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Regulating Web Tension in Tape Systems with Time-varying Radii

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***Abstract*— A tape system is time-varying as tape winds from one reel to the other. The variations in reel radii consist of two components: the nominal reel radii change due to tape winding and the reel eccentricities as a result of non-circular reels. These variations introduce disturbances in the tape tension. This paper presents a feedforward control that addresses the time-varying reel radius to regulate the tension. The algorithm is based on the fact that the change of the nominal radius is slow and hence the nominal plant in two consecutive revolutions can be considered as the same. The reel eccentricity disturbance is then repetitive in these two revolutions. We investigate the tension error in the previous revolution and calculate the compensation input that should have cancelled it. The compensation input is interpolated properly and then fed forward to the system in the current revolution. A new compensation input sequence is computed for the immediately following revolution. This computation pattern continues until the end of the tape winding process. Simulation results demonstrate and compare the performance of the proposed algorithm with other recently developed methods.**

I. Introduction

High density digital tape information systems are widely used in data centers for large volumes of data back-up storage because of historical and economical reasons. One of the main control objectives in tape systems is to increase as much as possible the tape transport velocity while simultaneously regulating the tension. Some work has been conducted on tension controls in tape systems [1][4][5] where the plant is assumed to be linear time-invariant (LTI). In [6], adaptive control is investigated to reject tape tension error where the disturbances are considered as either an input or an output disturbance to a LTI system. In [8], the authors combine the adaptive control algorithm with a gain scheduling algorithm to address the time-varying plant due to nominal variations in the radius.

Two feedforward control schemes in the discrete-time domain are discussed in [7] to attenuate tension error caused by reel eccentricities. The first algorithm introduces a time-varying feedforward filter for the reference input. It can achieve zero steady-state tension error when all the varying components in the radius are known. The second algorithm assumes the nominal reel radius is fixed and hence the tension loop is purely periodic. The tension error caused by unknown reel eccentricities in previous periods is investigated to calculate the compensation input that should have cancelled it. This input is then fed forward to the system in future periods. However, a real tape system is not purely periodic since the nominal reel radius keeps changing when the tape winds. Hence this method needs to be extended or combined with other techniques to address both the time-varying disturbances due to the slowly time-varying nominal reel radius and the pseudo-periodic disturbances caused by reel eccentricities.

In this paper, we propose a feedforward control to simultaneously take into account both the time-varying nominal radius and the unknown reel eccentricities. Since the tape is very thin, the variation in the nominal reel radius of two consecutive revolutions is small. Hence the nominal plant can be approximately considered as invariant during these two periods and the reel eccentricities are then nearly repetitive. We investigate the error in the previous revolution and compute the compensation input that should have cancelled it. This input is then fed forward to the current revolution with some modification and the tension error of the current revolution is then used to calculate the feedforward compensation input for the immediately following revolution. Thus, the compensation input is updated in every revolution for the entire tape winding process.

The rest of this paper is organized as follows. Section II reviews a prototypical tape system model, introduces the decoupled tape tension loop that we use in this research, and discusses the varying components in the reel radius. Section III first briefly reviews a recently developed algorithm [7] from which the proposed algorithm is extended; then derives the latter algorithm that attenuates tension errors caused by time-varying nominal radius and unknown reel eccentricities. In Section IV, the control algorithm is applied to the tension loop and simulation results and performance comparisons with other algorithms are presented. Finally, Section V summarizes the conclusions of this research and highlights areas of future work.

II. Time-varying Tape Tension Loop

The popular lumped-parameter model of a tape system is illustrated in Fig. 1 [1][2][3][5]. The tape winds from the source reel (reel 1) to the take-up reel. $J_i(t)$, $r_i(t)$, and $\omega_i(t)$ $(i = 1, 2)$ are the inertia, radius, and angular velocity of each reel, respectively. The unsupported tape between the two tangential points on the reels is modeled by a parallel dashpot and spring with damping coefficient D and spring constant K, respectively. Each reel is driven by a DC motor with motor friction viscosity coefficient β_i and torque constant K_{ti}. The current applied to each motor is $u_i(t)$. When the tape winds, both the reel radii r_i and the reel rotating inertia

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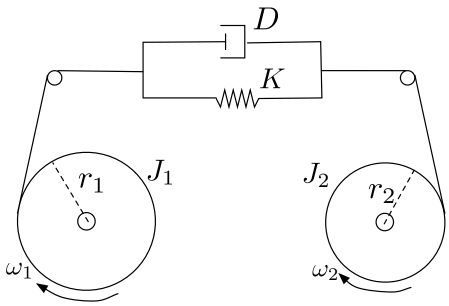

Fig. 1. Lumped-parameter model of a tape system.

J_i change. Other parameters such as D and K might also vary [1]. In this study, we aim at attenuating tension error directly caused by the time-varying radii and consider other parameters constant.

Define $T(t)$ as the tape tension and $V_i(t)$ as the tangential velocity of the tape at each reel. Choose $X = [T(t), V_1(t), V_2(t)]^\top$ as the state and $U = [u_1(t), u_2(t)]^\top$ as the input for the system. With $\eta = \frac{r_1^2(t)}{J_1(t)} + \frac{r_2^2(t)}{J_2(t)}$, a state-space equation of the tape system is [1]

$$\dot{X}(t) = \mathbf{A}(t)X(t) + \mathbf{B}(t)U(t),$$

with

$$\mathbf{A}(t) = \begin{bmatrix} -D\eta & -K+D\frac{\beta_1}{J_1(t)} & K-D\frac{\beta_2}{J_2(t)} \\ \frac{r_1^2(t)}{J_1(t)} & -\frac{\beta_1}{J_1(t)} & 0 \\ -\frac{r_2^2(t)}{J_2(t)} & 0 & -\frac{\beta_2}{J_2(t)} \end{bmatrix},$$

and

$$\mathbf{B}(t) = \begin{bmatrix} -D\frac{r_1(t)K_{t1}}{J_1(t)} & D\frac{r_2(t)K_{t2}}{J_2(t)} \\ \frac{r_1(t)K_{t1}}{J_1(t)} & 0 \\ 0 & \frac{r_2(t)K_{t2}}{J_2(t)} \end{bmatrix}.$$

At mid-pack, where the radii of both reels are equal, the tension loop can be perfectly decoupled from the velocity loop using the decoupling method discussed in [3]. The transfer function of the decoupled single-reel tension loop from the input current to the output tension at mid-pack is

$$G_T = \frac{2K_tDr_m(s+\frac{K}{D})}{Js^2 + (\beta + 2Dr_m^2)s + 2Kr_m^2},$$

where r_m is the reel radius at mid-pack. To simulate the time-varying tension loop in *Matlab*, we convert G_T to a state-space form with time-varying matrices $A_T(t)$ and $B_T(t)$. Using the observability canonical realization, we have

$$\begin{cases} \dot{X}_T(t) = A_T(t)X_T(t) + B_Tu(t) \\ Y(t) = CX(t) \end{cases}, \tag{1}$$

$$A_T(t) = \begin{bmatrix} -\frac{\beta+2Dr(t)^2}{J} & 1 \\ -\frac{2Kr(t)^2}{J} & 0 \end{bmatrix},$$

$$B_T(t) = \begin{bmatrix} \frac{2K_tDr(t)}{J} \\ \frac{2K_tKr(t)}{J} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The tension T is the first state and the second state is a function of the difference between the tangential velocities of the two reels. The current to the motor is the input. $A_T(t)$ and $B_T(t)$ are functions of the reel radius $r(t)$.

The variations in the time-varying radii consist of two components: (a) the nominal change as a result of web winding and (b) the reel eccentricities due to non-perfectly circular reels, also known as reel runout. The nominal reel radii, denoted as $r_{ni}(t)$, are

$$r_{ni}(t) = r_i(t_0) \mp \int_{t_0}^{t} \frac{\epsilon\omega_i(\tau)}{2\pi} d\tau,$$

where ϵ is the tape thickness. Denoting reel eccentricities as $r_{ri}(t)$, the radii are

$$r_i(t) = r_i(t_0) \mp \int_{t_0}^{t} \frac{\epsilon\omega_i(\tau)}{2\pi} d\tau + r_{ri}(t).$$

A basic full-state feedback K_f is applied to stabilize the tension loop (Fig. 2). In this figure, G_{cl} is the closed tension loop, T_d is the desired tension value in steady state, Y is the output tension, and $R(t)$ is a time-varying filter for the reference input to generate $\hat{u}$ that takes into account the time-varying nominal radius. C_{ff} is the feedforward controller to generate the compensation input $\tilde{u}$ to address reel eccentricities. e^{-sT_v} is a delay to the output Y where the delay constant T_v varies across revolutions. The input to the system, u, is the sum of $\hat{u}$ and $\tilde{u}$. In this paper, we use the same $R(t)$ as in [7] and focus on designing C_{ff}.

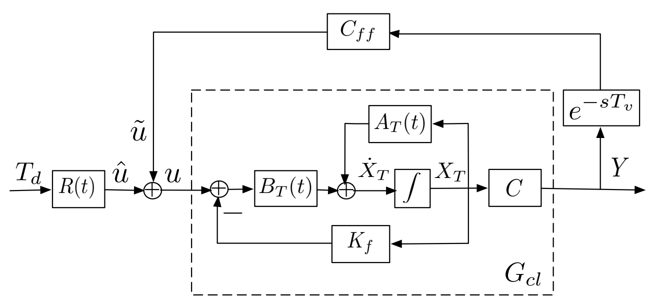

Fig. 2. The full-state feedback K_f stabilizes the tension loop. Time-varying $R(t)$ takes into account the time-varying nominal radius and $\tilde{u}$ is the feedforward input to address reel eccentricities.

As shown in [7], for a tape system with parameters such as those in Table I and the desired tension $T_d = 1$ N, reel runout on the order of 10^{-4} m causes tension error on the order of 10^{-3} N. The frequency of the tension error is the same as the varying rotating frequency and the magnitude depends on the varying radius.

III. FEEDFORWARD CONTROL

A. Feedforward Controller to Generate $\hat{u}$

Define $\{A_T(t_i), B_T(t_i), C\}$ as the state matrices of the system in Equation (1) at a specific time t_i. The feedforward controller to generate $\hat{u}$ at t_i is [7]

$$R(t_i) = K_f\gamma_{\mathbf{1}}(t_i) + \gamma_2(t_i),$$

TABLE I

TAPE SYSTEM PARAMETERS FOR SIMULATION

Parameter	Label	Value
t_ρ	Tape density	$1.6e3$ kg/m^3
ϵ	Thickness of the tape	$7.7e$-6 m
K_J	Tape pack inertia constant	20.14 kg/m^2
K_t	Motor torque constant	0.0189 N m/Amp
D	Dashpot constant	0.9 N sec/m
$r(t)$	Radius of the reel	0.014m to 0.028 m
r_m	Mid-pack radius of the reel	0.02389 m
J	Inertia of the reel	$3.05e$-5 kg m^2
K	Spring constant	600 N/m
β	Motor viscosity coefficient	$5.9828e$-5 N m sec/rad

where

$$\begin{bmatrix} \gamma_{\mathbf{1}}(t_i) \\ \gamma_2(t_i) \end{bmatrix} = \begin{bmatrix} A_T(t_i) & B_T(t_i) \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix},$$

$\gamma_{\mathbf{1}}(t_i)$ is the required state and $\gamma_2(t_i)$ is the required coefficient of the reference input at steady state so that the tension output reaches the desired value T_d.

In the discrete-time domain, we can compute one reference input filter R_k at each sampling step. If the state matrices are known, this feedforward filter will lead to zero steady-state tension error. In this study, the feedforward control

$$\hat{u}_k = R_k T_d$$

addresses the known variations in the nominal reel radius r_n.

B. Compensation Input $\tilde{u}$ to Address Reel Runout

The effects from the unknown reel eccentricities on the tension are attenuated by the compensation control $\tilde{u}$. Represent the discrete-time state-space form of the time-varying tension loop in Equation (1) as

$$\left\{ \begin{array}{rcl} X_{k+1} & = & A_k X_k + B_k u_k \\ Y_k & = & C X_k \end{array} \right. , \tag{2}$$

where A_k and B_k are the discrete-time state matrices at step k [2]. For the closed tension loop $\{\bar{A}_k, B_k, C\}(\bar{A}_k = A_k - B_k K_f)$, denote the desired output as y_d, the output error in Y_k as $\tilde{Y}_k = y_d - Y_k$, and the compensation input needed to cancel $\tilde{Y}_k$ as $\tilde{u}_{k-1}$, then

$$\tilde{u}_{k-1} = \frac{1}{CB_{k-1}} \left(\tilde{Y}_k - C \sum_{j=1}^{k-2} \prod_{i=j+1}^{k-1} \bar{A}_i B_j \tilde{u}_j \right).$$

if CB_{k-1} is non-singular (see [7] for details).

When the nominal radius is fixed, the system is periodic. Suppose the period is N_p, then the compensation input for revolution μ can be applied to revolution μ+1 to achieve the desired output tension y_d. Considering varying initial conditions, two periods of tension error after the transient process (starting at step k_0) are used to compute the compensation input series $\tilde{u}_p(i)(i = 1, 2, \cdots, 2N_p)$. In [7], the complete compensation input signal $\tilde{u}$ for the entire winding process then is constructed as:

$$\tilde{u}_k = \begin{cases} 0, & k - k_0 \leq 2N_p \\ \tilde{u}_p(k - k_0 - 2N_p), & 2N_p < k - k_0 \leq 3N_p \\ \tilde{u}_p\left((\text{mod } \frac{k-k_0}{N_p}) + N_p\right), & k - k_0 > 3N_p \end{cases}.$$

Thus, the same compensation input sequence repeats after the first compensated period.

Simulation results show that when the nominal radius is fixed, this algorithm reduces the tension error from an order of 10^{-3} N to 10^{-6} N in magnitude. There are several issues that this method should be extended to address. First, the tape tension loop is purely periodic when the nominal radius is fixed and ideally the steady-state tension error should be zero. The residual error in the simulations exists because the state matrices used to compute the compensation input are the nominal state matrices that do no include the (unknown) reel eccentricities in the radius. Thus the computed input is not ideally accurate. Second, a real tape system is not purely periodic as the nominal reel radius varies. This algorithm should be extended to address the slowly time-varying period of the reel runout disturbance.

Let m be the revolution index and n the step index in one revolution. Define the pair (m, n) as the index of the sampled nth step in the mth revolution; (m, n) may be in subscripts in some equations to save space. Define the compensation input for revolution m as $\tilde{u}(m, :)$. Denoting the tension error at step (m, n) as $\tilde{Y}(m, n)$, the additional input at step $(m, n-1)$

$$\tilde{v}_{m,n-1} = \frac{1}{CB_{m,n-1}} \left(\tilde{Y}_{m,n} - C \sum_{j=1}^{n-2} \prod_{i=j+1}^{n-1} \bar{A}_{m,i} B_{m,j} \tilde{v}_{m,j} \right).$$

should have canceled $\tilde{Y}(m, n)$. If the system is periodic or pseudo-periodic, applying $\tilde{v}_{m,n-1}$ in addition to $\tilde{u}(m, n-1)$ to the $n-1$ step in revolution $m+1$ will attenuate the tension error at step $(m+1, n)$. Thus the compensation input for step $(m+1, n)$ is the sum of $\tilde{u}(m, n-1)$ and $\tilde{v}_{m,n-1}$. Computing the additional input at every step in revolution m and then forwarding $\tilde{v}(m, :)$ to revolution $m + 1$, we can reduce the tension error in revolution $m + 1$ (Fig. 3).

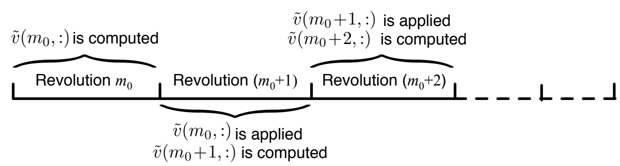

Fig. 3. The compensation input is updated every revolution.

In this figure, m_0 is the index of the revolution where the additional input sequence is computed for the first time. No compensation input is applied in this revolution. The additional input sequence $\tilde{v}(m_0, :)$ computed from revolution m_0 is the compensation input $\tilde{u}(m_0 + 1, :)$ for revolution $m_0 + 1$ in which $\tilde{v}(m_0 + 1, :)$ is calculated. Similarly, $\tilde{v}(m, :)$ is computed when $\tilde{u}(m, :)$ is applied to the system. Hence, the compensation input $\tilde{u}(m, :)$ is defined as

$$\tilde{u}(m, n) = \begin{cases} 0, & m \leq m_0 \\ \tilde{v}(m-1, n), & m = m_0 + 1 \\ \tilde{u}_{m-1,n} + \tilde{v}_{m-1,n}, & m > m_0 + 1 \end{cases}. \tag{3}$$

1) Nominal Radius Fixed: If the nominal radius is fixed, the system is purely periodic. If the tangential velocity of the tape is constant, the angular velocity of the reel is fixed. Then a sampling rate can be chosen so that the number of sampled points in every revolution is the same integer. The algorithm discussed above achieves the ideal compensation input sequence to cancel the periodic tension error after computing $\tilde{u}$ for a certain number of revolutions without knowing the state matrices accurately. The tension error in [7] can be eliminated.

2) Nominal Radius Varying: When the nominal radius is time-varying, the system is not purely periodic and no fixed sampling rate guarantees an integer number of sample points for every rotation of 2π radians. In this case, we consider a full revolution completed when the angular position of the sampling point crosses over that of the first sampling point in the same revolution. The angular position of the first sample of revolution m relative to that in revolution $m-1$ is defined as $\alpha_0(m)$ as shown in Fig. 4. The angles are not drawn to scale. For the first revolution, $\alpha_0(1)$ is zero.

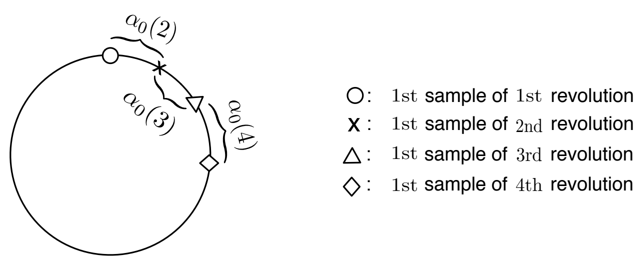

Fig. 4. A revolution is completed when the angular position of a sampling point crosses over that of the first sampling point in the same revolution.

Let $\theta(m,n)$ be the angle that the reel has rotated at the nth step in the mth revolution. The angle θ is reset to zero when a new revolution starts, i.e., $\theta(:,1)$ is always zero. Suppose the physical angular location on the reel at $\theta(m,n)$ lies between the angular positions of steps n_e and n_{e+1} in the previous revolution $m-1$. Instead of directly applying the compensation input from Equation (3) to the system, we linearly interpolate $\tilde{u}(m,n_e)$ and $\tilde{u}(m,n_e+1)$ to generate $\bar{\tilde{u}}(m,n)$, the compensation input at step (m,n). Specifically,

$$n_e = \left\lfloor \frac{\theta_{m,n} + \alpha_0(m)}{2\pi/N_{m-1}} \right\rfloor + 1$$

and

$$\bar{\tilde{u}}_{m,n} = \tilde{u}_{m,n_e} + (\theta_{m,n} - \theta_{m-1,n_e}) \cdot \frac{\tilde{u}_{m,n_e+1} - \tilde{u}_{m,n_e}}{2\pi/N_{m-1}},$$

where N_{m-1} is the total number of sampling points in revolution $m-1$ and $\lfloor\cdot\rfloor$ is the floor operator.

IV. Simulation Results

In the simulations, the tension loop is implemented as in Equation (1) with the parameter values listed Table I. Since we aim at observing the system's time-varying nature caused by variations in reel radius $r(t)$, the other parameters K_t, K, D, β, and J are considered as fixed constants.

The matrices of the tension loop state equation (1) are

$$A_T(t) = \begin{bmatrix} -1.95 + 5.87 \times 10^4 r(t)^2 & 1 \\ -3.9 \times 10^7 r(t)^2 & 0 \end{bmatrix}$$

and

$$B_T(t) = \begin{bmatrix} 1.11 \times 10^3 r(t) \\ 7.39 \times 10^5 r(t) \end{bmatrix}.$$

We freeze the parameters of the time-varying matrices at every time step and convert the continuous-time model to the discrete-time domain. The sampling rate is 12,000 Hz unless otherwise noted. The state feedback K_f is designed to be $[-0.7766, 0.0109]$ to guarantee the stability of the tension loop during the entire tape winding process. The desired tension and velocity values are $T_d = 1$ N and $V_d = 4$ m/sec, respectively. The actual tangential velocity of the tape in steady state is assumed to track V_d perfectly as a result of a separate velocity control loop such as in [1].

The simulations use reel runout data from an actual tape industry sample. The 48 discrete runout samples are linearly interpolated as shown in Fig. 5. The compensation input $\tilde{u}$ is applied to the system at the 10th revolution. Some of the simulation results are shown from 0.1 sec to avoid the beginning part of the transient process so that more details of the tension error in steady state can be seen.

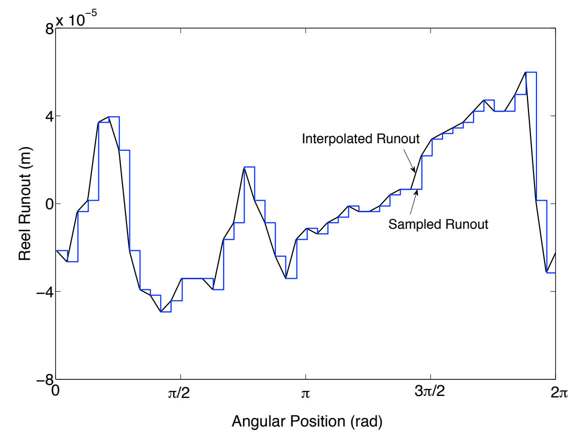

Fig. 5. Runout interpolation.

A. Fixed Nominal Radius; Unknown Reel Runout

Fig. 6 shows the simulated tension error caused by reel runout when the nominal radius is assumed constant. The magnitude of the runout is on the order of 10^{-4} m and causes tension error on the order of 10^{-3} N.

The algorithm in [7] uses the tension error in two revolutions to compute the compensation control input for all future periods. Assuming the nominal radius is fixed and the system is purely periodic, the control scheme attenuates the tension error by three orders of magnitude to 10^{-6} N as shown in Fig. 7 (a). The non-zero steady-state tension error is due to the inaccurate state matrices used to calculate the compensation input. Since the reel runout is unknown, only the nominal state matrices that do not include the reel eccentricities in the radius are available for computation. Thus, the periodic compensation input is not ideally accurate.

This insufficiency is addressed in the algorithm proposed in this paper by calculating the compensation input from the immediately preceding period for the entire process of tape winding. As shown in Fig. 7 (b), the tension error

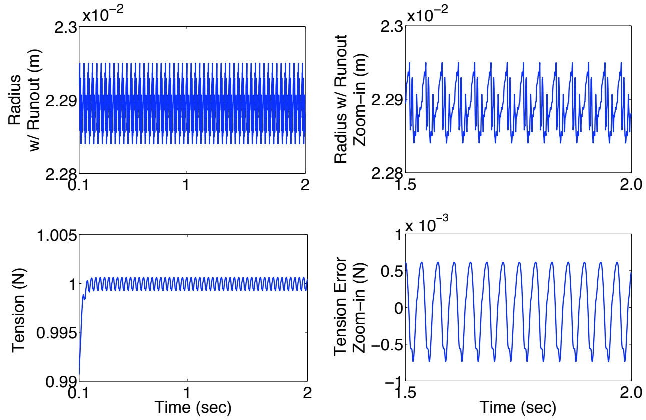

Fig. 6. The nominal radius is fixed and the reel runout data is from an actual industrial tape reel (Fig. 5). The tension error caused by the reel runout is on the order of 10^{-3} N.

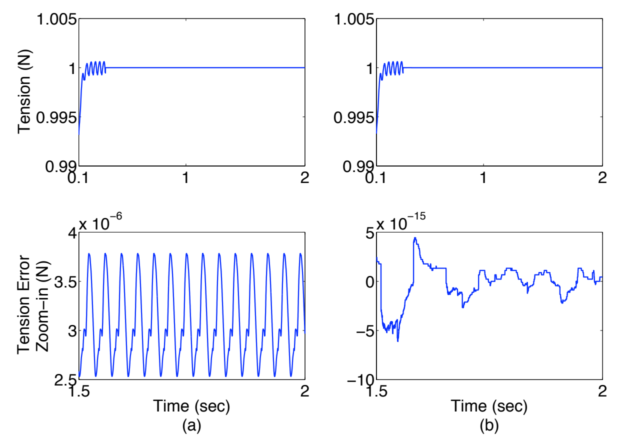

Fig. 7. Nominal radius fixed (purely periodic system): (a) the compensation input computed from tension errors in two revolutions reduces the error by three orders of magnitude; (b) the compensation input is updated every revolution based upon the tension error in the previous revolution, and the steady-state tension error is effectively zero.

is eliminated by the compensation input that is updated every revolution. The reference input filter R is fixed as the nominal radius does not change and $\hat{u}$ is the same in both cases. The difference in the compensation inputs $\tilde{u}$ between Fig. 7 (a) and (b) at steady-state is illustrated in Fig. 8. The sampling rate in these simulations is chosen to be $12,015$ Hz to guarantee an integer number of sampling points in every revolution so that the periodicity of the system is maintained.

B. Time-varying Nominal Radius; Unknown Reel Runout

In this simulation, the nominal radius varies as in a real tape system. Fig. 9 illustrates the simulated tension error caused by reel runout on a source reel. As tape winds to the take-up reel, the nominal radius of the source reel decreases. Without the compensation input to take into account the reel runout, the tension error is on the order of 10^{-3} N, similarly as in Fig. 6. Here, the time-varying reference input filter R addressing the known variation in the nominal radius is updated at every time step.

The compensation sequence calculated from two revolutions [7] reduces the tension error by one order of magnitude in the first two seconds of the simulation when the change in

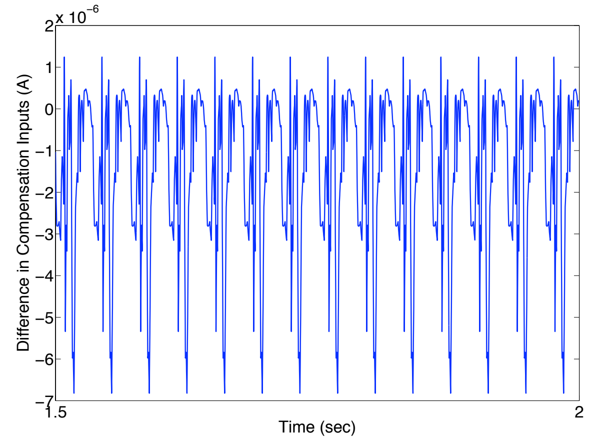

Fig. 8. The difference in the compensation input $\tilde{u}$ between the simulations in Fig. 7 (a) and (b).

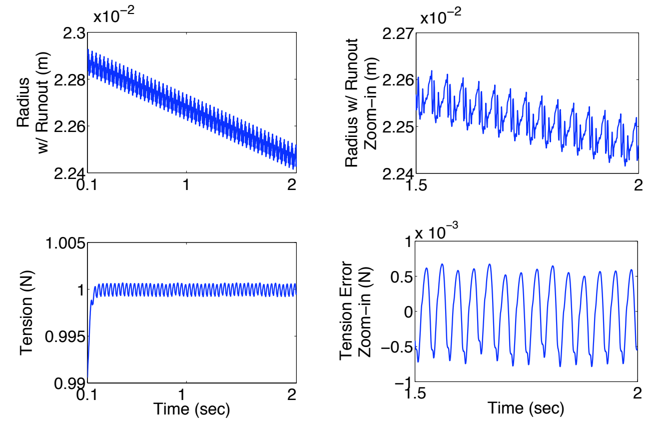

Fig. 9. The nominal radius of the source reel decreases. The reel runout data is from an actual industrial tape reel (Fig. 5). The tension error caused by the reel runout is on the order of 10^{-3} N.

the nominal radius is relatively small, as shown in Fig. 10 (a). However, as the nominal radius keeps decreasing, the tension error increases as the system is more different than when the $\tilde{u}$ sequence is computed. Fig. 11 (a) illustrates the tension error for a longer simulation duration (4 sec). When the compensation input sequence is updated every revolution, the tension error is reduced by two orders of magnitude to 10^{-5} all the time, as show in Fig. 10 (b) and Fig. 11 (b). The difference in the compensation inputs $\tilde{u}$ between the simulations in Fig. 11 (a) and (b) is shown in Fig. 12.

C. Sampling Frequency

The performance of the proposed algorithm depends on the sampling frequency of the system as shown in Table II. $T_{e_{max}}$ and $T_{e_{rms}}$ are the infinity norm and the 2-norm of the tension error T_e in steady state, respectively. A higher sampling frequency provides better performance while requiring more computational power. The simulation results shown above are achieved with a sampling frequency at $12,000$ Hz unless otherwise noted.

V. Conclusions, Discussion, and Future Work

This paper proposes a feedforward control scheme to regulate tension in a time-varying tape system where the

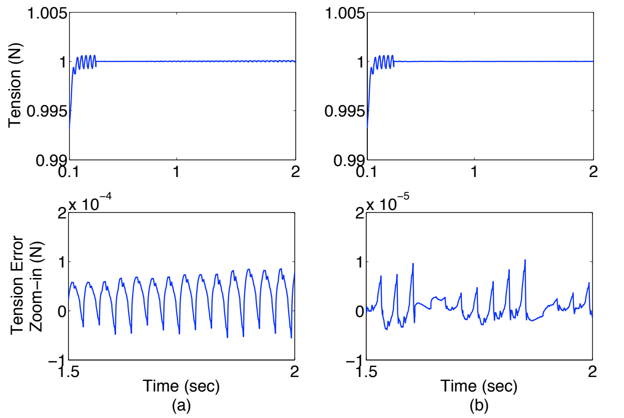

Fig. 10. Nominal radius varying: (a) in the first 2 sec of simulations, the compensation input reduces the tension error by 1 order of magnitude; (b) the compensation input is updated every revolution, and the steady-state tension error is reduced by 2 orders of magnitude.

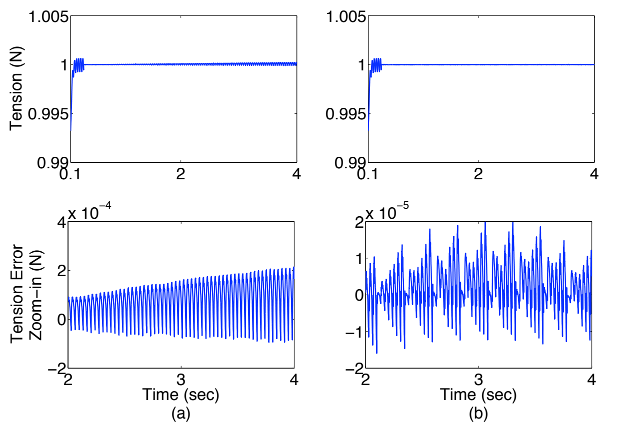

Fig. 11. Nominal radius varying: (a) the tension error gets larger with the repeated compensation input sequence as tape winds; (b) the proposed algorithm reduces the tension error from 10^{-3} N to 10^{-5} N.

variations in the reel radii are composed of nominal radii changes and reel eccentricities. The variations in the nominal radius are taken into account by a time-varying feedforward filter to the tension reference input. The reel eccentricities can be considered as nearly repetitive in two consecutive revolutions if the change in the nominal radius is negligible compared to the reel runout. The compensation input to address reel eccentricities is then computed from the tension error observed in the previous revolution and applied to the current revolution. For more precise compensation, the input computed from the previous revolution is interpolated properly for application to the current revolution. The input sequence is updated every revolution.

The algorithm is simulated in a tape tension loop using "unknown" reel eccentricities obtained from a sample reel from the tape industry. When the nominal radius is fixed and the reel eccentricity is purely periodic, the updated compensation input for each revolution shows superior performance over using a repeating compensation input from a previously developed method [7]. The tension error is completely eliminated to numerical precision limits in the steady state. When the nominal radius is time-varying, the

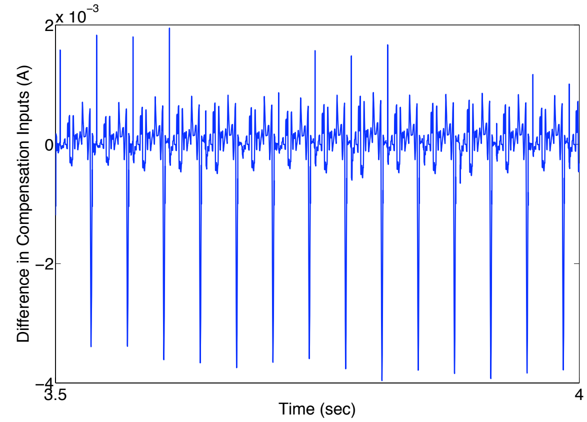

Fig. 12. The difference in the compensation input $\tilde{u}$ between the simulations in Fig. 11 (a) and (b).

TABLE II

SAMPLING FREQUENCY AND ALGORITHM PERFORMANCE

Sampling Freq. (Hz)	$T_{e_{max}}$ (N)	$T_{e_{rms}}$ (N)
2,667	8.7e-5	4.6e-5
4,000	4.6e-5	2.0e-5
8,000	1.6e-5	8.2e-6
12,000	5.7e-6	3.6e-6
30,000	4.5e-6	2.7e-6

algorithm attenuates the tension error by two orders of magnitude.

We have developed and evaluated a feedforward control scheme to simultaneously take into account unknown reel eccentricities and time-varying nominal radii for the entire tape winding process. One advantage of this algorithm is that it is independent of the characteristics of the reel eccentricity characteristics; the runout can be of any form. Future work includes combining the controller developed here with other controllers developed to address other issues in web-winding systems (e.g., [1]) to yield an overall controller that can effectively handle air entrainment, lateral motion, reel eccentricities, and other issues.

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