to $\mathcal{R}_2$ and $\mathcal{R}_5$ symmetric to $\mathcal{R}_3$, with respect to origin. The ellipsoidal covering of the state-space partitioning is

$$\begin{aligned} \epsilon_1 &= \{x \mid \| \begin{bmatrix} 0 & \frac{15}{\pi} & 0 \end{bmatrix} x + 0 \| \leqslant 1\} \\ \epsilon_2 &= \{x \mid \| \begin{bmatrix} 0 & \frac{15}{\pi} & 0 \end{bmatrix} x + 2 \| \leqslant 1\} \\ \epsilon_3 &= \{x \mid \| \begin{bmatrix} 0 & \frac{5}{\pi} & 0 \end{bmatrix} x + 2 \| \leqslant 1\} \end{aligned} \tag{32}$$

and the PWA slab model is

$\forall x \in \mathcal{R}_1$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \tag{33}$$

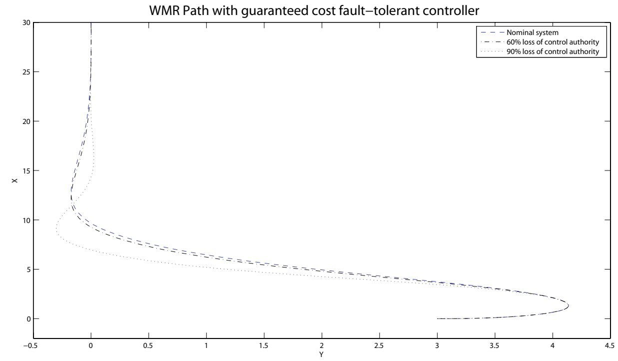

Fig. 2. WMR following path $y = 0$ using the guaranteed cost PWA fault-tolerant controller with $\psi_0 = \pi/2$ and $y_0 = 3$

$\forall x \in \mathcal{R}_2$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 0.907 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} -0.018 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \tag{34}$$

$\forall x \in \mathcal{R}_3$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 0.2891 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} -0.4061 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \tag{35}$$

A guaranteed cost fault-tolerant controller is designed for this system using SeDuMi/YALMIP [11]. An LQR controller is also designed for a linear model of the system (33) with the same weighting matrices in the cost function (8)

$$\begin{aligned} \Upsilon &= \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \\ \Xi &= \begin{bmatrix} 0.1 \end{bmatrix} \end{aligned} \tag{36}$$

The controller design is based on a maximum of 90% loss of effectiveness in the control authority. The resulting PWA controller is

$$\begin{aligned} K_1 &= \begin{bmatrix} -1.0408 & -5.2769 & -9.7329 \end{bmatrix} \\ K_2 = K_4 &= \begin{bmatrix} -1.2946 & -5.0972 & -10.1958 \end{bmatrix} \\ K_3 = K_5 &= \begin{bmatrix} -1.3287 & -5.1018 & -10.2580 \end{bmatrix} \end{aligned} \tag{37}$$

with

$$P = \begin{bmatrix} 0.7247 & 1.2181 & 1.3220 \\ 1.2181 & 3.9754 & 5.0744 \\ 1.3220 & 5.0744 & 10.2458 \end{bmatrix} \tag{38}$$

The LQR controller is

$$K_{LQR} = \begin{bmatrix} 1.0000 & 2.4142 & 2.4142 \end{bmatrix} \tag{39}$$

Simulations are performed for the nonlinear system in nominal, maximum fault (90%) and less severe (60%) fault cases. The resulting paths for the WMR using the guaranteed cost fault-tolerant PWA controller and the LQR controller are plotted in Figs. 2-4. The initial heading angle is $\psi_0 = \pi/2$ for Fig.2 and $\psi_0 = \pi$ for Fig.3 and Fig.4. It is observed that the guaranteed cost fault-tolerant PWA controller, which

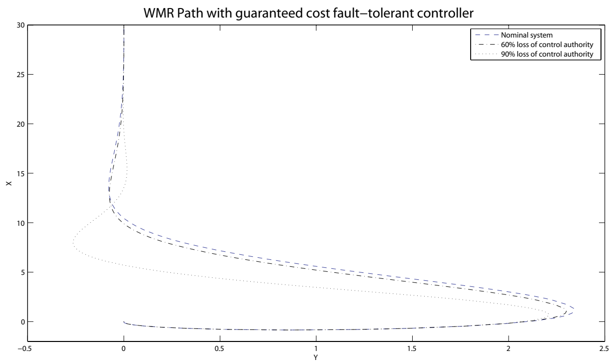

Fig. 3. WMR following path $y = 0$ using the guaranteed cost PWA fault-tolerant controller with $\psi_0 = \pi$ and $y_0 = 0$

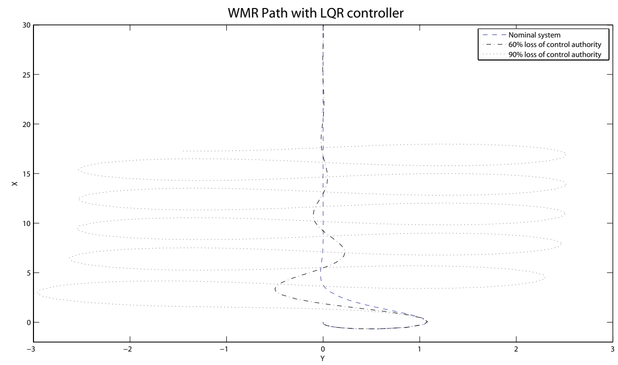

Fig. 4. WMR following path $y = 0$ using the LQR controller with $\psi_0 = \pi$ and $y_0 = 0$

is designed for a PWA model of the system, stabilizes the faulty nonlinear system up to 90% failure. It also keeps the performance of the faulty closed loop system the same as the performance of the nominal system. However, as it is observed in the simulations, the LQR controller fails to stabilize the nonlinear system at 90% failure.

VI. CONCLUSIONS

In this paper, guaranteed cost state feedback controllers are synthesized for PWA models of nonlinear systems to deal with partial loss of control authority. A quadratic common Lyapunov function is applied for stability analysis and controller synthesis for PWA models of nonlinear nominal and faulty systems. An upper bound of a quadratic cost function is minimized for both the nominal and faulty systems. The PWA controller is capable of stabilizing the nominal and faulty systems with guaranteed cost performance in large deviations from the equilibrium point. Simulation results of the guaranteed cost fault-tolerant controller synthesized for the PWA model of a WMR, work with up to %90 loss of control authority, whereas the LQR controller designed for a linear model of the nonlinear system cannot stabilize the system at this percentage of failure.

VII. ACKNOWLEDGMENTS

The authors would like to thank le Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT) and the Natural Sciences and Engineering Research Council of Canada (NSERC) for funding this research.

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