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Robust Control of Ship Fin Stabilizers Subject to Disturbances and Constraints

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Abstract— **This paper is concerned with constrained roll control of ship fin stabilizers operating in wave fields. Our approach is based on a robust control algorithm for linear discrete-time systems subject to bounded additive disturbances and a general class of input-state constraints. The proposed method is applied to control of fin stabilizers. Simulation results show that the proposed robust control method reduces the ship roll motion while satisfying the input and dynamic stall constraints. The proposed control algorithm does not involve on-line optimization, except for a linear program solved at initialization. On the theory side, we present a generalized form of our robust constrained control algorithm to the case of mixed state-input constraints.**

I. Introduction

Control of the roll motion of ships has been considered in the literature extensively (see [1]-[2] and references therein). As is elaborated in [2], large roll motions induced by ocean waves can severely affect the safety and performance of surface ships. To reduce the roll motion, different devices have been developed, including the "fin stabilizer" used for high speed ships [3], [4]. The fin stabilizer reduces roll motion by controlling the mechanical angle of the fin according to the ship roll angle and roll rate.

Another important consideration in the fin stabilization problem is to compensate the dynamic stall. Dynamic stall is a nonlinear phenomenon caused by unsteady hydrodynamic effects which lead to the loss of roll moment and therefore of the control authority when the angle of attack exceeds a certain threshold [5]. The dynamic stall can be prevented by imposing an input-state constraint in the form of a linear combination of the mechanical angle of the fin (input) and the roll rate (state).

In addition, there is a mechanical limitation for the fin angle which is captured as an input constraint (saturation). A common approach to deal with these constraints is to reduce the gain of the controller compared to the gain designed for the optimal performance for unconstrained conditions. Consequently, the overall performance, even for small signals, reduces. Recently, a Model Predictive Control (MPC) approach has been proposed for the roll stabilization subject to input-state inequality constraints and input saturation constraint [1]. While the MPC has certain inherent robust properties to small disturbances, the MPC strategy developed in [1] does not explicitly consider the disturbance effects induced by sea waves and therefore the constraints may be violated in presence of disturbances. In this paper, we address the problem with a robust control method applied to discrete time linear system model subject to bounded additive disturbances. The proposed method guarantees convergence of state to the target set in the presence of constraints without performing an optimization at each sampling instant.

The general problem of robust control of linear systems subject to additive bounded disturbance has been studied employing invariant set methods (see [7], [8] and references therein) and optimization-based control strategies such as MPC. MPC is known as an effective method to deal with constraints and uncertainties [9]. In the MPC context, one approach to address the problem is to rely on the inherent robustness of MPC and on the assumption that the open-loop system is sufficiently contractive [10]. The open-loop input control sequence MPC strategy proposed in [11], in which the control action is taken as the first element of an optimal control sequence, may cause uncertainties to spread over the horizon and therefore may result in a conservative domain of attraction in the presence of disturbances. A feedback MPC approach has been proposed in which the optimization is performed over feedback policies [12]. However, optimization over arbitrary feedback policies, in the presence of constraints, may be difficult. Therefore, affine feedback policies are employed where state feedback gain(s) are calculated off-line and optimization is performed over constant terms [13], [14], [15].

Another approach dealing with constrained control problems for systems with disturbances is based on the idea of tightening constraints on states and controls over the prediction horizon. It has been proposed initially in [16] and further developed in [13], [17], [19], [20]. The key idea is to retain a suitable margin over the prediction horizon so that feasibility is guaranteed for future iterations, in the presence of worst-case disturbances.

In this paper, we employ an extension of the robust control method proposed in [18] to the case where the system is subject to mixed input-state constraints. The proposed scheme, which is also based on the constraint tightening approach, has several special features. First, unlike the robust MPC approaches, our proposed method does not involve repeated online optimization to determine the control action. Second, if the input and state constraints over the prediction horizon are feasible, the proposed controller guarantees recursive feasibility for future iterations. Third, the minimal invariant

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set corresponding to the off-line calculated state feedback is an attractor, i.e., all trajectories will converge to this set. Forth, our approach does not require the terminal constraint set to be contained in the desired target set, which is a typical assumption made in the prior literature, except for [19]. In fact, the terminal constraint set, namely the set to which the final predicted state must belong, can be much larger than the target set. Finite-time convergence to the target set is guaranteed as long as the target set contains the minimal invariant set. We show that the roll motion of the ship in the presence of wave disturbance can be stabilized using the proposed algorithm while the input-state constraints and input saturation constraints can be effectively enforced and the recoverable domain of initial ship states can be very large.

II. Problem Statement

To present the proposed algorithm in a general framework, we consider a class of linear time-invariant, discrete-time systems described by

$$\begin{aligned} &x^+ = Ax + Bu + w, \\ &x(k) \in \mathbb{R}^n,\ u(k) \in \mathbb{R}^m,\ w(k) \in \mathbb{R}^n \end{aligned} \tag{1}$$

where x, u and w are, respectively, the state, control and disturbance vectors; x^+ denotes the successor state of x and $k \in \mathbb{N}$, where $\mathbb{N}$ is the set of non-negative integers.

We assume that the disturbance w belongs to a polytope W, the control and state are subject to hard constraints, i.e.,

$$(u, x) \in \Omega \subset \mathbb{U} \times \mathbb{X} \text{ and } w \in W, \tag{2}$$

where $\mathbb{U}$ and W are (convex, compact) polytopes, containing the origin in their interior, and $\mathbb{X}$ is a (convex) closed polyhedron. Finally, a target constraint set $\mathbb{X}_t$ is given by

$$\mathbb{X}_t = \{x \in \mathbb{R}^n | Yx \leq q\}, Y \in \mathbb{R}^{r\times n},\ q \in \mathbb{R}^r. \tag{3}$$

We assume that $\mathbb{X}_t$ is bounded and $0 \in int(\mathbb{X}_t)$. The control objective is to find u that steers the state into the target set $\mathbb{X}_t$. Moreover, we assume the existence of a feedback gain matrix $K \in \mathbb{R}^{m\times n}$ such that $A_K = A + BK$ is an exponentially stable matrix and the minimal robust invariant set[1] F_K for the system $x^+ = A_K x + w$, defined in [21], satisfies

$$F_K \subseteq \mathbb{X}_t. \tag{4}$$

Notations: Pontryagin difference [21] of two sets S and T is defined as $S \sim T = \{x | x + t \in S,\ \forall\ t \in T\}$.

III. Robust Control Algorithm

For any initial state $x \in \mathbb{X}$, the following control sequence

$$\mathbf{u}^*(x) := \{u_0^*(x), u_1^*(x), \cdots, u_{N-1}^*(x)\}$$

and associated state sequence

$$\mathbf{x}^*(x) := \{x_0^*(x), x_1^*(x), \cdots, x_N^*(x)\}$$

are feasible if they satisfy the set of constraints $C(x)$, defined as follows:

$$\begin{aligned} &x_0^*(x) = x, \\ &x_{i+1}^*(x) = Ax_i^*(x) + Bu_i^*(x),\ i = 0, \cdots, N-1, \end{aligned} \tag{5}$$

$$\begin{aligned} &\Omega_0 = \Omega, \\ &\Omega_{i+1} = \Omega_i \sim [\,K^T \quad I\,]^T A_K^i W,\ i = 0, \cdots, N-1, \\ &(u_i^*(x), x_i^*(x)) \in \Omega_i\ \ i = 0, \cdots, N-1, \end{aligned} \tag{6}$$

$$x_N^*(x) \in \mathbb{X}_f \tag{7}$$

where $\mathbb{X}_f$ is a robust invariant set for the system

$$x^+ = A_K x + w \tag{8}$$

with $w \in A_K^N W$, i.e.,

$$A_K \mathbb{X}_f + A_K^N W \subset \mathbb{X}_f \tag{9}$$

and

$$[\,K^T \quad I\,]^T \mathbb{X}_f \subset \Omega_N. \tag{10}$$

Moreover, for any set S which satisfies conditions (9) and (10), $S \subset \mathbb{X}_f$.

Remark 3.1: It should be noted that to achieve a large constrained domain of attraction, $\mathbb{X}_f$ should ideally be selected as the maximal robust invariant set.

Let us assume that for an initial state $x(0)$, $\mathbf{u}^*(x(0))$ and $\mathbf{x}^*(x(0))$ are feasible control and state sequences. Computing these initial sequences involve finding a point inside a polyhedron defined by (5)-(7); it can be performed using linear programming. Now we propose the following iterative algorithm, where at each time instant k, the feasible control sequence $\mathbf{u}^*(x(k))$ is constructed using the feasible control and state sequences $\mathbf{u}^*(x(k-1))$ and $\mathbf{x}^*(x(k-1))$, where $x(k)$ is the observed state at the time instant k:

$$\begin{aligned} &u_i^*(x(k)) = u_{i+1}^*(x(k-1)) + K(x_i^*(x(k)) - x_{i+1}^*(x(k-1))), \\ &\text{for } i = 0, \cdots, N-2, \\ &u_{N-1}^*(x(k)) = Kx_{N-1}^*(x(k)); \end{aligned} \tag{11}$$

$$\begin{aligned} &x_0^*(x(k)) = x(k), \\ &x_{i+1}^*(x(k)) = Ax_i^*(x(k)) + Bu_i^*(x(k)),\ i = 0, \cdots, N-1. \end{aligned} \tag{12}$$

At each time instant, the first element of the feasible control sequence is applied as the control signal, therefore we form the robust control law as

$$u(k) = \kappa_N^*(x(k)) := u_0^*(x(k)). \tag{13}$$

Theorem 3.1: Suppose the set of constraints $C(x(0))$ is satisfied with the feasible control, $\mathbf{u}^*(x(0))$, and state, $\mathbf{x}^*(x(0))$, sequences. Then the state and input trajectories of the system (1) with the control law

$$u(\cdot) = \kappa_N^*(x(\cdot)) \tag{14}$$

defined by (13) satisfy the input and state constraints (2). Furthermore, the set of constraints $C(x(k))$ is satisfied by the

[1]The robust invariant set F_K for the system $x^+ = A_K x + w$ is minimal if for all closed robust invariant sets X such that $A_K X + W \subset X$, it follows that $F_K \subset X$.

control and state sequences $\mathbf{u}^*(x(k))$ and $\mathbf{x}^*(x(k))$, defined by (11) and (12), for all $k > 0$.

Proof: Assume $\mathbf{u}^*(x)$ and $\mathbf{x}^*(x)$ are feasible control and state sequences for $C(x)$ and x^+ is the successor state defined in (1). Considering the state evolution (12) and control update (11), we have:

$$\begin{aligned} x^*_{i+1}(x^+) &= Ax^*_i(x^+) + Bu^*_i(x^+) \\ &= Ax^*_i(x^+) + Bu^*_{i+1}(x) + BK(x^*_i(x^+) - x^*_{i+1}(x)) \\ &= A_K(x^*_i(x^+) - x^*_{i+1}(x)) + x^*_{i+2}(x),\ i = 0, \cdots, N-2 \end{aligned} \tag{15}$$

where the last equality is achieved by adding and subtracting $A_K x^*_{i+1}(x)$ and using equation (12). From (12), we have

$$x^*_0(x^+) - x^*_1(x) = x^+ - Ax - Bu^*_0(x) = w_0 \in W, \tag{16}$$

and using (15) it can be easily shown that

$$x^*_i(x^+) - x^*_{i+1}(x) = A^i_K w_0 \in A^i_K W,\ i = 0, \cdots, N-1. \tag{17}$$

Moreover, from (11) and (17) we have

$$u^*_i(x^+) - u^*_{i+1}(x) = KA^i_K w_0 \in KA^i_K W,\ i = 0, \cdots, N-2. \tag{18}$$

From equations (17) and (18), we have

$$\begin{bmatrix} u^*_i(x^+) \\ x^*_i(x^+) \end{bmatrix} = \begin{bmatrix} u^*_{i+1}(x) \\ x^*_{i+1}(x) \end{bmatrix} + \begin{bmatrix} K \\ I \end{bmatrix} A^i_K w_0, \tag{19}$$

for $i = 0, \cdots, N-2$. Considering (6) and feasibility of $u^*(x)$, $x^*(x)$, (19) can be written as follows:

$$\begin{aligned} &\begin{bmatrix} u^*_i(x^+) \\ x^*_i(x^+) \end{bmatrix} \in \Omega_{i+1} + \begin{bmatrix} K \\ I \end{bmatrix} A^i_K W \\ &= (\Omega_i \sim \begin{bmatrix} K \\ I \end{bmatrix} A^i_K W) + \begin{bmatrix} K \\ I \end{bmatrix} A^i_K W \subseteq \Omega_i. \end{aligned} \tag{20}$$

From (11) and (17), where $i = N-1$, we have

$$\begin{aligned} \begin{bmatrix} u^*_{N-1}(x^+) \\ x^*_{N-1}(x^+) \end{bmatrix} &= \begin{bmatrix} K \\ I \end{bmatrix} x^*_{N-1}(x^+) \\ &= \begin{bmatrix} K \\ I \end{bmatrix} (x^*_N(x) + A^{N-1}_K w_0). \end{aligned} \tag{21}$$

Since (according to the terminal predicted state constraint (10)) $x^*_N(x) \in \mathbb{X}_f$, (21) implies that

$$\begin{bmatrix} u^*_{N-1}(x^+) \\ x^*_{N-1}(x^+) \end{bmatrix} \in \Omega_N + \begin{bmatrix} K \\ I \end{bmatrix} A^{N-1}_K W \subseteq \Omega_{N-1}, \tag{22}$$

where the last inclusion follows from (6).

On the other hand, from equations (11) and (12), we have

$$\begin{aligned} x^*_N(x^+) &= Ax^*_{N-1}(x^+) + Bu^*_{N-1}(x^+) \\ &= (A + BK)x^*_{N-1}(x^+) = A_K x^*_{N-1}(x^+). \end{aligned} \tag{23}$$

From (17), where $i = N-1$, we have

$$x^*_{N-1}(x^+) - x^*_N(x) \in A^{N-1}_K W. \tag{24}$$

Multiplying (24) by A_K and using (23), we have

$$x^*_N(x^+) \in \{A_K x^*_N(x)\} + A^N_K W. \tag{25}$$

Since $x^*_N(x) \in \mathbb{X}_f$ and the set $\mathbb{X}_f$ is a robust invariant set for the system (8) and disturbance set $A^N_K W$,

$$\{A_K x^*_N(x)\} + A^N_K W \subset A_K \mathbb{X}_f + A^N_K W \subset \mathbb{X}_f.$$

Thus, $x^*_N(x^+) \in \mathbb{X}_f$. This and (22) imply that $\mathbf{x}^*(x^+)$, $\mathbf{u}^*(x^+)$ satisfy constraints (6)-(7). ■

To investigate convergence properties of the controller (13), we first recall that [21]

$$F_K = \sum_{i=0}^{\infty} A^i_K W. \tag{26}$$

We need the following auxiliary results.

Lemma 3.1 (See [18]): Let $\mathbf{u}^*(x)$ and $\mathbf{x}^*(x)$ be feasible control and state sequences corresponding to state x, and let $\mathbf{u}^*(x^+)$ and $\mathbf{x}^*(x^+)$ be control and state sequences generated by (11) and (12), where x^+ is the successor state defined in (1). Then

$$d(x^*_i(x^+), A^i_K F_K) \leq d(x^*_{i+1}(x), A^{i+1}_K F_K),\ i = 0, \cdots, N–1. \tag{27}$$

Lemma 3.2: Let $x^+ = A_K x + w$, $w \in A^N_K W$, P be a Lyapunov matrix corresponding to the stable matrix A_K, i.e., $P \succ 0$ and $\exists Q \succ 0\ s.t.\ A^T_K P A_K - P = -Q$, and the norm $\| \cdot \|_p$ is defined as $\|x\|_p := \sqrt{x^T P x},\ x \in \mathbb{R}^n$. If the distance is defined in the normed space $(\mathbb{R}^n, \|\cdot\|_p)$ and $\|D\|_p$ denotes the induced norm of square matrix $D \in \mathbb{R}^{n \times n}$, then

$$\begin{aligned} &\exists\ 0 < \alpha < 1\ s.t.\ \|A_K\|_p \leq \alpha \\ \text{and}\quad &d(x^+, A^N_K F_K) \leq \alpha d(x, A^N_K F_K) \end{aligned} \tag{28}$$

Theorem 3.2: If for an initial state $x(0)$, there exist feasible control and state sequences satisfying the set of constraints $C(x(0))$, then the set F_K is robustly attractive (all trajectories converge to F_K despite disturbances) for the system

$$x^+ = Ax + B\kappa^*_N(x) + w, \tag{29}$$

where $w \in W$. Furthermore, the region of attraction is

$$R = \{x \in \mathbb{R}^n |\ C(x) \text{ is feasible}\}.$$

Remark 3.2: The important feature of the proposed method is that the attraction to F_K is achieved without involving any repeated on-line optimization or minimal robust invariant set approximation. In contrast, in the MPC based method ([13], [19]) attraction to F_K is achieved by solving online an optimization problem. The additional benefit of on-line optimization may, however, include transient response shaping through a cost function optimization. For our approach, the transient performance is tuned through the selection of K.

Remark 3.3: The proposed robust control method is based on tightening constraints, at each time instance over the prediction horizon, by $A^i_K W$, similar to [10], [20]. However, the advantage of the proposed method is that it does not require the final constraint set $\mathbb{X}_f$ to be a subset of the desired target set $\mathbb{X}_t$. In fact, the target set $\mathbb{X}_t$ is only required to contain the minimal robust invariant set F_K, i.e., $F_K \subset \mathbb{X}_t$, in order to be attractive.

In the next section we consider the problem of control of ship fin stabilizer as a practical case study for the proposed robust control method.

IV. Control of Ship Fin Stabilizer

For ships that normally operate above certain speeds, using fins is one of the most effective roll stabilization techniques [3]. Ship fin stabilizers consist of a pair of fins located approximately amidship on the bilge of the hull, as indicated in Figure 1. These fins have the freedom to rotate about a stock, and the control system changes the mechanical angle of the fins, α_m, according to a control algorithm that uses measurements of the roll angle, ϕ, and roll rate, p. Defining the angle of attack, α_e, as the angle of the flow with respect to the fin, hydrodynamic forces, proportional to the angle of attack, are induced on the fins. Due to the location of the fins on the hull, these forces produce a moment that reduces the wave-induced roll motion.

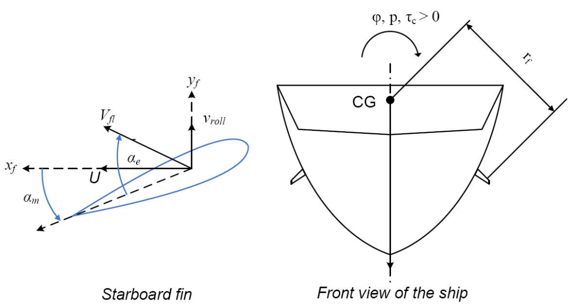

Fig. 1. Ship roll fin stabilizer.

Depending on the size of the ship and the severity of the sea state, the effectiveness of the fin stabilizer can be degraded due to nonlinear effects associated with unsteady hydrodynamics of the fin. This phenomenon is called dynamic stall. For a small angle of attack, the roll moment starts to increase linearly as a function of the angle of attack. When the angle of attack exceeds a certain degree, the roll moment generated by fin decreases nonlinearly, as the angle of attack increases. This gain reversal in the nonlinear hydrodynamic moment results in the loss of control in the fin stabilizer.

The dynamic stall depends on operation of the fins and their location on the hull. It usually occurs when a group of high waves appears over a short time interval and makes the angle of attack exceed a certain value, α_{stall} [24]. Under the dynamic stall condition, the control system becomes ineffective and as the result, the roll angle, in the presence of high waves, increases rapidly and significantly. A common approach to deal with these effects in practice is to reduce the gain of controller. Since the conditions for dynamic stall may not be always present, this conservative approach reduces the overall performance when dynamic stalls are not present. MPC is employed in [1] as an alternative approach to enforce input constraints associated with the mechanical angle of the fin as well as the output constraint associated with the effective angle of attack of the fin. In this paper, we consider the fin stabilizer control problem as a robust control problem, where the linear dynamics of the system are subject to bounded additive disturbance, and we employ the robust control algorithm proposed in Section III without repeated solving an optimization problem online.

A. Equations of motion

In this paper, we use the ship model from [1]. The following linear equations describe the roll motion expressed in a frame fixed at the center of gravity of the ship:

$$\begin{aligned} \dot{\phi} &= p, \\ I_{\phi\phi}\dot{p} + Dp + G\phi &= \tau_c + \tau_w, \end{aligned} \tag{30}$$

where ϕ is the roll angle, p is the roll rate, τ_c is the control moment produced by the fins, and τ_w is the wave excitation moment. Moreover, $I_{\phi\phi}$ is the total inertia in roll about the axis along the ship longitudinal direction, D is the equivalent linear damping (which accounts for potential and viscous effects), and G is the linear roll restoring coefficient [1], [6].

For a ship fin stabilizer, the effective angle of attack can be calculated as follows

$$\alpha_e = -\alpha_{pu} - \alpha_m \tag{31}$$

where α_m is the mechanical angle of the fin (control input) and α_{pu} is the flow angle induced by the combination of forward speed, U, and roll rate, p. It is calculated as follows

$$\alpha_{pu} = arctan(r_f p/U) \approx \frac{r_f}{U}p. \tag{32}$$

If the angle of attack is less than a certain value, i.e.,

$$\alpha_e < \alpha_{stall},$$

the roll moment generated by one fin is approximately proportional to the angle of attack as follows:

$$\tau_c \approx K_\alpha \alpha_e. \tag{33}$$

The above linear relation does not hold if the angle of attack (α_e) exceeds α_{stall} [24].

B. Constraints

We consider two set of constraints:

- Input constraint which reflects saturation of the mechanical angle of the fin:

$$|\alpha_m| \leq \alpha_{sat}, \tag{34}$$

- Input-state constraint that is aimed at preventing dynamic stall:

$$|\alpha_e| = |\frac{r_f}{U}p + \alpha_m| \leq \alpha_{stall}. \tag{35}$$

V. Controller design and Simulation results

To proceed with the controller design and performance evaluation of the proposed system, we use the vessel model introduced in [2]. The vessel has 15 kts forward speed, i.e., $U = 15\ kst$, with a magnitude constraint for the mechanical angle of the fin of $0.436\ rad$, and a magnitude constraint for the angle of attack of $0.401\ rad$. Moreover, the coefficients in (30) are:

$$\begin{aligned} I_{\phi\phi} &= 3.4263 \times 10^6\ Kgm^2/rad, \\ D &= 0.5 \times 10^6\ Kgm^2/(rad/sec), \\ G &= 3.57 \times 10^9\ Nm/rad,\ r_f = 4.22\ m. \end{aligned} \tag{36}$$

A discrete-time model of (30), with sampling period $T_s = 0.1\ sec$, is

$$x(k+1) = A_d x(k) + B_d u(k) + B_w \tau_w(k) \tag{37}$$

where $x = [\phi\ p]^T$, $u = \alpha_m$ and

$$A_d = \begin{bmatrix} 0.99 & 0.095 \\ -0.08 & 0.90 \end{bmatrix},\ B_d = \begin{bmatrix} -0.007 \\ -0.142 \end{bmatrix},\ B_w = \begin{bmatrix} 0.004 \\ 0.095 \end{bmatrix}. \tag{38}$$

Assuming $|\tau_w| \le 0.2 I_{\phi\phi}$, according to the general formulation (1), the disturbance set W is:

$$W = \{B_w w, |w| \le .2 I_{\phi\phi}\}.$$

The feedback gain $K = [-6.31\ \ -3.66]$ is designed using LQR technique with weight $R = 10$ for control input and the weight $Q = diag[10\ 2]$ for the states. With the designed feedback gain K, the corresponding minimal invariant set is a subset of the target set

$$\mathbb{X}_t = \{(\phi\ p) | \phi \in [-0.02\ 0.02],\ p \in [-0.06\ 0.06]\}$$

for the disturbance set W. Considering the constraints (34) and (35), the sets $\Omega_i, i = 1, \cdots, 10$ are defined as follows:

$$\Omega_i = \left\{ (u,x) \left| \begin{aligned} & |c_u u + c_x x| \le \alpha_{stall} - \sum_{j=0}^{i-1} h_{A_k^j W}(c_x + c_u K) \\ & |u| \le \alpha_{sat} - \sum_{j=0}^{i-1} h_{A_k^j W}(K) \end{aligned} \right. \right\} \tag{39}$$

where

$$c_u = 1,\ c_x = [0\ \frac{r_f}{U}] \tag{40}$$

and for a set $S \subset \mathbb{R}^n$, $h_S(\cdot)$ denotes its support function, see e.g. [21]. The value of $N = 10$ was chosen to provide large domain of attraction.

Moreover, for this example, the set $\mathbb{X}_f$ in the robust control algorithm is the maximal invariant set. This set is contained in the following set as shown in Figure 2,

$$\mathbb{X}_N := \left\{ x \left| \begin{aligned} & |(c_u K + c_x) x| \le \alpha_{stall} - \sum_{j=0}^{N-1} h_{A_k^j W}(c_x + c_u . K) \\ & |Kx| \le \alpha_{sat} - \sum_{j=0}^{N-1} h_{A_k^j W}(K) \end{aligned} \right. \right\}. \tag{41}$$

Given the sets $\mathbb{X}_f$ and Ω_i, the control law for the fin stabilizers is determined according to (11) and (12).

The simulation for the closed loop is performed based on a sinusoidal wave torque profile with period of $7sec$ and magnitude of $0.2 I_{\phi\phi}$.

Figure 2 shows the trajectory of the system with initial condition $[\phi\ p] = [0\ rad\ \ 0.45\ rad/sec]$. It can be seen that in the presence of sinusoidal wave disturbance, the ship roll motion is stabilized around the origin within a minimal invariant set characterized by the matrix A_k and the set W, while saturation constraints as well as the constraint on the angle of attack α_e are satisfied. Figure 3 and 4 show the roll angle and the angle of attack respectively. As one can see from these figures, the constraints are satisfied. The region of attraction of the proposed robust controller is shown in Figure 5.

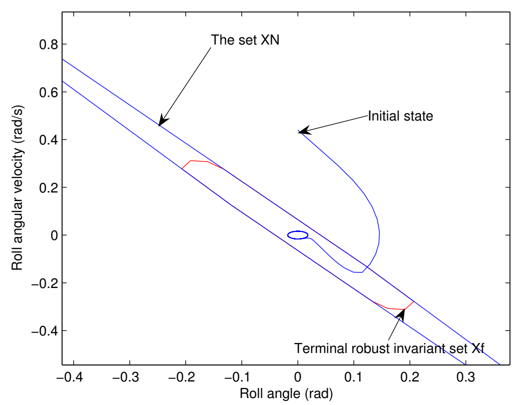

Fig. 2. Trajectory of the system with initial condition [ϕ p]=[0 rad, 0.45 rad/sec].

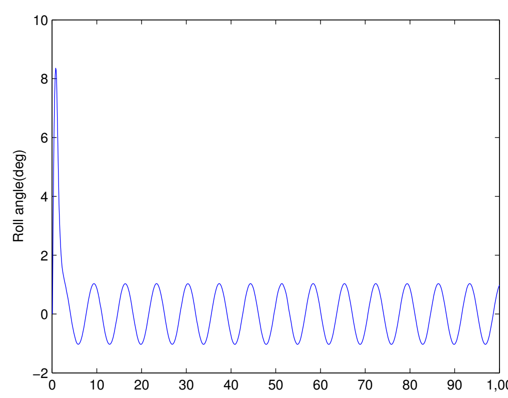

Fig. 3. Roll angle of the system with initial condition [ϕ p]=[0 rad, 0.45 rad/sec].

VI. Conclusion

This paper presented a robust control scheme for the roll motion of a high speed ship, which enforces the dynamic stall and fin saturation constraints. The proposed robust controller

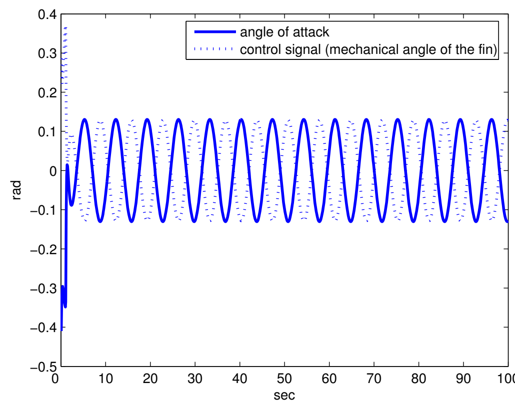

Fig. 4. Angle of attack and fin angle of the system with initial condition [ϕ p]=[0 rad, 0.45 rad/sec]. Angle of attack is constrained to $\pm$ 0.401 rad and fin is constrained to $\pm$ 0.436 rad.

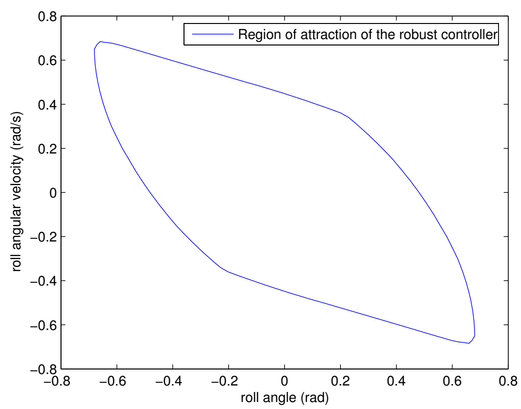

Fig. 5. Region of attraction of the proposed robust controller.

guarantees feasibility at each sampling time. Convergence to a desired target set in the presence of sea waves has been demonstrated. Simulation results were presented to show the effectiveness of the proposed method. On a theory side, motivated by this application, we generalized the robust control algorithm developed in [18] to the case of mixed state and input constraint. This extension can be applied to other robust constrained control problems. In particular, constrains on fin angle rates can be similarly handled.

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