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Local Minimum Time Trajectory Planning for Five-axis Machining with or without Deflection

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Abstract—**This paper presents the algorithms to obtain the local minimum-time trajectory planning for the five-axis machining with or without the tool deflection. The tool and workpiece are first combined as a closed-chain system of rigid bodies. The tool path and shape are simplified to the tool-workpiece system curve and tool line. The forward and inverse kinematics are applied to obtain the kinematic relation between the system path and the position of the five motors. Based on the kinematic equations, the velocity, acceleration, and jerk of each motor can be derived. The motion trajectory of the end-effector of this system can be described through a sequence of intervals. In order to guarantee the dimension accuracy, the quintic polynomial was applied to fit the curve on each individual interval. The genetic algorithm (GA) with one generation is applied to find the minimum time period at every interval on the system curve in the case that the system has no deflection on the reference path. When there is the deflection on the system path, an algorithm is developed to find its projection at the i^{th} interval on the reference path without deflection through the potential field method, and then the local minimum time periods at two new intervals composed with deflection, its projection, and the $(i+1)^{th}$ reference point, which is on the curve without deflection, respectively. The derived kinematic equations and the proposed algorithms are verified through the simulation.**

I. INTRODUCTION

FIVE-AXIS computer numerical controlled (CNC) machines are extensively used in milling for the realization of ruled surface or complex parts with large curvature radii, including turbine blades, impellers, and aircraft parts [1]-[5]. Comparing with the 3-axis machines 5-axis machines own two more degrees of freedom (DOF), which provide many advantages over the 3-axis machines, such as faster material removal rates, better tool accessibility, and improved surface finish [4]. In order to use the 5-axis machines, more complicated tool path planning needs to be solved.

A lot of efforts have been paid to tool path planning for the 5-axis machining. Rao et al. proposed the tool path planning through principal axis method to define the placement of the cutting tool at a single point on the workpiece surface and assume that a preferred feed direction will be maintained [7]. Sheng and Pan presented a non-uniform layered rough cut plan for B-spline surfaces using convex hull boxes [8]. Cho et al. developed a CNC tool path generation method for a multi-patch sculptured surface in the parametric plane to obtain a minimum number of cutter location points while maintaining the required machining accuracy [9]. Jun and Lee derived a methodology and algorithms of optimizing and smoothing the tool orientation control for 5-axis sculptured surface machining [4]. Affouard et al. studied a topical method for avoiding the tool to traverse singular position in 5-axis machining [3].

Based on the cutting process, we consider the workpiece and tool as a closed-chain system of rigid bodies for the trajectory planning, which is often used in the trajectory planning of multiple robotic arms [10-13]. The tool and workpiece deflections are important aspects for the dimension errors [6]. The tool path and shape are simplified as curve and line respectively. The tool path can be considered as a sequence of intervals with points. For the reduction of the dimension error and the motors control, the trajectory planning is need at every interval on the reference tool path. We assume the reference tool path with collision free has been obtained.

This paper deals with the methods for trajectory planning at every interval on the tool-workpiece system path with and without the deflection. The Forward and inverse kinematics are widely used in the geometric analysis of the robots [14] [15]. The problem of forward kinematics is to determine the position and orientation of the end effector given the values for the joint variables of the robot. The problem of inverse kinematics is to determine the values of the joint variables of the robot. In our case, the tool is assumed to be in contact with the workpiece curve all the time. After obtaining the position of motors, the corresponding posture of the tool-workpiece system can be found in the workpiece coordinate frame based on the forward kinematics. With the help of the tool-workpiece system position in workpiece coordinate frame, the rotation angles of two motors can be derived based on the tangent line at the system position, and then the other three values of linear motors can be obtained through the inverse kinematics. The velocity, acceleration, and jerk of the motors can be derived based on the kinematic relation between the motors and tool-workpiece system.

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The system path is assumed to be made up of $n-1$ intervals. The i^th interval is described by two adjacent points (P_i and $P_{i+1}, i = 1 \cdots n, n$: the number of points). In order to make the feed rate of tool-workpiece system, and the velocity and acceleration of motors smooth and continuous, the velocity and acceleration at every reference point on the curve are set in advanced. Based on Cho's paper [9], the parametric-based trajectory planning is applied on our research. The basic idea of the parametric-plane-based tool path generation is for the tool paths to be parallel straight lines on the parametric plane [9]. Thus, the quintic polynomial trajectory is chosen in our case to satisfy above requirement. Genetic algorithms (GAs) are population-based, stochastic, and global search methods. Their performance is better than that of some classical techniques, and they have been successfully used in the trajectory planning of industrial robotic manipulator [16]. GAs has the search ability that can provide the possibility of finding optimal solutions [16]. The disadvantage of the GAs is that it may need relative long time to find the global optimal solutions. Since the process of the trajectory planning for the system without deflection is off-line, the local minimum time period at every interval can be obtained through the simple genetic algorithm (GA) [16][17] with one generation under the constraints that the feed rate of the tool-workpiece system, and velocity, acceleration and jerk of motors must be in certain range.

During the cutting process, the deflection may appear on the path of the tool-workpiece system. In order to guarantee the workpiece dimension accuracy, an algorithm is developed in this paper to obtain the new path for the system through making it pass through the projection of the deflection on the reference curve without the deflection. The projection of the deflection can be found through the potential field method, which is commonly used for autonomous mobile robot path planning in the past decade [15][19-21]. The basic concept of the potential field method is to fill the robot workspace with an artificial potential field in which the robot is attracted to its target position and is repulsed away from the obstacles. After obtaining the projection of deflection at the i^{th} interval on the reference curve without deflection, the algorithm can find the local minimum time periods at two new intervals composed with deflection, its projection, and the $(i+1)^{th}$ reference point, which is on the curve without deflection, respectively.

In the next section, we first define the coordinate frames on one kind of the 5-axis machines. It owns two more DOF; namely A angle rotating around x-axis, and B angle rotating around y-axis, compared with the 3-axis machines. Secondly the kinematic relations between the motors position and tool-workpiece system path are obtained by the forward and inverse kinematics. Then the relation of velocity, acceleration, and jerk between the motors and system can be obtained based on the kinematic equations. Thirdly the trajectory planning algorithms are proposed to find the minimum time period at every interval on the system path with or without the deflection. The derived kinematic equations and algorithms are verified through a simulation case study in section III. The paper is ended by some conclusions.

II. Method

A. Coordinate Frames Definition

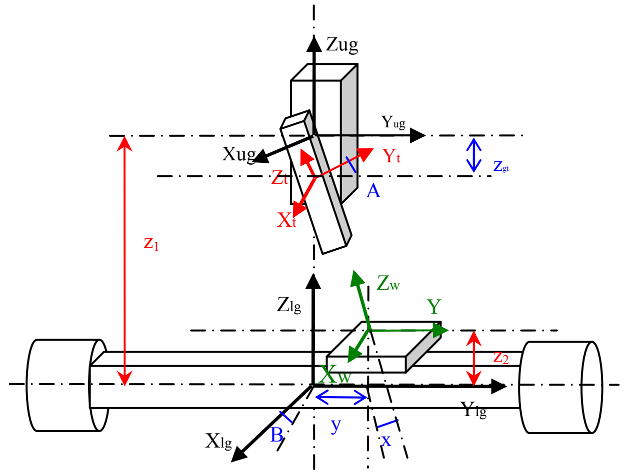

Fig. 1. The coordinate frames in the 5-axis machine

The structure of the 5-axis machine and the coordinate frames are shown in Fig. 1. Here the origin of the lower-ground coordinate frame is set at the center of the y-axis motor shaft; the origin of the upper-ground coordinate frame is set at the initialization point of the tool; the origin of tool coordinate frame is set on the head of tool; the origin of the workpiece coordinate frame is set at the center of the x-axis motor shaft; the direction of y-axis at the lower-ground, upper- ground, and workpiece coordinate frame is the same. The values of z_1 and z_2 are constants based on machine structure. The motors position corresponding to $x_{lg}-y_{lg}-z_{lg}$ axis is represented by x, y, and z_{gt}, and the motors angles rotating around x_{lg} and y_{lg} axis are called A and B respectively.

B. Forward and Inverse Kinematics

We simplify the tool path and shape to the tool-workpiece system curve and the tool line. After obtaining the position of motors, the corresponding posture of tool-workpiece system can be found in the workpiece coordinate frame based on the forward kinematics [14] [15].

For the tool in tool coordinate frame, the position of the nose of tool is $P_t(0,0,l_t,1)^T$, where l_t is the length of the tool. This point can be described in the lower-ground coordinate frame through the transformation matrix T_t^{lg}

$$T_t^{lg} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & -\sin A & 0 \\ 0 & \sin A & \cos A & z_{gt}+z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

So,

$$P_t^{lg} = T_t^{lg} \bullet P_t = \begin{bmatrix} 0 \\ l_t \sin A \\ -l_t \cos A + z_{gt} + z_1 \\ 1 \end{bmatrix} \quad (2)$$

The point in the lower-ground coordinate frame can be described in workpiece coordinate frame via the transformation matrix T_{lg}^{w}

$$T_{lg}^{w} = (T_w^{lg})^{-1} = \begin{bmatrix} \cos B & 0 & -\sin B & -x \\ 0 & 1 & 0 & -y \\ \sin B & 0 & \cos B & -z_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Thus, the tool can be described in the workpiece coordinate frame through the following equation

$$P_t^{w} = T_t^{w} \cdot P_t = \begin{bmatrix} -x - (z_1 + z_g - l_t \cdot \cos A) \cdot \sin B \\ l_t \cdot \sin A - y \\ (z_1 + z_{gt} - l_t \cdot \cos A) \cdot \cos B - z_2 \\ 1 \end{bmatrix} \quad (4)$$

Based on the cutting process, the tool is always in contact with the workpiece, which means $P_t^{w} = P_w$. Thus we can find the expression of $P_w(x_w, y_w, z_w, 1)^T$ with the motors position.

$$\begin{cases} x_w = -x - (z_1 + z_g - l_t \cdot \cos A) \cdot \sin B \\ y_w = l_t \cdot \sin A - y \\ z_w = (z_1 + z_{gt} - l_t \cdot \cos A) \cdot \cos B - z_2 \end{cases} \quad (5)$$

Next the motor position needs to be described after obtaining the value of P_w in the workpiece coordinate through the inverse kinematics.

The value of rotation angle A and B can be obtained by the method of Lin and Korean [22]. With the known A and B, the expression of x, y, and z_{gt} can be derived after knowing the position of P_w through Eq. (6).

$$\begin{cases} x = -x_w - (z_2 + z_w) \cdot \tan B \\ y = l_t \cdot \sin A - y_w \\ z_{gt} = (z_2 + z_w) \cdot \sec B + l_t \cdot \sin A - z_1 \end{cases} \quad (6)$$

With the differentiation of Eq. (6), we can obtain the velocity, acceleration, and jerk between the linear motors (represented by $x, y,$ and z_{gt}) and the system in the workpiece coordinate frame. The velocity, acceleration, and jerk of rotation motors can be interpolated with their known position.

C. The Trajectory Planning Algorithm without Deflection

The tool-workpiece system curve can be described by a sequence of intervals with points. For the motor control, the trajectory planning algorithm is required at the i^th interval with two adjacent points (P_i and $P_{i+1}, i = 1 \cdots n$) as the beginning and end points. The parametric-based trajectory planning is applied on our case [9]. Since the milling feed-rate and the velocity and acceleration of five motors should be smooth, continuous and under certain constraints, a parameter u and the quintic-polynomial trajectory are chosen based on following equations [18] [22]

Given $r(k), r(k)', r(k)'', r(k+1), r(k+1)',$ and $r(k+1)''$

$$r(u)_{k \to k+1} = C_0 + C_1 \cdot u + C_2 \cdot u^2 + C_3 \cdot u^3 + C_4 \cdot u^4 + C_5 \cdot u^5 \quad (7a)$$

where $u \in [0,1], r$ represents $x_w, y_w, z_w, A,$ or B.

Here

$$C_0 = r(k), C_1 = r(k)', C_2 = r(k)''/2, C_3 = 10D_1 - 4D_2 + D_3, \\ C_4 = -15D_1 + 7D_2 - 2D_3, C_5 = 6D_1 - 3D_2 + D_3 \quad (7b)$$

$$\text{where} \begin{cases} D_1 = r(k+1) - r(k) - C_1 - C_2 \\ D_2 = r(k+1)' - C_1 - 2C_2 \\ D_3 = r(k+1)'' - 2C_2 \end{cases}$$

The feed rate along the surface can be derived as follows [22]

$$v(t) = \left|\frac{ds}{dt}\right| = \left|\frac{ds}{du} \cdot \frac{du}{dt}\right| \quad (7c)$$

where

$$\frac{ds}{du} = \sqrt{(x_w')^2 + (y_w')^2 + (z_w')^2}, x_w' = dx_w/du, y_w' = dy_w/du,$$

$$z_w' = dz_w/du, du/dt = v(t)/\sqrt{(x_w')^2 + (y_w')^2 + (z_w')^2}$$

The relation between the parameter u and time period T_i can be obtained through Taylor's expansion [22]

$$u_{i+1} = u_i + T_i \cdot \dot{u}_i + (T_i^2/2) \cdot \ddot{u}_i + o(T_i) \quad (7d)$$

where

$$\dot{u}_i = \frac{du_i}{dt} = \frac{v(t)}{\sqrt{[x_w(i)']^2 + [y_w(i)']^2 + [z_w(i)']^2}}, T_i = t_{i+1} - t_i,$$

$$\ddot{u}_i = \frac{dv(t)}{dt} \frac{1}{\sqrt{[x_w(i)']^2 + [y_w(i)']^2 + [z_w(i)']^2}} - \frac{v(t)^2 \cdot [x_w(i)' \cdot x_w(i)'' + y_w(i)' \cdot y_w(i)'' + z_w(i)' \cdot z_w(i)'']}{\{[x_w(i)']^2 + [y_w(i)']^2 + [z_w(i)']^2\}^2}$$

The velocity, acceleration, and Jerk for five motors can be derived as follows

$$V(w) = w(u)' \cdot \dot{u}; a(w) = w(u)'' \cdot \dot{u}^2 + w(u)' \cdot \ddot{u}; \\ J(w) = w(u)^{(3)} \cdot \dot{u}^3 + 3 \cdot w(u)'' \cdot \dot{u} \cdot \ddot{u} \quad (7e)$$

where w represents $A, B, x, y, or\ z_{gt}$. V, a, and J represent velocity, acceleration and jerk, respectively.

Since the procedure of desired trajectory planning at every interval on the curve without deflection is off-line, the time period T_i at the i^{th} interval can be minimized through the simple genetic algorithm (GA) [16] [17] with one generation under the constraints that the feed rate of the system, and velocity acceleration and jerk of the motors must be in certain range. The fitness function is $T = \sum_{i=1}^{n-1} T_i$. The block diagram of the algorithm to obtain the local minimum time periods when the system has no deflection is shown in Fig. 2.

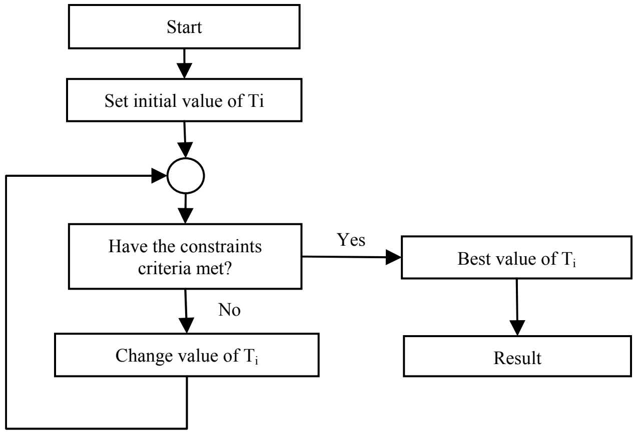

Fig. 2. Block diagram of the trajectory planning algorithm without deflection at the i^{th} interval

D. The Trajectory Planning Algorithm with Deflection

During the cutting process, the path of the tool-workpiece system may have the deflection as shown in Fig. 3.

Since the system should follow as many reference points as possible, an algorithm is proposed to find its projection at the i^{th} interval on the reference path without deflection through the potential field method firstly. Then it is easy to find the local minimum time periods at two new intervals composed with deflection, its projection, and the $(i+1)^{th}$ reference point, which is on curve without deflection, respectively based on the same algorithm in Section II. C.

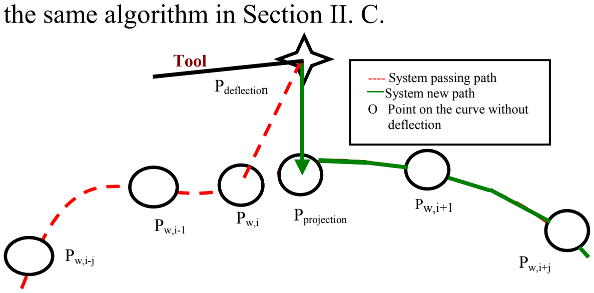

Fig. 3. Tool-workpiece path with the deflection

For the potential field method, the attractive potential is defined as a function of the relative distance between the robot and the target and their velocity [21]. The potential field function ca be described as following

$$U_{att}(p)=\alpha_p\left\|p_{tar}(\tau_i)-p_d(\tau_i)\right\|^m+\alpha_v\left\|v_{tar}(\tau_i)-v_d(\tau_i)\right\|^n \quad (8)$$
$$i=1\cdots n, \tau_i\in[0\ \ 1]$$

where p_d is the position of deflection in the workpiece coordinate; p_{tar} is the projection of the deflection at the i^{th} interval on the system reference curve without deflection; v_d and v_{tar} are corresponding velocity of deflection and projection respectively. The corresponding virtual attractive force is defined as the negative gradient of attractive potential in terms of position [21]

$$F_{att}(p)=-\nabla U_{att}(p)=-\frac{\partial U_{att}(p)}{\partial p} \quad (9)$$

For the trajectory between the deflection and projection, the feed rate of the tool-workpiece system, and the velocity and acceleration of five motors should be smooth, continuous and under certain constraints. So the velocity and acceleration at the deflection and projection are set to be the same. The quintic polynomial trajectory is chosen the same as Eq. (7). Then Eq. (8) can be simplified

$$U_{att}(p)=\alpha_p\left\|p_{tar}(\tau_i)-p_d(\tau_i)\right\|^m \text{ with } \alpha_p=1, m=2 \quad (10)$$

Through Eq. (9) and (10), the projection of deflection on the reference curve without deflection can be derived.

The steps of trajectory planning with deflection are shown as follows

(1) Find the projection of deflection at the i^{th} ($i = 1\cdot\cdot n-1$) interval on the system reference curve without deflection through the potential field method.
(2) The deflection and its projection are chosen as the new beginning and end position at the first new interval respectively with the same velocity, acceleration, and jerk to find the parameters in the quintic polynomial trajectory.
(3) The time period T_{dp} in this interval is then minimized through the GA algorithm in Section II.C.
(4) The next interval begins from projection, and ends at the reference point P_{i+1} on the curve without deflection, and its time period is minimized through repeating Step (2) and (3).

III. Simulation Study

The simulation is used to verify derived equations and algorithms. The tool-workpiece system is supposed to follow a curve with collision free in the workpiece coordinate frame. Here the simulation case is a quarter circle with center at (0.0707 m, 0.0707m, 0) and radius 0.1 m as shown in Fig. 4.

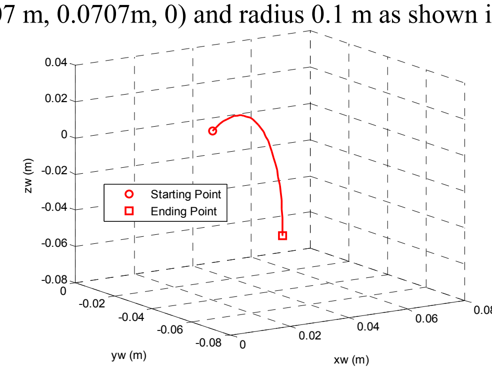

Fig. 4. The simulated system reference curve in workpiece coordinate frame

The length of tool is set as $l_t = 0.05$ m, and the structure parameters (z_1 and z_2) at the 5-axis machine are set as $z_1 = 0.3$ m and $z_2 = 0.03$ m. The tool-workpiece system curve in the workpiece coordinate frame is shown in Fig. 5. From this figure, we can find the tool-workpiece system follows the reference curve, which verifies the derived kinematics relation by Eqs. (1) through (6).

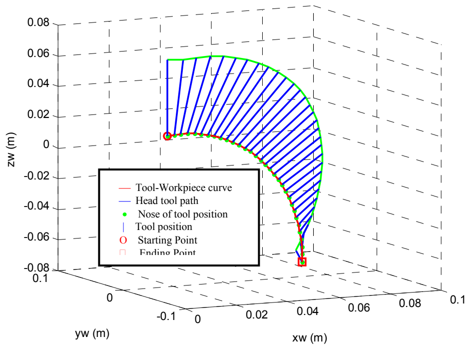

Fig. 5. The tool-workpiece system curve in the workpiece coordinate frame

Assume there are 31 intervals in a sequence to represent the system reference curve without deflection. Based on this condition, the velocity, acceleration, and jerk of motors, and the local minimum time period at every interval can be obtained through the quintic polynomial trajectory and GA algorithm in Section II. The constraints of feed rate, velocity, acceleration and jerk for the system and motors are shown in Eq. (11). The simulation results are shown in Fig. 6 and 7.

$$\begin{aligned} &V_{\text{feed rate}} = \sqrt{\dot{x}_w^{\,2} + \dot{y}_w^{\,2} + \dot{z}_w^{\,2}} \le F = 0.03 \text{ m/s} \\ &v_{x,y,z_{gt}} \le v_{x,y,z_{gt}\max} = 0.5 \text{ m/s}, \quad v_{A,B} \le v_{A,B\max} = 0.5 \text{ rad/s} \\ &a_{x,y,z_{gt}} \le a_{x,y,z_{gt}\max} = 20 \text{ m/s}^2, \quad a_{A,B} \le a_{A,B\max} = 20 \text{ rad/s}^2 \\ &J_{x,y,z_{gt}} \le J_{x,y,z_{gt}\max} = 200 \text{ m/s}^3, \quad J_{A,B} \le J_{A,B\max} = 200 \text{ rad/s}^3 \end{aligned} \tag{11}$$

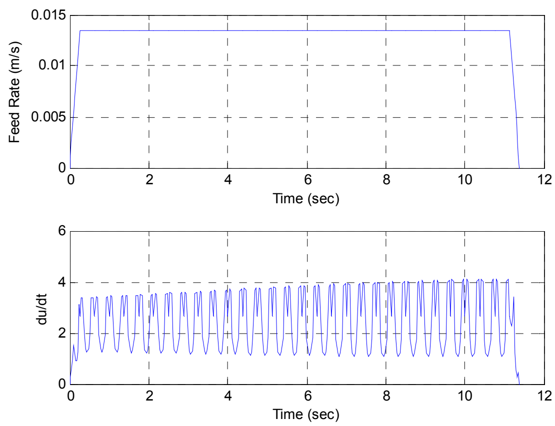

Fig. 6. Feed rate and du/dt for the system without deflection

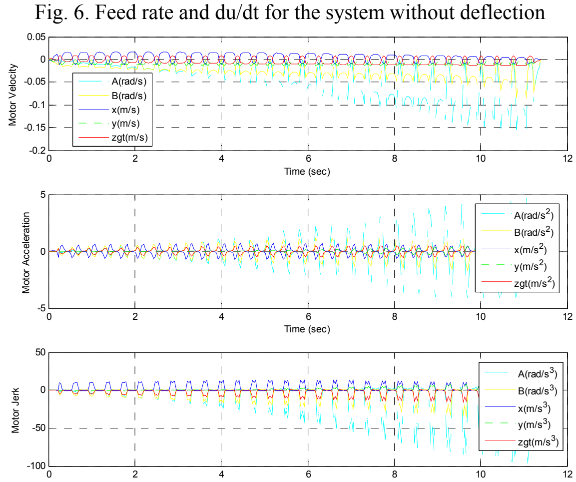

Fig. 7. Velocity, Acceleration, and Jerk of motors without deflection

In the results in Figs. 6 and 7, the feed rate of the system, and velocity, acceleration and jerk of the motors are all under the constraints shown in Eq. (11). So the proposed algorithm used to deal with the system trajectory planning without deflection can be considered as correct.

Assume the position of the deflection on the system path at $P_{deflection}(0.059, 0.0568, 0.0589)$. Through Eq. (9) and (10), the projection of the deflection $P_{projection}(0.058, 0.058, 0.0571)$ can be found at the 20^{th} interval on the reference curve without deflection as shown in Fig. 8. Set the constraints are the same as Eq. (11). The simulation using the proposed algorithm in Section II.D is shown in Figs. 9 and 10.

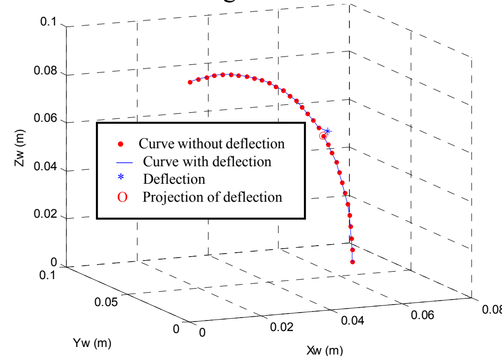

Fig. 8. The tool-workpiece system curves without and with deflection

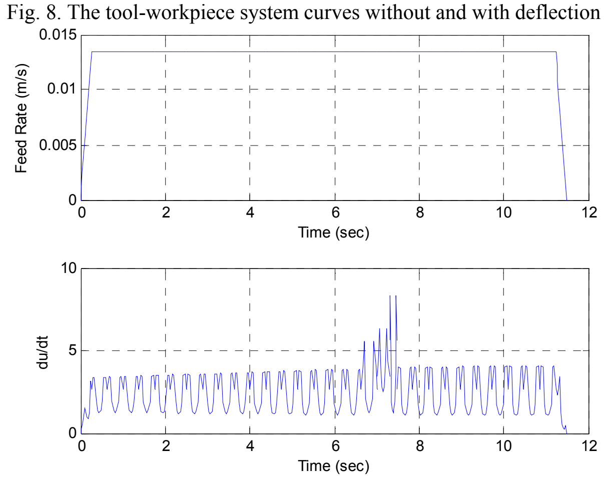

Fig. 9. Feed rate and du/dt for the system with deflection

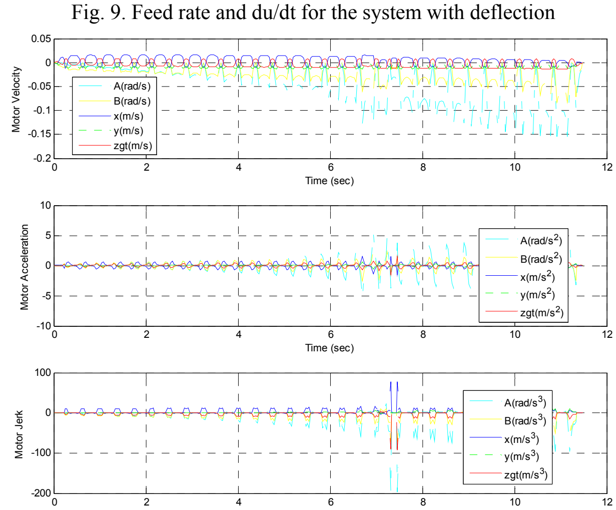

Fig. 10. Velocity, Acceleration and Jerk of motors with deflection

Through Fig. 9-10, the feed rate of the system, and velocity, acceleration and jerk of each motor all satisfy the constraints shown in Eq. (11). Thus the proposed algorithm can be applied on the system trajectory planning with deflection.

IV. CONCLUSIONS AND FUTURE WORK

In 5-axis machining, when the tool and workpiece were considered as a closed-chain system of rigid bodies, the kinematic relation between the system and five motors can be derived through the forward and inverse kinematics after the definition of coordinate frames on the machines. In our case, the tool path and shape were simplified to the tool-workpiece system curve and the tool line in the workpiece coordinate frame respectively. After knowing the position of five motors, the posture of the tool-workpiece system can be found through the forward kinematics. With the known position of the tool-workpiece system and the rotation angles (A and B) of two rotation motors, the position of three linear motors were obtained through the inverse kinematics. The velocity, acceleration, and jerk of each motor can be obtained through the differentiation on the derived kinematic equations.

Commonly a reference curve of the tool-workpiece system can be described by a series of intervals in the workpiece coordinate frame. For the motor control, the trajectory planning on every interval was required. The quintic polynomial trajectory planning was used to make the feed rate of the system, and velocity and acceleration of motors smooth and continuous. When the end-effector had no deflection, the off-line genetic algorithm was applied to find the minimum time period at every interval during the parametric-based trajectory planning.

When the deflection appears during the cutting process, the potential field method was adopted to find its projection at the i^{th} interval on the reference curve without deflection. For the dimensional accuracy of the workpiece surface, an algorithm was proposed to obtain the trajectory planning with local minimum time periods at two new intervals composed with deflection, its projection, and the $(i+1)^{th}$ reference point, which was on the reference curve without deflection, respectively.

The derived kinematic relation between the system and five motors was simulated successfully. Under the constraints at the feed rate of the system, and velocity, acceleration, and jerk of motors, the proposed algorithms for the local minimum time trajectory planning with or without deflection can find the suitable results in the simulation case.

In the future, the derived kinematic relation and algorithms will be verified through the experiments.

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