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Robust path planning for mobile robot based on fractional attractive force

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Abstract— **In path planning, potential fields introduce force constraints to ensure curvature continuity of trajectories and thus to facilitate path-tracking design. In previous works, a path planning design by fractional (or generalized) repulsive potential has been developed to avoid fixed obstacles: danger level of each obstacle was characterized by the fractional order of differentiation, and a fractional road was determined by taking into account danger of each obstacle. If the obstacles are dynamic, the method was extended to obtain trajectories by considering repulsive and attractive potentials taking into account position and velocity of the robot with respect to obstacles. Then, a new attractive force based on fractional potential was developed. The advantage of the generalized normalized force is the possibility to control its variation. The curve is continuously varying and depends only on one parameter, the non integer order of the generalized attractive potential. But, in case of robot parameter variations, these two previous attractive forces do not allow to obtain robust path planning. In this paper, a new fractional attractive force for robust path planning of mobile robot is defined. This method allows to obtain robust path planning despite robot mass variations. The robustness of the obtained trajectories is studied. A comparison between a classical method and the proposed approach is presented.**

Index Terms— **Robotics, Mobile robot, Robust Path planning, Fractional potential, Attractive force, Dynamic environment.**

I. INTRODUCTION

Path planning design is the elaboration of a strategy to define a trajectory which will reach a target by avoiding obstacles. The danger concept, which will modify smoothly the trajectory, is therefore of interest for the path planning of a mobile robot. For fixed polygonal obstacles, the connectivity of the robot free space can be captured in a network of nodes and arcs: the roadmap. A variety of methods can be used to obtained roadmaps for path planning. Local methods allowing a real time estimation of the trajectory have been introduced. The better known local methods in path planning are the fictitious potential methods. A fictitious force field is introduced to take into account the dynamics of the robot and to obtain realistic speeds. The concept of a fictitious potential field in path planning is the following [13][19][20] "The potential field concept considers the robot as a charged particle moving under the influence of repulsion potentials for the obstacles, and attraction potentials for the target". The attraction (negative) potential $U_{cible}(\mathbf{M})$ is associated to the target and the positive repulsive Coulomb type potential fields, $U_k(\mathbf{M})$, are defined for each obstacle. The smoothness of the curve obtained with potential field methods makes practical steering and speed control possible. In previous works, a path planning design by fractional (or generalized) repulsive potential has been developed to avoid fixed obstacles: danger level of each obstacle was characterized by the fractional order of differentiation, and a fractional road was determined by taking into account danger of each obstacle. If the obstacles are dynamic, the method was extended to obtain trajectories by considering repulsive and attractive potentials taking into account position and velocity of the robot with respect to obstacles [14][15][16][17][18]. The potential field method is a well known tool to study and to drive the robot motion along a convenient trajectory. It allows an efficient control of the robot speed when it moves from its initial position to its goal. The main idea is to assume that the robot is attracted to the goal and is repulsed away from the obstacles. Then it is guided by a force field system generated by both the goal and the obstacle. Many formula dealing with the force applied on the robot to define its trajectory are found on the literature [1][2][3][4][6][8] and [13]. Then, a new attractive force based on fractional potential was developed. The advantage of the generalized normalized force is the possibility to control its variation. The curve is continuously varying and depends only on one parameter, the non integer order of the generalized attractive potential. But, in case of robot parameter variations, these two previous attractive forces do not allow to obtain robust path planning. In this paper, a new fractional attractive force for robust path planning of mobile robot is defined. This method allows to obtain robust path planning despite robot mass variations. The robustness of the obtained trajectories is studied. A comparison between a classical method and the proposed approach is presented. Section 1 is an introduction on path planning. Section 2 presents fractional mathematical background. Section 3 deals with the fractional attractive force definition and presents a dynamic analysis. Section 4 presents the robustness analysis in frequency domain and the

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comparison between a classical method and the proposed approach. Section 5 presents simulation results. Finally a conclusion is given in Section 6.

II. FRACTIONAL MATHEMATICAL BACKGROUND

During the 19th century some mathematicians such as Abel, Liouville, Riemann and Cauchy were interested in the extension of classical integer differentiation to real orders. Some definitions and proprieties of this mathematical tool are now provided [22][23][24][25][26].

A. Fractional integration

Let $f(t)$ be a continuous real function. The fractional integral of a function $f(t)$ is defined by [22]:

$$(I_{a^+}^n f)(t) \triangleq \frac{1}{\Gamma(n)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-n}} d\tau \quad (1)$$

where $t > a$ and n is the real positive integration order, $\Gamma(n)$ is the Euler Gamma function:

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx. \quad (2)$$

When n is real, the integral in equation (4) is the area of the surface generated by $f(t)$ weighted with the factor:

$$\frac{1}{\Gamma(n)(t-\tau)^{1-n}}.$$

The Laplace transform of the integral of a function $f(t)$ is:

$$\begin{aligned} \mathrm{L}\left\{I_0^n f(t)\right\} &= \int_0^\infty \mathrm{e}^{-pt} \left(\tfrac{1}{\Gamma(n)} \int_0^t \tfrac{f(\tau)}{(t-\tau)^{1-n}} d\tau \right) dt \\ &= \tfrac{1}{s^n} F(s), \end{aligned} \quad (3)$$

where $F(s)$ is the Laplace transform of $f(t)$.

B. Fractional differentiation

The Riemann-Liouville fractional derivative of order n of $f(x)$ is defined as [23]:

$$D_{t_0}^n f(t) \triangleq \left(\frac{d}{dt}\right)^{n+1} \left(I_{t_0}^{1-n} f(t)\right). \quad (4)$$

Second definition (Grünwald's definition) is:

$$D_{t_0}^n f(t) = \lim_{h \to 0} \frac{1}{h^n} \sum_j^{(t-t0)/h} (-1)^j \binom{n}{j} f(t-jh) \quad (5)$$

where

$$\binom{n}{j} = \frac{\Gamma(n+1)}{\Gamma(j+1)\Gamma(n-j+1)}.$$

The two definitions (4) and (5) are equivalent. From equation (5), we note that fractional differentiation is not a local operator. The value of the fractional derivative function at t depends on the whole past of the function. However, in the case where the differentiation order is an integer value, the derivative function depends only on some local points. For example when n = 1:

$$D_0^1 f(t) = \lim_{h \to 0} \frac{f(t) - f(t-h)}{h}. \quad (6)$$

When

$$f(t_0) = D_{t_0}^1 f(t_0) = ... = D_{t_0}^\infty f(t_0) = 0 \quad (7)$$

the Laplace transform is [24]:

$$L\left\{D_0^n f(t)\right\} = s^n F(s). \quad (8)$$

This result is coherent with the classical case where n is an integer. Consequently, it is easy to define a symbolic representation of a dynamic system, such as a transfer function representation.

III. FRACTIONAL ATTRACTIVE FORCE DEFINITION

A. Attractive force definition

Conventionally, the attractive potential is defined as a function of the relative distance between the robot and the target only when the target is a fixed point in space [1][13]. The force applied on the robot is given by:

$$F_{rob} = m_{rob}.a_{rob} \quad (9)$$

with m_{rob} and a_{rob}, the robot mass and acceleration. On an other way, this force is given by:

$$F_{rob} = F_{tar} + F_{att} \quad (10)$$

with

$$F_{tar} = m_{tar}.a_{tar} \quad (11)$$

with F_{tar} the target attractive force, m_{tar} and a_{tar}, the target mass and acceleration. The robot is moving in the XY plane; in the next section the principles and the demonstrations are the same for the two axis; so, they are presented only on x axis.

B. Ge and Cui attractive force

The Ge and Cui method [5][6][7] allows to obtain trajectories in real time by considering repulsive and attractive potentials taking into account position and velocity of the robot with respect to obstacles. The Ge and Cui virtual attractive force is defined by:

$$F_{att} = \alpha_p.(X_{tar} - X_{rob}) + \alpha_v.(V_{tar} - V_{rob}) \quad (12)$$

where X_{tar} and X_{rob} denote the target and robot positions at time t, V_{tar} and V_{rob} denote the target and robot velocities at time t, and α_p and α_v are scalar positive parameters.

By taking m_{tar} equal to m_{rob}, substituting (9) and (11) into (10) gives:

$$m_{rob}.a_{rob} = m_{rob}.a_{tar} + \alpha_p.(X_{tar} - X_{rob}) + \alpha_v.(V_{tar} - V_{rob}) \quad (13)$$

or

$$m_{rob}.(a_{tar} - a_{rob}) + \alpha_v.(V_{tar} - V_{rob}) + \alpha_p.(X_{tar} - X_{rob}) = 0. \quad (14)$$

With

$$\begin{cases} e(t) = X_{tar} - X_{rob} \\ \frac{de(t)}{dt} = V_{tar} - V_{rob} \\ \frac{d^2e(t)}{dt^2} = a_{tar} - a_{rob} \end{cases} \quad (15)$$

the previous relation can be written as a differential equation which gives the x axis robot movement:

$$\frac{d^2e(t)}{dt^2} + \frac{\alpha_v}{m_{rob}}.\frac{de(t)}{dt} + \frac{\alpha_p}{m_{rob}}e(t) = 0. \quad (16)$$

In Laplace domain, the relation becomes:

$$s^2E(s) + \frac{\alpha_v}{m_{rob}}sE(s) + \frac{\alpha_p}{m_{rob}}E(s) = 0. \quad (17)$$

This can be interpreted as a classical control scheme given by Fig. 1, where α_p and α_v are the parameters of a PD controller. The corresponding open loop transfer function $\beta(s)$ is given

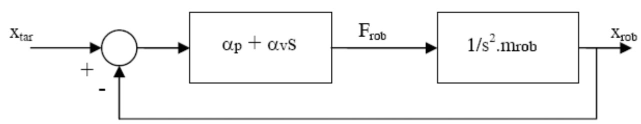

Fig. 1. Dynamic interpretation of Ge and Cui attractive force

by:

$$\beta(s) = \frac{\alpha_v s + \alpha_p}{m_{rob}s^2}. \quad (18)$$

The closed loop transfer function $H(s)$ is deduced:

$$H(s) = \frac{\beta(s)}{1+\beta(s)}, \quad (19)$$

that is to say:

$$H(s) = \frac{\alpha_v s + \alpha_p}{m_{rob}s^2 + \alpha_v s + \alpha_p}, \quad (20)$$

or

$$H(s) = \frac{(\frac{\alpha_v}{\alpha_p})s + 1}{(\frac{m_{rob}}{\alpha_p})s^2 + (\frac{\alpha_v}{\alpha_p})s + 1}. \quad (21)$$

The characteristic equation of the system is given by:

$$E_c(s) = (\frac{m_{rob}}{\alpha_p})s^2 + (\frac{\alpha_v}{\alpha_p})s + 1 = 0. \quad (22)$$

By analogy with the second order system one, the relation (22) can be written:

$$E_c(s) = \frac{s^2}{w_n^2} + \frac{2\xi}{w_n}s + 1 = 0, \quad (23)$$

where w_n is the natural frequency and ξ the damping factor, with:

$$\begin{cases} w_n^2 = \frac{\alpha_p}{m_{rob}} \\ \xi = \frac{\alpha_v}{2\alpha_p}.w_n = \frac{\alpha_v.\sqrt{\alpha_p}}{2\alpha_p\sqrt{m_{rob}}}, \end{cases} \quad (24)$$

lading to:

$$\begin{cases} w_n = \sqrt{\frac{\alpha_p}{m_{rob}}} \\ \xi = \frac{\alpha_v}{2\sqrt{\alpha_p m_{rob}}}. \end{cases} \quad (25)$$

So, the dynamic system behavior depends of the choice of the parameters α_p, α_v and m_{rob}.

- For $\xi > \frac{\sqrt{2}}{2}$, the system is damping an there is not oscillation.
- For $0 < \xi \leq \frac{\sqrt{2}}{2}$, the system has a little damping factor, and the robot converges to the target, but with oscillations.
- For a damping factor $\xi = \frac{\sqrt{2}}{2} = 0.707$, the robot mass is given by:

$$m_{rob} = (\frac{\alpha_v}{2.\xi})^2.\frac{1}{\alpha_p} = \frac{\alpha_v^2}{4\alpha_p.(\frac{\sqrt{2}}{2})^2} = \frac{\alpha_v^2}{2\alpha_p}. \quad (26)$$

So, the condition to avoid oscillation $\xi > \frac{\sqrt{2}}{2}$, leads to the maximal mass defined by $m_{rob} \leq 0.5\frac{\alpha_v^2}{\alpha_p}$.
This last relation shows that the damping factor is dependent of the mass robot m_{rob}. So the obtained trajectory is not robust in front of the mass robot variations. For example, for $m_{rob} = 1$, $\alpha_p = 0.005$, $\alpha_v = 0.1$ are the parameters values chosen in previous example by Ge and Cui to satisfied this relation. In Section V, the Ge and Cui parameters chosen are $\alpha_p = 0.002$, $\alpha_v = 0.8$, and for m_{rob}=160 kg there is not oscillations. So, this dynamic analysis allows to interpret the influence of the Ge and Cui parameters. It also introduces methodology to determine these parameters.

C. Fractional attractive force

The proposed attractive force is based on velocity fractional derivative. It is defined by:

$$F_{att} = \alpha_p.(X_{tar} - X_{rob}) + \alpha_v.\frac{d^n(V_{tar} - V_{rob})}{dt^n} \quad (27)$$

where α_p and α_v are scalar positive parameters.

By taking m_{tar} equal to m_{rob}, the relation gives:

$$m_{rob}.(a_{tar} - a_{rob}) + \alpha_v.\frac{d^n(V_{tar} - V_{rob})}{dt^n} + \alpha_p.(X_{tar} - X_{rob}) = 0. \quad (28)$$

With

$$\begin{cases} e(t) = X_{tar} - X_{rob} \\ \frac{de(t)}{dt} = V_{tar} - V_{rob} \\ \frac{d^2e(t)}{dt^2} = a_{tar} - a_{rob} \end{cases} \quad (29)$$

the previous relation can be written as a differential equation which gives the x axis robot movement:

$$\frac{d^2e(t)}{dt^2} + \frac{\alpha_v}{m_{rob}}.\frac{d^n(e(t))}{dt^n} + \frac{\alpha_p}{m_{rob}}e(t) = 0. \quad (30)$$

In Laplace domain, the relation becomes:

$$s^2E(s) + \frac{\alpha_v}{m_{rob}}s^nE(s) + \frac{\alpha_p}{m_{rob}}E(s) = 0. \quad (31)$$

This can be interpreted as a classical control scheme given by Fig. 2, where α_p and α_v are the parameters of a fractional PD controller. Note here that, taking into account the viscous friction force, changes only the parameters α_v. In this case,

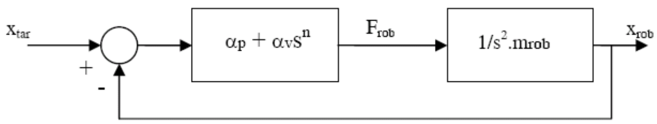

Fig. 2. Dynamic interpretation of the fractional attractive force

the corresponding open loop transfer function $\beta(s)$ is given by:

$$\beta(s) = \frac{\alpha_v s^n + \alpha_p}{m_{rob} s^2}. \tag{32}$$

For $\omega \gg \omega_c = (\frac{\alpha_p}{\alpha_v})^{1/n}$, $\beta(s)$ can be approximated by:

$$\beta(s) \approx \frac{\alpha_v s^n}{m_{rob} s^2} \tag{33}$$

or

$$\beta(s) = \frac{1}{(\frac{m_{rob}}{\alpha_v}) s^{(2-n)}} \tag{34}$$

lading to:

$$\beta(s) = (\frac{\omega_u}{s})^{n'} \tag{35}$$

with:

$$\begin{cases} n' = 2 - n \\ \omega_u = (\frac{\alpha_v}{m_{rob}})^{\frac{1}{(2-n)}}. \end{cases} \tag{36}$$

In the case of a fractional integrator, the resonant factor and the damping factor can be deduced [26]:

$$Q = \frac{1}{sin(2-n')\frac{\pi}{2}} \tag{37}$$

and

$$\xi(n') = -cos(\frac{\pi}{n'}). \tag{38}$$

This last relation shows that the damping factor is independent of the mass robot. This illustrates the robustness of the obtained trajectory with the proposed fractional attractive force.

IV. ROBUSTNESS ANALYSIS

If the mass of the robot is varying, the crossover frequency ω_{cg} and the stability degree are varying, and the path is modified. A comparison of the open loop Nichols diagrams obtain with Ge and Cui and fractional methods is presented. The mass of the robot m_{rob} is equal to [110, 150, 190, 250, 400], the nominal mass is 150 kg, α_p=0.002 and α_v= 0.8, and the fractional order n=0.7 (which leads to a phase margin equal to 60°).

Figure 3 presents the Ge and Cui open loop Nichols diagram. When the robot mass m_{rob} is varying from 110 to 400 kg, respectively, ω_{cg} is varying from 0.0077 rad/s to 0.0027 rad/s, the phase margin from 72° to 48° and the damping factor ξ from 0.85 to 0.45. In particular, for the nominal mass m_{rob}=150 kg, ω_{cg}=0.0058 rad/s, the phase margin is 67°, and the damping factor ξ=0.73. Figure 4 presents the fractional open loop Nichols diagram with same α_p and α_v than Ge and Cui method. In this case, the open loop is characterized by a vertical frequency template in the Nichols diagram, the phase margin is constant, and so the stability degree and the damping factor are also constant. That means that the robot mass variation leads to a very small phase variation (phase margin robustness). In Crone control [26], such robustness leads to a robust damping factor and a robust first overshoot. When the robot mass m_{rob} is varying from 110 to 400 kg, respectively, ω_{cg} is varying from 0.023 rad/s to 0.0086 rad/s, the phase margin from 61° to 59° and the damping factor ξ is quasi constant and equal to 0.7. In particular, for m_{rob}=150 kg, ω_{cg}=0.0184 rad/s, and the phase margin is 61°. In order to compare with the same rapidity, Figure 5 presents the fractional open loop Nichols diagram with same ω_{cg} than Ge and Cui for the nominal robot mass m_{rob}=150 kg, ω_{cg}=0.0058 rad/s. For this, α_p=0.002x0.22 and α_v= 0.8x0.22. When the robot mass m_{rob} is varying from 110 to 400 kg, respectively, ω_{cg} is varying from 0.0073 rad/s to 0.0028 rad/s, the phase margin from 59° to 54° and the damping factor ξ is quasi constant and equal to 0.7. For the nominal robot mass m_{rob}=150 kg, the phase margin is 58°. So, for the same Ge and Cui α_p and α_v parameters, the fractional method leads to a constant stability degree, a constant damping factor, that is to say a robust trajectory and a faster dynamic. In the same way, for the same Ge and Cui ω_{cg} or rapidity, the fractional attractive force leads to a constant stability degree, a constant damping factor, and a robust path planning. Time domain simulations are presented in the next section.

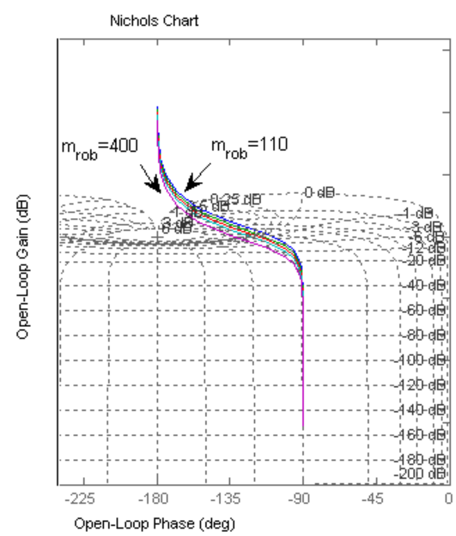

Fig. 3. Ge and Cui open loop Nichols diagram

V. SIMULATION RESULTS

A comparison between the Ge and Cui and the proposed approach is presented. In this simulation, there is not obstacle in order to evaluate the performances of the attractive forces. The mass of the robot m_{rob} is equal to [110, 150, 190, 250,

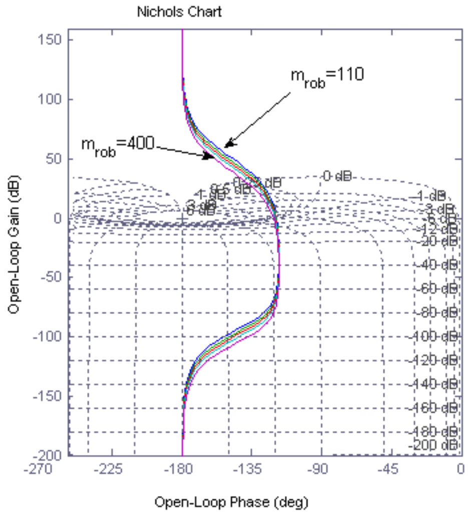

Fig. 4. Fractional open loop Nichols diagram with same α_p and α_v than Ge and Cui method

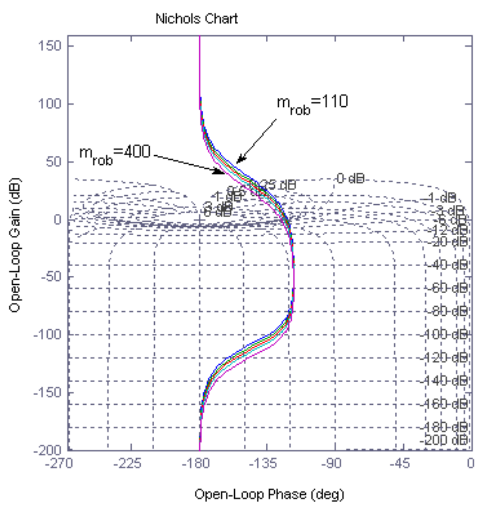

Fig. 5. Fractional open loop Nichols diagram with same ω_{cg} than Ge and Cui method

400], α_p=0.002 and α_v= 0.8 and the fractional order n=0.7. First, the mobile target is moving from point (20 140) and a constant velocity (0.1, 0). The initial position of the robot is (20 20) and its initial velocity (0 0). The simulations (Fig. 6, Fig. 7 and Fig. 8) show that the fractional attractive force allows to obtain a constant damping factor and a constant overshoot, and thus a robust path planning despite robot mass variation. Moreover, the robot reaches faster the target.

In the second example, the mobile target is decreasing from point (20 140) with a constant velocity (0.1, -0.01). The initial position of the robot is (20 20) and its initial velocity (0 0). In a same way, the simulations (Fig. 9, Fig. 10 and Fig. 11) show a robust and faster path planning despite robot mass variation.

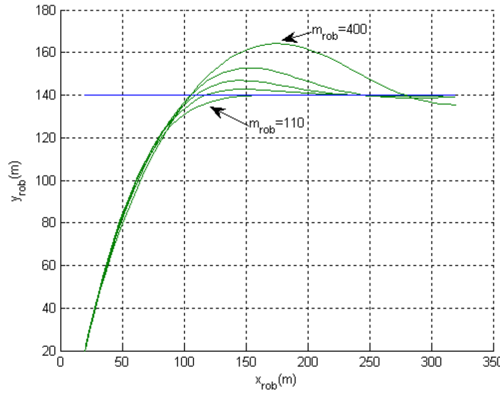

Fig. 6. Robot and Target paths obtained with Ge and Cui approach, with robot mass variation

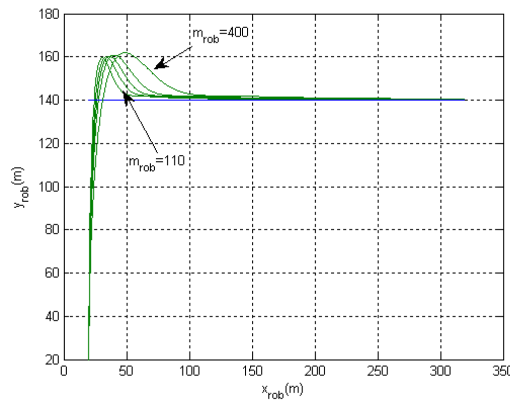

Fig. 7. Robot and Target paths obtained with the proposed fractional approach, with same α_p and α_v than Ge and Cui method, with robot mass variation

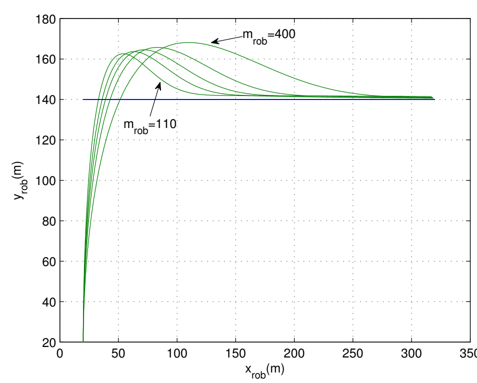

Fig. 8. Robot and Target paths obtained with the proposed fractional approach, with same ω_u than Ge and Cui method, with robot mass variation

VI. CONCLUSION

In this paper, a new fractional attractive force for robust path planning of mobile robot is presented. This method allows to obtain robust and faster path planning despite robot mass variations. Future works concern the use of this attractive force in dynamic environment with mobile target and obstacles.

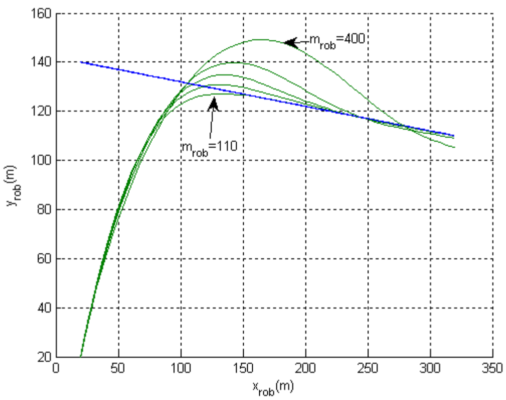

Fig. 9. Robot and Target paths obtained with Ge and Cui approach, with robot mass variation

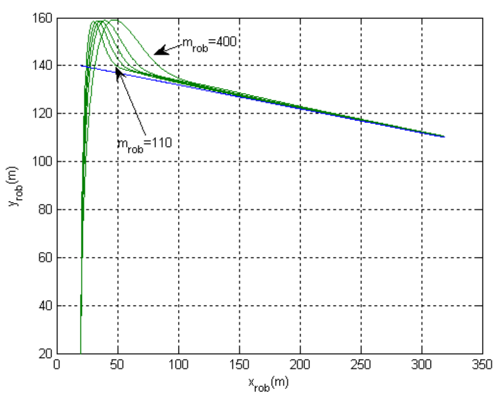

Fig. 10. Robot and Target paths obtained with the proposed fractional approach, with same α_p and α_v than Ge and Cui method, with robot mass variation

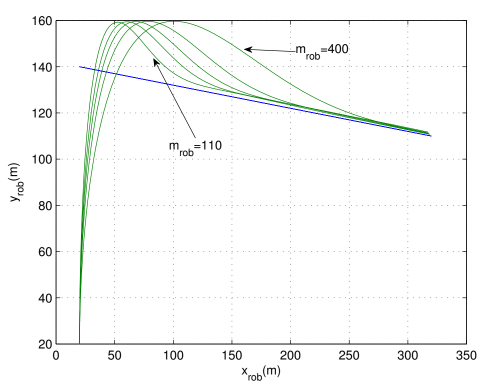

Fig. 11. Robot and Target paths obtained with the proposed fractional approach, with same ω_u than Ge and Cui method, with robot mass variation

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