

# Extremum Seeking Control for Efficient and Reliable Operation of Air-Side Economizers

Pengfei Li, Yaoyu Li, *Member, IEEE*, and John E. Seem, *Member, IEEE*

**Abstract:** Economizers have been recognized as a class of energy-saving devices for heating, ventilating and air conditioning (HVAC) systems that may increase the energy efficiency by taking advantage of outdoor air during cool or cold weather. There has been a tremendous demand for reducing energy consumption of HVAC systems in commercial buildings. However, many economizers do not operate in the expected manner and waste even more energy than before installation, mostly traceable to the unreliable sensors and actuators in practice. In this paper, an extremum seeking control (ESC) based self-optimizing strategy is proposed to minimize the energy consumption. Rather than depending on the unreliable temperature and humidity measurements, the proposed strategy is based on the feedback of chilled-water supply command. The mechanical cooling load is minimized by seeking the optimal outdoor-air damper opening in real time. Such scheme does not need temperature and humidity sensors and depends much less on the knowledge of the economizer model. Simulation was performed on a Modelica based transient model of a single-duct air-handling unit (AHU) developed with Dymola and the AirConditioning Library. The simulation results demonstrated the potential of using ESC to achieve minimal mechanical cooling load in a self-optimizing manner. In addition, an anti-windup ESC scheme is proposed to overcome the ESC windup issue due to actuator (damper) saturation. The simulation results validated the effectiveness of the proposed anti-windup ESC.

## I. INTRODUCTION

**B**UILDINGS are responsible for a large portion of electricity and natural gas demand. Significant amount of energy consumption for buildings is due to the heating, ventilation and air conditioning (HVAC) systems. Improving the efficiency of building HVAC system is thus critical for energy and environmental sustainability. The economizers have been developed as a class of energy saving devices that may increase the energy efficiency by taking advantage of outdoor air during cool or cold weather [1]. Figure 1 is a schematic diagram of a typical single-duct air-handling unit (AHU) and controller. The AHU has a supply fan, three (outdoor air, relief air and mixed air) dampers for controlling airflow between the AHU and the outdoors, heating and cooling coils for conditioning the air, a filter for removing airborne particles, various sensors and

actuators, and a controller that receives sensor measurements (inputs) and computes and transmits new control signals (outputs). The air economizer moves the dampers to let in 100% outdoor air when it is cool but not extremely cold outside. When it is hot outside, the dampers are controlled to provide the minimum amount of outdoor air required for ventilation.

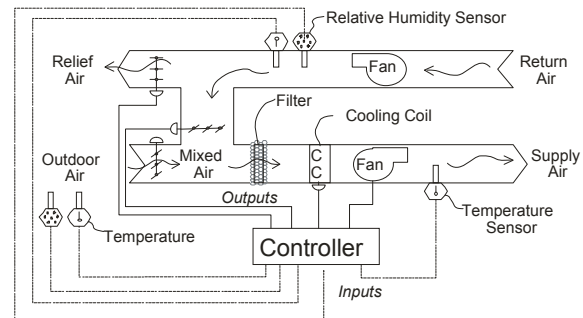


Fig. 1: Single duct air handling unit

The American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE) recommends using economizers based on the cooling capacity size and weather characteristics for the building location [2], as described in the Appendix. ASHRAE [3] describes several control strategies for transitioning between 100% outdoor air and the minimum outdoor air required for ventilation. The control strategies are called “high limit shutoff control for air economizer.” However, in practice, many economizers do not operate as expected and waste even more energy than before installation [4]. Temperature and RH sensor errors can have a large impact on the energy savings or possible penalty of economizer strategies. The NBCIP [5] performed long term performance tests on 20 RH sensors from six manufacturers. Nine of the 20 RH sensors failed during the testing. All of the remaining sensors had many measurements outside of specifications. The largest mean error was 10% RH, and the largest standard deviation of the error was 10.2%. The best performing sensor had a mean error of  $-2.9\%$  RH and a standard deviation of 1.2%. The specifications for the best performing sensor were  $\pm 3\%$ . Control strategies not relying on RH measurement would greatly enhance the reliability of economizer operation.

Modeling and optimal control of air-handling units and economizers have been previously studied [6, 7]. Seem and House [8] describe two model-based strategies for economizer control and use simulations to investigate the energy performance of the strategies in comparison to traditional economizer strategies. While the model-based

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strategies achieved modest energy savings over the traditional strategies for perfect sensors, the performance of all the strategies suffered when sensor errors were introduced. In this study, an on-line self-optimizing control approach is described for air economizers. This approach is considered more robust than other model-based approaches in part because it does not require sensor input to achieve optimal control. This research investigates the application of the extremum seeking control (ESC) [9-13] to optimize the use of outdoor air so as to minimize the energy consumption. The input and output of the proposed ESC framework are the damper opening and power consumption (or equivalently, the chilled water flow rate), respectively. This approach does not rely on the use of RH sensor and accurate model of the economizer for optimal operation. Therefore, it provides a more reliable control strategy for economizer operation. The proposed ESC scheme works as part of a three-state economizer control strategy, as shown in the state diagram in Fig. 2. State 1 uses heating to maintain the supply air temperature. In state 2, outside air is mixed with the return air to maintain the supply air at a given setpoint. In state 3, the ESC is used to control the dampers to minimize the mechanical cooling load. Also, the dampers must be controlled to guarantee enough outdoor air inflow to satisfy the ventilation requirement for the rooms. Figure 3 shows the control regions for different outside air conditions on a psychometric chart. The return air condition was 75 °F and 50% RH, the cooling coil was ideal, and the minimum fraction of outdoor air to supply air was 0.3. The heating region is for state 1, the free cooling region is for state 2, and the three regions that need mechanical cooling are combined into state 3.

In addition to the standard ESC for economizer control, an enhancement on the ESC is proposed: an anti-windup ESC scheme against damper (actuator) saturation. Due to the inherent integral action incorporated in the ESC loop, the integral windup due to the damper saturation would disable the ESC, as to be shown in Section 3. The back-calculation scheme is applied to the ESC loop to achieve the anti-windup capability.

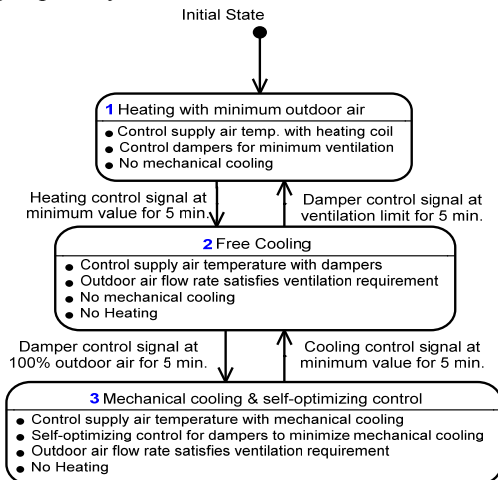


Fig. 2: State transition diagram for the proposed control strategy

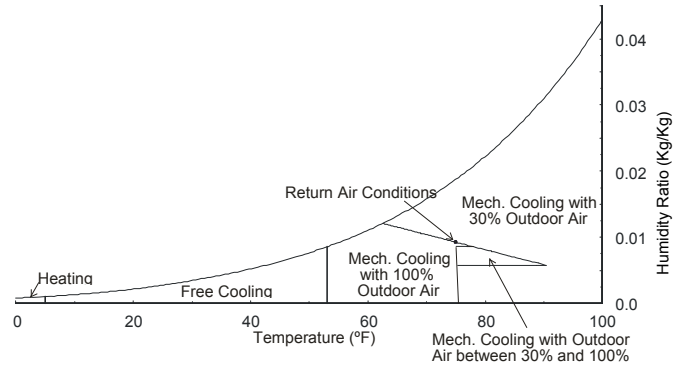


Fig. 3: Control states for different outside air conditions for an ideal coil with return conditions 75 °F and 50% RH

In order to design and simulate the proposed control strategy, a quality dynamic model of economizer is needed. In this study, an economizer simulation model was developed in Modelica [14]. Modelica, as an object-oriented language for physical modeling, has demonstrated its great capability for simulating multi-physical dynamic systems. Compared to most HVAC simulation tools based on steady-state modeling, Modelica based transient modeling has great advantage. In this study, a dynamic model of a single-duct air-side economizer is developed using Dymola (Version 6.1) developed by Dynasim [15], the Modelica Fluid Library (MFL) and the AirConditioning Library (ACL) developed by Modelon [16]. As the ACL has been developed strongly oriented to the automotive air-conditioning systems, some improvements have been made to fit the building HVAC applications. Figure 4 shows the economizer model that we have developed in Dymola [17], which includes air ducts, air mixing box, fans, cooling coil, and a room space.

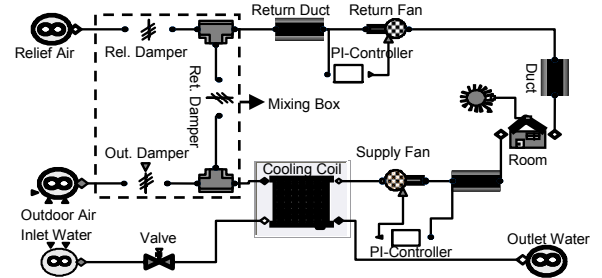


Fig. 4: Dymola layout of the air-side economizer model

## II. EXTREMUM SEEKING CONTROL (ESC) OF ECONOMIZER OPERATION

### A. Overview of ESC

The extremum seeking control deals with the on-line optimization problem of finding an optimizing input  $u_{opt}(t)$  for the generally unknown and/or time-varying cost function  $l(t, u)$ , where  $u(t) \in \mathbb{R}^m$  is the input parameter vector, i.e.

$$u_{opt}(t) = \arg \min_{u \in \mathbb{R}^m} l(t, u). \quad (1)$$

Figure 5 shows the block diagram for a typical ESC system [18]. The measurement of the cost function  $l(t, u)$ , denoted by  $y(t)$ , is corrupted by noise  $n(t)$ . The transfer function  $F_l(s)$  denotes the linear dynamics of the mechanism

that command the control or optimization parameter vector  $u(t)$ .  $F_O(s)$  denotes the transfer function of the sensor dynamics that measure the cost function, which is often a low-pass filter for removing noise from the measurement. The basic components of the ESC loop are defined as follows. The dithering and demodulating signals are denoted by  $d_2^T(t) = [a_1 \sin(\omega_1 t + \alpha_1) \cdots a_m \sin(\omega_m t + \alpha_m)]$  and  $d_1^T(t) = [\sin(\omega_1 t) \cdots \sin(\omega_m t)]$ , respectively, where  $\omega_i$  are the dithering frequencies for each input parameter channel, and  $\alpha_i$  are the phase angles introduced intentionally between the dithering and demodulating signals. The signal vector  $d_2(t)$  contains the perturbation or dither signals used to extract the gradient of the cost function  $l(t, u)$ . These signals work in conjunction with the high-pass filter  $F_{HP}(s)$ , the demodulating signal  $d_1^T(t) = [\sin(\omega_1 t) \cdots \sin(\omega_m t)]$  and the low-pass filter  $F_{LP}(s)$ , to produce a vector-valued signal proportional to the gradient  $\frac{\partial l}{\partial u}(\hat{u})$  of the cost function at the

input of the multivariable integrator, where  $\hat{u}$  is the control input based on the gradient estimation. By integrating the gradient signal, asymptotic stability of the closed loop system will make the gradient vanish, i.e., achieving the optimality. Adding compensator  $K(s)$  may enhance the transient performance by compensating the input/output dynamics. For a detailed explanation of ESC, consult references [12, 13, 18].

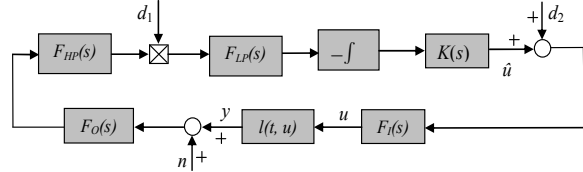
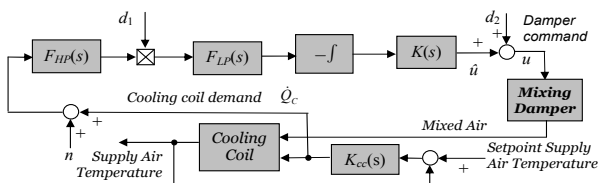


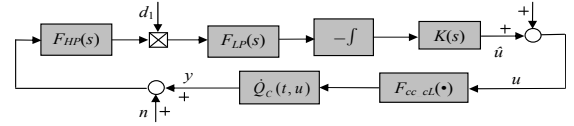
Fig. 5: Block diagram of ESC

### B. ESC for Energy Efficient Operation of Economizers

The proposed ESC based economizer control is illustrated in Fig. 6. This control strategy can be considered as a dual-loop structure. The inner loop is the supply air temperature control for the cooling coil, which has faster dynamics. The outer loop is the damper opening tuning for minimizing the cooling coil demand, which is realized with an ESC framework. The nonlinear performance mapping is from the outdoor air damper (OAD) opening to the cooling coil demand, and the input dynamics are effectively the closed loop dynamics for supply air temperature control. In the three-state economizer operation scheme, as described in Section 1, the ESC is used for state 3 where mechanical cooling is required.



(a) Detailed block diagram



(b) Simplified block diagram

Fig. 6: ESC based economizer control

### C. Extremum Seeking Controller Design

Typical ESC design needs to determine the following parameters: the dither amplitude  $a$ , the dither frequency  $\omega_d$  and phase  $\alpha$ , the high pass filter  $F_{HP}(s)$ , the low pass filter  $F_{LP}(s)$ , and the dynamic compensator  $K(s)$ . Based on averaging analysis, the dither frequency should be relatively large with respect to the adaptation gain, but should not be too large to trigger unmodeled dynamics and make the system more sensitive to measurement noise. Also, if the dither frequency is well out of the bandwidth of the input dynamics, the roll-off in the magnitude response will slow down the convergence [13]. Therefore, dither frequency  $\omega$  is typically chosen to be just a moderate value smaller than the cut-off frequency of the input dynamics as long as it is enough to separate the time scales of the dither signal and the inner loop dynamics. Generally, the dynamic compensator should be designed based on the dither signal, adaptation gain and the frequency responses of the input dynamics. Particularly, a proper proportional-derivative (PD) action can increase the phase margin of the input dynamics and thus make the inner loop more stable. However, extreme values of the adaptation gain, especially the derivative gain, will make the system more affected by noise and thus destabilize the system. Further design guidelines are summarized as follows.

- 1) The dither frequency should be in the passband of the high pass filter and in the stopband of the low pass filter, and it should be below the cut-off frequency of the input dynamics of the respective channel.
- 2) The dither amplitude should be chosen to be sufficiently small to avoid large oscillation of output, and meanwhile sufficiently large to overcome the noise effect.
- 3) The dither phase angle should be chosen such that  $\theta = \angle F_i(j\omega) + \angle F_{HP}(j\omega) + \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , and it is desirable to make  $\theta$  close to 0.

### D. Anti-windup ESC

Actuator saturation is often encountered in control systems. To our best knowledge, the issue of actuator saturation has not been discussed for extremum seeking control. For the economizer control, the actuator saturation will happen when it is cool or hot outside. For instance, when the outdoor air is around 53°F, the outdoor air damper will be positioned fully open to allow 100% outdoor air to enter the AHU. When it is warmer than 100 °F, the damper will be closed to a minimum opening which only maintain the lowest ventilation for indoor air quality [19]. In other words, the optimal reference input is not inside the saturation limit, but rather at either limit point. Transition

between the ESC operation and the non-ESC operation is affected by the saturation issue. The averaging analysis of ESC [18] showed that, at a large time scale, the ESC can be deemed as a linear system regulating the gradient signal with a PI controller. When saturation presents in the ESC loop, integrator wind-up is unavoidable, and in consequence, leads to the undesirable windup phenomena. Later in the simulation section, results will show that, due to the windup issue, the ESC action may be totally disabled even when the air condition changes to a point demanding its re-activation. It is thus necessary to modify the standard ESC structure in order to avoid integrator windup.

There has been much work reported in the field of anti-windup control (AWC) [20, 21]. In order to keep the simple nature of ESC, a back-calculation method is proposed as shown in Fig. 7, following the spirit of the references [21-23]. The difference between the input and output of the actuator is fed back to the input end of the integrator through some gain factor. Our simulation results have demonstrated that this method works well to prevent the integrator windup in ESC system. Future research needs to investigate the design guidelines for the proposed anti-windup ESC. The analysis will be based on combining the existing method for back-calculation AWC and the averaging analysis [24, 25].

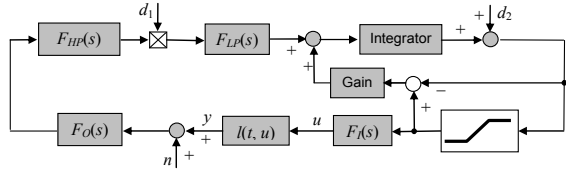


Fig. 7: Block diagram for the anti-windup ESC

### III. SIMULATION STUDY

#### A. ESC with Standard Design

As previously stated, the control objective in this study is to minimize the chilled water flow rate of the cooling coil by tuning the OAD opening. The following second-order linear model was used to approximate the input dynamics:

$$F_I(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (2)$$

where  $\omega_n$  is the undamped natural frequency and  $\zeta$  is the damping ratio. The input dynamics from the OAD opening to the chilled water flow rate was approximated based on several inner loop simulations. Time-domain system identification was used based on the step response based parameter estimation [26]. Fast (3 seconds) ramp input was used to approximate step input in order to remove the output jitter due to the inner loop PID control. The  $\omega_n$  was estimated from 10% to 90% rise time  $T_r = \frac{1.8}{\omega_n}$  [27].

A suitable  $\omega_n$  could be then obtained by further manually tuning the responses based on the above approximation. A group of tests indicate that  $\omega_n$  ranged from 0.105 to 0.121 rad/sec. As an average approximation,  $\omega_n$  was chosen to be 0.117 rad/sec. The damping ratio was estimated to be 1. To

properly separate the dither signal and plant dynamics, the dither frequency  $\omega$  is selected as approximately one-third of the inner loop cutoff frequency. Next, the high pass filter  $F_{HP}(s)$  was selected as:

$$F_{HP}(s) = \frac{s^2}{s^2 + 2 \cdot 0.65 \cdot 0.008s + 0.008^2} \quad (3)$$

which has unit gain at  $\omega_d$ . The low pass filter was designed as

$$F_{LP}(s) = \frac{0.0095^2}{s^2 + 2 \cdot 0.65 \cdot 0.0095s + 0.0095^2} \quad (4)$$

which has approximately 10dB and 20dB attenuation at  $\omega_d$  and  $2\omega_d$ , respectively. The dither amplitude was designed to have 5% opening variation. To compensate for the phase lag and phase lead from the input dynamics  $F_I(s)$  and the high pass filter  $F_{HP}(s)$ , the dither phase  $\alpha$  was selected as  $-0.0083$  radian, which makes  $\theta = \angle F_I(j\omega) + \angle F_{HP}(j\omega) + \alpha \approx 0^\circ$ .

The designed ESC controller was first tested with a fixed operating condition. The initial air temperature and RH were set to be 297.15K and 35%, respectively. To be consistent with standard economizer design conditions, the supply air temperature is controlled at 286K (55°F) and the return air temperature is maintained around 297K (75°F) by providing a constant heat input to the indoor space. A minimal OAD opening of 30% is assumed to ensure adequate indoor air quality. In addition, the indoor humidity gain is assumed to be generated by people so that the RH of the return air is maintained around 50%. The static mapping from the OAD opening to the chilled water flow rate was approximated with the steady-state simulation data shown in Fig. 8. As can be observed, the optimal OAD opening and chilled water flow rate are around 51% and 6.436Kg/s. Figure 9 shows the time histories of the optimized chilled water flow rate and OAD opening. The system started at the minimal OAD opening, i.e. 30%. The ESC controller was turned on at about  $t = 2000$  seconds, and the system output was brought to the optimum with the settling time of about 750 seconds. At steady state, the mean values of OAD opening and chilled water flow rate were 50.37% and 6.45Kg/s, respectively, which are off from the optimum in Fig. 8 by only 0.63% and 0.014Kg/s, respectively.

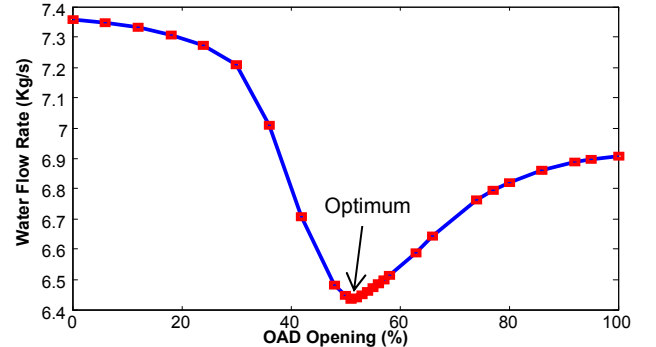


Fig. 8: Static map from OAD opening to chilled water flow rate



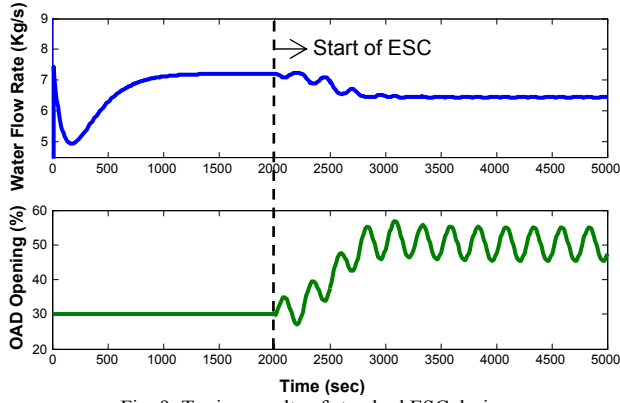


Fig. 9: Tuning results of standard ESC design

A further study was conducted to test the ESC tunings with different initial OAD openings. Table 1 summarizes the tuning results of different initial conditions based on the same outdoor air condition as above. Note that under different initial damper positions, especially when the initial OAD opening is greater than 80%, the settling time increased considerably, most likely due to particular shape of the performance map. Work is under way to improve the transient performance.

TABLE I:  
SUMMARY OF ESC TUNING RESULTS BASED ON DIFFERENT INITIAL OUTDOOR DAMPER POSITIONS

Test No.	Initial OAD (%)	$t_{s,2\%}$ (sec)	Steady-State OAD (%)	Steady-State Flow Rate (Kg/s)
1	20	1780	50.36	6.45
2	25	1038	50.37	6.46
3	40	268	50.36	6.45
4	65	870	50.36	6.45
5	80	2130	50.38	6.45
6	90	4120	50.42	6.45

### B. Anti-Windup ESC

Simulation study was also conducted to verify the effectiveness of the proposed anti-windup ESC. Assume that a 30% damper opening is still the minimum requirement for indoor air quality, and this was set as the lower saturation limit. The upper saturation limit was 100%. The initial outdoor air damper opening was again set at 30%, same as the lower saturation limit. The change of outdoor air conditions is represented on the Psychrometric chart shown in Fig. 10. The initial air temperature and RH were again set to be 297.15K and 35% (State 1), respectively. Figure 12 shows the integrator windup phenomenon when the standard ESC scheme was applied. Driven by the ESC, the damper opening first reached 50.37%, which was supposed to be the first optimum. Then the outdoor air temperature was suddenly (100 seconds ramp) increased to 305.15K (89.6°F) at 5000s (State 2). Figure 11 shows the static map of chilled water flow rate to OAD opening at state 2. As shown in Fig. 12, the ESC was able to detect such change and the damper opening was decreased from 50.37% to 30% which was the corresponding achievable optimal setting. However, when another 100 seconds ramp signal was applied to bring the

outdoor air temperature and RH (State 3) back to the initial settings at 8000s, the new optimal opening was supposed to be switched back to the first optimum, i.e. 50.37%. However, the results show that the standard ESC was unable to respond to such change by increasing the damper opening. Rather the damper appeared “stuck” at the previous position. In comparison, as shown in Fig. 13, applying the back-calculation based anti-windup ESC starting from 2000s effectively solved this problem. Therefore, the proposed anti-windup ESC scheme is shown to be capable of handling the saturation windup problem. Note that the OAD opening kept oscillating in Fig. 13. Such behavior is acceptable for durability concerns. Since in the actual implementation, the ESC tuning will stop once the optimum is reached. For building HVAC operation, optimum variations traceable to changes from thermal responses in the conditioned space and/or changes from the outdoor air conditions are usually slow. In addition, the period of the dither signal was set to be 250 seconds in this study, which could hardly damage the dampers.

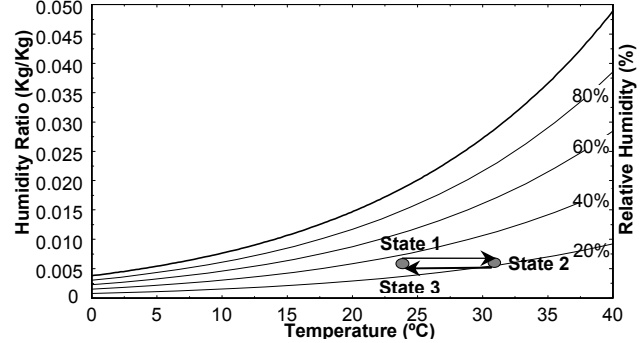


Fig. 10: Change of outdoor air conditions on the psychrometric chart

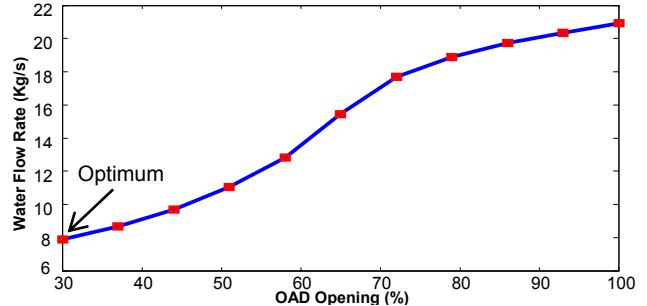


Fig. 11: Static Map from OAD opening to water flow rate (state 2)

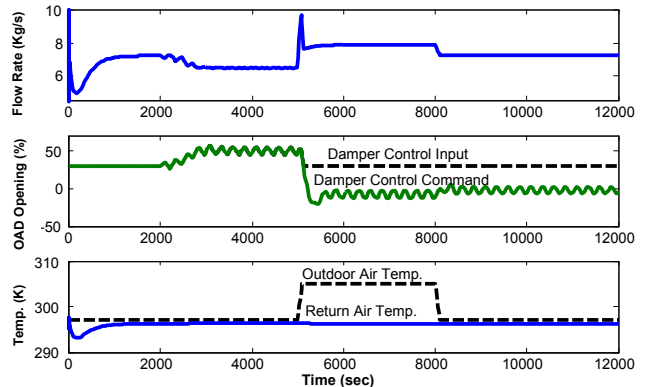


Fig. 12: Integral windup of standard ESC under actuator saturation

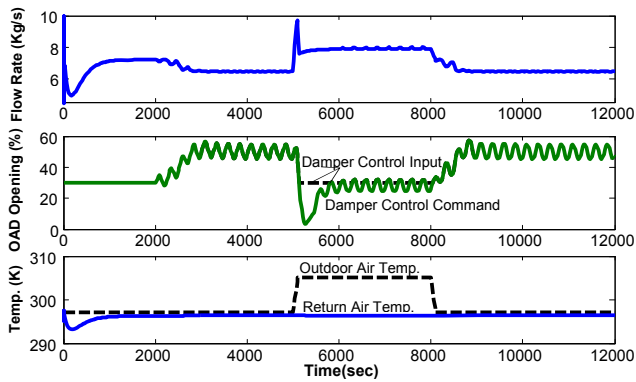


Fig. 13: Anti-windup ESC under damper saturation

#### IV. CONCLUSION

Extremum seeking control has been proposed as a self-optimizing strategy to achieve more efficient and reliable operation for air-side economizers. The mechanical cooling load can be minimized via searching for the optimal outdoor air damper opening. Simulation study has been conducted on a Modelica based transient model of a single-duct AHU. Simulation results have demonstrated that the optimal damper opening can be obtained based on the feedback of chilled water flow command. Due to the inherent integral element in ESC, the windup phenomenon has been observed through simulation, which may disable ESC during practical operation. An anti-windup ESC strategy has been proposed. Its effectiveness was evaluated with simulation. The proposed ESC strategy indicates a perspective of “sensor-free” operation for economizers, which would greatly enhance the efficiency and reliability for such devices, and also reduce the cost of system operation and energy consumption.

#### REFERENCES

- [1] EPA, 2000, "Energy Cost and IAQ Performance of Ventilation Systems and Controls," EPA Report, EPA-4-2-S-01-001.
- [2] ASHRAE, 2004, "Energy Standard for Buildings except Low-Rise Residential Buildings," Technical Report, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 1791 Tullie Circle NE, Atlanta, GA 30329.
- [3] ASHRAE, 2004, "90.1 User's Manual ANSI/ASHRAE/IESNA Standard 90.1-2004, Technical Report, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- [4] AEC, 2007, "Advanced Automated Hvac Fault Detection and Diagnostics Commercialization Program," Technical Report No. 500-03-030, California Energy Commission, Sacramento, CA.
- [5] NBCIP, 2005, "Product Testing Report Supplement: Duct-Mounted Relative Humidity Transmitters," Technical Report, National Building Controls Information Program.
- [6] Song, L., and Liu, M., 2004, "Optimal Outside Airflow Control of an Integrated Air-Handling Unit System for Large Office Buildings," *Journal of Solar Energy*

- Engineering, Transactions of the ASME*, 126(1), pp. 614-619.
- [7] Guo, C., Song, Q., and Cai, W., 2007, "A Neural Network Assisted Cascade Control System for Air Handling Unit," *IEEE Transactions on Industrial Electronics*, 54(1), pp. 620-628.
- [8] Seem, J. E., and House, J. M., 2009, "Using Models and Optimization to Control Air Economizers," *Applied Energy*, submitted for publication.
- [9] Blackman, P. F., 1962, *An Exposition of Adaptive Control*, Pergamon Press, Extremum-Seeking Regulators.
- [10] Sternby, J., 1980, "Extremum Control Systems: An Area for Adaptive Control?" San Francisco, CA.
- [11] Åström, K. J., and Wittenmark, B., 1995, *Adaptive Control*, Addison-Wesley, Reading, Mass.
- [12] Krstić, M., and Wang, H.-H., 2000, "Stability of Extremum Seeking Feedback for General Nonlinear Dynamic Systems," *Automatica*, 36(4), pp. 595-601.
- [13] Krstić, M., 2000, "Performance Improvement and Limitations in Extremum Seeking Control," *Systems and Control Letters*, 39(5), pp. 313-326.
- [14] Modelica, 2009, <http://www.modelica.org/>
- [15] Dynasim, 2007, <http://www.dynasim.se/dynasim.htm>
- [16] Modelon, 2007, <http://www.modelon.se/>
- [17] Li, P., Li, Y., and Seem, J. E., 2008, "Dynamic Modeling and Control of an Air-Side Economizer Using Modelica," University of Applied Sciences Bielefeld, Germany, pp. 447-464.
- [18] Rotea, M. A., 2000, "Analysis of Multivariable Extremum Seeking Algorithms," *Proceedings of the American Control Conference*, 1, pp. 433-437.
- [19] ASHRAE, 2001, *ASHRAE Standard : Ventilation for Acceptable Indoor Air Quality*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, GA.
- [20] Åström, K. J., and Wittenmark, B., 1997, *Computer-Controlled Systems: Theory and Design*, Prentice Hall Information and System Sciences Series, Prentice Hall, Upper Saddle River, N.J.
- [21] Åström, K. J., and Rundqwist, L., 1989, "Integrator Windup and How to Avoid It," pp. 1693-1698.
- [22] Fertik, H. A., and Ross, C. W., 1967, "Direct Digital Control Algorithms with Anti-Windup Feature," *ISA Transactions*, pp. 317-328.
- [23] Åström, K. J., 1987, *Advanced Control in Computer Integrated Manufacturing. Proceedings of the Thirteenth Annual Advanced Control Conference. Advanced Control Methods – Survey and Assessment of Possibilities.*
- [24] Sanders, J. A., Verhulst, F., and Murdock, J. A., 2007, *Averaging Methods in Nonlinear Dynamical Systems*, Springer, New York.
- [25] Khalil, H. K., 2002, *Nonlinear Systems*, Prentice Hall, Upper Saddle River, N.J.
- [26] Astrom, K. J., and Wittenmark, B., 1996, *Computer-Controlled Systems: Theory and Design*, Prentice Hall.
- [27] Franklin, G., 2005, *Feedback Control of Dynamic Systems*, Prentice Hall.