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Dynamic fault detection and accommodation for dissipative distributed processes

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Abstract— **This paper proposes a nonlinear detection observer for the component fault detection and accommodation of nonlinear distributed processes. Specifically, the proposed results constitute an extension to previous work which utilized a linear observer for both detection and diagnosis of component faults for the same class of nonlinear distributed processes. An advantage of the proposed nonlinear observer is not simply the inclusion of nonlinear dynamics terms in the detection and diagnostic observers, but also the inclusion of such additional information in the expression for the dynamic residual threshold. Such an improved version of a dynamic threshold contributes to the robustness of the detection scheme and minimizes fault detection time. Finally, an adaptive diagnostic observer is proposed that is subsequently utilized in an automated control reconfiguration scheme that accommodates the component faults.**

I. Introduction

Recently, there has been a renewed interest on many aspects of fault detection and diagnosis for various classes of infinite dimensional systems. Many of the most recent works revisit the finite dimensional literature, see [24], [2] and references therein, which has matured in the last three decades, and either extend the various notions of fault detection and accommodation to infinite dimensional systems, or work on concepts that are specific to infinite dimensional systems.

Earlier attempts to address the problem of fault detection, diagnosis and accommodation using model-based methods along with an adaptive detection observer, were presented in [7], [8] for different types of component faults in general infinite dimensional systems. An adaptive scheme along with robust modifications allowed for the on-line diagnosis of the component fault parameters. Closer to the proposed work, the concept of a time-varying threshold that minimized the fault detection time in [11], [12], [13], [16]. Actuator faults with fault tolerant controller design were considered in [9], [17], [18], [19], [20].

In this work, we extend our previous work on fault identification and accommodation of component faults [10]. We consider transport-reaction processes that can be modeled by a class of infinite dimensional nonlinear systems which can be decomposed into a finite dimensional slow and an infinite dimensional fast subsystems. Using concepts from singularly perturbed systems and utilizing the time-scale separation of the infinite dimensional representation of transport processed we are able to employ techniques for finite dimensional systems. Utilizing then a Beard-Jones *nonlinear* detection filter for the slow subsystem along with a time-varying threshold similar to the one presented in [16], the component fault can be detected and the detection time can be reduced over the case of using a constant threshold. Using an adaptive diagnostic observer, the component fault is accommodated via the use of a control reconfiguration that utilizes on-line estimates of the fault parameters.

The mathematical formulation for the class of dissipative distributed processes is summarized in Section II while the proposed nonlinear detection and diagnostic observers are presented in Section III. The fault accommodating scheme is also presented in Section III and conclusions follow in Section IV.

II. Mathematical formulation

We consider transport-reaction processes that can be modeled by a class of nonlinear distributed processes, represented by an evolution equation in an abstract space

$$\dot{x}(t) = \mathcal{A}x + f(x) + \mathcal{N}(x) + \mathcal{B}u(t) + \beta_c(t)\Xi(x) \qquad (1)$$

where $x(t) \in D(\mathcal{A})$ denotes the state of the system expressed in an appropriate Sobolev subspace $D(\mathcal{A}) \subset W^{2,n}$ [25], $\mathcal{A}$ is the spatial differential operator of order n [6], $u(t) \in \mathbb{R}^m$ is the vector of manipulated variables. The locally Lipschitz terms $f(x)$ and $\mathcal{N}(x)$ denote the nonlinear dynamics and unmodeled dynamics respectively, and $\mathcal{B}$ denotes the control input operator. The *time profile* of the component fault may represent both abrupt and incipient profiles and is given by $\beta_c(t)) = [1-\exp(-\lambda(t-T_c))]H(t-T_c)$, where $H(\cdot)$ represents the standard Heaviside function and T_c is the time instant when the fault takes place. Note that $0<\lambda<\infty$ for incipient fault profiles and $\lambda = \infty$ for abrupt fault profiles. The component fault is represented by the nonlinear term $\Xi(x)$.

Using rather standard assumptions on the class of nonlinear systems under consideration, it is assumed that the infinite dimensional state can be decomposed as

$$x(t) = x_s(t) + x_f(t) = \mathcal{P}_s x + \mathcal{P}_f x, \qquad (2)$$

where x_s represents the state of the finite dimensional *slow* (and possibly unstable) subsystem and x_f denotes the state of the infinite dimensional *fast* subsystem. It is furthermore assumed that there is a time-scale separation between the slow and fast dynamics of the system. Dissipative systems such as systems with strongly elliptic spatial operators have

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that property [3], [22]. Under these assumptions, the above system can subsequently be written in the following singularly perturbed form [4]

$$\begin{aligned} \dot{x}_s &= \mathcal{A}_s x_s + \mathcal{B}_s u + f_s(x_s,x_f) + \mathcal{N}_s(x_s,x_f) + \beta_c(t)\Xi_s(x_s,x_f) \\ \varepsilon\dot{x}_f &= \mathcal{A}_f x_f + \mathcal{B}_f u + f_f(x_s,x_f) + \mathcal{N}_f(x_s,x_f) + \beta_c(t)\Xi_f(x_s,x_f), \end{aligned} \tag{3}$$

where ε_f signifies the time scale separation due to the spatial operator and has values $\varepsilon_i = |\max\lambda\{\mathcal{A}_s\}/\max\lambda\{\mathcal{A}_f\}|$ where $\lambda\{\cdot\}$ signifies the eigenvalue of the respective projection of the spatial operator [22]. Before we proceed with the design of the robust adaptive detection observer, we provide state bounds in the fault-free case. These bounds are necessary in the design of dynamic thresholds used in the fault detection stage via the use of residual signals. Use of such time-varying thresholds minimize the detection time, i.e. time it takes to declare a fault since its occurrence.

Employing concepts from singularly perturbed systems [4], we may neglect the fast infinite dimensional x_f subsystem to obtain the m_s-dimensional slow system

$$\begin{cases} \dot{x}_s = \mathcal{A}_s x_s + \mathcal{B}_s u + f_s(x_s,x_f) + \mathcal{N}_s(x_s,x_f) + \beta_c(t)\Xi_s(x_s,x_f) \\ x_f \equiv 0. \end{cases} \tag{4}$$

Following on the earlier work [10] we provide a list of objectives, which while similar in nature, differ in the manner at which they are accomplished.

Objective: The primary objective is to design a robust monitoring filter that would provide information on the time occurrence of the fault and attempt to diagnose the nature of the component fault. The secondary objective is to provide an automated control reconfiguration policy based on the estimates of the component fault, and which would be able to accommodate the component faults.

Remark 2.1: Unlike earlier efforts addressing the above objectives in [10], we now consider a *nonlinear* filter to achieve both detection and diagnosis. This significantly affects the residual signal, since it is based on the detection filter. Such a residual takes into account the nonlinear term $f(x)$, an approach that was not used in the earlier effort [10] which only utilized the linearized dynamics of the process.

III. MAIN RESULTS

For the detection of a component fault, a nonlinear detection observer is utilized. Such a detection observer consists of a copy of the slow dynamics plus an output injection term required for the stability analysis of the associated detection error. A dynamic threshold, based on the fault-free dynamics, is utilized by the detection scheme to declare a component fault in the system when the said threshold is exceeded by the residual. Subsequently, an on-line nonlinear diagnostic observer is used to provide information on the type of the fault and finally, fault accommodation is attained via the appropriate control reconfiguration.

A. Design of nonlinear detection observer

We propose to design a *nonlinear* observer and an on-line identification scheme based on the slow dynamics of the system of (4). Towards this end, we propose a nonlinear detection observer of the form

$$\frac{d}{dt}\widehat{x} = \mathcal{A}_s\widehat{x} + f_s(\widehat{x},0) - \mathcal{L}(\widehat{x}-x_s) + \mathcal{B}_s u, \quad \widehat{x}(0) = \widehat{x}_0, \tag{5}$$

where $\mathcal{L}_s$ is an appropriately chosen linear filter gain used to ensure local stability of the error dynamics, in the sense of making $\mathcal{A}_s - \mathcal{L}_s$ "more" stable. We assume that the initial state estimates *may* or *may not* be correct $\widehat{x}_0 \neq x_s(0)$. The state detection error further serves to generate the residual signal used in fault detection/diagnosis. The state detection error $e_s(t) = x_s(t) - \widehat{x}(t)$ is governed by

$$\begin{aligned} \dot{e}_s =& (\mathcal{A}_s - \mathcal{L}_s)e_s + \Big(f(e_s+\widehat{x},x_f) - f(\widehat{x},0)\Big) \\ &+ \mathcal{N}_s(e_s+\widehat{x},x_f) + \beta_c(t)\Xi_s(e_s+\widehat{x},x_f) \\ e_s(0) =& x_s(0) - \widehat{x}(0) \neq 0, \end{aligned} \tag{6}$$

Remark 3.1: The system of (3) can be used to derive nonlinear observers of higher dimensionality, or alternatively the design of nonlinear dynamic observers with nonlinear filters. The design of such filters is investigated in a follow-up extension to this conference proceedings paper.

Remark 3.2: It should be noted that the proposed observer is based on (4) which neglects the infinite dimensional fast dynamics, *but* the stability analysis involving the estimation error uses the fully nonlinear system (3).

B. Nonlinear detection observer and dynamic threshold

The proposed nonlinear detection observer is subsequently designed based on the error dynamics of (6) and employing information concerning the process unmodeled dynamics.

The state estimation error will be nonzero prior to the fault occurrence ($t < T_c$), even when one sets $\widehat{x}_s(0) = x_s(0)$. This is due to the presence of the nonlinear dynamics terms $f_s(e_s+\widehat{x},x_f) - f_s(\widehat{x},0)$ and the unmodelled dynamics term $\mathcal{N}_s(x_s(t),x_f(t))$. Following the finite dimensional analogue regarding false alarms [23], to avoid such false declaration of faults, one considers a threshold which, when exceeded, designates the occurrence of the component fault in the system. To find the threshold, one considers the above state detection error in the fault-free case (i.e. $\Xi \equiv 0$)

$$\begin{aligned} \dot{e}_s =& (\mathcal{A}_s - \mathcal{L}_s)e_s + \Big(f_s(e_s+\widehat{x},x_f) - f_s(\widehat{x},0)\Big) \\ &+ \mathcal{N}_s(e_s+\widehat{x},x_f) \\ e_s(0) =& x_s(0) - \widehat{x}(0) \neq 0, \end{aligned} \tag{7}$$

Employing concepts from singularly perturbed systems, the above system can be approximated by a finite dimensional differential equation system [22], [3]. Such concept in essence utilizes Tychonov's theorem for infinite dimensional dissipative systems and their finite dimensional approximation.

$$\begin{aligned} \dot{e}_s =& (\mathcal{A}_s - \mathcal{L}_s)e_s + \Big(f_s(e_s+\widehat{x},0) - f_s(\widehat{x},0)\Big) \\ &+ \mathcal{N}_s(e_s+\widehat{x},0) \\ e_s(0) \neq& 0, \end{aligned} \tag{8}$$

Its solution is given by

$$
\begin{aligned}
e_s(t) = & e^{(\mathcal{A}_s-\mathcal{L}_s)t}e_s(0) \\
& + \int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\Big(f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\Big)\,d\tau \\
& + \int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\mathcal{N}_s(x_s(\tau),0)\,d\tau.
\end{aligned} \tag{9}
$$

The dynamic threshold, which is based on the nonlinear detection observer, is then given by the norm bound of (9) and not (7)

$$
\begin{aligned}
\|e_s(t)\| = & \Big\| e^{(\mathcal{A}_s-\mathcal{L}_s)t}e_s(0) \\
& + \int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\Big(f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\Big)\,d\tau \\
& + \int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\mathcal{N}_s(x_s(\tau),0)\,d\tau\Big\| \\
& \le \|e^{(\mathcal{A}_s-\mathcal{L}_s)t}e_s(0)\| \\
& + \Big\|\int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\Big(f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\Big)\,d\tau\Big\| \\
& + \Big\|\int_0^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\mathcal{N}_s(x_s(\tau),0)\,d\tau\Big\| \\
& \le e^{-\alpha t}\|e_s(0)\| \\
& + \int_0^t e^{-\alpha(t-\tau)}\|f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\|\,d\tau \\
& + \int_0^t e^{-\alpha(t-\tau)}\|\mathcal{N}_s(x_s(\tau),0)\|\,d\tau,
\end{aligned} \tag{10}
$$

where α is the spectrum bound of the matrix $(\mathcal{A}_s-\mathcal{L}_s)$. Using the assumption of Lipschitz continuity for the nonlinear term

$$\|f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0))\| \le K_f|e_s|$$

Assuming a maximum value for the error, the term corresponding to the nonlinear terms becomes

$$
\begin{aligned}
\int_0^t \|e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\|\|f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\|\,d\tau \\
\le \int_0^t \|e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\|\,K_f\sup_t|e_s|\,d\tau.
\end{aligned}
$$

Using the *a priori* L_∞ bound on the unmodelled dynamics $\|\mathcal{N}_s(x_s(\tau),0)\| \le \mathcal{N}_0$ for all x_s in a compact set in $\mathbb{R}^m$, we obtain the *time varying threshold*

$$r_0(t) = e^{-\alpha t}\varepsilon_{s0} + \frac{1-e^{-\alpha t}}{\alpha}\Big(e_{max}+\mathcal{N}_0\Big), \qquad t \ge 0, \tag{11}$$

where ε_{s0} is the bound in $\|e_s(0)\| \le \varepsilon_{s0}$ and $e_{max} = K_f\sup_t|e_s|$. We now proceed with the following result on robust fault detection scheme.

Lemma 3.1 (Nonlinear detection observer): Consider the approximate finite dimensional slow subsystem with incipient component faults

$$
\begin{aligned}
\dot{x}_s(t) &= \mathcal{A}_s x_s(t) + \mathcal{B}_s u(t) + f_s(x_s(t),x_f(t)) \\
&\quad + \mathcal{N}_s(x_s(t),x_f(t)) + \beta_c(t-T_c)\Xi_s(x_s(t),x_f(t)) \\
x_s(0) &= x_{s0}.
\end{aligned}
$$

A fault is declared when the residual signal, given by the norm of the state detection error $e_s(t) = x_s(t)-\widehat{x}(t)$ and governed by

$$
\begin{aligned}
\dot{e}_s &= (\mathcal{A}_s-\mathcal{L}_s)e_s + \Big(f_s(e_s+\widehat{x},x_f)-f_s(\widehat{x},0)\Big) \\
&\quad + \mathcal{N}_s(e_s+\widehat{x},x_f) + \beta_c(t)\Xi_s(e_s+\widehat{x},x_f) \\
e_s(0) &= e_{s0} \ne 0,
\end{aligned}
$$

exceeds the dynamic threshold $r_0(t)$. The norm of the state detection error is bounded by $r_0(t)$ for all $t \le T$. Fault is declared at the detection time t_d:

$$t_d = \arg\left(\|e_s(t)\| \ge r_0(t)\right).$$

Proof: Prior to the fault, the state detection error is

$$
\begin{aligned}
\dot{e}_s &= (\mathcal{A}_s-\mathcal{L}_s)e_s + \Big(f_s(e_s+\widehat{x},x_f)-f_s(\widehat{x},0)\Big) \\
&\quad + \mathcal{N}_s(e_s+\widehat{x},x_f) \qquad\qquad 0 \le t \le T. \\
e_s(0) &= e_{s0} \ne 0,
\end{aligned}
$$

Its norm bound will be bounded above by the threshold $r_0(t)$. After the unknown fault occurrence, the state detection error is governed by

$$
\begin{aligned}
\dot{e}_s(t) &= (\mathcal{A}_s-\mathcal{L}_s)e_s(t) + \Big(f_s(e_s+\widehat{x},x_f)-f_s(\widehat{x},0)\Big) \\
&\quad + \mathcal{N}_s(x_s(t),x_f(t)) + \beta_c(t-T_c)\Xi_s(x_s(t),x_f(t)) \\
e_s(T) &= e_{sT} \ne 0,
\end{aligned}
$$

having a solution

$$
\begin{aligned}
e_s(t) &= e^{(\mathcal{A}_s-\mathcal{L}_s)(t-T)}e_s(T) \\
&\quad + \int_T^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}\Big(f_s(e_s(\tau)+\widehat{x}(\tau),x_f(\tau))-f_s(\widehat{x}(\tau),0)\Big)\,d\tau. \\
&\quad + \int_T^t e^{(\mathcal{A}_s-\mathcal{L}_s)(t-\tau)}(\mathcal{N}_s(x_s(\tau),x_f(\tau))\,d\tau \\
&\quad + \int_T^t \beta_c(t-T_c)\Xi_s(x_s(\tau),x_f(\tau)))\,d\tau.
\end{aligned}
$$

Using the following

$$
\begin{aligned}
f_s(e_s+\widehat{x},x_f)) - f_s(\widehat{x},0) &= f_s(e_s+\widehat{x},x_f) - f_s(e_s+\widehat{x},0) \\
&\quad + f_s(e_s+\widehat{x},0) - f_s(\widehat{x},0) \\
&= \Big(f_s(e_s+\widehat{x},0)-f_s(\widehat{x},0)\Big) \\
&\quad + \Big(f_s(e_s+\widehat{x},x_f)-f_s(e_s+\widehat{x},0)\Big)
\end{aligned}
$$

along with the same algebraic manipulation applied to the unmodelled dynamics term, then the residual signal $r(t)$, given by the norm of $e_s(t)$, is

$$
\begin{aligned}
r(t) &= \|e_s(t)\|_\infty \le e^{-\alpha(t-T)}\|e_s(T)\| \\
&+ \int_T^t e^{-\alpha(t-\tau)}\|f_s(e_s(\tau)+\hat{x}(\tau),x_f(\tau)) - f(\hat{x}(\tau),x_f(\tau))\|\, d\tau \\
&+ \int_T^t e^{-\alpha(t-\tau)}\|\mathcal{N}_s(x_s(\tau),x_f(\tau))\|\, d\tau \\
&+ \int_T^t e^{-\alpha(t-\tau)}\beta_c(\tau-T_c)\|\Xi_s(x_s(\tau),x_f(\tau))\|\, d\tau \\
&\le e^{-\alpha(t-T)}\|e_s(T)\| \\
&+ \int_T^t e^{-\alpha(t-\tau)}\|f_s(e_s(\tau)+\hat{x}(\tau),0) - f_s(\hat{x}(\tau),0)\|\, d\tau \\
&+ \int_T^t e^{-\alpha(t-\tau)}\|\mathcal{N}_s(x_s(\tau),0)\|\, d\tau + \int_T^t e^{-\alpha(t-\tau)}\ \text{(H.O.T.)}\ d\tau \\
&+ \int_T^t e^{-\alpha(t-\tau)}\beta_c(\tau-T_c)\|\Xi_s(x_s(\tau),x_f(\tau))\|\, d\tau \\
&\le r_0(t) + \|F_{H.O.T.}(x_f)\| \\
&\quad + \left| \int_T^t e^{-\alpha(t-\tau)}\beta_c(\tau-T_c)\|\Xi_s(x_s(\tau),x_f(\tau))\|\, d\tau \right|
\end{aligned}
$$

The term $F_{H.O.T.}(x_f)$ denotes the cumulative error due to the fast dynamics state in the mismatch in the f_s and $\mathcal{N}_s$ terms which is dominated by $e^{-\alpha t}$. The presence of a fault forces the norm of e_s to exceed the threshold, and hence a fault is declared. □

C. On-line nonlinear diagnostic observer

After the fault declaration, one may subsequently activate a diagnostic observer in order to diagnose the component faults. It is assumed that the unknown component fault term admits the following parametrization

$$
\begin{aligned}
\Xi_s(x_s,x_f) &= \Xi_s(x_s,0) + \Big(\Xi_s(x_s,x_f) - \Xi_s(x_s,0)\Big) \\
&= \Theta\, g(x_s) + \Big(\Xi_s(x_s,0) - \Theta\, g(x_s)\Big) \\
&\quad + \Big(\Xi_s(x_s,x_f) - \Xi_s(x_s,0)\Big) \\
&= \Theta\, g(x_s) + \nu(x_f)
\end{aligned}
\tag{12}
$$

where the $m \times m$ constant matrix Θ is assumed unknown and desired to be identified, and the regressor vector $g(x_s)$ is assumed known. The term

$$
\nu(x_f) = \Big(\Xi_s(x_s,x_f) - \Xi_s(x_s,0)\Big) + \Big(\Xi_s(x_s,0) - \Theta\, g(x_s)\Big)
$$

denotes the modeling error due to the presence of the fast dynamics when one uses (4) to approximate (3) and the mismatch due to the linear approximation of the on-line approximator $\Theta g(x_s)$. In fact, the unknown constant matrix Θ is found as the matrix that minimizes the L_2 norm distance between $\Xi_s(x_s,0)$ and $\Theta g(x_s)$ over all x. This constant matrix is an artificial quantity in the sense that it provides a linearly parameterized component fault term and is only used for analysis purposes.

The integrated on-line diagnostic scheme consists of a nonlinear diagnostic observer and a parameter learning scheme given by

$$
\begin{aligned}
\dot{\hat{x}}_d(t) &= \mathcal{A}_s\hat{x}_d(t) + f_s(\hat{x}_d(t),0) - \mathcal{L}_d(\hat{x}_d(t) - x_s(t)) \\
&\quad + \mathcal{B}_s u(t) + \widehat{\Theta}(t)\, g(x_s(t)) \\
\dot{\widehat{\Theta}}(t) &= \mathcal{P}\left\{\Gamma g(x_s(t)) e^T(t)\Pi\right\}
\end{aligned}
\tag{13}
$$

where $\hat{x}_d(t) \in \mathbb{R}^m$ is the *estimate state vector*, $e = x_s - \hat{x}_d$ is the state diagnostic error, $\widehat{\Theta}(t) g(x_s(t))$ is the adaptive fault approximator model with $\widehat{\Theta}(t) \in \mathbb{R}^{m\times m}$ a matrix of (recursively) *adjustable parameters* and $\mathcal{L}_d$ a constant $m \times m$ observer gain matrix that satisfies the following Lyapunov equation

$$
(\mathcal{A}_s - \mathcal{L}_d)\Pi + \Pi(\mathcal{A}_s - \mathcal{L}_d)^T = -Q \tag{14}
$$

with $\Pi = \Pi^T > 0$ and $Q > 0$. The projection operator $\mathcal{P}[\cdot]$ constrains the parameter estimate $\widehat{\Theta}(t)$ to an a priori selected compact convex region $\mathcal{M}$ of the parameter space $\mathcal{Q}$. When $\widehat{\Theta}(t)$ is not in $\mathcal{M}$, the adaptation is ceased. This is described in detail in [15], [10], and when applied to the current case, is given by

$$
\dot{\widehat{\Theta}}(t) = \mathcal{P}\Big\{\Gamma g(x_s(t)) e^T(t)\Pi\Big\}
= \begin{cases} \Gamma g(x_s) e^T(t)\Pi & \text{if } \widehat{\Theta} \in \mathcal{M}^0 \text{ or if } \widehat{\Theta} \in \partial\mathcal{M} \text{ and} \\ & \widehat{\Theta} \text{ tends to move towards } \mathcal{M} \\ 0 & \text{otherwise} \end{cases}
$$

To examine the stability properties of the diagnostic observer in (13), we use a lemma similar to the one presented in [10] for the linear case.

Lemma 3.2 (On-line nonlinear diagnostic observer): Consider the post-fault finite dimensional subsystem

$$
\begin{aligned}
\dot{x}_s(t) &= \mathcal{A}_s x_s(t) + \mathcal{B}_s u(t) + f_s(x_s(t),x_f(t)) \\
&\quad + \mathcal{N}_s(x_s(t),x_f(t)) + \beta_c(t-T_c)\Xi_s(x_s(t),x_f(t)), \\
x_s(T_c) &= x_{sT}, \quad t \ge T_c.
\end{aligned}
\tag{15}
$$

Once the fault is declared via the adaptive detection observer (5), then the following diagnostic observer plus on-line approximator

$$
\begin{aligned}
\dot{\hat{x}}_d(t) &= \mathcal{A}_s\hat{x}_d(t) + f_s(\hat{x}_d(t),0) - \mathcal{L}_d(\hat{x}_d(t) - x_s(t)) \\
&\quad + \mathcal{B}_s u(t) + \widehat{\Theta}(t) g(x_s(t)), \qquad \hat{x}_d(t_d) = \hat{x}_{d0}, \\
\dot{\widehat{\Theta}}(t) &= \mathcal{P}\left\{\Gamma g(x_s(t)) e^T(t)\Pi\right\}, \qquad \widehat{\Theta}(t_d) = \widehat{\Theta}_d,
\end{aligned}
\tag{16}
$$

for $t \ge t_d > T_c$, guarantees that all signals are bounded. Similar to the linear case, when both unmodelled and nonlinear dynamics term being zero, one obtains convergence of the *state diagnostic error* to zero and with the additional assumption of persistence of excitation convergence of $\widehat{\Theta}(t)$ to Θ (parameter convergence) [1], [5].

Proof: Most of the steps taken to demonstrate the stability properties are similar to the linear case in [10]. We thus summarize the key points and include the contributions due to the presence of the nonlinear dynamics in both the plant and observer equations, which essentially take advantage of the assumption on Lipschitz continuity. Therefore the *state diagnostic error* $e(t) = x_s(t) - \widehat{x}_d(t)$ is governed by

$$\begin{aligned}
\dot{e} =& (\mathcal{A}_s - \mathcal{L}_d)e + f_s(x_s, x_f) - f_s(\widehat{x}_d, 0) \\
&+ \mathcal{N}_s(x_s, x_f) + \beta_c(t - T_c)\Xi_s(x_s, x_f) - \widehat{\Theta}(t)g(x_s) \\
=& \mathcal{A}_{so}e + f_s(x_s, x_f) - f_s(\widehat{x}_d, 0) + \mathcal{N}_s(x_s, x_f) \\
&+ \beta_c(t - T_c)\Theta g(x_s) + \beta_c(t - T_c)\nu(x_f) - \widehat{\Theta}(t)g(x_s) \\
=& \mathcal{A}_{so}e + f_s(x_s, x_f) - f_s(\widehat{x}_d, 0) + \mathcal{N}_s(x_s, x_f) \\
&- [I - \beta_c(t - T_c)]\Theta g(x_s) \\
&+ \Theta g(x_s) - \widehat{\Theta}(t)g(x_s) + \beta_c(t - T_c)\nu(x_f) \\
=& \mathcal{A}_{so}e + f_s(x_s, x_f) - f_s(\widehat{x}_d, 0) + \mathcal{N}_s(x_s, x_f) \\
&- \Phi(t)\Theta g(x_s) - \widetilde{\Theta}(t)g(x_s) + \beta_c(t - T_c)\nu(x_f) \\
=& \mathcal{A}_{so}e + f_s(e + \widehat{x}_d, 0) - f_s(\widehat{x}_d, 0) + \mathcal{N}_s(x_s, 0) \\
&- \Phi(t)\Theta g(x_s) - \widetilde{\Theta}(t)g(x_s) + \nu'(x_f),
\end{aligned} \tag{17}$$

where $\mathcal{A}_{so} \triangleq \mathcal{A}_s - \mathcal{L}_d$, $\Phi(t) \triangleq I - \beta_c(t - T_c)$, $\widetilde{\Theta}(t) \triangleq \Theta(t) - \Theta$ and

$$\begin{aligned}
\nu'(x_f) \triangleq\; & \beta_c(t - T_c)\nu(x_f) + \Big(f_s(x_s, x_f) - f_s(x_s, 0)\Big) \\
& + \Big(\mathcal{N}_s(x_s, x_f) - \mathcal{N}_s(x_s, 0)\Big).
\end{aligned}$$

We use the following Lyapunov function for examining the stability of the diagnostic scheme

$$V = \frac{1}{2}e^T\Pi e + \frac{1}{2}\widetilde{\Theta}\Gamma^{-1}\widetilde{\Theta} + \frac{1}{2}\text{tr }\{\lambda\Phi R^{-1}\Phi\}, \tag{18}$$

where λ is a positive constant. The parameter error, used for the stability analysis below, is given by

$$\dot{\widetilde{\Theta}}(t) = \mathcal{P}\left\{\Gamma g(x_s(t))e^T(t)\Pi\right\}. \tag{19}$$

The time derivative evaluated along the trajectories of the state diagnostic and parameter error equation is given by

$$\begin{aligned}
\dot{V} =& \tfrac{1}{2}e^T(\mathcal{A}_{so}^T\Pi + \Pi\mathcal{A}_{so})e + e^T\Pi\Big(f_s(x_s, x_f) - f_s(x_s, 0)\Big) \\
&+ e^T\Pi\mathcal{N}_s(x_s(t), 0) - e^T\Pi\Phi(t)\Theta g(x_s(t)) + e^T\Pi\nu'(x_f) \\
&- e^T\Pi\widetilde{\Theta}(t)g(x_s(t)) + \widetilde{\Theta}\Gamma^{-1}\dot{\widetilde{\Theta}} - \text{tr }\{\lambda\Phi\dot{\Phi}\} \\
=& -\tfrac{1}{2}e^TQe_s + e^T\Pi\Big(f_s(x_s, x_f) - f_s(x_s, 0)\Big) \\
&+ e^T\Pi\mathcal{N}_s(x_s(t), 0) - e^T\Pi\Phi(t)\Theta g(x_s(t)) + e^T\Pi\nu'(x_f) \\
&- e^T\Pi\widetilde{\Theta}(t)g(x_s(t)) + \widetilde{\Theta}\Gamma^{-1}\dot{\widetilde{\Theta}} - \text{tr }\{\lambda\Phi\dot{\Phi}\},
\end{aligned}$$

where we used the fact that $\mathcal{A}_{so}^T\Pi + \Pi\mathcal{A}_{so} = -Q$ and $\dot{\Phi} = -R\Phi$. Following a similar analysis in [14], one uses the fact that the projection modification makes the derivative of the Lyapunov function "more" negative. Using the assumption of the uniform boundedness of x_s, we have that $\sup_{t \geq T_c}\Xi(x_s(t), 0) = \sup_{t \geq T_c}\Theta g(x_s(t)) = c_1$. The Lipschitz continuity of the nonlinear term results in

$$\|f_s(x_s, x_f) - f_s(x_s, 0)\| \leq K_f|e|.$$

Additionally, using the fact that the Frobenius norm of a matrix is bounded below by its L_2 norm, $\|\Phi\|_2 \leq \|\Phi\|_F = \text{tr}\{\Phi\Phi\}$, we arrive at

$$\begin{aligned}
\dot{V} \leq& -\tfrac{1}{2}\lambda_{min}(Q)|e|^2 + \|\Pi\|_2K_f|e|^2 \\
&+ \|\Pi\|_2|e||\mathcal{N}_s(x_s(t), 0)| + c_1\|\Pi\|_2|e|\|\Phi\|_2 \\
&- \lambda\|\Phi\|_2^2 + \|\Pi\|_2|e||\nu'(x_f)|.
\end{aligned} \tag{20}$$

Using twice the inequality $2\alpha\beta \leq \alpha^2/\varepsilon + \varepsilon\beta^2$ for some $\varepsilon > 0$ in the expression above, we then have

$$\begin{aligned}
\dot{V} \leq& -\Big(\tfrac{1}{4}\lambda_{min}(Q)|e|^2 + \tfrac{\lambda}{2}\|\Phi\|_2^2\Big) + \|\Pi\|_2K_f|e|^2 \\
&+ c_1\|\Pi\|_2|e|\|\Phi\|_2 - \tfrac{1}{4}\lambda_{min}(Q)|e|^2 - \tfrac{\lambda}{2}\|\Phi\|_2^2 \\
&+ \|\Pi\|_2|e|\Big(|\mathcal{N}_s(x_s(t), 0)| + |\nu'(x_f)|\Big) \\
\leq& -\big(\tfrac{1}{4}\lambda_{min}(Q) - \|\Pi\|_2K_f\big)|e|^2 - \tfrac{\lambda}{2}\|\Phi\|_2^2 \\
&+ c_2\Big(|\mathcal{N}_s(x_s(t), 0)|^2 + |\nu'(x_f)|^2\Big)
\end{aligned} \tag{21}$$

where $\lambda = 4c_1^2\|\Pi\|_2^2/\lambda_{max}(Q)$, $c_2 = 4\|\Pi\|_2^2/\lambda_{max}(Q)$. Provided that the nonlinear term, via its Lipschitz constant K_f, is such that $0 < \frac{1}{4}\lambda_{min}(Q) - \|\Pi\|_2K_f = c_3$, then when

$$(c_3|e_s|^2 + \frac{\lambda}{2}\|\Phi\|_2^2) > c_2(|\mathcal{N}_s(x_s(t), 0)|^2 + |\nu'(x_f)|^2),$$

we have $\dot{V} \leq 0$.

The uniform boundedness assumption of $\mathcal{N}_s(x_s(t), 0)$ implies the uniform boundedness of e_s and $\widehat{\Theta}$. Following the analysis in [14], one can infer that the extended L_2 norm of the state diagnostic error over any finite time interval is, at most, of the same order as the extended L_2 norm of the unmodelled/nonlinear dynamics $\mathcal{N}_s(x_s(t), 0)$. In the absence of the unmodelled/nonlinear dynamics, one can easily show, via the application of Barbălat's lemma [21], that the state diagnostic error converges to zero asymptotically $\lim_{t\to\infty}|e(t)| = 0$. In the latter case, when the additional condition of persistence of excitation is imposed, then parameter convergence is guaranteed $\lim_{t\to\infty}\widehat{\Theta}(t) = \Theta$. □

Remark 3.3: It is worth mentioning that the same combination of detection-plus-diagnostic observer for the current nonlinear observer design approach is valid as in the linear observer design case [10]. Specifically, all that is required is to ensure that during the detection stage with no fault present, no adaptation takes place. This ensures that any possible contribution to the state detection error due to a falsely estimated component fault is not realized. During the pre-fault stage, the state error is described by

$$\begin{aligned}
\dot{e}(t) =& \mathcal{A}_{so}e(t) + \Big(f_s(e + \widehat{x}_{so}, 0) - f_s(\widehat{x}_{so}, 0)\Big) \\
&+ \mathcal{N}_s(x_s(t), 0) + \nu(x_f) - \widehat{\Theta}(t)g(x_s(t))
\end{aligned}$$

which leads to

$$\begin{aligned}
r(t) =& \|e_s(t)\| \leq e^{-\alpha(t)}|\|e_s(t)\| \\
&+ \int_0^t e^{-\alpha(t-\tau)}\Big(f_s(e(\tau) + \widehat{x}_{so}(\tau), 0) - f_s(\widehat{x}_{so}(\tau), 0)\Big)d\tau \\
&+ \int_0^t e^{-\alpha(t-\tau)}\Big(\|\mathcal{N}_s(x_s(\tau), 0)\| + |\nu(x_f(\tau))|\Big)d\tau \\
&+ \int_0^t e^{-\alpha(t-\tau)}\widehat{\Theta}(\tau)\|g(x_s(\tau))\|d\tau \\
\leq& r_0(t) + \int_0^t e^{-\alpha(t-\tau)}\widehat{\Theta}(\tau)\|g(x_s(\tau))\|d\tau.
\end{aligned}$$

The same arguments follow as in the linear case. Prior to the fault occurrence ($t \leq T_c$), one will have $r(t) \geq r_0(t)$ if no measures are taken to avoid false declarations of faults.

D. Fault accommodation

Accommodation of the component fault takes the form of control reconfiguration, once the diagnostic observed is activated following fault declaration.

The control reconfiguration takes the form of an additive signal to the controller for the nominal, fault-free case as follows. Denote by $u_0(t)$ the control signal for the fault-free system

$$\dot{x}_s(t) = \mathcal{A}_s x_s(t) + \mathcal{B}_s u(t) + f_s(x_s(t),0) + \mathcal{N}_s(x_s(t),0),$$

and given by

$$u_0(t) = -\mathcal{B}_s^{-1} f_s(x_s(t),0) - \mathcal{K}_s x_s(t), \qquad (22)$$

where the feedback gain $\mathcal{K}_s$ is an appropriately chosen gain that satisfies certain performance and stability criteria. The above control signal yields the following closed loop fault-free slow subsystem

$$\dot{x}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)x_s(t) + \mathcal{N}_s(x_s(t),0).$$

The linearized part $(\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)$ should be designed, via the choice of the linear feedback gain $\mathcal{K}_s$, to dominate the unmodeled part $\mathcal{N}_s(x_s(t),0)$ in order to yield closed loop stability. When component faults are present in the slow subsystem

$$\begin{aligned}\dot{x}_s(t) =& \mathcal{A}_s x_s(t) + \mathcal{B}_s u(t) + f_s(x_s(t),0) \\ &+ \mathcal{N}_s(x_s(t),0) + \Xi_s(x_s(t),0)\end{aligned}$$

then the fault accommodating controller

$$u_{accom}(t) = u_0(t) - \mathcal{B}_s^{-1}\Xi_s(x_s(t),0)$$

will cancel the effects of the component fault and yield

$$\dot{x}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)x_s(t) + \mathcal{N}_s(x_s(t),0).$$

Of course such a fault accommodating controller cannot be implemented as the knowledge of the additional dynamics due to the component fault are not known. In this case, one simply replaces them by their adaptive estimates

$$u_{accom}(t) = u_0(t) - \mathcal{B}_s^{-1}\widehat{\Theta}(t)g(x_s(t)) \qquad (23)$$

to arrive at the closed loop system

$$\dot{x}_s(t) = (\mathcal{A}_s - \mathcal{B}_s\mathcal{K}_s)x_s(t) + \mathcal{N}_s(x_s(t),0) - \widetilde{\Theta}(t)g(x_s(t)). \quad (24)$$

The closed loop stability can be studied when considering both the diagnostic observer (16) and the slow subsystem (8). The difference with the linear counterpart in [10] is that we now take advantage of the knowledge of the Lipschitz continuity of the nonlinear dynamics.

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