2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Passivity Based Control of Drum Boiler

Chengtao Wen and B. Erik Ydstie

***Abstract*— This paper proposes a novel state space model for the drum boilers with natural recirculation. This model uses the total mass and energy inventories of the boiler as the state variables, and has an affine structure in the control variables. A passivity based inventory controller is developed, which ensures the asymptotic stability of the closed-loop boiler system. Numerical simulations show the performance and efficiency of the passivity based controller design technique.**

I. INTRODUCTION

The modeling and control of a drum boiler with natural circulation is quite important because they are widely used in power generation and account for an important part of the overall fuel consumption in a power station. A number of models are suggested in literature to study the dynamic response of natural circulation boilers[1-5]. These models are developed based on the mass, energy and momentum balance laws. In order to map the terms in balance equations to the intensive variables, the thermodynamic or kinetic relationships have to be introduced, which often leads to a set of nonlinear differential functions. Although such models are important for plant design and simulations, they are usually not easy to use for control design because of their complexity.

The availability of steam at proper thermal conditions and rates is one of the critical features in the operation of every thermoelectric power plant. Many control schemes have proposed in achieving this goal [6-8]. Unfortunately, it is still an open problem to develop a systematic approach to boiler control design that grantees the stability of the closed-loop system.

Passivity theory provides an effective method to control a wide range of process systems. The main advantage of passivity is that it allows to develop controllers for the process without detailed modeling. The central feature of the passivity-based control is that the extensive variables, such as inventories, are used as the process state variables for controller design. According to the inventory control theory, a quadratic error function between the inventories and their ideal objective values, is a suitable storage function for passivity design. This choice ensures the passivity of the whole process system, whose inventories will converge to their set points when we use strictly passive feedback [9-11].

Corresponding author: B. Erik Ydstie Tel. +1-412-2682261. Fax. +1-412-2687139.

B. Erik Ydstie is with the Faculty of the Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, 15213, U. S. A. ydstie@andrew.cmu.edu

C. Wen is with the Department of Chemical Engineering, Carnegie Mellon University, Pittsburgh, 15213, U. S. A. chengtao@andrew.cmu.edu

In this paper, a state-space model of drum boiler is developed, whose states are the extensive variables, i.e. the total mass and energy inventories of the boiler system. The model is affine in the control inputs, which facilitates the design of a passivity-based inventory controller that ensures asymptotic set-point tracking.

II. MODEL PHILOSOPHY

We consider the modeling and control of a nonlinear system

$$\frac{dx}{dt} = f(x) + g(x, d, m) \tag{1}$$

where x is the called microscope state, d the disturbances, m the control variables. Following [9], we define the inventories $Z_i(x), i = 1, \cdots, n$ to be any C^1 function, so that $Z_i(x) \geq 0$ for any x. From continuity, we can write

$$\frac{dZ_i}{dt} = \frac{\partial Z_i}{\partial x}\frac{dx}{dt} = p_i(x) + \phi_i(x, d, m) \tag{2}$$

where $p_i(x) = \frac{\partial Z_i}{\partial x} f(x)$ and $\phi_i(m, x, d) = \frac{\partial Z_i}{\partial x} g(x, d, m)$. An inventory is said to be invariant if the drift $p(x) = 0$. If $p(x) \geq 0$, the inventory satisfies the Clausius-Planck inequality, and if $p(x) \leq 0$, it satisfies the dissipative property.

The state Z of a single component thermodynamic system is defined by the vector of extensive variables, i.e. the internal energy, volume and mass.

$$Z = [U, V, M] \tag{3}$$

The inventories U, V, M of a single component system are invariant so that $p_i(x) = 0$ in (2) holds for $i = 1, 2, 3$. A fundamental result in thermodynamics states that there exist an inventory $S(Z)$, which satisfies the Clausius-Planck property. It follows that $\frac{\partial S}{\partial x} f(x) \geq 0$ holds for all x, so that we have

$$p_i(x) \geq 0 \tag{4}$$

in (2). Inequality (4) is called the second law of thermodynamics. By using the fact that the state is determined by the vector Z defined in (3), we can define a vector of dual variables called potentials, so that $w = \frac{\partial S}{\partial Z}$. The potentials w are functions of the temperature, pressure and chemical potential, i.e. $w = [\frac{1}{T}, \frac{P}{T}, \frac{\mu}{T}]$. Sometimes the potentials are also called observables, since they can be measured directly, whereas the extensive variables often has to be inferred indirectly.

978-1-4244-4524-0/09/$25.00 ©2009 AACC

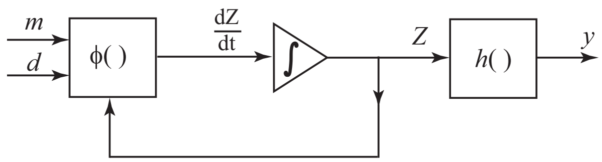

Fig. 1. Block diagram of passivity based process model.

A generic macroscopic model of a single component process system with the microscopic model defined by (1) can be written in terms of its invariant as follows:

$$\begin{cases} \frac{dZ}{dt} &= \phi(Z, d, m) \\ y &= h(Z) \end{cases} \tag{5}$$

The net transport $\phi(Z, d, m)$ can be decomposed into n flows so that $\phi = \sum f_i$, where f_i denote the flows of mass, energy and volume with $i = 1, \cdots, n$. We note that the outputs y are intensive variables, e.g. the temperature, pressure and water level. The function $h(Z)$ defines the measurement strategy. The measurement have to be chosen so that the state Z is observable from y. Fig. 1 shows a block diagram of the general process model.

III. Boiler Model with Extensive States

A. *Åström-Bell Boiler Model*

In 2000, Åström and Bell proposed a fourth-order drum boiler model, which relies on a mixture of extensive and intensive variables. This model captures the key dynamical properties of drum boilers over a wide operating range. The Åström-Bell model also pays particular attention to modeling the drum water level dynamics. It is an important objective in boiler control design to maintain a steady water level, because the level mismanagement contributes to a significant number of emergency shutdowns [5]. A schematic picture of a boiler system is shown in Fig.2.

The Åström-Bell model consists of four state variables: the total water volume V_{wt}, drum pressure p, steam quality at the riser outlet α_r, and steam volume under the liquid level in the drum V_{sd}. Denote $\dot{m}$ as the mass flow rate, V as the volume, A as intersection area, h as the enthalpy, and ρ as the density. The subscripts fw, s, w, c, d, r, dc represent feed water, steam, water, condenser, drum, riser, downcomer, respectively. The Åström-Bell model is formulated as

$$\begin{aligned} e_{11}\frac{dV_{wt}}{dt} + e_{12}\frac{dp}{dt} &= \dot{m}_{fw} - \dot{m}_s \\ e_{21}\frac{dV_{wt}}{dt} + e_{22}\frac{dp}{dt} &= Q + \dot{m}_{fw}h_{fw} - \dot{m}_s h_s \\ e_{32}\frac{dp}{dt} + e_{33}\frac{d\alpha_r}{dt} &= Q - \alpha_r h_c \dot{m}_{dc} \\ e_{42}\frac{dp}{dt} + e_{43}\frac{d\alpha_r}{dt} + e_{44}\frac{dV_{sd}}{dt} &= \frac{\rho_s(V_{sd}^* - V_{sd})}{T_d} \\ &+ \frac{(h_f - h_w)\dot{m}_{fw}}{h_c} \end{aligned} \tag{6}$$

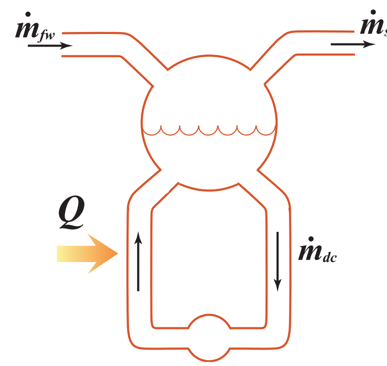

Fig. 2. Schematic picture of a drum boiler.

The drum water level is calculated as

$$\ell = \frac{V_{wd} + V_{sd}}{A_d} \tag{7}$$

where the drum water volume V_{wd} and the steam volume fraction $\hat{\alpha}_v$ are

$$V_{wd} = V_{wt} - V_{dc} - (1 - \hat{\alpha}_v)V_r \tag{8}$$

$$\hat{\alpha}_v = \frac{\rho_w}{\rho_w - \rho_s}\left[1 - \frac{\rho_s}{(\rho_w - \rho_s)\alpha_r}\ln(1 + \frac{\rho_w - \rho_s}{\rho_s}\alpha_r)\right] \tag{9}$$

Refer to Appendix for detail descriptions of the coefficients in (6-9).

B. *State Space Isomorphism*

Theorem 1: Let V_t be the boiler volume, M, U be the mass and energy inventories in a drum boiler. For any given M, U, there exists one and only one set of V_{wt} and p, such that the following inventory equations hold.

$$M = \rho_w V_{wt} - \rho_s(V_t - V_{wt}) \tag{10}$$

$$U = \rho_w V_{wt} h_w - \rho_s(V_t - V_{wt})h_s - pV_t + m_t C_p t_s \tag{11}$$

where t_s is the steam temperature, m_t and C_p are the mass and specific heat of boiler wall.

Proof: Multiplying (10) by h_s and subtracting the result from (11) gives

$$U - Mh_s = \rho_w h_w(h_w - h_s) - pV_t + m_t C_p t_s \tag{12}$$

It follows from (10) that $M = (\rho_w - \rho_s)V_{wt} - \rho_s V_t$. Normally, the total volume V_t has the same magnitude order as the water volume V_{wt}. Furthermore, the water density is greatly larger than that of the steam, i.e. $\rho_w \gg \rho_s$. It leads that $\rho_w - \rho_s \approx \rho_w$ and $\rho_w V_{wt} \gg \rho_s V_t$. Accordingly, (10) can be simplified as

$$M = \rho_w V_{wt} \tag{13}$$

Substituting (13) into (12), we can get

$$U = Mh_w - V_t p + m_t C_p t_s.$$

Note that h_w, t_s are functions of the pressure p, i.e. $h_w = f_1(p), t_s = f_3(p)$. According to the Gibbs phase law, both f_1 and f_3 are bijections. By denoting $p = f_2(p)$ and defining

$$U = F(p) = Mf_1 - V_t f_2 + m_t C_p f_3 \tag{14}$$

we can obtain that F is also a bijection because it is a linear combination of three bijections f_1, f_2 and f_3. Therefore, for any given U, there must exist one and only one p such that (14) holds. In addition, the pressure p uniquely determines the water density ρ_w. It follows from (13) that a unique state $V_{wt} = M/\rho_w$ is specified .

Here it is proved that for any given M, U, there exist one and only one set of V_{wt}, p satisfying the inventory equations. The total water volume V_{wt} specifies the distribution of water and steam in the plant, and the pressure p determines all the intensive state variables. Hence M, U can uniquely specify all the states of the water-steam system in a drum boiler. This completes the proof of Theorem 1. ∎

Theorem 1 shows that the state space spanned by V_{wt} and p is isomorphic to that by M and U. This lays the theoretical foundations for the development of boiler model with the inventories as the state variables.

C. State Space Model Based on Inventory

According to (3) and (6), we have the following mass and energy balances

$$\begin{aligned} \frac{dM}{dt} &= \dot{m}_{fw} - \dot{m}_s \\ \frac{dU}{dt} &= Q + \dot{m}_{fw}h_{fw} - \dot{m}_s h_s \end{aligned}$$

Denoting $Z_1 = [M, U]^T$ and $u = [Q, \dot{m}_{fw}, \dot{m}_s]^T$, we can get

$$\frac{dZ_1}{dt} = \phi_1(Z_1)u \tag{15}$$

where $\phi_1(Z_1) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & h_{fw} & h_s \end{bmatrix}$ with $\phi_1(Z_1)$ is a functional matrix because both h_f and h_s are functions of Z_1 due to Theorem 1 and the Gibbs state law.

Using (6) and noting $e_{44} = \rho_s$ (see Appendix), we can obtain

$$\begin{aligned} \frac{d\alpha_r}{dt} &= -\frac{h_c\dot{m}_{dc}}{e_{33}}\alpha_r + \frac{1}{e_{33}}Q - \frac{e_{32}}{e_{33}}\frac{dp}{dt} && (16) \\ \frac{dV_{sd}}{dt} &= -\frac{1}{T_d}(V_{sd} - V_{sd}^*) + \frac{h_f - h_w}{h_c\rho_s}\dot{m}_{fw} && (17) \\ &- \frac{e_{43}}{\rho_s}\frac{d\alpha_r}{dt} - \frac{e_{42}}{\rho_s}\frac{dp}{dt} \end{aligned}$$

It follows from Theorem 1 that p is a function of M and U. Then the following equations hold.

$$\frac{dp}{dt} = \frac{\partial p}{\partial M}\frac{dM}{dt} + \frac{\partial p}{\partial U}\frac{dU}{dt} = \left[\frac{\partial p}{\partial M}, \frac{\partial p}{\partial U}\right]\phi_1(Z_1)u \tag{18}$$

By substituting (16) and (18) into (17), we can delete the derivative terms of $d\alpha_r/dt$ and dp/dt. By denoting $Z_2 = [\alpha_r, V_{sd}]^T$, (16) and (17) can be rewritten as

$$\frac{dZ_2}{dt} = \phi_{21}(Z_1)Z_2 + \phi_{22}(Z_1)u + \phi_{23}(Z_1)$$

where

$$\begin{aligned} \phi_{21}(Z_1) &= -\begin{bmatrix} \frac{h_c\dot{m}_{dc}}{e_{33}} & 0 \\ 0 & \frac{1}{T_d} \end{bmatrix} \\ \phi_{22}(Z_1) &= -\begin{bmatrix} \frac{e_{32}}{e_{33}} \\ \frac{e_{43}e_{32} - e_{42}e_{33}}{\rho_s e_{33}} \end{bmatrix}\left[\frac{\partial p}{\partial M}, \frac{\partial p}{\partial U}\right]\phi_1(Z_1) \\ &+ \begin{bmatrix} \frac{1}{e_{33}} & 0 & 0 \\ \frac{e_{43}}{e_{33}\rho_s} & \frac{h_{fw} - h_w}{h_c\rho_s} & 0 \end{bmatrix} \\ \phi_{23}(Z_1) &= \begin{bmatrix} 0 \\ \frac{V_{sd}^*}{T_d} \end{bmatrix} \end{aligned}$$

Let $y = [p, \ell]^T$ be the outputs. According to Theorem 1 and the Gibbs state law, the water level ℓ and pressure p are functions of Z_1 and Z_2, i.e. $y = g(Z_1, Z_2)$. Therefore, we formulate the final state space model for the drum boiler as follows

$$\begin{cases} \frac{dZ_1}{dt} &= \phi_1(Z_1)u \\ \frac{dZ_2}{dt} &= \phi_{21}(Z_1)Z_2 + \phi_{22}(Z_1)u + \phi_{23}(Z_1) \\ y &= g(Z_1, Z_2) \end{cases} \tag{19}$$

D. Decouple of State Variables

Theorem 2: In the state space model of (19), Z_2 will asymptotically converge to some constants after the convergence of Z_1 to their respective setpoints.

Proof: It follows from the convergence of Z_1 that $dM/dt = 0$ and $dU/dt = 0$. Using Theorem 1, we further have $dV_{wt}/dt = 0$ and $dp/dt = 0$. Accordingly, (16) can be simplified as

$$\frac{d\alpha_r}{dt} = -\frac{h_c\dot{m}_{dc}}{e_{33}}\alpha_r + \frac{1}{e_{33}}Q \tag{20}$$

According to the Appendix, we can get

$$e_{33} = K\frac{\partial\hat{\alpha}_v}{\partial\alpha_r} \tag{21}$$

with $K = [(1 - \alpha_r)\rho_s + \alpha_r\rho_w]h_c V_r$. It is evident that $K \geq 0$ because $1 \geq \alpha_r \geq 0$ and $\rho_w, \rho_s, h_c, V_r > 0$. Therefore, the signal of e_{33} is solely determined by $\partial\hat{\alpha}_v/\partial\alpha_r$.

It follows from (9) that

$$\frac{\partial\hat{\alpha}_v}{\partial\alpha_r} = -\frac{\rho_w\rho_s}{(\rho_w - \rho_s)^2}\frac{d}{d\alpha_r}\left[\frac{\ln(1 + \frac{\rho_w - \rho_s}{\rho_s}\alpha_r)}{\alpha_r}\right]$$

Notice that

$$\begin{aligned} &\frac{d}{d\alpha_r}\left[\frac{\ln(1 + \frac{\rho_w - \rho_s}{\rho_s}\alpha_r)}{\alpha_r}\right] \\ = \; &\frac{1}{\alpha_r^2}\left[\frac{\frac{\rho_w - \rho_s}{\rho_s}\alpha_r}{1 + \frac{\rho_w - \rho_s}{\rho_s}\alpha_r} - \ln(1 + \frac{\rho_w - \rho_s}{\rho_s}\alpha_r)\right] \end{aligned}$$

we can obtain the following simplified equation

$$\frac{\partial\hat{\alpha}_v}{\partial\alpha_r} = -\frac{\rho_w\rho_s}{(\rho_w - \rho_s)^2\alpha_r^2}f(x) \tag{22}$$

where $f(x) = \frac{x}{1+x} - \ln(1 + x)$ with $x = \frac{\rho_w - \rho_s}{\rho_s}\alpha_r$. It is evident that $x \geq 0$.

Here the only thing left is to prove the signal of $f(x)$. Note that

$$\frac{df(x)}{dx} = \frac{1}{(1+x)^2} - \frac{1}{1+x} = -\frac{x}{(1+x)^2}$$

we can get

$$\begin{cases} f(x) = 0, & x = 0 \\ \frac{df(x)}{dx} < 0, & x > 0 \end{cases}$$

It is easy to see that $f(x)$ is a monotonically decreasing function provided that $x > 0$. Therefore, $f(x) < 0$ holds for any $x > 0$. Using (21) and (22), we obtain $e_{33} \geq 0$. It immediately follows that $-h_c\dot{m}_{dc}/e_{33} < 0$. Therefore, due to (20), the state variable α_r asymptotically converge.

It follows from the convergence of α_r that $d\alpha_r/dt = 0$. Then (17) can be rewritten as

$$\frac{dV_{sd}}{dt} = -\frac{1}{T_d}V_{sd} + \left(\frac{h_f - h_w}{h_c\rho_s}\dot{m}_{fw} + \frac{1}{T_d}V_{sd}^*\right)$$

Notice that T_d is the time constant, V_{sd}^* is the volume constant, and the parameters $h_f, h_w, \rho_s, h_c, \dot{m}_{fw}$ will be constants after the convergence of M and U. The V_{sd} will converge asymptotically because the coefficient $-\frac{1}{T_d} \leq 0$. This completes the proof of Theorem 2. ■

Theorem 2 shows that the state variables can be decoupled in the stability analysis and controller synthesis of the boiler systems. The state Z_1 dominates the boiler model's dynamics. This conclusion can facilitate the control design. The stability of the model with four intensive states will grantee provided that we design a suitable controller to stabilize only two extensive states. This can lead to a much simplified controller design process, because the mass and energy inventories are passive, such that they can be efficiently controlled by simple inventory controllers.

IV. Passivity Based Inventory Control

It is proved in [9] that the synthetic input and output pair (u, e_v) of the controlled part of system (5) is passive

$$\begin{cases} u &= \phi(Z, d, m) + \frac{dZ_1^*}{dt} \\ e_Z &= Z - Z_1^* \end{cases}$$

and that a control can be calculated if $\phi(Z, d, m)$ is invertible with respect to m. Note that m does not have to be unique. Z_1^* is the desired setpoint for Z_1. Therefore, the inventory control law can be written in the form:

$$u = -\mathbf{C}(e_Z) = \phi(Z, d, m) + \frac{dZ_1^*}{dt} \tag{23}$$

This control strategy ensures that the closed-loop system asymptotically tracks the desired set point. The operator $\mathbf{C}$, which maps errors into synthetic controls, should be strictly input passive, e.g. the PID controller, adaptive feedforward controllers, optimal controllers and many gain scheduling controllers [10].

Now we consider the boiler follow mode. In this mode, the mass flow rate of steam out of the boiler is set by the demand for the steam in the turbine. Then the water flow rate into the drum $\dot{m}_{fw}$ has to be set so that the total mass inventory is kept constant, and the heat flow rate Q (i.e. fuel flow rate) has to be controlled so that the total energy inventory is maintained at a given setpoint.

Denote $m = [\dot{m}_{fw}, Q]$ as the manipulated variables, and $d = \dot{m}_s$ as the disturbance. We can rewrite (15) as follows

$$\frac{dZ_1}{dt} = \phi_{11}(Z_1)m + \phi_{12}(Z_1)d$$

with $\phi_{11}(Z_1) = \begin{bmatrix} 1 & 0 \\ 1 & h_{fw} \end{bmatrix}$ and $\phi_{12}(Z_1) = [-1, h_s]^T$.

According to the passivity based controller design theory, we have the following equations.

$$\frac{dZ_1}{dt} = \begin{bmatrix} -\mathbf{C}_1(M - M^*) \\ -\mathbf{C}_2(U - U^*) \end{bmatrix} + \frac{dZ_1^*}{dt}$$

where $\mathbf{C}_1$ and $\mathbf{C}_2$ are PI controllers, M^* and U^* are the setpoints for M and U, respectively. By letting Z_1^* be constant, we have $\frac{dZ_1^*}{dt} = 0$.

Note that $\phi_{11}(Z_1)$ is a lower triangular matrix. We have $|\phi_{11}(Z_1)| \neq 0$. Therefore, we can get

$$m = -\phi_{11}(Z_1)^{-1}\left(\begin{bmatrix} \mathbf{C}_1(M - M^*) \\ \mathbf{C}_2(U - U^*) \end{bmatrix} + \phi_{12}(Z_1)d\right)$$

Finally, we have the following passivity-based control system

$$\begin{bmatrix} \dot{m}_f \\ Q \end{bmatrix} = \begin{bmatrix} \dot{m}_s - \mathbf{C}_1(M - M^*) \\ (h_s - h_w)\dot{m}_s - h_w\mathbf{C}_1(M - M^*) - \mathbf{C}_2(U - U^*) \end{bmatrix} \tag{24}$$

This control structure is the classical combination of feedback and feed-forward control, i.e.

$$m = \underbrace{-\phi_{11}(Z_1)^{-1}\begin{bmatrix} \mathbf{C}_1(M - M^*) \\ \mathbf{C}_2(U - U^*) \end{bmatrix}}_{Feedback} + \underbrace{\phi_{11}(Z_1)^{-1}\phi_{12}(Z_1)d}_{Feedforward}$$

Since the feedforward term cancels the nonlinearities, this method is therefore also referred to as input-output linearization.

V. Step Response to Mass Inventory

To illustrate the performance of the passivity based inventory controllers, we simulate responses to step changes in the total mass inventroy. The boiler's structure parameters used are those from the Swedish power plant [5]. The inventory controllers are used as stated in (24). The thermodynamic properties are calculated using the Xsteam package [12]. This package is also used to bridge the inventories with the measured variables, e.g. the steam temperature and pressure.

Fig. 3 shows the profiles of the inventories and control variables. It is easy to see from Fig. 3 (1)-(2) that all the mass and energy inventories are controlled around their

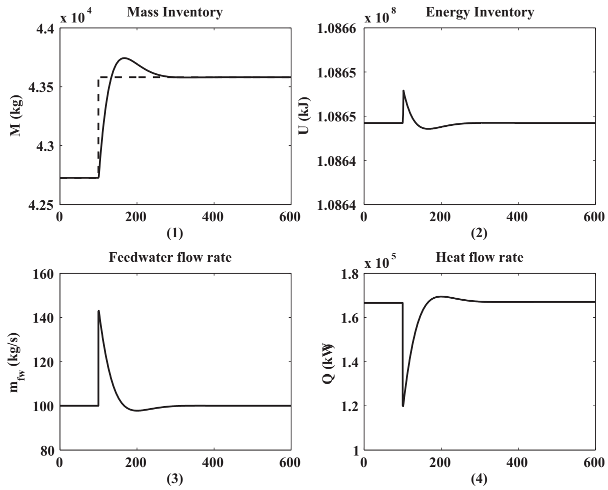

Fig. 3. Profiles of the inventories and control variables to the step change in mass.

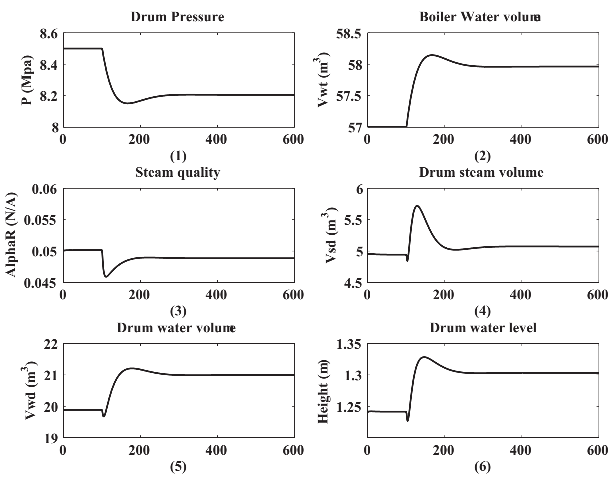

Fig. 4. Profiles of key state variables in the Åström-Bell model to the step change in mass.

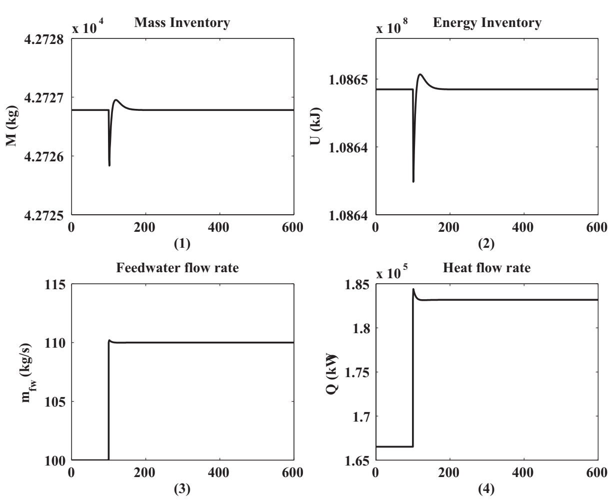

Fig. 5. Dynamic responses of the mass, energy inventories and the profiles of control variables.

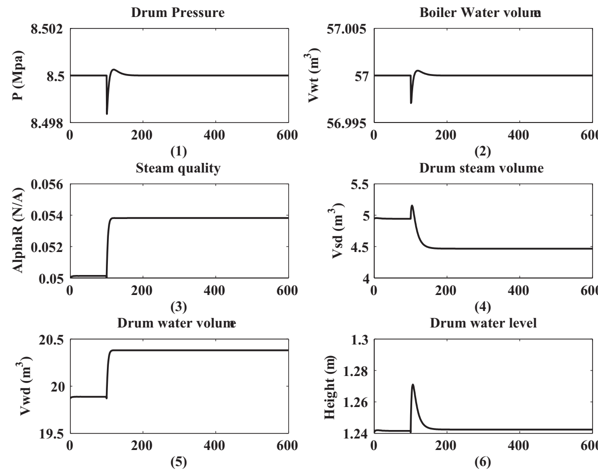

Fig. 6. Dynamic response of key state variables in the Åström-Bell model to a steam flow change.

setpoints. In particular, the mass inventory M tracks the new setpoint value after a small overshoot. This overshoot is caused by the PI controllers.

Fig. 4 demonstrates the step response of some key state variables in the Åström-Bell model. In the initial stage, more water is feeded into the drum because of an increased setpoint of the mass inventory. Higher feedwater flow rate will cause more steam condensation. This leads to the increase of the water volume V_{wt} and the decrease of drum pressure p. Meanwhile, higher feedwater bring more energy, then the heat flow rate will decrease to maintain a constant energy inventory. This leads to the decrease of the steam quality α_r in the riser. Lower drum pressure will cause more water evaporation. This leads to the increase of the steam volume V_{sd} in drum. In the second stage, the inlet mass flow of the feedwater will decrease after the maximum overshoot. This reverses all the plot patterns of the state variables.

The water level presents a typical swell and shrink phenomena. The water level decreases initially due to the decreased evaporation in the riser caused by the decreasing heat flow rate. Then the feedwater flow dominates the dynamics of the water level. Accordingly, the drum water level increases and decreases before it is stabilized at the initial value. Here we can see that the inventory controllers perform quite well.

VI. Step Response to Steam Flow

In this example, the drum boiler model is subjected to a step increase in the outlet steam flow rate equivalent to 10 kg/s. Fig. 5 shows the dynamic responses of the inventories and the state variables. Fig. 6 shows the step response of some key state variables in the Åström-Bell model. It is not hard to see that not only the mass and energy inventories, but also the drum pressure and total water volume track their setpoint accurately throughout the simulation time. Although the two states α_r and V_{sd} are not controlled directly, they also converge to the corresponding constant values.

During the transient period, the state α_r increases, while the V_{sd} decreases. The reason for this is that more heat and feed water is required to remedy the step increase of the outlet steam flow rate in order to maintain the mass and energy inventories to be constant. More heat to the riser will evaporate more water, this leads to the increase of the steam quality α_r. At the same time, more feed water will condensate more steam under the water level in the drum. This causes the decrease of the steam volume V_{sd} in the drum. Finally, the feed water and heat will balance the mass and energy flow of the outlet steam. Then all the states and inventories are stabilized.

The dynamic response of water level is complicated and depends on a combination of the dynamics of water and steam in the drum. Two competing mechanisms contribute to the response in water level. The drum water volume decrease first due to the decrease of the total mass inventories. Then the rapid initial response of steam leads to the swell of the water level. The increase of the average steam quality $\hat{\alpha}_v$ causes the drum volume V_{wd} to increase. After that, the steam volume V_{sd} will dominate the level response. Therefore, the water level decrease with the decrease of V_{sd} until both of them are stabilized.

VII. Conclusion

This paper proposes a novel state space model for drum boilers. This model has an affine structure in the control variables. This facilitates the design of a passivity based inventory controller, which ensures the closed-loop stability and good control performances. The affine structure in control variables is derived from the mass, energy and momentum balances. Then this proposed modeling and passivity-based control scheme are promising to find applications in a wide range of process systems governed by constitution laws.

Acknowledgment

The authors would like to acknowledge Mr. Richard Kephart, Charles Menten and Dr. Xu Cheng (Emerson Process Management) for the valuable suggestions and helpful technical supports. We also thank Dr. Kwong Ho Chan Keyu Li and Alex B. Smith (CMU) for the helpful discussions.

References

[1] K. L. Chien, E. I. Ergin, C. Ling & A. Lee. "Dynamic analysis of a boiler." *ASME Transactions*, vol. 80: 1809-1819, 1958.

[2] H. W. Kwan & J. H. Anderson. "A mathematical model of a 200 MW boiler." *International Journal of Control*, vol. 12: 977-998, 1970.

[3] F. T. Thompson. "A dynamic model of a drum-type boiler system." *IEEE Transactions on Power Apparatus and Systems* PAS-86: 625-635, 1967.

[4] A. Tysso. "Modelling and parameter estimation of a ship boiler." *Automatica*, vol. 17: 157-166, 1981.

[5] K. J. Åström & R. D. Bell. "Drum-boiler dynamics." *Automatica*, vol. 36: 363-378, 2000.

[6] J. P. McDonald & H. G. Kwatny. "Design and analysis of boiler-turbine-generator controls using optimal linear regulator Theory." *IEEE Transactions on Automatic Control*, Vol. AC-18: 202-209, 1973.

[7] A. Tysso, J. C. Brembo & K. Lind. "The design of a multivariable control system for a ship boiler," *Automatica*, Vol. 12: 211-224, 1976.

[8] B. W. Hogg & N. M. Rabaie. "Multivariable generalized predictive control of a boiler system." *IEEE Transactions on Energy Conversion*, Vol. 6: 282-288, 1991.

[9] C. A. Farschman, K. P. Viswanath & B. E. Ydstie. "Process systems and inventory control." *AIChE Journal*, Vol. 44:1841-1857, 1998.

[10] M. Ruszkowski, V. Garcia-Osorio & B. E. Ydstie. "Passivity based control of transport reaction systems." *AIChE Journal* Vol.51: 3147-3166, 2005.

[11] A. Alonso & B. E. Ydstie. "Stabilization of distributed systems using irreversible thermodynamics." *Automatica*, vol. 37: 1739-1755, 2001.

[12] M. Holmgren. http://www.x-eng.com

Appendix

The coefficients e_{ij} in (5) are calculated as

$$
\begin{aligned}
e_{11} &= \rho_w - \rho_s \\
e_{12} &= V_{wt}\frac{\partial \rho_w}{\partial p} + V_{st}\frac{\partial \rho_s}{\partial p} \\
e_{21} &= \rho_w h_w - \rho_s h_s \\
e_{22} &= V_{wt}\left(h_w\frac{\partial \rho_w}{\partial p} + \rho_w\frac{\partial h_w}{\partial p}\right) \\
&+ V_{st}\left(h_w\frac{\partial \rho_s}{\partial p} + \rho_w\frac{\partial h_s}{\partial p}\right) - V_t + m_t C_p\frac{\partial t_s}{\partial p} \\
e_{32} &= \left(\rho_w\frac{\partial h_w}{\partial p} - \alpha_r h_c\frac{\partial \rho_w}{\partial p}\right)(1-\hat{\alpha}_v)V_r \\
&+ \left[(1-\alpha_r)h_c\frac{\partial \rho_s}{\partial p} + \rho_s\frac{\partial h_s}{\partial p}\right]\hat{\alpha}_v V_r - V_r \\
&+ [\rho_s + (\rho_w - \rho_s)\alpha_r]h_c V_r\frac{\partial \hat{\alpha}_v}{\partial p} + m_r C_p\frac{\partial t_s}{\partial p} \\
e_{33} &= [(1-\alpha_r)\rho_s + \alpha_r\rho_w]h_c V_r\frac{\partial \hat{\alpha}_v}{\partial \alpha_r} \\
e_{42} &= V_{sd}\frac{\partial \rho_s}{\partial p} + \frac{1}{h_c} \\
&\quad \left(\rho_s V_{sd}\frac{\partial h_s}{\partial p} + \rho_w V_{wd}\frac{\partial h_w}{\partial p} - V_{sd} - V_{wd} + m_d C_p\frac{\partial t_s}{\partial p}\right) \\
&+ \alpha_r(1+\beta)V_r\left[\hat{\alpha}_v\frac{\partial \rho_s}{\partial p}(1-\hat{\alpha}_v)\frac{\partial \rho_w}{\partial p} + (\rho_s-\rho_w)\frac{\partial \hat{\alpha}_v}{\partial p}\right] \\
e_{43} &= \alpha_r(1+\beta)(\rho_s-\rho_w)V_r\frac{\partial \hat{\alpha}_v}{\partial \alpha_r} \\
e_{44} &= \rho_s
\end{aligned}
$$

with $h_c = h_s - h_w$ is the condenser heat of the steam. The steam table is used to evaluate the thermodynamic features $h_s, h_w, \rho_s, \rho_w, t_s$ and the partial derivatives $\partial\rho_s/\partial p, \partial\rho_w/\partial p, \partial h_s/\partial p, \partial h_w/\partial p, \partial t_s/\partial p$.

Finally, the downcomer mass flow rate $\dot{m}_{dc}$ is given by

$$\dot{m}_{dc} = \sqrt{\frac{2\rho_w A_{dc}(\rho_w - \rho_s)g\hat{\alpha}_v V_r}{k}}$$