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Fault Detection and Identification for Bimodal Piecewise Affine Systems

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***Abstract*— This paper presents for the first time a fault detection and identification technique for bimodal piecewise affine (PWA) systems. A Luenberger-based observer structure is applied to the state estimation problem of the PWA system. The unknown value of the fault parameter is estimated by an observer equation obtained from a Lyapunov function. The design procedure is formulated as a set of linear matrix inequalities (LMIs) and guarantees global asymptotic stability of the estimation error, provided the norm of the input is upper and lower bounded by positive constants. The proposed method is applied to estimation of the amount of partial loss in control authority for a PWA model of a wheeled Mobile Robot (WMR).**

I. Introduction

Increasing reliability of modern complex systems has received much attention for the past two decades [1], [6]. This interest has spurred a growing demand for fault detection and identification (FDI) in complex systems. Most of the present FDI methods can only address linear systems [1], [6]. However, most of complex dynamical systems exhibit nonlinear behavior. Unfortunately, the FDI methods which are synthesized for linear models of nonlinear systems are valid only within a small range around the equilibrium point about which the system is linearized. This creates the need for FDI methods that can work at a more global scale. In this paper, complex nonlinear systems are approximated by PWA models.

The theory of continuous-time PWA systems has been applied to several different systems, such as, production systems [2], aerospace systems [3], wheeled robots [4] and electric circuits [5]. PWA systems are a class of hybrid systems and are a good modeling framework for nonlinear phenomena. Each mode of the PWA system approximates the nonlinear phenomena by linear or affine dynamics when the switching state is in a certain range. Using a PWA model of a complex nonlinear system enables the designer to have a global approximation and to use it for detection and identification of a fault. The type of fault which is studied in this paper is partial loss of control authority, which is widely used to model the faults in actuators [6]. State observer design for general PWA systems was first considered in [7] and later addressed for PWA bimodal systems in [8]. Our paper builds on these previous methods and proposes for the first time a state and fault parameter observer for bimodal PWA systems. The observer design is cast as a set of Linear Matrix Inequalities (LMIs) and solved with SeDuMi/YALMIP [9]. The design technique is applied to an FDI problem for a WMR. It is observed that the occurrence of a fault is detected and the amount of fault is estimated accurately.

The paper is organized as follows. First the system and observer structure are introduced. Then, the observer design method is developed. Finally, a numerical example is presented, followed by conclusions.

II. System and Observer Structure

Consider a bimodal PWA representation for a system with partial loss of control authority and the corresponding state space partitioning:

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 \rho u(t) + m_1 & \forall x \in \mathcal{R}_1 \\ y(t) = C_1 x(t) \end{cases}$$
$$\begin{cases} \dot{x}(t) = A_2 x(t) + B_2 \rho u(t) + m_2 & \forall x \in \mathcal{R}_2 \\ y(t) = C_2 x(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^k$ is the input to the system. The vector $m_i \ \forall\, i \in \{1, 2\}$ is the affine term for each PWA model. The actual input might be reduced by a coefficient matrix ρ due to faults in the system. The diagonal matrix ρ is composed of unknown values of partial loss of control authority for $j \in \{1, 2, ..., k\}$ actuators in the system yielding a faulty input

$$u_j^F(t) = \rho_j u_j(t)$$
$$\rho = diag[\rho_1, \rho_2, ..., \rho_k] \quad (2)$$

PWA slab systems switch among affine models based on variations of one state variable in the system. State space partitioning can be described by ellipsoidal cell boundings for PWA slab systems as

$$\epsilon_i = \{x \mid \| E_i x + f_i \| < 1\} \quad (3)$$

More precisely, if $\mathcal{R}_i = \{x \mid d_1 < c_i^T x < d_2\}$, then the associated ellipsoidal covering is described by $E_i = 2c_i^T/(d_2 - d_1)$ and $f_i = -(d_2 + d_1)/(d_2 - d_1)$. The structure of the proposed observer is as follows,

$\forall \hat{x} \in \mathcal{R}_1$

$$\begin{cases} \dot{\hat{x}}(t) = A_1 \hat{x}(t) + B_1 \hat{\rho}(t) u(t) + m_1 + G_1(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C_1 \hat{x}(t) \end{cases}$$

$\forall \hat{x} \in \mathcal{R}_2$

$$\begin{cases} \dot{\hat{x}}(t) = A_2 \hat{x}(t) + B_2 \hat{\rho}(t) u(t) + m_2 + G_2(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C_2 \hat{x}(t) \end{cases} \quad (4)$$

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Depending on the initial conditions of the system and the observer they might or might not work in the same mode. Therefore, the dynamics of the estimation error of the observer $e(t) = \hat{x}(t) - x(t)$ can be divided in four different cases,

Case 1: $\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_1$

$$\dot{e}(t) = (A_1 + G_1 C_1) e(t) + \sum_{j=1}^{k} b_{1j}\ \tilde{\rho}_j(t)\ u_j(t) \tag{5}$$

Case 2: $\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_2$

$$\begin{aligned}\dot{e}(t) =& (A_2 + G_2 C_2) e(t) \\ &+ m_2 - m_1 + [(A_2 - A_1) + G_2(C_2 - C_1)]x(t) \\ &+ (B_2 - B_1)\rho u(t) + \sum_{j=1}^{k} b_{2j}\ \tilde{\rho}_j(t)\ u_j(t)\end{aligned} \tag{6}$$

Case 3: $\forall x \in \mathcal{R}_2, \quad \forall \hat{x} \in \mathcal{R}_1$

$$\begin{aligned}\dot{e}(t) =& (A_1 + G_1 C_1) e(t) \\ &+ m_1 - m_2 + [(A_1 - A_2) + G_1(C_1 - C_2)]x(t) \\ &+ (B_1 - B_2)\rho u(t) + \sum_{j=1}^{k} b_{1j}\ \tilde{\rho}_j(t)\ u_j(t)\end{aligned} \tag{7}$$

Case 4: $\forall x \in \mathcal{R}_2, \quad \forall \hat{x} \in \mathcal{R}_2$

$$\dot{e}(t) = (A_2 + G_2 C_2) e(t) + \sum_{j=1}^{k} b_{2j}\ \tilde{\rho}_j(t)\ u_j(t) \tag{8}$$

where $\tilde{\rho}(t) = \hat{\rho}(t) - \rho$ and b_{ji} corresponds the i^th column of the B_j matrix.. The next theorem presents a result on the stability of the state and fault estimation errors.

Theorem 1: Assume $\epsilon < |u(t)| < \bar{u}$ for all t where $\epsilon, \bar{u} > 0$ and that the real amount of fault does not change during the estimation procedure. The observer state estimation error and fault estimation error $\tilde{\rho}(t) = \hat{\rho}(t) - \rho$ are globally asymptotically stable if there exist positive definite matrices $P = P^T > 0$ and $Q = Q^T > 0$, observer gains $G_1,\ G_2$, multipliers $\lambda_1,\ \lambda_2 > 0$ and fixed positive constants $l_i = 1, \ldots, k$ such that the following matrix inequalities are satisfied and the estimation laws for the fault parameter have the following structure (for each i^{th} actuator):
$\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_1$

$$\begin{aligned}&(A_1 + G_1 C_1)^T P + P(A_1 + G_1 C_1) + Q \leq 0 \\ &\dot{\hat{\rho}}_i(t) = -l_i e^T(t) P b_{1i} u_i(t)\end{aligned} \tag{9}$$

$\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_2$

$$\begin{bmatrix} \Pi_2 + Q & \begin{matrix}-P\Xi_2 \\ +\frac{1}{2}\lambda_1 E_2^T E_1\end{matrix} & \begin{matrix}-P\Delta m \\ +\frac{1}{2}\lambda_1 E_2^T f_1\end{matrix} \\ \begin{matrix}-\Xi_2^T P \\ +\frac{1}{2}\lambda_1 E_1^T E_2\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_1 E_1^T E_2 \\ +\frac{1}{2}\lambda_1 E_2^T E_1\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_1 E_1^T f_2 \\ +\frac{1}{2}\lambda_1 E_2^T f_1\end{matrix} \\ \begin{matrix}-\Delta m^T P \\ +\frac{1}{2}\lambda_1 f_1^T E_2\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_1 f_1^T E_2 \\ +\frac{1}{2}\lambda_1 f_2^T E_1\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_1 f_1^T f_2 \\ +\frac{1}{2}\lambda_1 f_2^T f_1 \\ -1\end{matrix} \end{bmatrix} \leq 0 \tag{10}$$

$$\dot{\hat{\rho}}_i(t) = -l_i e^T(t) P b_{2i} u_i(t) - l_i e^T(t) P \Delta b_{2i} \rho_i \tilde{\rho}_i^{-1}(t) u_i(t)$$

$\forall x \in \mathcal{R}_2, \quad \forall \hat{x} \in \mathcal{R}_1$

$$\begin{bmatrix} \Pi_1 + Q & \begin{matrix}P\Xi_1 \\ +\frac{1}{2}\lambda_2 E_1^T E_2\end{matrix} & \begin{matrix}P\Delta m \\ +\frac{1}{2}\lambda_2 E_1^T f_2\end{matrix} \\ \begin{matrix}\Xi_1^T P \\ +\frac{1}{2}\lambda_2 E_2^T E_1\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_2 E_2^T E_1 \\ +\frac{1}{2}\lambda_2 E_1^T E_2\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_2 E_2^T f_1 \\ +\frac{1}{2}\lambda_2 E_1^T f_2\end{matrix} \\ \begin{matrix}\Delta m^T P \\ +\frac{1}{2}\lambda_2 f_2^T E_1\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_2 f_2^T E_1 \\ +\frac{1}{2}\lambda_2 f_1^T E_2\end{matrix} & \begin{matrix}\frac{1}{2}\lambda_2 f_2^T f_1 \\ +\frac{1}{2}\lambda_2 f_1^T f_2 \\ -1\end{matrix} \end{bmatrix} \leq 0 \tag{11}$$

$$\dot{\hat{\rho}}_i(t) = -l_i e^T(t) P b_{1i} u_i(t) - l_i e^T(t) P \Delta b_{1i} \rho_i \tilde{\rho}_i^{-1}(t) u_i(t)$$

$\forall x \in \mathcal{R}_2, \quad \forall \hat{x} \in \mathcal{R}_2$

$$\begin{aligned}&(A_2 + G_2 C_2)^T P + P(A_2 + G_2 C_2) + Q \leq 0 \\ &\dot{\hat{\rho}}_i(t) = -l_i e^T(t) P b_{2i} u_i(t)\end{aligned} \tag{12}$$

where $\Pi_2 = (A_2 + G_2 C_2)^T P + P(A_2 + G_2 C_2)$, $\Xi_2 = (\Delta A + G_2 \Delta C)$, $\Delta A = A_1 - A_2$, $\Delta B = B_1 - B_2$, $\Delta C = C_1 - C_2$.

Proof:
Assuming that the real amount of fault does not change during the estimation procedure then $\dot{\tilde{\rho}}(t) = \dot{\hat{\rho}}(t)$. We consider a candidate Lyapunov function of the form

$$V = e^T(t) P e(t) + \sum_{i=1}^{k} \frac{\tilde{\rho}_i^2(t)}{l_i} \tag{13}$$

This function is positive definite because $P > 0$ and $l_i > 0,\ i = 1, \ldots, k$. Therefore, to prove asymptotic stability, it suffices to show that the derivative of V with respect to time is negative semi-definite and then we use La Salle's argument. To do this, we enforce that

$$\dot{V} + e^T(t) Q e(t) \leq 0, \tag{14}$$

where $Q > 0$ is a tuning performance parameter, yielding

$$\dot{e}^T(t) P e(t) + e^T(t) P \dot{e}(t) + 2 \sum_{i=1}^{k} \frac{\dot{\tilde{\rho}}_i(t) \tilde{\rho}_i(t)}{l_i} + e^T(t) Q e(t) \leq 0 \tag{15}$$

Replacing the estimation error dynamics (5), (6), (7) and (8) into (15) yields the following four cases:

Case 1: $\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_1$

$$\begin{aligned} & e^T(t)[(A_1+G_1C_1)^TP + P(A_1+G_1C_1)+Q]e(t) \\ & +2\sum_{i=1}^{k} e^T(t)Pb_{1i}\tilde{\rho}_i(t)u_i(t) + 2\sum_{i=1}^{k}\frac{\dot{\tilde{\rho}}_i(t)\tilde{\rho}_i(t)}{l_i} \le 0 \end{aligned} \tag{16}$$

The last two terms in equation (16) cancel out each other if $\dot{\tilde{\rho}}_i(t)$ has the following structure:

$$\dot{\tilde{\rho}}_i(t) = -l_i e^T(t)Pb_{1i}u_i(t) \tag{17}$$

This yields

$$[(A_1+G_1C_1)^TP + P(A_1+G_1C_1)+Q] \le 0 \tag{18}$$

Case 2: $\forall x \in \mathcal{R}_1, \quad \forall \hat{x} \in \mathcal{R}_2$

$$\begin{aligned} & e^T(t)[(A_2+G_2C_2)^TP + P(A_2+G_2C_2)+Q]e(t) \\ & + x^T(t)[-\Delta A - G_2\Delta C]^TPe(t) \\ & + e^T(t)P[-\Delta A - G_2\Delta C]x(t) \\ & - \Delta m^TPe(t) - e^T(t)P\Delta m + 2\sum_{i=1}^{k}\frac{\dot{\tilde{\rho}}_i(t)\tilde{\rho}_i(t)}{l_i} \\ & + 2\sum_{i=1}^{k} e^T(t)Pb_{2i}\tilde{\rho}_i(t)u_i(t) - 2\sum_{i=1}^{k} e^T(t)P\Delta b_i\rho_i u_i(t) \end{aligned} \tag{19}$$

The suggested structure for $\dot{\tilde{\rho}}_i(t)$ in Case 2 is:

$$\dot{\tilde{\rho}}_i(t) = -l_i e^T(t)Pb_{2i}u_i(t) + l_i e^T(t)P\Delta b_i\rho_i u_i(t) \tag{20}$$

From the fact that for case 2, the state of the system x is in $\mathcal{R}_1$ and the state of the observer $\hat{x}$ is in $\mathcal{R}_2$, one obtains

$$\frac{1}{2}(E_1x+f_1)^T(E_2\hat{x}+f_2) + \frac{1}{2}(E_2\hat{x}+f_2)^T(E_1x+f_1) < 1 \tag{21}$$

The S-procedure method is now applied yielding the following matrix inequality:

$$\begin{bmatrix} \Pi_2 + Q & \begin{array}{c}-P\Xi_2 \\ +\frac{1}{2}\lambda_1E_2^TE_1\end{array} & \begin{array}{c}-P\Delta m \\ +\frac{1}{2}\lambda_1E_2^Tf_1\end{array} \\ \begin{array}{c}-\Xi_2^TP \\ +\frac{1}{2}\lambda_1E_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1E_1^TE_2 \\ +\frac{1}{2}\lambda_1E_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1E_1^Tf_2 \\ +\frac{1}{2}\lambda_1E_2^Tf_1\end{array} \\ \begin{array}{c}-\Delta m^TP \\ +\frac{1}{2}\lambda_1f_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1f_1^TE_2 \\ +\frac{1}{2}\lambda_1f_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1f_1^Tf_2 \\ +\frac{1}{2}\lambda_1f_2^Tf_1 \\ -1\end{array} \end{bmatrix} \le 0 \tag{22}$$

$\dot{\tilde{\rho}}_i(t) = -l_i e^T(t)Pb_{2i}u_i(t) - l_i e^T(t)P\Delta b_{2i}\rho_i\tilde{\rho}_i^{-1}(t)u_i(t)$ The procedure for the proof of matrix inequalities (11) and (12) is similar to the proof of (9) and (10), respectively.

This yields $\dot{V} \le 0$. Using La Salle's theorem, we can see that the trajectories will converge to the largest invariant set for which $\dot{V} = 0$. But $\dot{V} = 0$ if and only if $e = 0$. If $e = 0$, since the control input has bounded norm, then the equations for $\dot{\tilde{\rho}}$ in the four different cases yield $\dot{\tilde{\rho}} = 0$, which implies $\tilde{\rho}$ will be constant. This constant value must be zero from the error dynamics (5), (6), (7) and (8) because $0 < \epsilon < |u(t)|$, or otherwise, $e \neq 0$, which would be a contradiction to La Salle's argument.

□

Remark: Notice that one needs the assumption that the control input will not be zero while detecting the fault. This assumption is needed for La Salle's argument in the proof and it physically corresponds to a persistent excitation.

In order to represent the matrix inequalities (9), (10), (11) and (12) in a convex form, new variables W_1 and W_2 are defined as:

$$\begin{aligned} W_1 &= PG_1 \\ W_2 &= PG_2 \end{aligned} \tag{23}$$

The new variables for the 4 cases yield the following LMIs

Case 1:

$$A_1^TP + PA_1 + W_1C_1 + C_1^TW_1^T + Q \le 0 \tag{24}$$

Case 2:

$$\begin{bmatrix} \begin{array}{c}A_2^TP + PA_2 \\ +W_2C_2 + C_2^TW_2^T \\ +Q\end{array} & \begin{array}{c}-P\Delta A \\ +\frac{1}{2}\lambda_1E_2^TE_1\end{array} & \begin{array}{c}-P\Delta m \\ +\frac{1}{2}\lambda_1E_2^Tf_1\end{array} \\ \begin{array}{c}-\Delta A^TP \\ +\frac{1}{2}\lambda_1E_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1E_1^TE_2 \\ +\frac{1}{2}\lambda_1E_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1E_1^Tf_2 \\ +\frac{1}{2}\lambda_1E_2^Tf_1\end{array} \\ \begin{array}{c}-\Delta m^TP \\ +\frac{1}{2}\lambda_1f_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1f_1^TE_2 \\ +\frac{1}{2}\lambda_1f_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_1f_1^Tf_2 \\ +\frac{1}{2}\lambda_1f_2^Tf_1 \\ -1\end{array} \end{bmatrix} \le 0 \tag{25}$$

Case 3:

$$\begin{bmatrix} \begin{array}{c}A_1^TP + PA_1 \\ +W_1C_1 + C_1^TW_1^T \\ +Q\end{array} & \begin{array}{c}P\Delta A \\ +\frac{1}{2}\lambda_2E_1^TE_2\end{array} & \begin{array}{c}P\Delta m \\ +\frac{1}{2}\lambda_2E_1^Tf_2\end{array} \\ \begin{array}{c}\Delta A^TP \\ +\frac{1}{2}\lambda_2E_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_2E_2^TE_1 \\ +\frac{1}{2}\lambda_2E_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_2E_2^Tf_1 \\ +\frac{1}{2}\lambda_2E_1^Tf_2\end{array} \\ \begin{array}{c}\Delta m^TP \\ +\frac{1}{2}\lambda_2f_2^TE_1\end{array} & \begin{array}{c}\frac{1}{2}\lambda_2f_2^TE_1 \\ +\frac{1}{2}\lambda_2f_1^TE_2\end{array} & \begin{array}{c}\frac{1}{2}\lambda_2f_2^Tf_1 \\ +\frac{1}{2}\lambda_2f_1^Tf_2 \\ -1\end{array} \end{bmatrix} \le 0 \tag{26}$$

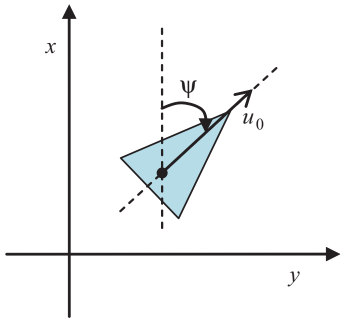

Fig. 1. Schematic of the Wheeled Mobile Robot (WMR)

Case 4:

$$A_2^T P + P A_2 + W_2 C_2 + C_2^T W_2^T + Q \leq 0 \quad (27)$$

To design the observer gains, the following convex problem will be solved.

Definition 2.1: (**Design Problem**) For fixed $\epsilon > 0$,

$$\begin{array}{ll} \min & \eta \\ s.t. & \eta > 0,\ \epsilon I < P < \eta \epsilon I \\ & (24),\ (25),\ (26),\ (27) \end{array}$$

From the solution to this problem one gets $G_1 = P^{-1} W_1$ and $G_2 = P^{-1} W_2$.

III. Example: Fault Detection and Identification in a WMR

In this section, a dynamical model of a WMR is used as an example. The WMR is shown in Fig. 1 and is assumed to be rigid and to be driven by a torque T to control the heading angle ψ of the WMR. The forward velocity u_0 is in the direction of the X-body axis and it is assumed to be already made constant by the proper design of a cruise controller. The heading angle of the WMR ψ is measured from the positive X-axis in the inertial frame. The kinematic equations of the WMR are

$$\begin{aligned} \dot{y} &= u_0 \ \sin\psi \\ \dot{\psi} &= R \end{aligned} \quad (28)$$

The dynamic equation of the WMR is

$$\dot{R} = \frac{1}{I} T \quad (29)$$

where T is the torque generated by the DC motors and is the input to the system. The moment of inertia of the WMR with respect to the center of mass is represented by $I = 1\ kg.m^2$ In this paper, it is desired that the WMR follows the path $y = 0$. The above differential equations are cast in matrix form as follows

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} u_0 \ \sin\psi \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \quad (30)$$

Piecewise-affine models of the system are derived for the following state-space partitioning

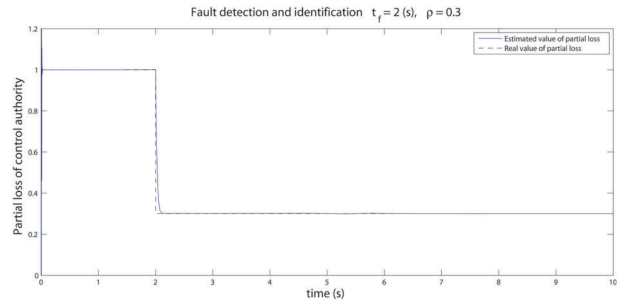

Fig. 2. Fault detection and identification in WMR actuator $t_f = 2(s), \rho = 0.3$

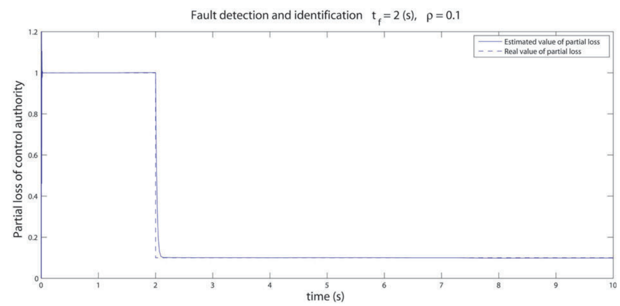

Fig. 3. Fault detection and identification in WMR actuator $t_f = 2(s), \rho = 0.1$

$$\begin{aligned} \mathcal{R}_1 &= \{x \in \mathbb{R}^3 \mid x_2 \in (-\frac{\pi}{2}, \frac{\pi}{2})\} \\ \mathcal{R}_2 &= \{x \in \mathbb{R}^3 \mid x_2 \in (\frac{\pi}{2}, \frac{3\pi}{2})\} \end{aligned} \quad (31)$$

The ellipsoidal covering of the state-space partitioning is:

$$\begin{aligned} \epsilon_1 &= \{x \mid \| \begin{bmatrix} 0 & \frac{2}{\pi} & 0 \end{bmatrix} x + 0 \| \leqslant 1\} \\ \epsilon_2 &= \{x \mid \| \begin{bmatrix} 0 & \frac{2}{\pi} & 0 \end{bmatrix} x - 2 \| \leqslant 1\} \end{aligned} \quad (32)$$

Thus, the PWA models before the fault occurrence are $\forall x \in \mathcal{R}_1$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \quad (33)$$

$\forall x \in \mathcal{R}_2$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & -0.6366 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T \quad (34)$$

After fault occurrence at $t = t_f$ the PWA model becomes $\forall x \in \mathcal{R}_1$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rho T \quad (35)$$

$\forall x \in \mathcal{R}_2$

$$\begin{bmatrix} \dot{y} \\ \dot{\psi} \\ \dot{R} \end{bmatrix} = \begin{bmatrix} 0 & -0.6366 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \psi \\ R \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rho T \quad (36)$$

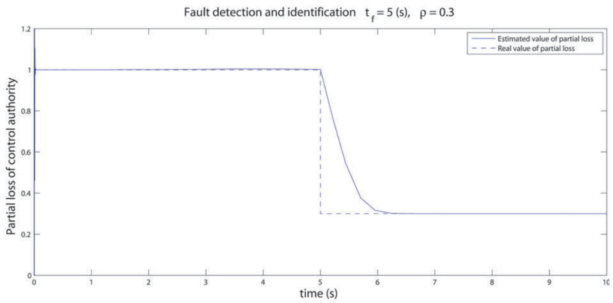

Fig. 4. Fault detection and identification in WMR actuator $t_f = 5(s), \rho = 0.3$

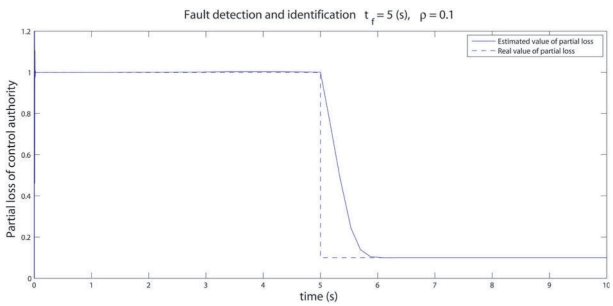

Fig. 5. Fault detection and identification in WMR actuator $t_f = 5(s), \rho = 0.1$

where the amount of fault ρ must be estimated by the observer. An LQR controller is designed for a linear model of the system as shown in (33) and the signal from this controller is used as the input to the system in the simulations. The weighting matrices for the LQR controller are

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$R = 0.1$$

The gain of the LQR controller is

$$K_{LQR} = \begin{bmatrix} 1.00 & 2.4142 & 2.4142 \end{bmatrix} \tag{37}$$

It is assumed that $G_1 = G_2 = G$. A solution to the design problem 2.1 is sought and obtained by SeDuMi/YALMIP [9] as

$$G = \begin{bmatrix} -986.4253 & -65.9326 & -66.4396 \\ -57.1708 & -784.1655 & -90.7489 \\ -57.6103 & -90.7488 & -789.7087 \end{bmatrix} \tag{38}$$

$$P = \begin{bmatrix} 3992.0 & 0 & 0 \\ 0 & 4603.8 & -0.0037 \\ 0 & -0.0037 & 4603.8 \end{bmatrix} \tag{39}$$

Figures 2-6 show FDI results for different values of partial loss of control and different fault occurrence times. It is observed that the FDI method works accurately while the control input to the system is large enough for rapid updating of the estimated amount of fault.

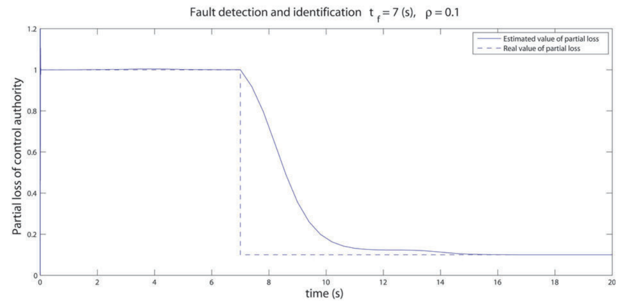

Fig. 6. Fault detection and identification in WMR actuator $t_f = 7(s), \rho = 0.1$

IV. CONCLUSIONS

In this paper a fault detection and identification technique for bimodal PWA systems is proposed for the first time. The proposed method enables the FDI algorithm to precisely estimate the unknown amount of fault based on a PWA representation of the system. The unknown value of the fault parameter is estimated by a law obtained from the Lyapunov function of the system. Global asymptotic stability of the estimation error is guaranteed provided the input norm is upper and lower bounded by positive constants. The proposed method is applied successfully to estimation of the amount of partial loss of control authority for a WMR.

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REFERENCES

[1] D. Wang, K.-Y. Lum, "Adaptive Unknown Input Observer Approach for Aircraft Actuator Fault detection and Isolation", *International Journal of Adaptive Control and Signal Processing*, Vol. 21, Issue 1, Pages 31-48, 2007.

[2] L. Rodrigues and E.-K. Boukas, "Piecewise-linear H_∞ controller synthesis with applications to inventory control of switched production systems," *Automatica*, vol. 42, pp. 1245–1254, 2006.

[3] S. Shehab and L. Rodrigues, "UAV Path Following Using a Mixed Piecewise-Affine and Backstepping Control Approach," *European Control Conference*, Kos, Greece, pp.301-306, 2007.

[4] S. LeBel and L. Rodrigues, "Piecewise-affine parameter-varying control of wheeled mobile robots", *American Control Conference*, Seattle, WA,pp.195–200 June 2008.

[5] S. LeBel and L. Rodrigues, "PWL and PWA H_∞ controller synthesis for uncertain PWA slab systems: LMI approach", *International Journal of Control*, vol. 82, issue 3, pp. 482–492, 2009.

[6] D.Ye, G.-H. Yang, "Adaptive Fault-Tolerant Tracking Control Against Actuator Faults With Application to Flight Control", *IEEE Trans. on Control Systems Technology*, Vol. 14, No. 6, Pages 1088-1096, 2006.

[7] L. Rodrigues and J. How, "Observer-based control of piecewise-affine systems", *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, FL, USA, pp. 1366–1371, 2001.

[8] A.Lj. Juloski, W.P.M.H. Heemels, S. Weiland, "Observer Design for a Class of Piece-wise Affine Systems", *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, USA, 2002.

[9] J. Lofberg, "YALMIP: A Toolbox for Modeling and Optimization in MATLAB", *In Proc. of the CACSD Conference*, Taipei, Taiwan, 2004.

[10] L. Rodrigues and S. Boyd, "Piecewise-affine state feedback for piecewise-affine slab systems using convex optimization", *Systems & Control Letters*, Vol. 54, Issue 9, Pages 835-853, September 2005.