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Neural Network Control of Quadrotor UAV Formations[1]

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Abstract—**In this paper, a novel framework for leader-follower formation control is developed for the control of multiple quadrotor unmanned aerial vehicles (UAVs) based on spherical coordinates. The control objective for the follower UAV is to track its leader at a desired- separation, angle of incidence, and a bearing by using an auxiliary velocity control. Then, a novel neural network (NN) control law for the dynamical system is introduced to learn the complete dynamics of the UAV including unmodeled dynamics like aerodynamic friction. Additionally, the interconnection dynamic errors between the leader and its followers are explicitly considered, and the stability of the entire formation is demonstrated using Lyapunov theory. Numerical results verify the theoretical conjectures.**

Index Terms—**Formation Control, Quadrotor UAV, Neural Networks, Lyapunov Stability**

I. Introduction

In recent years, quadrotor helicopters have become a popular unmanned aerial vehicle (UAV) platform, and their control has been undertaken [1] by many researchers. However, a team of UAVs working together is often more effective than a single UAV in scenarios like surveillance, search and rescue, and perimeter security. Therefore, the formation control of UAVs [2-6] has been proposed.

The authors in [2] present a modified leader-follower framework with a model predictive nonlinear control algorithm without proof of convergence and stability. In [3], a kinematic-based formation control law is proposed while ignoring the individual dynamics and the formation dynamics of UAVs; proof of stability is not provided. The work [4] offers an algorithm for perimeter security using UAVs without considering the UAV and formation dynamics.

On the other hand in [5], cylindrical coordinates and contributions from wheeled mobile robot formation control [8] are considered for leader-follower based formation control scheme by assuming dynamics are known. In [6] experimental results are provided by using a dynamic model and assuming that measured position and velocity of the leader to its followers are communicated. In [7], a robust formation controller is proposed based on higher order sliding mode controllers in the presence of disturbances.

By contrast, in this work, a new leader-follower formation control framework for quadrotor UAVs based on spherical coordinates is introduced where the desired position of a follower UAV is specified using a desired separation, s_d, a desired- angle of incidence, α_d and bearing, β_d. Then, a new control law is derived using neural networks (NN) to learn the complete dynamics of the UAV online, including unmodeled dynamics like aerodynamic friction and in the presence of bounded disturbances. Although a quadrotor UAV is underactuated, a novel virtual NN control input scheme is proposed which allows all six degrees of freedom of the UAV to be controlled using only four control inputs.

II. Background

A. Quadrotor UAV Dynamics

Consider a quadrotor UAV with six DOF defined in the inertial coordinate frame , E^a, as $[x, y, z, \phi, \theta, \psi]^T \in E^a$ where $\rho = [x, y, z]^T \in E^a$ are the position coordinates of the UAV and $\Theta = [\phi, \theta, \psi]^T \in E^a$ describe its orientation referred to as roll, pitch, and yaw, respectively. The translational and angular velocities are expressed in the body fixed frame attached to the center of mass of the UAV, E^b, and the dynamics of the UAV in the body fixed frame can be written as [1]

$$M\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix} = \bar{S}(\omega)\begin{bmatrix}v\\ \omega\end{bmatrix} + \begin{bmatrix}N_1(v)\\ N_2(\omega)\end{bmatrix} + \begin{bmatrix}G(R)\\ 0_{3x1}\end{bmatrix} + U + \tau_d, \qquad (1)$$

where $U = \begin{bmatrix}0 & 0 & u_1 & u_2^T\end{bmatrix}^T \in \Re^6$, $M = diag\{mI_3, J\} \in \Re^{6x6}$, m is a positive scalar that represents the total mass of the UAV, $J \in \Re^{3x3}$ represents the positive definite inertia matrix, $v(t) = [v_{xb}, v_{yb}, v_{zb}]^T \in \Re^3$ represents the translational velocity, $\omega(t) = [\omega_{xb}, \omega_{yb}, \omega_{zb}]^T \in \Re^3$ represents the angular velocity, $N_i(\bullet) \in \Re^{3x1}, i = 1,2$, are the nonlinear aerodynamic effects, $\bar{S}(\omega) = diag\{-mS(\omega), S(J\omega)\} \in \Re^{6x6}$, $u_1 \in \Re^1$ provides the thrust along the z-direction, $u_2 \in \Re^3$ provides the rotational torques, $\tau_d = [\tau_{d1}^T, \tau_{d2}^T]^T \in \Re^6$ and $\tau_{di} \in \Re^3$, $i = 1,2$ represents unknown, but bounded disturbances such that $\|\tau_d\| < \tau_M$ for all time t, with τ_M being an unknown positive constant, $I_{nxn} \in \Re^{nxn}$ is an nxn identity matrix, and $0_{mxl} \in \Re^{mxl}$ represents an mxl matrix of all zeros. Furthermore, $G(R) \in \Re^3$ represents the gravity vector and $S(\bullet) \in \Re^{3x3}$ is the general form of a skew symmetric matrix defined in [1].

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The matrix $R(\Theta) \in \Re^{3x3}$ is the translational rotation matrix which is used to relate a vector in the body fixed frame to the inertial coordinate frame defined as [1]

$$R(\Theta) = R = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \quad (2)$$

where the abbreviations $s_{(\bullet)}$ and $c_{(\bullet)}$ have been used for $\sin(\bullet)$ and $\cos(\bullet)$, respectively. It is important to note that $\|R\|_F = R_{\max}$ for a known constant $R_{\max}$, $R^{-1} = R^T$, $\dot{R} = RS(\omega)$ and $\dot{R}^T = -S(\omega)R^T$. It is also necessary to define a rotational transformation matrix from the fixed body to the inertial coordinate frame as in [1]

$$T(\Theta) = T = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{bmatrix} \quad (3)$$

where the abbreviation $t_{(\bullet)}$ has been used for $\tan(\bullet)$. The transformation matrix T is bounded as long as $-(\pi/2) < \phi < (\pi/2)$, $-(\pi/2) < \theta < (\pi/2)$ and $-\pi \le \psi \le \pi$. These regions will be referred to as the *stable operation regions* of the UAV, and under these flight conditions, it is observed that $\|T\|_F < T_{\max}$ for a known constant $T_{\max}$.

Finally, the kinematics of the UAV can be written as

$$\dot{\rho} = Rv, \qquad \dot{\Theta} = T\omega \quad (4)$$

B. Neural Networks

In this work, two-layer NNs are considered consisting of one layer of randomly assigned constant weights $V_N \in \Re^{axL}$ in the first layer and one layer of tunable weights $W_N \in \Re^{Lxb}$ in the second with a inputs, b outputs, and L hidden neurons. The sigmoid activation function is considered here. Furthermore, on any compact subset of $\Re^n$, the target NN weights are bounded by a known positive value W_M such that $\|W_N\|_F \le W_M$ [9]. For complete details of the NN and its properties, see [9].

C. Three Dimensional UAV Formation Framework

Throughout the development, the follower UAVs will be denoted with a subscript *'j'* while the formation leader will be denoted by the subscript *'i'*. To begin, an alternate reference frame is defined by rotating the inertial coordinate frame about the z-axis by the yaw angle, ψ_j, and denoted by E_j^a. In order to relate a vector in E^a to E_j^a, the transformation matrix is given by

$$R_{aj} = \begin{bmatrix} \cos(\psi_j) & \sin(\psi_j) & 0 \\ -\sin(\psi_j) & \cos(\psi_j) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where $R_{aj}^T = R_{aj}^{-1}$. The objective of the proposed leader-follower formation control is for the follower UAV to

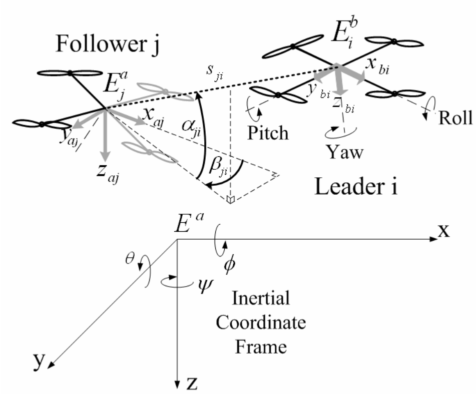

Fig. 1. UAV leader-follower formation control

maintain a desired separation, $s_{jid} \in \Re$, at a desired angle of incidence, $\alpha_{jid} \in E_j^a$, and bearing, $\beta_{jid} \in E_j^a$, with respect to its leader. The incidence angle is measured from the $x_{aj}-y_{aj}$ plane of follower *j* while the bearing angle is measured from the positive x_{aj}-axis as shown in Fig. 1. It is important to observe that each quantity is defined relative to the follower *j* instead of the leader *i* [5],[8]. To specify a unique configuration of follower *j* with respect to its leader, the desired yaw of follower *j* is selected to be the yaw angle of leader *i*, $\psi_i \in E^a$ as in [2]. Then, the measured separation between follower *j* and leader *i* is written as

$$\rho_i - \rho_j = R_{aj}^T s_{ji} \Xi_{ji} \quad (6)$$

where

$$\Xi_{ji} = [\cos(\alpha_{ji})\cos(\beta_{ji}) \;\; \cos(\alpha_{ji})\sin(\beta_{ji}) \;\; \sin(\alpha_{ji})]^T \quad (7)$$

Thus, to solve the leader-follower formation control problem in the proposed framework, a control velocity must be derived to ensure

$$\left. \begin{array}{l} \lim_{t\to\infty}(s_{jid} - s_{ji}) = 0, \; \lim_{t\to\infty}(\beta_{jid} - \beta_{ji}) = 0, \\ \lim_{t\to\infty}(\alpha_{jid} - \alpha_{ji}) = 0, \; \lim_{t\to\infty}(\psi_{jd} - \psi_j) = 0 \end{array} \right\}. \quad (8)$$

Throughout the development, the desired separation, angle of incidence and bearing s_{jid}, α_{jid} and β_{jid}, respectively, will be taken as constants, while it is assumed that each UAV has knowledge of its own constant total mass, $m_{(\bullet)}$, where $(\bullet)$ is i for the leader and j for the follower. Additionally, it will be assumed that reliable communication between the leader and its followers is available [4],[6], and the leader communicates its measured orientation and angular rate vector, Θ_i and ω_i, respectively, and its *desired* states, $\psi_{id}, \dot{\psi}_{id}, \ddot{\psi}_{id}$, $v_{id}, \dot{v}_{id}$. Future work will relax this assumption.

It is worth noting that communicating the desired states as opposed to the measured states does not necessarily reduce the amount of communication overhead. The benefit of considering the desired states of the leader in the design of the follower UAVs' control laws becomes significant when compensating for the formation dynamics which become incorporated in the follower UAVs dynamic controller design. Further, considering the desired states reduces the reliance on noisy sensor measurements.

III. Leader-Follower Formation Tracking Control

A. Follower UAV Control Law

Given a leader i subject to the dynamics and kinematics (1), and (4), respectively, define a reference trajectory at a desired separation s_{jid}, at a desired angle of incidence, α_{jid}, and bearing, β_{jid} for follower j given by

$$\rho_{jd} = \rho_i - s_{jid} R_{ajd}^T \Xi_{jid} \tag{9}$$

where R_{ajd} is defined as in (5) but written in terms of ψ_{jd}, and Ξ_{jid} is defined as in (7) but written in terms of the desired angle of incidence and bearing, $\alpha_{jid}, \beta_{jid}$, respectively. Next, using (6) and (9), define the position tracking error as

$$e_{j\rho} = \rho_{jd} - \rho_j = s_{ji} R_{aj}^T \Xi_{ji} - s_{jid} R_{ajd}^T \Xi_{jid} \in E^a \tag{10}$$

which can be measured using local sensor information. To form the position tracking error dynamics, it is convenient to rewrite (10) as $e_{j\rho} = \rho_i - \rho_j - R_{ajd}^T s_{jid} \Xi_{jid}$ revealing

$$\dot{e}_{j\rho} = R_i v_i - R_j v_j - \dot{R}_{ajd}^T s_{jid} \Xi_{jid}. \tag{11}$$

Next, select the desired translational velocity of follower j $v_{jd} = [v_{jdx} \;\; v_{jdy} \;\; v_{jdz}]^T \in E^b$, to stabilize (11) is written as

$$v_{jd} = R_j^T (R_i v_{id} - s_{jid} \dot{R}_{ajd}^T \Xi_{jid} + K_{j\rho} e_{j\rho}) \tag{12}$$

where $K_{j\rho} = diag\{k_{j\rho x}, k_{j\rho y}, k_{j\rho z}\} \in \Re^{3x3}$ is a diagonal positive definite design matrix all with positive design constants. Next, the translational velocity tracking error systems for follower j and leader i is defined as

$$e_{jv} = [e_{jvx} \;\; e_{jvy} \;\; e_{jvz}]^T = v_{jd} - v_j \tag{13}$$

and $e_{iv} = v_{id} - v_i$, respectively. Applying (12) to (11) while observing $v_j = v_{jd} - e_{jv}$ and similarly $v_i = v_{id} - e_{iv}$, reveals the closed loop position error dynamics to be rewritten as

$$\dot{e}_{j\rho} = -K_{j\rho} e_{j\rho} + R_j e_{jv} - R_i e_{iv}. \tag{14}$$

Next, the translational velocity tracking error dynamics are developed. Differentiating (13), observing

$$\begin{aligned}\dot{v}_{jd} = &-S(\omega_j) v_{jd} + R_j^T \left(R_i S(\omega_i) v_{id} + R_i \dot{v}_{id} - \ddot{R}_{ajd}^T s_{jid} \Xi_{jid} \right) \\ &+ R_j^T K_{j\rho} \left(R_i v_i - R_j v_j - \dot{R}_{ajd}^T s_{jid} \Xi_{jid} \right)\end{aligned},$$

adding and subtracting $R_j^T (K_{j\rho} (R_i v_{id} + R_j v_{jd}))$, and substituting the translational velocity dynamics in (1) allows the velocity tracking error dynamics to be written as

$$\begin{aligned}\dot{e}_{jv} = \dot{v}_{jd} - \dot{v}_j = &-N_{j1}(v_j)/m_j - S(\omega_j) e_{jv} - G(R_j)/m_j \\ &- u_{j1} E_{jz}/m_j - \bar{\tau}_{jd1} - R_j^T K_{j\rho} (R_i e_{iv} - R_j e_{jv}) + \\ &R_j^T \left(R_i S(\omega_i) v_{id} + R_i \dot{v}_{id} - \ddot{R}_{ajd} s_{jid} \Xi_{jid} + K_{j\rho} \left(R_j e_{jv} - K_{j\rho} e_{j\rho} \right) \right)\end{aligned}. \tag{15}$$

Remark 1: Examining the velocity tracking error dynamics (15), it is observed that the derivative of the leader's control velocity, $\dot{v}_{id}$, is required as a result of using v_{id} in (12). If the measured velocity of the leader, v_i, had been used instead of v_{id} in (12), the tracking error dynamics (15) would be dependent on $\dot{v}_i$ which are considered to be unknown by the follower j in this work. In the following development, a NN is introduced to learn the unknown quantities of (15); however, to effectively approximate the leader's dynamics, $\dot{v}_i$, terms like the leader's control thrust and rotational torques would be required to be communicated to each follower in addition to the leader's measured linear and angular velocities so that the terms could be included in the NN input of the follower.

Moving on, the velocity v_{jzb} is directly controllable with the thrust input. However, in order to control the translational velocities v_{jxb} and v_{jyb}, the pitch and roll must be controlled, respectively, thus redirecting the thrust. Thus, we now seek to find expressions for the desired pitch, θ_{jd}, and roll, ϕ_{jd}. Moreover, it is desirable to specify the maximum desired pitch and roll angles to be tracked by the follower UAV.

To accomplish these design objectives, we first define the *scaled* desired orientation vector, $\bar{\Theta}_{jd} = [\bar{\theta}_{jd} \;\; \bar{\phi}_{jd} \;\; \psi_{jd}]^T$ where $\bar{\theta}_{jd} = \pi\theta_{jd} / (2\theta_{d\max})$, $\bar{\phi}_{jd} = \pi\phi_{jd} / (2\phi_{d\max})$, where $\theta_{d\max} \in (0, \pi/2)$ and $\phi_{d\max} \in (0, \pi/2)$ are design constants used to specify the maximum desired roll and pitch, respectively. Next, we rewrite translational rotation matrix (2) in terms of $\bar{\Theta}_{jd}$, and define $R_{jd} = R_j(\bar{\Theta}_{jd})$.

Then, add and subtract $G(R_{jd})/m_j$ and $R_{jd}^T \Lambda_j$ with $\Lambda_j = R_i \dot{v}_{id} - \ddot{R}_{ajd}^T s_{jid} \Xi_{jid} + K_{j\rho} R_j e_{jv} - K_{j\rho} e_{j\rho}$ to (15) to yield

$$\begin{aligned}\dot{e}_{jv} = &-S(\omega_j) e_{jv} - G(R_{jd})/m_j + R_{jd}^T (\Lambda_j + f_{cj1}(x_{cj1})) \\ &- u_{j1} E_{jz}/m_j - K_{j\rho} R_i e_{iv} - \bar{\tau}_{jd1}\end{aligned} \tag{16}$$

where

$$\begin{aligned}f_{cj1}(x_{cj1}) = &R_{jd} \left(G(R_{jd})/m_j - G(R_j)/m_j + R_j^T(\Lambda_j) - R_{jd}^T(\Lambda_j) \right) \\ &+ R_{jd} \begin{pmatrix} K_{j\rho} R_j e_{jv} - N_{j1}(v_j)/m_j + R_j^T R_i S(\omega_i) v_{id} \\ + R_j^T K_{j\rho} (1 - K_{j\rho}) e_{j\rho} \end{pmatrix}\end{aligned} \tag{17}$$

is an unknown function which can be rewritten as $f_{jc1}(x_{jc1}) = [f_{jc11} \;\; f_{jc12} \;\; f_{jc13}]^T \in \Re^3$. In the forthcoming development, the approximation properties of NN will be utilized to estimate the unknown function $f_{jc1}(x_{jc1})$ by bounded ideal weights W_{jc1}^T, V_{jc1}^T such that $\|W_{jc1}\|_F \le W_{Mc1}$ for an unknown constant W_{Mc1}, and written as $f_{jc1}(x_{jc1}) = W_{jc1}^T \sigma(V_{jc1}^T x_{jc1}) + \varepsilon_{jc1}$ where $\varepsilon_{jc1} \le \varepsilon_{Mc1}$ is the bounded NN approximation error where ε_{Mc1} is a known constant. The NN estimate of f_{jc1} is written as $\hat{f}_{jc1} = \hat{W}_{jc1}^T \sigma(V_{jc1}^T x_{jc1}) = \hat{W}_{jc1}^T \sigma_{jc1} = [\hat{W}_{jc11}^T \sigma_{jc1} \;\; \hat{W}_{jc12}^T \sigma_{jc1} \;\; \hat{W}_{jc13}^T \sigma_{jc1}]^T$

where $\hat{W}_{jc1}^T$ is the NN estimate of W_{jc1}^T, $\hat{W}_{jc1i}^T, i = 1,2,3$ is the i^{th} row of $\hat{W}_{jc1}^T$, and x_{jc1} is the NN input $x_{jc1} = [1\ \Theta_j^T\ \Theta_i^T\ \omega_i^T\ \Lambda_j^T\ v_{jd}^T\ v_{id}^T\ \dot{v}_{id}^T\ \psi_{jd}\ \dot{\psi}_{jd}\ \ddot{\psi}_{jd}\ \omega_j^T\ v_j^T\ e_{jv}^T\ e_{j\rho}^T]^T$.

Next, the virtual control inputs θ_{jd} and ϕ_{jd} are identified to control to control the translational velocities v_{jxb} and v_{jyb}, respectively. The key step in the development is identifying the *desired* closed loop velocity tracking error dynamics. For convenience, the *desired* translational velocity closed loop system is selected as

$$\dot{e}_{jv} = -S(\omega_j)e_{jv} - K_{jv}e_{jv} - \bar{\tau}_{jd1} - K_{j\rho}R_i e_{iv} \tag{18}$$

where $K_{jv} = diag\{k_{jv1}\cos(\bar{\theta}_{jd}), k_{jv2}\cos(\bar{\phi}_{jd}), k_{v3}\}$ is a diagonal positive definite design matrix with each $k_{vi} > 0$, $i = 1,2,3$, and $\bar{\tau}_{jd1} = \tau_{jd1}/m_j$. In the following development, it will be shown that $\bar{\theta}_d \in (-\pi/2, \pi/2)$, $\bar{\phi}_d \in (-\pi/2, \pi/2)$; therefore, it is clear that $K_v > 0$. Then, equating (16) and (18) while considering only the first two velocity error states reveals

$$-g\begin{bmatrix} -s_{\bar{\theta}jd} \\ c_{\bar{\theta}jd}s_{\bar{\phi}jd} \end{bmatrix} + \begin{bmatrix} c_{\bar{\theta}jd}k_{jv1}e_{jvx} \\ c_{\bar{\phi}jd}k_{jv2}e_{jvy} \end{bmatrix} + \begin{bmatrix} c_{\bar{\theta}jd}c_{\psi jd} & c_{\bar{\theta}jd}s_{\psi jd} & -s_{\bar{\theta}jd} \\ s_{\bar{\phi}jd}s_{\bar{\theta}jd}c_{\psi jd} - c_{\bar{\phi}jd}s_{\psi jd} & s_{\bar{\phi}jd}s_{\bar{\theta}jd}s_{\psi jd} + c_{\bar{\phi}jd}c_{\psi jd} & s_{\bar{\phi}jd}c_{\bar{\theta}jd} \end{bmatrix} \times [\Lambda_{j1} + f_{jc11}\ \ \Lambda_{j2} + f_{jc12}\ \ \Lambda_{j3} + f_{jc13}]^T = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

where $\Lambda_j = [\Lambda_{j1}\ \Lambda_{j2}\ \Lambda_{j3}]^T$ was utilized. Then, applying basic math operations, the first line of (19) can be rewritten as

$$c_{\bar{\theta}jd}(c_{\psi jd}(\Lambda_{j1} + f_{jc11}) + s_{\psi jd}(\Lambda_{j2} + f_{jc12}) + k_{jv1}e_{jvx}) = s_{\bar{\theta}jd}(\Lambda_{j3} + f_{jc13} - g). \tag{20}$$

Similarly, the second line of (19) can be rewritten as

$$c_{\bar{\phi}jd}(c_{\psi jd}(\Lambda_{j2} + f_{jc12}) - s_{\psi jd}(\Lambda_{j1} + f_{jc11}) + k_{jv2}e_{jvy}) = s_{\bar{\phi}jd}\begin{pmatrix} gc_{\bar{\theta}jd} - s_{\bar{\theta}jd}c_{\psi jd}(\Lambda_{j1} + f_{jc11}) \\ - s_{\bar{\theta}jd}s_{\psi jd}(\Lambda_{j2} + f_{jc12}) - c_{\bar{\theta}jd}(\Lambda_{j3} + f_{jc13}) \end{pmatrix} \tag{21}$$

Next, (20) is solved for the desired pitch θ_{jd} while (21) can be solved for the desired roll ϕ_{jd}. Using the NN estimates, $\hat{f}_{cj1}$, the desired pitch θ_{jd} can be written as

$$\theta_{jd} = (2\theta_{\max}/\pi)a\tan\left(N_{\theta jd}/D_{\theta jd}\right) \tag{22}$$

where

$N_{\theta jd} = c_{\psi jd}(\Lambda_{j1} + \hat{f}_{jc11}) + s_{\psi jd}(\Lambda_{j2} + \hat{f}_{jc12}) + k_{jv1}e_{jvx}$ and $D_{\theta jd} = \Lambda_{j3} + \hat{f}_{jc13} - g$. Similarly, the desired roll angle, ϕ_{jd}, is found to be

$$\phi_{jd} = (2\phi_{\max}/\pi)a\tan\left(N_{\phi jd}/D_{\phi jd}\right) \tag{23}$$

where

$N_{\phi jd} = c_{\psi jd}(\Lambda_{j2} + \hat{f}_{jc12}) - s_{\psi jd}(\Lambda_{j1} + \hat{f}_{jc11}) + k_{jv2}e_{jvy}$

and

$D_{\phi jd} = gc_{\bar{\theta}jd} - s_{\bar{\theta}jd}c_{\psi jd}(\Lambda_{j1} + \hat{f}_{jc11}) - s_{\bar{\theta}jd}s_{\psi jd}(\Lambda_{j2} + \hat{f}_{jc12}) - c_{\bar{\theta}jd}(\Lambda_{j3} + \hat{f}_{jc13})$.

Remark 2: The expressions for the desired pitch and roll in (22) and (23) will always produce desired values in the *stable operation regions* of the UAV since $a\tan(\bullet)$ approaches $\pm\pi/2$ as its argument increases. Thus, introducing the scaling factors in $\bar{\theta}_{jd}$ and $\bar{\phi}_{jd}$ results in $\theta_{jd} \in (-\theta_{\max}, \theta_{\max})$ and $\phi_{jd} \in (-\phi_{\max}, \phi_{\max})$, and the aggressiveness of the UAVs maneuvers can be managed. Further, if the un-scaled desired orientation vector were used in the development of (16), the maximum desired pitch and roll would still remain within the stable operating regions.

Now that the desired orientation has been found, next define the attitude tracking error as

$$e_{j\Theta} = \Theta_{jd} - \Theta_j \in E^a \tag{24}$$

where dynamics are found using (4) to be $\dot{e}_{j\Theta} = \dot{\Theta}_{jd} - T_j\omega_j$. In order to drive the orientation errors (24) to zero, the desired angular velocity, ω_{jd}, is selected as

$$\omega_{jd} = T_j^{-1}(\dot{\Theta}_{jd} + K_{j\Theta}e_{j\Theta}) \tag{25}$$

where $K_{j\Theta} = diag\{k_{j\Theta1}, k_{j\Theta2}, k_{j\Theta3}\} \in \Re^{3x3}$ is a diagonal matrix of positive design constants. Define the angular velocity tracking error as

$$e_{j\omega} = \omega_{jd} - \omega_j, \tag{26}$$

and observing $\omega_j = \omega_{jd} - e_{j\omega}$, the closed loop orientation tracking error system can be written as

$$\dot{e}_{j\Theta} = -K_{j\Theta}e_{j\Theta} + T_j e_{j\omega}. \tag{27}$$

Examining (25), calculation of the desired angular velocity requires knowledge of $\dot{\Theta}_{jd}$; however, $\dot{\Theta}_{jd}$ is not known in view of the fact $\dot{\Lambda}_j$ and $\dot{\hat{f}}_{jc1}$ are not available. Further, development of u_{j2} in the following section will reveal $\dot{\omega}_{jd}$ is required which in turn implies $\ddot{\Lambda}_j$ and $\ddot{\hat{f}}_{jc1}$ must be known. Since these requirements are not practical, the *universal approximation property* of NN is invoked to estimate ω_{jd} and $\dot{\omega}_{jd}$ [1].

To begin the NN virtual control development, we rearrange (25) to observe the dynamics of the ideal virtual controller to be

$$\begin{aligned} \dot{\Theta}_{jd} &= T_j(\omega_{jd} - T_j^{-1}K_{j\Theta}e_{j\Theta}) \\ \dot{\omega}_{jd} &= \dot{T}_j^{-1}(\dot{\Theta}_{jd} + K_{j\Theta}e_{j\Theta}) + T_j^{-1}(\ddot{\Theta}_{jd} + K_{j\Theta}\dot{e}_{j\Theta}) \end{aligned} \tag{28}$$

For convenience, we define a change of variable as $\Omega_{jd} = \omega_{jd} - T_j^{-1}K_{j\Theta}e_{j\Theta}$, and the dynamics (28) become

$$\begin{aligned} \dot{\Theta}_{jd} &= T_j\Omega_{jd} \\ \dot{\Omega}_{jd} &= \dot{T}_j^{-1}\dot{\Theta}_{jd} + T_j^{-1}\ddot{\Theta}_{jd} = f_{j\Omega}(x_{j\Omega}) = f_{j\Omega} \end{aligned} \tag{29}$$

Defining the estimates of Θ_{jd} and Ω_{jd} to be $\hat{\Theta}_{jd}$ and $\hat{\Omega}_{jd}$, respectively, and the estimation error $\widetilde{\Theta}_{jd} = \Theta_{jd} - \hat{\Theta}_{jd}$, the dynamics of the proposed NN virtual control inputs become

$$\begin{aligned} \dot{\hat{\Theta}}_{jd} &= T_j \hat{\Omega}_{jd} + K_{j\Omega 1}\widetilde{\Theta}_{jd} \\ \dot{\hat{\Omega}}_{jd} &= \hat{f}_{j\Omega 1}(\hat{x}_{j\Omega}) + K_{j\Omega 2} T_j^{-1}\widetilde{\Theta}_{jd} \end{aligned} \tag{30}$$

where $K_{j\Omega 1}$ and $K_{j\Omega 2}$ are positive constants. The estimate $\hat{\omega}_{jd}$ is then written as

$$\hat{\omega}_{jd} = \hat{\Omega}_{jd} + T_j^{-1} K_{j\Theta} e_{j\Theta} + K_{j\Omega 3} T_j^{-1}\widetilde{\Theta}_{jd} \tag{31}$$

where $K_{j\Omega 3}$ is another positive constant. Observing

$$\widetilde{\omega}_{jd} = \omega_{jd} - \hat{\omega}_{jd} = \widetilde{\Omega}_{jd} - K_{j\Omega 3} T_j^{-1}\widetilde{\Theta}_{jd}, \tag{32}$$

subtracting (30) from (29), as well as adding and subtracting $T_j^T \widetilde{\Theta}_{jd}$ and $K_{j\Omega 3}\dot{T}_j^{-1}\widetilde{\Theta}_{jd}$, the virtual controller estimation error dynamics are found to be

$$\begin{aligned} \dot{\widetilde{\Theta}}_{jd} &= T_j \widetilde{\omega}_{jd} - (K_{j\Omega 1} - K_{j\Omega 3})\widetilde{\Theta}_{jd} \\ \dot{\widetilde{\Omega}}_{jd} &= f_{j\Omega 1}(x_{j\Omega}) - \hat{f}_{j\Omega 1}(\hat{x}_{j\Omega}) - K_{j\Omega 2} T_j^{-1}\widetilde{\Theta}_{jd} - T_j^T \widetilde{\Theta}_{jd} + K_{j\Omega 3}\dot{T}_j^{-1}\widetilde{\Theta}_{jd} \end{aligned} \tag{33}$$

where $f_{j\Omega 1}(x_{j\Omega}) = f_{j\Omega} + T_j^T \widetilde{\Theta}_{jd} - K_{j\Omega 3}\dot{T}_j^{-1}\widetilde{\Theta}_{jd}$ is an unknown function.

In (30), *universal approximation property* of NN has been utilized to estimate the unknown function $f_{j\Omega 1}(x_{j\Omega})$ by bounded ideal weights $W_{j\Omega}^T, V_{j\Omega}^T$ such that $\|W_{j\Omega}\|_F \le W_{M\Omega}$ for a known constant $W_{M\Omega}$, and written as $f_{1j\Omega}(x_{j\Omega}) = W_{j\Omega}^T \sigma\left(V_{j\Omega}^T x_{j\Omega}\right) + \varepsilon_{j\Omega}$ where $\varepsilon_{j\Omega}$ is the bounded NN approximation error such that $\|\varepsilon_{j\Omega}\| \le \varepsilon_{\Omega M}$ for a known constant $\varepsilon_{\Omega M}$. The NN estimate of $f_{j\Omega}$ is written as $\hat{f}_{j\Omega}(\hat{x}_{j\Omega}) = \hat{f}_{j\Omega} = \hat{W}_{j\Omega}^T \sigma\left(V_{j\Omega}^T \hat{x}_{j\Omega}\right) = \hat{W}_{j\Omega}^T \hat{\sigma}_{j\Omega}$ where $\hat{W}_{j\Omega}^T$ is the NN estimate of $W_{j\Omega}^T$ and $\hat{x}_{j\Omega}$ is the NN input written in terms of the virtual control estimates, desired trajectory, and the UAV velocity. The NN input is chosen to take the form of $\hat{x}_{j\Omega} = [1\ \Lambda_j^T\ \ \Theta_{jd}^T\ \ \hat{\Omega}_{jd}^T\ \ v_j^T\ \ \omega_j^T]^T$.

Next, differentiating (32), using (33) as well as adding and subtracting $W_{j\Omega}^T \hat{\sigma}_{j\Omega}$ reveals

$$\begin{aligned} \dot{\widetilde{\omega}}_{jd} &= -K_{j\Omega 3}\widetilde{\omega}_d + \widetilde{f}_{j\Omega 1}(\hat{x}_{j\Omega}) - T_j^T \widetilde{\Theta}_{jd} \\ &\quad - T_j^{-1}(K_{j\Omega 2} - K_{j\Omega 3}(K_{j\Omega 1} - K_{j\Omega 3}))\widetilde{\Theta}_{jd} + \xi_{j\Omega} \end{aligned} \tag{34}$$

where $\widetilde{\Omega}_{jd} = \Omega_{jd} - \hat{\Omega}_{jd}$, $\widetilde{f}_{j\Omega} = \widetilde{W}_{j\Omega}^T \hat{\sigma}_{j\Omega}$, $\widetilde{W}_{j\Omega}^T = W_{j\Omega}^T - \hat{W}_{j\Omega}^T$, $\xi_{j\Omega} = \varepsilon_{j\Omega} + W_{j\Omega}^T \widetilde{\sigma}_{j\Omega}$, and $\widetilde{\sigma}_{j\Omega} = \sigma_{j\Omega} - \hat{\sigma}_{j\Omega}$. Furthermore, $\|\xi_{j\Omega}\| \le \xi_{\Omega M}$ with $\xi_{\Omega M} = \varepsilon_{\Omega M} + 2W_{M\Omega}\sqrt{N_\Omega}$ a computable constant with N_Ω the constant number of hidden layer neurons in the virtual control NN and the fact $\|\sigma_{j\Omega}\| \le \sqrt{N_\Omega}$ was used. Examination of (33) and (34) reveals $\widetilde{\Theta}_{jd}^b = 0$, $\widetilde{\omega}_{jd} = 0$, and $\widetilde{f}_{j\Omega} = 0$ to be equilibrium points of the estimation error dynamics when $\|\xi_{j\Omega}\| = 0$.

To this point, the desired translational velocity for follower j has been identified to ensure the leader-follower objective (8) is achieved. Then, the desired pitch and roll were derived to drive $v_{jxb} \to v_{jdx}$ and $v_{jyb} \to v_{jdy}$, respectively. Then, the desired angular velocity was found to ensure $\Theta_j \to \Theta_{jd}$. What remains is to identify the UAV thrust to guarantee $v_{jzb} \to v_{jdz}$ and rotational torque vector to ensure $\omega_j \to \omega_{jd}$. First, the thrust is derived.

Consider again the translational velocity tracking error dynamics (16), as well as the *desired* velocity tracking error dynamics (18). Equating (16) and (18) and manipulating the third error state, the required thrust is found to be

$$\begin{aligned} u_{j1} &= m_j\left(c_{\bar{\phi}jd} s_{\bar{\theta}jd} c_{\psi jd} + s_{\bar{\phi}jd} s_{\psi jd}\right)\left(\Lambda_{j1} + \hat{f}_{jc11}\right) + m_j k_{jvz} e_{vj3} + \\ &\quad m_j c_{\bar{\theta}jd} c_{\bar{\phi}jd}\left(\Lambda_{j3} + \hat{f}_{jc13} - g\right) + m_j\left(c_{\bar{\phi}jd} s_{\bar{\theta}jd} s_{\psi jd} - s_{\bar{\phi}jd} c_{\psi jd}\right)\left(\Lambda_{j2} + \hat{f}_{jc12}\right) \end{aligned} \tag{35}$$

where $\hat{f}_{jc1}$ is the NN estimate in (17) previously defined. Substituting the desired pitch (22), roll (23), and the thrust (35) into the translational velocity tracking error dynamics (16) yields

$$\dot{e}_{jv} = -K_{jv} e_{jv} + R_{jd}^T \widetilde{W}_{jc1}^T \sigma_{jc1} - K_{j\rho} R_i e_{iv} + \xi_{jc1}, \tag{36}$$

with $\xi_{jc1} = R_{jd}^T \varepsilon_{jc} - \bar{\tau}_{jd1}$, $\widetilde{W}_{jc1} = W_{jc1} - \hat{W}_{jc1}$ and, $\|\xi_{jc1}\| \le \xi_{Mc1}$ for a computable constant $\xi_{Mc1} = R_{\max}\varepsilon_{Mc1} + \tau_M / m_j$. Additionally, in the formulation of (36), the expressions for the desired pitch and roll (22) and (23), respectively, were first written in the form of (20) and (21), so that sine and cosine of the angles could be substituted as opposed to substituting the arctangent expressions directly into the sine or cosine function.

Next, the rotational torque vector, u_{j2}, will be addressed. First, multiply the angular velocity tracking error (26) by the constant inertial matrix J_j, take the first derivative with respect to time, add and subtract $T_j^T e_{j\Theta}$, and substitute the UAV dynamics (1) to reveal

$$J_j \dot{e}_{j\omega} = f_{jc2}(x_{jc2}) - u_{j2} - T_j^T e_{j\Theta} - \tau_{jd2} \tag{37}$$

with $f_{jc2}(x_{jc2}) = J_j \dot{\omega}_{jd} - S(J_j \omega_j)\omega_j - N_{j2}(\omega_j) + T_j^T e_{j\Theta}$.

Examining $f_{jc2}(x_{jc2})$, it is clear that the function is nonlinear and contains unknown terms; therefore, the *universal approximation property* of NN is utilized to estimate the function $f_{jc2}(x_{jc2})$ by bounded ideal weights W_{jc2}^T, V_{jc2}^T such that $\|W_{jc2}\|_F \le W_{Mc2}$ for a known constant W_{Mc2} and written as $f_{jc2}(x_{jc2}) = W_{jc2}^T \sigma(V_{jc2}^T x_{jc2}) + \varepsilon_{jc2}$ where ε_{jc2} is the bounded NN functional reconstruction error such that $\|\varepsilon_{jc2}\| \le \varepsilon_{Mc2}$ for a known constant ε_{Mc2}. The NN estimate of f_{jc2} is given by $\hat{f}_{jc2}(\hat{x}_{jc2}) = \hat{W}_{jc2}^T \sigma(V_{jc2}^T \hat{x}_{jc2}) = \hat{W}_{jc2}^T \hat{\sigma}_{jc2}$ where $\hat{W}_{jc2}^T$ is the NN estimate of W_{jc2}^T and

$\hat{x}_{jc2} = [1\ \omega_j^T\ \dot{\hat{\Omega}}_{jd}^T\ \ \widetilde{\Theta}_{jd}^{b\ T}\ \ e_{j\Theta}^T]^T$ is the input to the NN written in terms of the virtual controller estimates. By the construction of the virtual controller, $\dot{\hat{\omega}}_{jd}$ is not directly available; therefore, observing (31), the terms $\dot{\hat{\Omega}}_{jd}^T$, $\widetilde{\Theta}_{jd}^{b\ T}$, and $e_{j\Theta}^T$ have been included instead.

Next, using the desired angular velocity (31), we define the estimated angular velocity tracking error as $\hat{e}_{j\omega} = \hat{\omega}_{jd} - \omega_j$. Now, using the NN estimate $\hat{f}_{jc2}$ and $\hat{e}_{j\omega}$, the rotational torque control input is written as

$$u_{j2} = \hat{f}_{jc2} + K_{j\omega}\hat{e}_{j\omega}, \tag{38}$$

and substituting the control input (38) into the angular velocity dynamics (37) reveals

$$J_j\dot{e}_{j\omega} = f_{jc2} - \hat{f}_{jc2} - K_{j\omega}\hat{e}_{j\omega} - T_j^T e_{j\Theta} - \tau_{jd2} \tag{39}$$

Now, adding and subtracting $W_{jc2}^T\hat{\sigma}_{jc}$ and observing $\hat{e}_{j\omega} = e_{j\Omega} - \widetilde{\omega}_{jd}$, the closed loop dynamics (39) become

$$J_j\dot{e}_{j\omega} = -K_{j\omega}e_{j\omega} + \widetilde{W}_{jc2}^T\hat{\sigma}_{jc2} + K_{j\omega}\widetilde{\omega}_{jd} - T_j^T e_{j\Theta} + \xi_{jc2} \tag{40}$$

where $\widetilde{W}_{jc2}^T = W_{jc2}^T - \hat{W}_{jc2}^T$, $\xi_{jc2} = \varepsilon_{jc2} + W_{jc2}^T\widetilde{\sigma}_{jc} - \tau_{jd2}$, and $\widetilde{\sigma}_{jc2} = \sigma_{jc2} - \hat{\sigma}_{jc2}$. Further, $\|\xi_{jc2}\| \le \xi_{Mc2}$ for a computable constant $\xi_{Mc2} = \varepsilon_{Mc2} + 2W_{Mc2}\sqrt{N_{c2}} + \tau_{dM}$ where N_{c2} is the number of hidden layer neurons.

As a final step, we define the augmented variables $\hat{e}_{jD} = [e_{jv}^T\ \ \hat{e}_{j\omega}^T]^T$, $\widetilde{W}_{jc} = [\widetilde{W}_{jc1}\ 0;\ 0\ \widetilde{W}_{jc2}]$ and $\hat{\sigma}_{jc} = [\hat{\sigma}_{jc1}^T\ \ \hat{\sigma}_{jc2}^T]^T$. In the following theorem, the stability of the follower j is shown while considering $e_{iv} = 0$. In other words, the position, orientation, and velocity tracking errors are considered along with the estimation errors of the virtual controller and the NN weight estimation errors of each NN for follower j while ignoring the interconnection errors between the leader and its followers. This assumption will be relaxed later.

Theorem 1: (Follower UAV System Stability) Given the dynamic system of follower j in the form of (1), let the desired translational velocity for follower j to track be defined by (12) with the desired pitch and roll defined by (22) and (23), respectively. Let the NN virtual controller be defined by (30) and (31), respectively, with the NN update law given by

$$\dot{\hat{W}}_{j\Omega} = F_{j\Omega}\hat{\sigma}_{j\Omega}\widetilde{\Theta}_{jd}^T - \kappa_{j\Omega}F_{j\Omega}\hat{W}_{j\Omega} \tag{41}$$

where $F_{j\Omega} = F_{j\Omega}^T > 0$ and $\kappa_{j\Omega} > 0$ are design parameters. Let the dynamic NN controller for follower j be defined by (35) and (38), respectively, with the NN update given by

$$\dot{\hat{W}}_{jc} = F_{jc}\hat{\sigma}_{jc}\left(A_{jd}\hat{e}_{jD}\right)^T - \kappa_{jc}F_{jc}\hat{W}_{jc} \tag{42}$$

where $A_{jd} = [R_{jd}\ 0_{3x3};0_{3x3}\ I_{3x3}] \in \Re^{6x6}$, and $F_{jc} = F_{jc}^T > 0$ and $\kappa_{jc} > 0$ are constant design parameters. Then there exists positive design constants $K_{j\Omega1}, K_{j\Omega2}, K_{j\Omega3}$, and positive definite design matrices $K_{j\rho}, K_{j\Theta}, K_{jv}, K_{j\omega}$, such that the virtual controller estimation errors $\widetilde{\Theta}_{jd}^b, \widetilde{\omega}_{jd}$ and the virtual control NN weight estimation errors, $\widetilde{W}_{j\Omega}$, the position, orientation, and translational and angular velocity tracking errors, $e_{j\rho}, e_{j\Theta}, e_{jv}, e_{j\omega}$, respectively, and the controller NN weight estimation errors, $\widetilde{W}_{jc}$, are all *semi-globally uniformly ultimately bounded (SGUUB)*.

Proof: Consider the following positive definite Lyapunov candidate

$$V_j = K_{j\omega Max}^2 V_{j\Omega} + V_{jc} \tag{43}$$

where $K_{j\omega Max}^2$ is the maximum singular value of $K_{j\omega}$ and

$$V_{j\Omega} = \frac{1}{2}\widetilde{\Theta}_{jd}^T\widetilde{\Theta}_{jd} + \frac{1}{2}\widetilde{\omega}_{jd}^{\ T}\widetilde{\omega}_{jd} + \frac{1}{2}tr\{\widetilde{W}_{j\Omega}^T F_{j\Omega}^{-1}\widetilde{W}_{j\Omega}\}$$

$$V_{jc} = \frac{1}{2}e_{j\rho}^T e_{j\rho} + \frac{1}{2}e_{j\Theta}^T e_{j\Theta} + \frac{1}{2}e_{jv}^T e_{jv} + \frac{1}{2}e_{j\omega}^T J e_{j\omega} + \frac{1}{2}tr\{\widetilde{W}_{jc}^T F_{jc}^{-1}\widetilde{W}_{jc}\}$$

whose first derivative with respect to time is given by $\dot{V}_j = K_{j\omega Max}^2\dot{V}_{j\Omega} + \dot{V}_{jc}$. Considering first, $\dot{V}_{j\Omega}$ and substituting the closed loop virtual control estimation error dynamics (33) and (34) as well as the NN tuning law (41), yields

$$\begin{aligned}\dot{V}_j \le & -K_{j\omega Max}^2\left(K_{j\Omega1} - K_{j\Omega3} - \frac{N_\Omega}{\kappa_{j\Omega}}\right)\|\widetilde{\Theta}_{jd}\|^2 - \frac{K_{j\omega Max}^2\kappa_{j\Omega}}{4}\|\widetilde{W}_{j\Omega}\|_F^2 \\ & - \frac{K_{j\omega Max}^2}{2}\left(K_{j\Omega3} - \frac{2N_\Omega}{\kappa_{j\Omega}} - \frac{3N_c}{2\kappa_{jc}K_{j\omega Max}^2} - \frac{3}{2K_{j\omega Min}}\right)\|\widetilde{\omega}_{jd}\|^2 \\ & - \left(K_{j\rho Min} - \frac{3R_{Max}^2}{4K_{jvMin}}\right)\|e_{j\rho}\|^2 - K_{j\Theta Min}\|e_{j\Theta}\|^2 - \frac{K_{jvMin}}{3}\|e_{jv}\|^2 \\ & - \frac{K_{j\omega Min}}{3}\|e_{j\omega}\|^2 - \frac{\kappa_{jc}}{3}\|\widetilde{W}_{jc}\|_F^2 + \eta_j\end{aligned} \tag{44}$$

where $\eta_j = \eta_{jc}/4 + K_{j\omega Max}^2\eta_{j\Omega}$, $\eta_{j\Omega} = \kappa_{j\Omega}W_{M\Omega}^2 + \xi_{\Omega M}^2/(2K_{j\Omega3})$, and $\eta_{jc} = 3\kappa_{jc}W_{cM}^2 + 3\xi_{Mc1}^2/K_{jvMin} + 3\xi_{Mc2}^2/K_{j\omega Min}$. Finally, (44) is less than zero provided

$$K_{j\Omega1} > K_{j\Omega3} + N_\Omega/\kappa_{j\Omega},\quad K_{j\Omega3} > \frac{2N_\Omega}{\kappa_{j\Omega}} + \frac{3N_c}{2\kappa_{jc}K_{j\omega Max}^2} + \frac{3}{2K_{j\omega Min}}$$

$$K_{j\rho Min} > 3R_{Max}^2/(4K_{jvMin}) \tag{45}$$

and the following inequalities hold:

$$\|\widetilde{\omega}_{jd}\| > \sqrt{\frac{\eta_j}{K_{j\omega Max}^2\left(\frac{K_{j\Omega3}}{2} - \frac{N_\Omega}{\kappa_{j\Omega}} - \frac{3N_c}{4\kappa_{jc}K_{j\omega Max}^2} - \frac{3}{4K_{j\omega Min}}\right)}} \tag{46}$$

or $\|e_{j\rho}\| > \sqrt{\dfrac{\eta_j}{(K_{j\rho Min} - 3R_{Max}^2/(4K_{jvMin}))}}$ or $\|\widetilde{W}_{j\Omega}\|_F > \sqrt{\dfrac{4\eta_j}{K_{j\omega Max}^2\kappa_{j\Omega}}}$

or $\|e_{j\Theta}\| > \sqrt{\dfrac{\eta_j}{K_{j\Theta Min}}}$ or $\|e_{jv}\| > \sqrt{\dfrac{3\eta_j}{K_{jvMin}}}$ or $\|\widetilde{W}_{jc}\|_F > \sqrt{\dfrac{3\eta_j}{\kappa_{jc}}}$

or $\|e_{j\omega}\| > \sqrt{\dfrac{3\eta_j}{K_{j\omega Min}}}$ or $\|\widetilde{\Theta}_{jd}\| > \sqrt{\dfrac{\eta_j}{K_{j\omega Max}^2(K_{j\Omega1} - K_{j\Omega3} - N_\Omega/\kappa_{j\Omega})}}$

Therefore, it can be concluded using standard extensions of Lyapunov theory [9] that $\dot{V}_j$ is less than zero outside of a compact set, revealing the virtual controller estimation errors, $\widetilde{\Theta}_{jd}^b, \widetilde{\omega}_{jd}$, and the NN weight estimation errors, $\widetilde{W}_{j\Omega}$, the position, orientation, and translational and angular velocity tracking errors, $e_{j\rho}, e_{j\Theta}, e_{jv}, e_{j\omega}$, respectively, and the dynamic controller NN weight estimation errors, $\widetilde{W}_{jc}$, are all *SGUUB*.

B. Formation Leader Control Law

The dynamics and kinematics for the formation leader are defined similarly to (1) and (4), respectively. In our previous work [1], an output feedback control law for a single quadrotor UAV was designed to ensure the robot tracks a desired path, $\rho_{id} = [x_{id}, y_{id}, z_{id}]^T$, and desired yaw angle, ψ_{id}. Using the design methods described in this work as well as in [1], a state feedback control law for the formation leader can be derived which ensures the leader's error systems are SGUUB.

Next, the stability of the formation consisting of 1 leader and N followers is considered in the following theorem while considering the interconnection errors between the leader and its followers.

C. Quadrotor UAV Formation Stability

Before proceeding, it is convenient to define the following augmented error systems consisting of the position and translational velocity tracking errors of leader i and N follower UAVs as

$$e_\rho = [e_{i\rho}^T \; e_{j\rho}^T\big|_{j=1} \ldots e_{j\rho}^T\big|_{j=N}]^T \in \Re^{3(N+1)}$$

$$e_v = [e_{iv}^T \; e_{jv}^T\big|_{j=1} \ldots e_{jv}^T\big|_{j=N}]^T \in \Re^{3(N+1)}.$$

Next, the transformation matrix (2) is augmented as

$$R_F = diag\{R_i, R_j\big|_{j=1}, \ldots, R_j\big|_{j=N}\} \in \Re^{3(N+1)x3(N+1)}$$

while the NN weights for the translational velocity error system are augmented as

$$\hat{W}_{c1} = diag\{\hat{W}_{ic1}, \hat{W}_{jc1}\big|_{j=1}, \ldots, \hat{W}_{jc1}\big|_{j=N}\} \in \Re^{(N \cdot N_{jc1} + N_{ic1})}$$

$$\hat{\sigma}_{c1} = [\hat{\sigma}_{ic1}^T \; \hat{\sigma}_{jc1}^T\big|_{j=1}, \ldots, \hat{\sigma}_{jc1}^T\big|_{j=N}]^T \in \Re^{(N \cdot N_{jc1} + N_{ic1})}$$

Now, using the augmented variables above, the augmented closed loop position and translational velocity error dynamics for the entire formation are written as

$$\dot{e}_\rho = -K_\rho e_\rho + (I - G_F) R_F \, e_v$$

$$\dot{e}_v = -K_v e_v + A_{dF} \widetilde{W}_{c1}^T \hat{\sigma}_{c1} - K_\rho G_F R_F e_v + \xi_c$$

where $A_{dF} = diag\{A_{id}, \; A_{jd}\big|_{j=1}, \ldots, A_{jd}\big|_{j=N}\}$ with A_{id} defined similarly to A_{jd} in terms of $\overline{\Theta}_{id}$, ξ_c is an appropriately defined vector consisting of $\xi_{ic1}, \xi_{jc2}\big|_{j=1}$, etc.,

$$K_\rho = diag\{K_{i\rho}, K_{j\rho}\big|_{j=1}, \ldots, K_{j\rho}\big|_{j=N}\}$$

$$K_v = diag\{K_{iv}, K_{jv}\big|_{j=1}, \ldots, K_{jv}\big|_{j=N}\},$$

G_F is a constant matrix relating to the formation interconnection errors defined as

$$G_F = [0 \;\, 0; \; F_T \;\; 0] \in \Re^{N+1xN+1}$$

and $F_T \in \Re^{NxN}$ is dependent on the specific formation topology. For instance, in a string formation where each follower follows the UAV directly in front of it, follower *1* tracks leader *i*, follower *2* tracks follower *1*, etc., and F_T becomes the identity matrix.

Theorem 2: (UAV Formation Stability) Given the leader-follower criterion of (8) with 1 leader and N followers, let the hypotheses of *Theorem 1* hold. Let the virtual control system for the leader i be defined similarly to (30) and (31) with the virtual control NN update law defined similarly to (41). Let the control velocity, desire pitch and roll long with the thrust and rotation torque vector for the leader be given by [1] using state feedback, and let the control NN update law be defined identically to (42). Then, the position, orientation, and velocity tracking errors as well as the virtual control estimation errors for the entire formation are all *SGUUB*.

Proof: Proof of *Theorem 2* is omitted whereas it follows as in Theorem 1.

IV. Conclusions

A new framework for quadrotor UAV leader-follower formation control was presented along with a novel NN formation control law which allows each follower to track its leader without the knowledge of dynamics. All six DOF are successfully tracked using only four control inputs while in the presence of unmodeled dynamics and bounded disturbances. Lyapunov analysis guarantees *SGUUB* of the entire formation, and numerical results, although not shown, confirm the theoretical conjectures.

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