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NONLINEAR CONTROL OF ASSOCIATIONS INCLUDING SYNCHRONOUS MOTORS AND AC/DC/AC CONVERTERS
A formal analysis of speed regulation and power factor correction

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Abstract— **We are considering the problem of controlling synchronous motors driven through AC/DC rectifiers and DC/AC inverters. The control objectives are threefold: (i) forcing the motor speed to track a reference signal, (ii) regulating the DC Link voltage, (iii) enforcing power factor correction (PFC) with respect to the power supply net. First, a nonlinear model of the whole controlled system is developed in the Park-coordinates. Then, a nonlinear multi-loop controller is synthesized using the backstepping design technique. A formal analysis based on Lyapunov stability and average theory is developed to describe the control system performances. In addition to closed-loop global asymptotic stability, it is proved that all control objectives (motor speed tracking, DC link voltage regulation, and unitary power factor) are asymptotically achieved up to small harmonic errors (ripples).. The above results are confirmed by simulations which, besides, show that the proposed regulator is quite robust with respect to uncertain changes of load torque.**

I. Introduction

PERAMANENT magnet synchronous (PMS) motors are more suitable for electric traction compared with induction motors. Indeed, they possess a better mass/power ratio, develop a much higher power level and present a more satisfactory efficiency. In effect, the Joule losses in PMS motors are much less important as these involve no field and rotor currents. The spectacular development of power electronics technology, over the last recent years, has resulted in reliable power electronic converters which make it possible to drive synchronous machines in varying speed mode. Indeed, speed variation can only be achieved for these machines by acting on the supply net frequency. Until the recent development of modern power electronics, there was no effective solution to AC machine speed control because there was no simple way to vary the net frequency. On the other hand, in the electric traction domain, the used power nets are either DC or AC but mono-phase. Therefore, three-phase DC/AC inverters turn out to be the only possible interface (between railway nets and 3-phase AC motors) due to their important capability to ensure a flexible voltage and frequency variation. The above considerations illustrate the major role of modern power electronics in the recent development of electrical traction applications.

As mentioned above, a three-phase DC/AC inverter used in traction is supplied by a power net that can be either DC or mono-phase AC. In the case of AC supply, the (mono-phase) net is connected to the three-phase DC/AC inverter through a transformer and AC/DC rectifier (Fig 1). The connection line between the rectifier and the inverter is called DC link.

The system consisting of the AC/DC converter, the DC/AD inverter and the PMS motor has to be controlled to achieve varying speed reference tracking. The point is that such system behaves as a nonlinear load vis-à-vis to the AC supply line. Then, undesirable current harmonics are likely to be generated in the AC line. These harmonics reduce the rectifier efficiency, induce voltage distortion in the AC supply line and cause electromagnetic compatibility problems. The pollution caused by the converter may be reduced resorting to additional protection equipments (transformers, condensers…) and/or over-dimensioning the converter and net elements. However, this solution is costly and may not be sufficient. To overcome this drawback, the control problem must have as objective not only motor speed control but also rejection of current harmonics. The last objective is referred to power factor correction (PFC), [1].

Previous works on synchronous machine speed control simplified the control problem neglecting the dynamics of the AC/DC rectifier and so making the focus only on the set 'DC/AC inverter - Motor'. A wide range of control solutions have thus been proposed. These involved as well simple techniques such as field-oriented control (FOC) [7] and NL techniques such as feedback linearization (FL) [10], direct torque control (DTC) [8] or sliding mode (SM) [9]. Ignoring the AC/DC rectifier in the development of a control strategy, is criticized at least from two viewpoints. First, such development relies on the assumption that the DC voltage provided by the AC/DC rectifier is perfectly regulated. The problem is that a perfect regulation of the rectifier output voltage can not be met ignoring the rectifier load which is nothing other than the set 'DC/AC inverter - Motor'. The second drawback of the previous control strategy lies in the entire negligence of the PFC requirement. It is not judicious, from a control viewpoint, to consider separately the association 'inverter - Motor', on one hand, and the rectifier, on the other hand.

In the present work, we are developing a new multiloop control strategy that deals simultaneously with both controlled subsystems: the AC/DC converter and the combination 'DC/AC inverter - Motor'. The main feature of our control design is threefold:

i. A input current loop is first designed so that the

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coupling between the power supply net and the AC/DC rectifier operates with a unitary power factor;

ii. A second loop is designed to regulate the output voltage of AC/DC rectifier so that the DC link between the rectifier and inverter operates with a constant voltage;

iii. A bi-variable loop is designed to enforce the motor velocity to track its varying reference value and to regulate the d-component of stator current to zero in order to optimize the absorbed stator current.

All loops are designed using the backstepping technique and Lyapunov design, [5]. A theoretical analysis will prove that the four-loop controller thus described actually stabilizes (globally and asymptotically) the controlled system and does achieve its tracking objectives with a good accuracy. More precisely, it is shown that the steady-state tracking errors corresponding to rectifier input current and rectifier output voltage, motor speed and stator current d-component are harmonic signals and their amplitudes depend on the supply net frequency: the larger the net frequency the smaller the error amplitudes. It follows in particular that the motor regulation objective and the PFC requirement are actually ensured, up to harmonic errors of insignificant amplitude, provided the net frequency is large enough. This formally establishes the existence of the so-called ripples, which are usually observed in similar practical applications, and proves why this phenomenon is generally insignificant. These theoretical results are obtained making a suitable use of different automatic control tools e.g. averaging theory and Lyapunov stability [2]. The paper also includes a simulation study confirming the above theoretical results and, besides, shows that the controller compensates well to disturbing effects due load changes.

The paper is organized as follows: the controlled system (including the AC/DC/AC converter and the synchronous motor) is modeled and given a state space representation; the control objectives in Section 2; the controller design and the closed-loop system analysis are presented in Section 3; the controller performances and robustness are illustrated Section 4 through numerical simulations; a conclusion and a reference list end the paper. To alleviate the paper presentation, a list of notations is given hereafter.

Notation list	
L	*stator winding inductance*
R	*resistance of the stator windings*
i_d, i_q	*d- and q- axis currents*
v_d, v_a	*d- and q- axis voltages*
ω	*angular velocity of the rotor*
p	*number of pole pairs*
T_L	*load torque*
J	*combined inertia of rotor and load*
f	*combined viscous friction of rotor and load*
K_M	*flux motor constant*

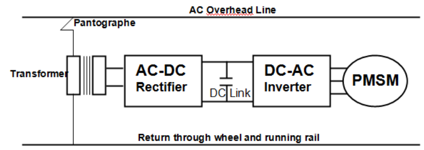

Fig.1. Schematic representation of single phase AC supply powering 3-phase AC motor

II. Modeling the association AC/DC/AC converter-Synchronous motor

The controlled system is illustrated by Fig 2. It includes an AC/DC boost rectifier, on one hand, and a combination 'DC/AC converter-synchronous' motor on the other hand. The circuit operates according to the well known Pulse Width Modulation (PWM) principle.

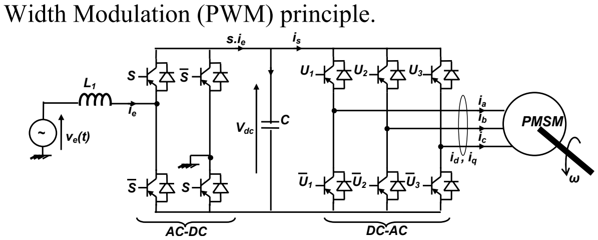

Fig.2. AC/DC/AC drive circuit with three-level inverter

A. *Modeling the PMW AC/DC Rectifier*

The transformer secondary is connected to a H-bridge converter which consists of four IGBT's with anti-parallel diodes for bidirectional power flow arrangement. This subsystem is described by the following set of differential equations:

$$\frac{di_e}{dt} = \frac{v_e}{L_1} - \frac{1}{L_1} s\, v_{dc} \quad (1.a)$$

$$\frac{dv_{dc}}{dt} = \frac{1}{C} s\, i_e - \frac{1}{C} i_s \quad (1.b)$$

where i_e is the current in inductor L_1, v_{dc} denotes the voltage in capacitor C, i_s designates the input current inverter, v_e is the supply net sinusoidal voltage ($v_e = \sqrt{2}.E.\cos(\omega_e t)$) and s is the switch position function taking values in the discrete set $\{-1, 1\}$. Specifically:

$$s = \begin{cases} 1 & \text{if } S \text{ is } ON \text{ and } \overline{S} \text{ is } OFF \\ -1 & \text{if } S \text{ is } OFF \text{ and } \overline{S} \text{ is } ON \end{cases}$$

It is not suitable for control design due to the switched nature of the control input s. As a matter of fact, existing nonlinear control approaches apply to systems with continuous control inputs. Therefore, control design for the above inverter will be based upon the following average version of (1.a-b):

$$\frac{dx_1}{dt} = \frac{v_e}{L_1} - \frac{1}{L_1} u_1 x_2 \quad (2.a)$$

$$\frac{dx_2}{dt} = \frac{1}{C} u_1 x_1 - \frac{1}{C} \bar{i}_s \quad (2.b)$$

where $x_1 = \bar{i}_e$, $x_2 = \bar{v}_{dc}$ and $u_1 = \bar{s}$ denote, respectively, the average values of i_e, v_{dc} and s over cutting periods.

B. Modeling the combination PMW DC/AC converter-synchronous motor

Such modeling is generally performed in the d-q rotating reference frame because the components i_d and i_q then turn out to be DC currents. According to [5], the model of the synchronous motor, expressed in the d-q coordinates, is given by:

$$\frac{d\omega}{dt} = -\frac{F}{J}\omega + \frac{3K_M}{2J}i_q - \frac{T_L}{J} \tag{3.a}$$

$$\frac{di_q}{dt} = -\frac{R}{L}i_q - p\,\omega i_d + \frac{K_M}{L}\omega + \frac{1}{L}v_q \tag{3.b}$$

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + p\,\omega i_q + \frac{1}{L}v_d \tag{3.c}$$

The inverter d- and q-voltage can be controlled independently. To this end, these voltages are expressed in function of the corresponding control action (see e.g. [4]):

$$v_q = v_{dc}\,u_2 \tag{4.a}$$

$$v_d = v_{dc}\,u_3 \tag{4.b}$$

$$i_s = 3(u_2\,i_q + u_3\,i_d)/2 \tag{4.c}$$

where $u_2 = \bar{u}_q, u_3 = \bar{u}_d$ are the average modulation indexes in the d- and q-axis, respectively. Similarly, let us introduce the state variables $x_3 = \omega$, $x_4 = i_q$, $x_5 = i_d$. Then, substituting (4a-b) in (3a-c) yields the following state space representation of the combination 'inverter-synchronous motor':

$$\frac{dx_3}{dt} = -\frac{F}{J}.x_3 + \frac{3K_M}{2J}x_4 - \frac{T_L}{J} \tag{5.a}$$

$$\frac{dx_4}{dt} = -\frac{R}{L}.x_4 - p.x_3.x_5 + \frac{K_M}{L}.x_3 + \frac{1}{L}u_2.x_2 \tag{5.b}$$

$$\frac{dx_5}{dt} = -\frac{R}{L}.x_5 + p.x_3.x_4 + \frac{1}{L}u_3.x_2 \tag{5.c}$$

The state space equations obtained up to now constitute a state-space model of the whole system including the AC/DC/AC converters combined with the synchronous motor:

$$\frac{dx_1}{dt} = \frac{v_e}{L_1} - \frac{1}{L_1}u_1\,x_2 \tag{6.a}$$

$$\frac{dx_2}{dt} = \frac{1}{C}u_1\,x_1 - \frac{3}{2C}(u_3x_5 + u_2x_4) \tag{6.b}$$

$$\frac{dx_3}{dt} = -\frac{F}{J}.x_3 + \frac{3K_M}{2J}x_4 - \frac{T_L}{J} \tag{6.c}$$

$$\frac{dx_4}{dt} = -\frac{R}{L}x_4 - p\,x_3x_5 + \frac{K_M}{L}x_3 + \frac{1}{L}u_2x_2 \tag{6.d}$$

$$\frac{dx_5}{dt} = -\frac{R}{L}x_5 + p\,x_3x_4 + \frac{1}{L}u_3x_2 \tag{6.e}$$

III. CONTROLLER DESIGN

A. Control objectives

The first control objective is to force the speed ω to track a reference signal ω_{ref}. The second objective is to constrain the input current rectifier to be sinusoidal and in phase with the AC supply voltage (PFC). But, there are three control inputs at hand, namely u_1, u_2 and u_3. Then, we will further seek two additional control objectives. Specifically:

- controlling the continuous voltage v_{dc} so as it tracks a given reference signal v_{dcref} (generally constant, equal to the nominal voltage entering the inverter)
- regulating the current i_d to a reference value i_{dref}, equal to zero in order to guarantee the absence of d-axis stator current

The last requirement is explained by the fact that the developed torque is given by the relation $T = 3.p(K_M i_q + (L_d - L_q)i_d i_q)/2$ (see e.g. [6]). Accordingly, torque control should be performed acting on both i_d and i_q. But, for the surface-magnet synchronous motor, the large effective airgap means that $L_d \approx L_q = L$ i.e. i_d does not really influence T and so it is sufficient to regulate it to zero.

B. Control loop design for current i_e

The PFC objective means that the input current rectifier should be sinusoidal and in phase with the AC supply voltage. We therefore seek a regulator that enforces the current x_1 to tack a reference signal x_1^* of the form:

$$x_1^* = k\,v_e \tag{7}$$

At This point k is any positive (time-varying) parameter. Introduce the current tracking error:

$$z_1 = x_1 - x_1^* \tag{8}$$

In view of (6.a), the above error undergoes the following equation:

$$\dot{z}_1 = \frac{v_e}{L_1} - \frac{1}{L_1}.u_1.x_2 - \dot{x}_1^* \tag{9}$$

To get a stabilizing control law for this first-order system, consider the quadratic Lyapunov function $V_1 = 0.5z_1^2$. It can be easily checked that the time-derivative $\dot{V}_1$ is a negative definite function of z_1 if control input is chosen to be:

$$u_1 = L_1\left(c_1.z_1 + (v_e/L_1) - \dot{x}_1^*\right)/x_2 \text{ with } c_1 > 0 \tag{10}$$

***Proposition 1**. Consider the control subsystem (6.a) and the control law (10). The reference x_1^* is assumed available and derivative. The inner closed-loop system undergoes the following equation: $\dot{z}_1 = -c_1.z_1$ with $c_1 > 0$. It is clearly seen that the error z_1 converges exponentially fast to zero, whatever the initial conditions.*

C. Control loop design for the voltage v_{dc}

The aim of the outer loop is to generate a tuning law for the ratio *k* so that the output voltage v_{dc} be regulated to a given reference value v_{dcref}.

1) Relation between k and x_2

The first step in designing such a loop is to establish the relation between the ratio *k* (control input) and the output voltage x_2. This is the object of the following proposition.

Proposition 2. *Consider the power converter described by (6.a-b) and the i_e control loop defined by (10). One has the following properties:*

1) The output voltage x_2 varies, in response to the tuning ratio k, according to the equation:

$$\frac{dx_2}{dt} = \frac{1}{C x_2}(k v_e^2 + z_1 v_e) - \frac{3}{2C}(u_3 x_5 + u_2 x_4) \quad (11)$$

2) The squared voltage ($y = x_2^2$) varies, in response to the tuning ratio k, according to the equation:

$$\frac{dy}{dt} = \frac{2}{C} k v_e^2 + f(x,t) \quad (12)$$

where

$$f(x,t) = \frac{2}{C}(z_1 v_e - \frac{3}{2} x_2 (u_3 x_5 + u_2 x_4)) \quad (13)$$

Proof. 1) The input power in the AC/DC side is expressed by $P_e = x_1 v_e$. The delivered power at the load (capacity and inverter) is given by $P_{trans} = u_1.x_1.x_2$. Using the power conservation argument ($P_{trans} = P_e$), one has:

$$x_1.v_e = u_1.x_1.x_2 \quad (14)$$

Using (14) and the fact that the input current expression is $x_1 = k.v_e + z_1$, yields : $u_1\, x_1 = (k\, v_e^2 + z_1\, v_e)/x_2$, which together with (6.b) establishes (11)

2) Lets introduce the variable change $y = x_2^2$ in (11). Deriving y with respect to time and using (11) yields the model (12) and completes the proof of proposition 1.

2) Squared DC-link voltage regulation

The ratio k stands up as a virtual control input in the system (12). The reference signal $y_{ref} \hat{=} v_{dcref}^2$ of the squared output capacitor voltage ($x_2 = v_{dc}$) is chosen to be constant, equal to the nominal input voltage of the inverter. Then, it follows from (12) that the tracking error $z_2 = y - y_{ref}$ undergoes the following equation:

$$\dot{z}_2 = \frac{2}{C} E^2 k + \frac{2}{C} E^2 k \cos(2\omega_e t) + f(x,t) - \dot{y}_{ref} \quad (15)$$

To get a stabilizing control law for this system, consider the following quadratic Lyapunov function:

$$V_2 = 0.5 z_2^2 \quad (16)$$

It is easily checked that the time-derivative $\dot{V}_2$ can be made a negative definite function of the state z_2 by letting:

$$k\, E^2 + k\, E^2 \cos(2\omega_e t) = \frac{C}{2}\left(- c_2 z_2 - f(x,t) + \dot{y}_{ref}\right) \quad (17)$$

where $c_2 > 0$ is a design parameter.

An approximate simple solution is:

$$k = \frac{C}{2}\left(- c_2 z_2 - f(x,t) + \dot{y}_{ref}\right)/ E^2 \quad (18)$$

In view of such choice, it follows from (18), (17) and (15) that z_2 undergoes the differential equation:

$$\dot{z}_2 = -c_2 z_2 + \frac{2}{C} k\, E^2 \cos(2\omega_e t) \quad (19)$$

Remark. The signal k is treated by a prefilter to obtain its derivative signal (then the time-derivative of x_1^* is available).

D. Control loop design for motor speed ω

A control law for the remaining (actual) control input, namely u_2, will now be determined based on equations (6c-d) in order to guarantee speed reference tracking. To this end, let z_3 denote the speed tracking error:

$$z_3 = x_3 - \omega_{ref} \quad (20)$$

In view of (6.c), the above error undergoes the following equation:

$$\dot{z}_3 = -\frac{F}{J}.x_3 + \frac{3K_M}{2J}.x_4 - \frac{T_L}{J} - \dot{\omega}_{ref} \quad (21)$$

In (21), the quantity $\alpha = (3K_M / 2J).x_4$ stands up as a (virtual) control input for the z_3-dynamics. Let α^* denote the desired trajectory (yet to be determined) of α. It is easily seen from (21) that if $\alpha = \alpha^*$ with:

$$\alpha^* = -c_3 z_3 + \frac{F}{J}.x_3 + \frac{T_L}{J} + \dot{\omega}_{ref} \quad (22)$$

Then one would get $\dot{z}_3 = -c_3 z_3$ with $c_3 > 0$ is a design parameter. This would clearly ensures asymptotic stability of (21) with respect the Lyapunov function:

$$V_3 = 0.5 z_3^2 \quad (23)$$

In effect, the time derivative of V_3 would then be:

$$\dot{V}_3 = z_3 \dot{z}_3 = -c_3 z_3^2 < 0 \quad (24)$$

As $\alpha = (3K_M / 2J).x_4$, is a virtual control input, one can not set $\alpha = \alpha^*$. Nevertheless, the above expression of the desired trajectory is retained and a new error is introduced:

$$z_4 = \alpha - \alpha^* \quad (25)$$

Using (23)-(25), it follows from (21) that the z_3-dynamics undergoes the following equation:

$$\dot{z}_3 = -c_3 z_3 + z_4 \quad (26)$$

The next step consists in determining the control input u_2 so that the errors (z_3, z_4) vanish asymptotically. The trajectory of the error z_4 is obtained by operating a time-derivation on (25), that is:

$$\dot{z}_4 = (3K_M / 2J).\dot{x}_4 - \dot{\alpha}^* \quad (27)$$

Using (22) and (6c-d) in (27) yields:

$$\dot{z}_4 = \beta(x) + \gamma + c_3 \dot{z}_3 + \frac{3K_M}{2JL} u_2 x_2 \quad (28)$$

where

$$\beta(x) = -\frac{3K_M}{2J}\left(\frac{R}{L}.x_4 + p.x_3.x_5 - \frac{K_M}{L}.x_3\right) + \left(\frac{F^2}{J^2}.x_3 - \frac{3FK_M}{2J^2}.x_4\right) \quad (29)$$

$$\gamma(t) = \frac{FT_L}{J^2} - \frac{\dot{T}_L}{J} - \ddot{\omega}_{ref} \quad (30)$$

Then, the error equation (26) and (28) can be rewritten in a more compact form:

$$\dot{z}_3 = -c_3 z_3 + z_4$$

$$\dot{z}_4 = \beta(x) + \gamma + c_3 \dot{z}_3 + \frac{3K_M}{2JL} u_2 x_2 \quad (31)$$

To determine a stabilizing control law for (31), let us consider the quadratic Lyapunov function candidate:

$$V_4 = V_3 + 0.5 z_4^2 \quad (32)$$

Using (26), the time derivative of V_4 can be rewritten as:

$$\dot{V}_4 = -c_3 z_3^2 + z_3 z_4 + z_4 \dot{z}_4 \quad (33)$$

This shows that, for the z_3, z_4-system to be globally asymptotically stable, it is sufficient to choose the control u_2 so that $\dot{V}_4 = -c_3 z_3^2 - c_4 z_4^2$ (with $c_4 > 0$). In view of (33), this amounts to let:

$$\dot{z}_4 = -c_4 z_4 - z_3 \quad (34)$$

Comparing (34) and (31) yields the following backstepping control law:

$$u_2 = -(2JL/3K_M)\left((c_3 + c_4) z_4 - (c_3^2 - 1) z_3 + \beta(x) + \gamma\right)/x_2 \quad (35)$$

E. d-axis current loop design

The d-axis current x_5 undergoes equation (6.e) in which the following quantity:

$$v = p.x_3.x_4 + u_3.x_2 / L \quad (36)$$

acts as a virtual input. As the reference signal i_{dref} is zero, it follows that the tracking error $z_5 = x_5$ undergoes the equation:

$$\dot{z}_5 = -(R/L) z_5 + v \quad (37)$$

To get a stabilizing control signal for this first-order system, consider the following quadratic Lyapunov function:

$$V_5 = 0.5 z_5^2 \quad (38)$$

It can be easily checked that the time-derivative $\dot{V}_5 = -c_5 z_5^2$ is a negative definite function of z_5 if the (virtual) control input is let to be:

$$v = -(-(R/L) + c_5) z_5 \text{ with } c_5 > 0 \quad (39)$$

Now, it is readily observed that the actual control input u_3 is obtained substituting (39) in (36) and solving the resulting equation. Doing so, one gets:

$$u_3 = \left(-c_5 z_5 + \frac{R}{L} z_5 - p x_3 x_4\right) \frac{L}{x_2} \quad (40)$$

Proposition 3. *Consider the control system consisting of the subsystem (6c-e) and the control laws (35) and (40). The resulting closed-loop system undergoes, in the (z_3, z_4, z_5)-coordinates, the following equation:*

$$\begin{pmatrix} \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} -c_3 & 1 & 0 \\ -1 & -c_4 & 0 \\ 0 & 0 & -c_5 \end{pmatrix} \begin{pmatrix} z_3 \\ z_4 \\ z_5 \end{pmatrix} \quad (41)$$

It is readily seen that (41) is globally asymptotically stable with respect to the Lyapunov function $V = 0.5(z_3^2 + z_4^2 + z_5^2)$. As (41) is linear, then the error vector (z_3, z_4, z_5) converges exponentially fast to zero, whatever the initial conditions.

Theorem. *Consider the system including the AC/DC/AC power converters and the synchronous motor connected in tandem, as shown in Fig.2. For control design purpose, the system is represented by its average model (6a-e). Let the reference signals v_{dcref}, ω_{ref} and i_{dref} be selected such that $v_{dcref} > 0$, $\omega_{ref} \geq 0$ and $i_{dref} = 0$. Consider the controller defined by equations (10), (35) and (40) where all design parameters, namely c_1, c_2, c_3, c_4 and c_5 are positive. Introduce the notations: $z = [z_1 \; z_2 \; z_3 \; z_4 \; z_5]^T$; $\varepsilon = 1/2\omega_e$*

Then, one has the following properties:

1) The resulting closed-loop system undergoes the following equation:

$$\dot{z}(t) = A\, z(t) + g(z,t) \quad (42)$$

with

$$A = \begin{pmatrix} -c_1 & 0 & 0 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & 0 \\ 0 & 0 & -c_3 & 1 & 0 \\ 0 & 0 & -1 & -c_4 & 0 \\ 0 & 0 & 0 & 0 & -c_5 \end{pmatrix}; \quad g(z,t) = \begin{pmatrix} 0 \\ \frac{2}{C}.k.E^2 \cos(2\omega_e t) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2) The tracking errors z_1, z_3, z_4 and z_5 vanish asymptotically

3) Let the reference signals v_{dcref} and ω_{ref} be nonnegative and periodic with period $N\pi/\omega_e$, where N is any positive integer. Then exists a positive real ε^* and ξ such that for all $0 < \varepsilon < \varepsilon^*$: the tracking error z_2 is a harmonic signal that continuously depend on ε and:

$$\|z(t,\varepsilon)\| < \xi\varepsilon \text{ Consequently: } \lim_{\varepsilon \to 0} z_2(t,\varepsilon) = 0 \quad (43)$$

Proof. In order to prove the theorem, consider the equation (42) and introduce the time-scale change $\tau = 2\omega_e t$ and the state variable $w(\tau) = z(t)$ which implies $\dot{w}(\tau) = \dot{z}(t)/2\omega_e$. Then, equation (42) gives:

$$\dot{w}(\tau) = \varepsilon\, A\, w(\tau) + \varepsilon\, g_1(\tau, w, \varepsilon) \quad (44a)$$

with:

$$g_1(\tau, w, \varepsilon) = \left(0 \quad (2/C)\, k\, E^2 \cos(\tau) \quad 0 \quad 0 \quad 0\right)^T \quad (44b)$$

The stability of system (44) will be now be analyzed with using tools from the averaging theory [2]. As v_{dcref} and ω_{ref} are periodic with period $N_1\pi/\omega_e$ and $N_2\pi/\omega_e$ respectively, with N_1 and N_2 are any positive integer numbers. The average system is essentially obtained averaging the function $g_1(\tau, \bullet, \bullet)$ with respect to its first argument, over the interval [0, 2π]. From (44b) it is readily seen that the average value of $g_1(\tau, \bullet, \bullet)$ is precisely equal to zero. Hence, the average version of (44a) is:

$$\dot{\overline{w}}(\tau) = \varepsilon\, A\, \overline{w}(\tau) \quad (46)$$

In order to get stability results regarding the system of interest, i.e. (44a), it is sufficient to analyze the linear average system (45). It is clear that matrix A is Hurwitz (all its eigenvalues have negative real parts) because the coefficients c1 to c5 are positive. Then, the origin $\overline{w}=0$ is an equilibrium point of the average system (45). Now, invoking averaging theory (see e.g. Theorem 10.4 in [2]), we conclude that there exists a positive real constants ε^* and ξ such that, for all $0<\varepsilon<\varepsilon^*$, (42) has a unique,

exponentially stable, 2π-periodic solution $\overline{w}(z,\varepsilon)$ with the property $\|\overline{w}(z,\varepsilon)\| = \|z(t,\varepsilon)\| \le \xi\varepsilon$. This proves the theorem.

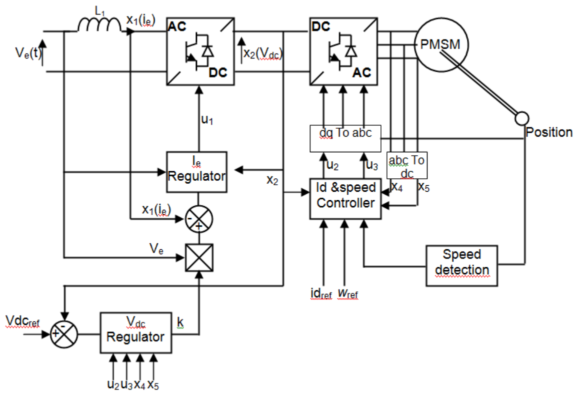

Fig.3. Control system including AC/DC/AC converter and a PMSM

IV. Simulations

The experimental setup, described by Fig. 3, has been simulated in Matlab/Simulink environment. The involved elements have the following characteristics:

. Supply network: $v_e(t)=\sqrt{2}.E\cos(\omega_e t)$; $E=220v/50Hz$

. AC/DC/AC converters: $L_1=15mH$; $C=4.5mF$;

. Synchronous motor: $L=9.4mH$; $R=0.6\Omega$; $K_M=0.29$; $J=0.000765Nm/rd/s^2$, $F=0.003819Nm/rd/s$; $p=2$.

The reference values of the state variables are chosen as:

$v_{dcref}=500V$; ω_{ref} steps from 0 to 100 *rad/s* at $t=0.3s$; a constant load torque of *15 Nm* is applied to the drive at $t=0.5s$ and then back to *10 Nm* at $t=0.7s$. $i_{dref}=0$.

The following values of the controller parameters turned out to be suitable: $c_1=1000$, $c_2=50$, $c_3=80$, $c_4=900$, $c_5=800$.

The controller performances are illustrated by Figs 4 to 6. Fig 4 shows that a unitary power factor is achieved after a transient period following each change in reference values or load torque. Figs 5 and 6 show that the tracking quality is quite satisfactory for all controlled variables (v_{dc}, ω, i_d). The response time is less than 0.05 s. The disturbing effect, due to load torque change, is also well compensated by the regulator.

V. Conclusion

In this paper we have considered the problem of controlling the power electronic AC/DC/AC converters with synchronous motor load. The system dynamics have been described by the averaged 5th order nonlinear state-space model (6a-e). Based on such a model, the Lyaponuv stability and averaging theory are used to establish the multiloop nonlinear controller. Presented approach guarantees (Theorem) well line side power quality, controllable and stable (average) DC-link voltage (v_{dc}) as well as, the power factor at AC input mains is close to unity (PFC) in the entire operating range of the drive. The convergence of the rotor speed and d-axis current, towards their references values, is guaranteed. These results have been confirmed by a simulation study which, further, showed the robustness of controller performances with respect to load changes.

References

[1] Singh B., G. Bhuvaneswari, and V. Garg, "Improved Power Quality AC-DC Converter for Electric Multiple Units in Electric Traction", Power India Conference, 2006 IEEE, 6 pp

[2] Khalil H, "NONLINEAR SYSTEMS", Prentice Hall, 2002.

[3] Senthil Kumar N., Saravanan K., "Speed control of PMSM drive using VSI", Industrial Electronic Conference (IECON), Busan, South Korea, pp.888- 895 Vol. 1 , 2004

[4] Michael J, Ryan D and Rik W, "Modeling of Sinewave Inverters: A Geometric Approach", IECON, Proceedings of the 24th Annual Conference of the IEEE, pp.396 - 401 vol.1 1998.

[5] Wallmark O., "On control of permanent-Magnet synchronous Motors in Hybrid-Electric Vehicle Application", Technical Report, School of Electrical Engineering, Chalmers University of technology Goteborg, Sweden, 2004.

[6] Muhammad H, Rashid, "Power electronics handbook", Academic press, 2001.

[7] M. Jasinski, M. Cichowlas, M. P. Kazmierkowski " Direct control for AC/DC/AC converter-fed induction motor with active filtering function", Compel, The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Vol.25, pp. 235-242, 2006.

[8] Saleh, K.I.; Mohammed, O.A.; Badr, M.A."Field oriented vector control of synchronous motors with additional field winding", Energy Conversion, IEEE Transaction on Vol 19, pp. 95 – 101, 2004.

[9] Yang Z.P., M.H. Wang, C.L. Liu, Hou, Y.B. Cai, "Variable structure control with sliding mode for self-controlled synchronous motor drive speed regulation", IEEE International Symposium on Industrial Electronics, vol.2, pp 620 – 624, 1992.

[10] Kuroe Y., K. Okamura, H. Nishidai, T. Maruhashi, "Optimal speed control of synchronous motors based on feedback linearization", International Conference on Power Electronics and Variable-Speed Drives, pp 328 – 331, 1988.

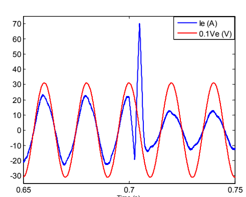

Fig.4. Input current and voltage waveform

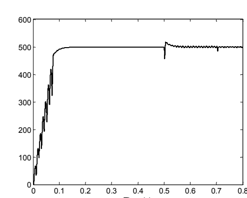

Fig. 5. Dynamic response of the DC-link voltage :v_{dc}

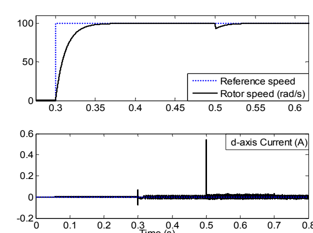

Fig. 6. Speed ω(rad/s) and d-axis current id(A)