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Exploiting Information Content in Relative Motion

Dhananjay Raghunathan and John Baillieul

***Abstract*—We present a formal protocol for communication based on modulating the relative motion between mobile agents. This type of communication is typically dependent on context in that a particular motion or gesture will indicate something related to the current activity. The focus of this article is on enabling such motion-based signaling between non-holonomic mobile agents moving in $\mathbb{R}^2$. A Control law that enables such signaling is presented and analyzed. Properties of signals that are amenable to such signaling are presented, as are error bounds to the sensing of such motion by the agent receiving the signal. We conclude with a presentation of potential applications and current technological challenges towards enabling these applications.**

I. Introduction

Gesturing is a common means for signaling for animals and humans alike. Ranging from explicit hand gestures in an effort to communicate with another individual across a noisy hall, to the more subtle yet effective facial expressions, gesturing is indeed an art form that most of us have learned to use effectively. There are three important aspects to any gesture: the ability to effect motion, the ability to perceive motion and, the ability to comprehend an observed motion. Mobile robots of today have the ability to move, and to sense motion. Further, most have on-board computers and hence, the ability to process sensed information. Thus, with appropriate control laws, mobile robots should be capable of motion-based communication. This article presents control laws that enable motion-based information exchange between mobile robots.

There is considerable advantage to motion-based signaling. One can avoid using wireless modes of signaling such as 802.11, and benefit from the stealth. This mode of signaling can also be used by formations of agents to signal between each other. Finally, by formalizing the information theory of such signaling, we may gain useful insight into the limits of our cognitive ability to communicate through gestures.

The actuation bandwidth of the electro-mechanical systems that propel and control the motion of autonomous vehicles tend to be in the range of tens of hertz, and this bandwidth limits the data rate. Thus there are limitations on the types of messages that can be transmitted by means of motions. The expectation is that the proposed mode of communication has applications in niche areas such as signaling through formations and augmenting communications through existing wireless networking technologies.

There is considerable interest today in the creating and maintaining rigid formations of unmanned ground and aerial vehicles (see for instance [**?**][**?**][**?**][**?**].) A stable control law for tracking a reference frame is presented in [**?**] based on global positioning information. [**?**] tackles a similar problem with a local coordinate frame. Trajectory tracking for non-holonomic vehicles has been studied as well (for instance [**?**][**?**][**?**][**?**].) Algorithms that enable wireless connectivity between mobile autonomous agents by exploiting their mobility have been studied as well (see for instance [**?**][**?**].) *The wireless connectivity of the agents enables information to be exchanged between agents over the wireless medium, but the motion of the agents in itself is not used to communicate information. The distinguishing feature of our work is that we use motion itself to communicate information between agents.* We extend the notion of static formations of robots, to formations where the agents follow a signaling pattern such that the relative motion they observe between each other contains useful information. The participating agents can be considered to be moving in *dynamic* formation. We present control laws by which signaling patterns can be achieved and observed by participating agents.

The sequel is organized as follows. We start out by describing the notion of signaling using relative motion and formulate the problem for non-holonomic robots moving in $\mathbb{R}^2$. We then discuss properties of signals that can be exchanged given the constraints of the robotic motion. A control law is presented and analyzed to achieve this signaling, and bounds on the errors are computed. We present properties of viable *codebooks* for signaling using relative motion. Simulations and one detailed protocol are presented. We conclude by presenting open challenges to bringing this idea into a full-fledged technology that can be exploited by mobile robots.

II. Signaling Using Relative Motion in $\mathbb{R}^2$

Figure **??** shows two robot agents moving in the plane, with R_2 executing a motion that conveys information to R_1. The information is encoded in a simple function, $r(s)$, and the information is exchanged by having R_1 measure the position (ideally $(s, r(s))$) of R_2 relative to its own position $(s, 0)$. Because R_1 and R_2 control their motions independently, each will have its own local coordinate s_1 and s_2 recording its location in the x-axis direction of the coordinate frame. In general it will not be possible to exactly maintain $s_1 \equiv s_2$ throughout the signaling maneuver, and part of the challenge in implementing this simple motion-based signaling protocol is to ensure that $|s_1 - s_2|$ remains small. We can suppose that the parameter s_1 is just the arclength along agent $R1$'s path along the x-axis. Assume R_1 moves with unit velocity, and hence $s_1(t) = t$. The agent R_2 traces the path $(s_2, r(s_2))$, with the goal of keeping $|s_2(t) - t|$ small. To the extent that

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The authors are with the Mechanical Engineering Department at Boston University, Boston MA, USA.({rdjay, johnb}@bu.edu)

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this regulation can be done exactly, we have the speed of R_2 expressed by $\sqrt{1+r'(s_2)^2}$. This provides the first of a number of constraints on this type of motion-based signaling—*the slope of the function $r(s)$ cannot be so large as to exceed the velocity limits of R_2.*

It is also the case that any mismatch between $s_1(t) = t$ and $s_2(t)$ will introduce the possibility of error in detecting the transmitted function. While R_2 transmits a message encoded as $(s_2, r(s_2))$, R_1 receives the message as $(t, g(t))$, where $g(t) = r(s_2(t))$. Hence, we shall want to choose encoding functions $r(\cdot)$ with the property that $|r(s) - r(s+\epsilon)|$ is uniformly small when ϵ is sufficiently small.

There will be several additional mild assumptions underlying our discussion. Most relative distance sensors (sonar, ladar, etc.) operate over prescribed ranges. Hence, we assume that there are parameters d_{min} and d_{max} such that $0 < d_{min} < r(s) < d_{max}$ over the range of interest $0 \leq s \leq L$. We also assume that there are no obstacles to the motion of the robot in the plane.

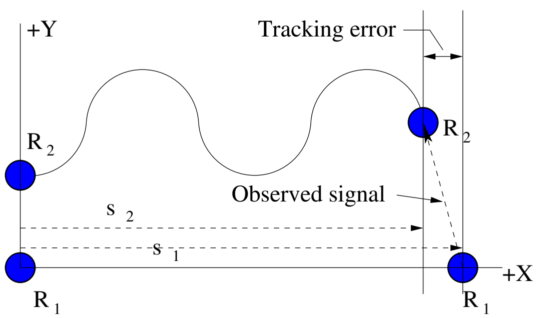

Fig. 1. Robot R_2 is signaling to robot R_1 by controlling its relative distance with R_1 in coordination with the path that R_1 is traversing.

We now present a mathematical model for relative motion signaling like the illustration of Figure **??**.

A. A Non-holonomic Robot Model

Figure **??** shows a non-holonomic vehicle in the plane, with a description of the coordinate system (x, y, θ). The wheels W_1 and W_2 are assumed not to have any slip with the ground, and hence, this robot is constrained to instantaneously move only in a direction perpendicular to the wheel axis. This non-holonomic constraint is expressed by

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

and the consequent equations of motion can be written as

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega$$

where (v, ω) represent, respectively, the velocity of the mid-point of the robot and the angular velocity of the robot. The pair (v, ω) serves as the control input to the robot. Such a vehicle is commonly called a *unicycle*. A *Dubins vehicle* is a unicycle that travels with a constant speed $v = v_c > 0$ and a constrained angular velocity ω satisfying $(|\omega|/v_c) < \kappa_{max}$. We define a ***Variable Speed Dubins Vehicle*** as a *unicycle*

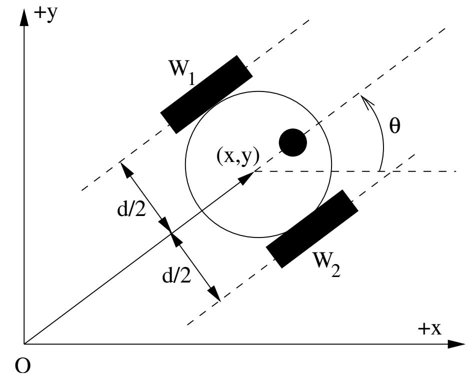

Fig. 2. A global coordinate system for a Dubins vehicle. The wheels W_1 and W_2 are separated by a distance d. The black dot represents the front of the robot, and the robot, as shown in the figure, will be moving further out in the first quadrant.

for which the control pair (v, ω) satisfy (a) $v > 0$, and, (b) a curvature constraint $(|\omega|/v) < \kappa_{max}$, $\kappa_{max} > 0$. It follows from this definition that a Dubins vehicle as well as a Variable Speed Dubins Vehicle have a minimum turn radius $1/\kappa_{max}$.

III. Relative Motion Signaling and Control

We start by analyzing the case of two Variable Speed Dubins Vehicles in the plane attempting to signal to each other. As has been shown in Figure **??**, we constrain the receiver robot R_1 to traverse a straight-line trajectory along the positive x-axis. It is possible for R_1 to follow a more general trajectory, but we shall restrict our consideration to the straight-line trajectory for this article. Since the planar curve that R_2 describes in the plane is sensed by R_1 with respect to R_1's own trajectory, it seems natural to parameterize the planar curve that R_2 wishes to signal by a variable that is representative of the *path* of R_1. We choose to use the *path length* s_1 of R_1 as the parameter and hence $\mathcal{C} = \mathcal{C}(s_1, \beta(s_1)) \in \mathbb{R}^2$. Let $r(s_1)$ represent the distance between R_1 and R_2 when R_1 has moved s_1 along its trajectory. Then, we can say that $(s_1, r(s_1))$ *represents the **observation model** of the observer R_1.*

We say that curve $\mathcal{C}$ is a ***feasible trajectory** for a Variable Speed Dubins Vehicle* if there exists a control law pair (u, ω) which can steer it along $\mathcal{C}$ with no error. We say that a finite extent curve $\mathcal{C}$ is ***observable under an observation model*** if one can reconstruct $\mathcal{C}$ perfectly after observing it using the observation model for a finite duration. Figure **??** illustrates curves that are and are not *observable* under an observation model $(s, r(s))$.

Let $\dot{x}_i = u_i\cos\theta_i, \dot{y}_i = u_i\sin\theta_i, \dot{\theta}_i = \omega_i,\ i = 1, 2$ represent the kinematics of R_1 and R_2 respectively. Since R_1 is traveling along a straight line, $\omega_1 = 0$. We can now state the following problem:

Problem 1. *Given*

1) *R_1 and R_2 are Variable Speed Dubins Vehicles,*
2) *$\omega_1 = 0$, and R_1 is moving along the +x-axis with $v_1 > 0$,*
3) *a planar curve $\mathcal{C}$, and,*

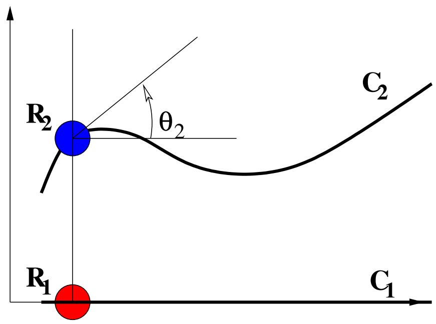

Fig. 3. Robot R_1 moves to the right along the x-axis. Robot R_2 moves along a curve, with the tangent to the curve C_2 subtending an angle θ_2 with the horizontal. This may also be interpreted as a relative angle of θ_2 with respect to the trajectory C_1 of R_1, for which $\theta_1(t) \equiv 0 \ \forall t \geq 0$. Figure **??** illustrates this for R_1 moving in a more general trajectory.

4) *observer* R_1 *with observation model* $(s_1, r(s_1)) \equiv (x_1, r(x_1))$,

find

1) *if* $\mathcal{C}$ *is a feasible trajectory for* R_2, *and if so,*
2) *if* $\mathcal{C}$ *is observable to* R_1, *and, if so,*
3) *control laws* (v_2, ω_2) *and* $(v_1, \omega_1 = 0)$ *such that the observer can faithfully register* $\mathcal{C}$.

Such control laws $R_2 : (v_2, \omega_2)$, $R_1 : (v_1, 0)$ represent a **protocol** for the signaling of the curve $\mathcal{C}$ by R_2 to R_1.

A. Properties and Requirements of $\mathcal{C}$

The requirements of Problem **??** make it necessary for $\mathcal{C}$ to satisfy a few properties. Clearly, for the observation model $(x_1, r(x_1))$ to be identical to $\mathcal{C}$, $\mathcal{C}$ should be representable in a parametric form $\mathcal{C} \sim (s, \beta(s))$ where $\beta(s)$ is a twice differentiable function. We thus require $\beta(s)$ to be a *function* of s. In order for such a $\mathcal{C}$ to be *feasible*, the curvature $\mathcal{K}(\mathcal{C})$ needs to be bounded by the largest curvature trajectory κ_{max} that a Variable Speed Dubins Vehicle can describe.

$$|\mathcal{K}(\mathcal{C}(s, \beta(s)))| = \frac{|\beta''(s)|}{(1+\beta'^2)^{3/2}} \leq \frac{2}{d} \qquad (1)$$

where d is the distance between the wheels (Figure **??**.) Figure **??** illustrates some of these constraints. As the relative positions of the two vehicles change over time, we are interested in how the heading of vehicle R_2 is changing with respect to the heading of R_1. Figure **??** illustrates this when R_1 moves in a straight line along the x-axis. Consider a curve parameterized as $\mathcal{C} \equiv \mathcal{C}(s, \beta(s))$, with $\beta(s)$ being twice differentiable. We have $|\beta'(s)| < +\infty$, which in turn implies the absolute slope $|\tan\theta_2| = |\beta'(s)| < +\infty \implies \theta_2 \in [0, \pi/2)$ (see Figure **??**.) Figure **??** shows signaling by R_2 to R_1 when R_1 moves along a more general trajectory. The trajectory slope that R_2 needs to over lay on R_1 in order to transmit a message is $\theta_2 - \theta_1$, as shown in the figure. For the case of R_1 moving in a straight line, $\theta_1(t) \equiv 0$. In general, our interest is in the *relative slope*; since this article deals with the case that $\theta_1(t) \equiv 0$, we will interchangeably use *slope* and *relative slope*.

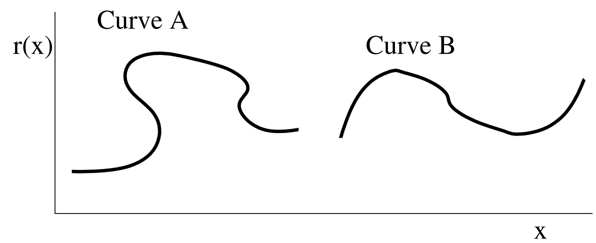

Fig. 4. Curve A is not representable as a single valued function $r(x)$ and hence, is not observable under the prescribed observation model. Curve B is a function and hence observable. The feasibility needs to be independently tested, based on the maximal curvature for this curve.

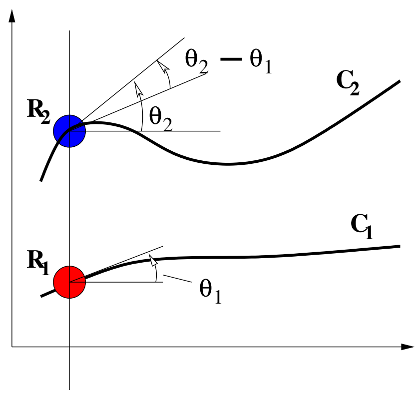

Fig. 5. C_i represents the trajectory of R_i. θ_i represents the tangent angle to the trajectories C_i respectively at the corresponding R_i positions.

Before we conclude this section, we compute the time derivative for a parameterized curve $\mathcal{C}(s, \beta(s))$ with respect to the corresponding tangent angle θ_2 in Figure **??**:

$$\begin{aligned} \tan\theta_2 &= \frac{d\beta(s)}{d(s)} = \frac{d\beta}{ds} \\ \Lambda(s) &= \frac{\beta''(s)}{(1+\beta'^2(s))^2} \\ \frac{d\theta_2}{dt} &= \frac{ds}{dt}\Lambda(s) \end{aligned} \qquad (2)$$

We thus have a relation between the curvature of the planar curve $\mathcal{C}$ and a desired control on the angular velocity of the Variable Speed Dubins Vehicle in order to achieve trajectory tracking of this planar curve. We will use this result for signaling.

B. Control for Relative Motion Signaling

We now present a control law for signaling when R_1 follows a constant velocity zero curvature motion. The equations of motion of R_1 are augmented with an integrator for the path length s_1. The equations motion motion of R_2 are augmented with an integrator for the *projected* path length s_2, projected along the trajectory of R_1. From Figure **??**, the velocity of R_2 along s_2 is $v_2 \cos\theta_2$. R_1 moves along the positive x-axis with $(v_1, \omega_1) = (v_c > 0, 0), \theta_1(0) = 0 \implies \theta_1(t) \equiv 0$. We use a simple nonlinear proportional control law to regulate s_2

relative to s_1 (**??**), and a regulator for the angular velocity based on (**??**).

$$\dot{x_1} = v_c \quad , \quad \dot{x_2} = v_2 \cos\theta_2 \tag{3}$$
$$y_1(t) \equiv 0 \quad , \quad \dot{y_2} = v_2 \sin\theta_2 \tag{4}$$
$$\theta_1(t) \equiv 0 \quad , \quad \dot{\theta_2} = \omega_2 \tag{5}$$
$$\dot{s_1} = v_1 = v_c \quad , \quad \dot{s_2} = v_2 \cos\theta_2 \tag{6}$$
$$(v_2, \omega_2) = (K_p(s_1 - s_2), v_2\Lambda(s_2)) \tag{7}$$

Theorem **??** derives a bound on the projected tracking error $e_{trk}(t) = s_1(t) - s_2(t)$ (Figure **??**.) Since the curves we are interested in are parameterized as $\mathcal{C}(s, \beta(s))$, with $\beta(s)$ being twice differentiable, we have, from Section **??**, that $|\theta_2| \in [0, \pi/2)$. We denote the upper bound on $|\theta_2|$ as $\theta_{max} > 0$.

Theorem III.1. *Consider the system* (**??**)-(**??**). *Let* $\epsilon > 0, K_p > 0$. *Let* θ_{max} *represent an upper bound on the absolute slope of a feasible and observable curve* $\mathcal{C}(s, \beta(s))$. *Then, for initial configurations of the two vehicles sufficiently close to* $(0,0)$ *(for* R_1*) and* $(0, \beta(0))$ *(for* R_2*), the tracking error will remain bounded and satisfy*

$$|e_{trk}(t)| < \frac{v_1}{K_p \cos\theta_{max}} + \epsilon \; \forall \; t \geq 0$$

Proof: Define a Lyapunov function $V = e_{trk}^2/2$ so that $\dot{V} = e_{trk}\dot{e}_{trk} = -K_p e_{trk}^2 \cos\theta_2 + e_{trk}v_1$. Clearly, $\dot{V} > 0$ for $|e_{trk}| < v_1/(K_p \cos\theta_2) \leq v_1/(K_p \cos\theta_{max})$, and $\dot{V} \leq 0$ otherwise. Thus the tracking error stays bounded. ∎

We now define as **signaling error** e_{sig} the error between the *desired* signal $(x_1, \beta(x_1))$ and the *observed* signal.

$$e_{sig} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - \beta(s_1) \tag{8}$$

We now show that that for bounded tracking error, the signaling error is *uniformly bounded*. In what follows, $d_{min} > 0$ represents the a minimum prescribed separation between two signaling robots (Section **??**.)

Theorem III.2. *Consider system* (**??**)-(**??**)*, with* R_2 *tracking a feasible curve* $\mathcal{C}(s, \beta(s))$ *that has maximal relative slope* $\theta_{max} \in [0, \pi/2)$. *If* $\epsilon > 0, \delta_1 > 0, K_p > 0$*, and,*

$$\frac{v_1}{K_p \cos\theta_{max}} + \epsilon \leq e_{trk}(0) \leq \delta_1 < d_{min},$$

then $e_{sig}(t)$ *is bounded for all* $t \geq 0$.

Proof: Since $\|x_1 - x_2\| = \|s_1 - s_2\| = |e_{trk}|$, we use Theorem **??** to write

$$\begin{aligned}
|e_{sig}| &= \left|\sqrt{e_{trk}^2 + \beta(x_2)^2} - \beta(x_1)\right| \\
&\leq \left|\sqrt{\delta_1^2 + \beta(x_2)^2} - \beta(x_1)\right| \\
&= \left|\beta(x_2)\sqrt{1 + \frac{\delta_1^2}{\beta^2(x_2)}} - \beta(x_2 + \delta_1)\right| \\
&= \left|\beta(x_2)\left(1 + O\left(\frac{\delta_1^2}{\beta^2(x_2)}\right)\right) - \beta(x_2 + \delta_1)\right| \\
&= \left|\beta(x_2) - \beta(x_2 + \delta_1) + \beta(x_2) O\left(\frac{\delta_1^2}{\beta^2(x_2)}\right)\right| \\
&\leq |\beta(x_2) - \beta(x_2 + \delta_1)| + \\
&\quad \max_{x_2} |\beta(x_2)| \cdot \left| O\left(\frac{\delta_1^2}{(\min_{x_2} \beta(x_2))^2}\right)\right| \\
&= |\beta(x_2) - \beta(x_2 + \delta_1)| + \\
&\quad \max_{x_2} |\beta(x_2)| \cdot \left| O\left(\frac{\delta_1^2}{d_{min}^2}\right)\right|
\end{aligned}$$

The last expression follows from the requirement that R_1 and R_2 are separated by a minimum approach distance of d_{min}. Also, $\max_{x2} \beta(x_2) < +\infty$ as $\mathcal{C}$ is twice differentiable. For the same reason, $\beta(s)$ is *Lipschitz* and hence

$$|\beta(x_2) - \beta(x_2 + \delta_1)| \leq \mathcal{L}\delta_1$$

where $\mathcal{L}$ is the *Lipschitz constant*. Hence we have

$$|e_{sig}| \leq \mathcal{L}\delta_1 + \max_{x2} |\beta(x_2)| \cdot \left| O\left(\frac{\delta_1^2}{d_{min}^2}\right)\right| < +\infty$$

for $0 < \delta_1 < d_{min}$. ∎

The error bounds derived so far are helpful in encoding and decoding messages that R_2 can transmit to R_1. One can thus derive a codebook of messages for communication, and this is discussed in the following subsection.

C. Developing A Codebook for Signaling

We have already presented a few constraints on the nature of the signals that can be transmitted: constraints from Section **??**, and the effect of the sensor working range (d_{min}, d_{max}) and the separation of R_1 and R_2. Sensor noise and the error in trajectory detection due to tracking error can be used to determine a codebook of symbols that can be transmitted with zero error probability. If some error probability can be tolerated, clearly the set will be larger.

For signals transmitted for a finite duration, one can define a measure for the *distance* between two curves. This *distance* distinguishes one signal (message) in a codebook from another. Let $\mathcal{B}$ represent a codebook of signals. The L_2 norm is one possible choice of metric. Given two elements $(s, \beta_1(s)), (s, \beta_2(s)) \in \mathcal{B}$ defined on a common interval $s \in [0, L]$, one can define the distance between the two elements as

$$\begin{aligned}
\mathcal{D}((s, \beta_1(s)), (s, \beta_2(s))) &\equiv \|\beta_1(s) - \beta_2(s)\|_2 \\
&= \sqrt{\int_0^L |\beta_1(s) - \beta_2(s)|^2 \, ds}
\end{aligned}$$

Based on the proof of Theorem **??**, we define

$$\mathcal{L}_{max} = \max_{(s,\beta(s))\in\mathcal{B}} |\mathcal{L}| \tag{9}$$

which represents the largest slope magnitude of all possible slopes of signals in $\mathcal{B}$.

Owing to the errors in tracking a curve in $\mathcal{B}$, one needs to determine a sufficient spread for the signals in $\mathcal{B}$ so that they are distinguishable by R_1, even with perfect observation (i.e. negligible observation sensor noise.) The following represents a conservative lower bound between the distance $\mathcal{D}(\cdot,\cdot)$ between two signals $\mathcal{C}(s,\beta_1(s))$ and $\mathcal{C}(s,\beta_2(s))$:

$$\begin{aligned}\|\beta_1(s)-\beta_2(s)\|_2 &\geq \left\|\max_{s\in[0,L]} |e_{sig}|\right\|_2 = \left(\max_{s\in[0,L]} |e_{sig}|\right) L^{\frac{1}{2}} \\ &= \mathcal{L}_{max}\delta_1 L^{\frac{1}{2}} \text{ [from Theorem ??]}\end{aligned}$$

Thus, if for all $(s,\beta_i(s)),(s,\beta_j(s))\in\mathcal{B}, i\neq j$,

$$\mathcal{D}((s,\beta_i(s)),(s,\beta_j(s))) \geq \mathcal{L}_{max}\delta_1 L^{\frac{1}{2}}, \tag{10}$$

*then the receiver can identify a signal in the codebook $\mathcal{B}$ to within $O(\delta_1^2/d_{min}^2)$ (Theorem **??**) under perfect observation and imperfect but bounded tracking control.*

Example III.1. *Consider a codebook $\mathcal{B}$ composed of signals from the sequence of curves $(s_n,\beta_n(s_n)) \equiv \{(x, a + b\cos 2\pi nx)\}, x\in[0,1), n\in\mathbb{Z}^+$, and $a,b\in\mathbb{R}^+$. Clearly $\beta_n(s_n)$ is twice differentiable. The elements of $\mathcal{B}$ need to be feasible curves for R_2 and observable curves by R_1. In order to ensure that (a) R_2 does not bump into R_1, and (b) the $R_1 - R_2$ separation is always within an appropriate sensor range for R_1, we require elements of $\mathcal{B}$ to satisfy $d_{max}\geq a\geq d_{min}, b=\min(a-d_{min}, d_{max}-a)$. The curvature of the set of curves in $\mathcal{B}$ is given by*

$$\mathcal{K}((s,\cos(2\pi nx))) = -\frac{4\pi^2 n^2 b\cos(2\pi nx)}{(1+4\pi^2n^2b^2\sin^2(2\pi nx))}.$$

$|\mathcal{K}((s,\cos(2\pi nx)))|$ is maximized for $x = r/(2n), r\in\mathbb{Z}, n\neq 0$, and the maximal value is $4\pi^2n^2b$. Since we need to satisfy the feasibility condition for the Variable Speed Dubins Vehicle to be able to track signals in $\mathcal{B}$ without error, the maximum curvature is bounded by $2/d$ (d is the wheel separation), and hence we have

$$4\pi^2n^2b \leq \frac{2}{d} \implies n^2 \leq \frac{1}{2\pi^2 bd} \implies n_{max} = \left\lfloor\sqrt{\frac{1}{2\pi^2 bd}}\right\rfloor \tag{11}$$

*Equation (**??**) gives us a bound n_{max} on n based on the feasibility for a Variable Speed Dubins Vehicle to track a desired curve from $\mathcal{B}$. We now investigate the observability of the curves in $\mathcal{B}$. For any two elements $(x,\beta_p(x)),(x,\beta_q(x))\in\mathcal{B}, p\neq q, p\neq 0, q\neq 0$, we have*

$$\mathcal{D}((x,\beta_p(x)),(x,\beta_q(x))) = \|\cos(2\pi px)-\cos(2\pi qx)\|_2 = 1 \tag{12}$$

*The slope for the element q in $\mathcal{B}$ is $d\beta_q(s)/ds = -b2\pi q\sin(2\pi qx)$, and has a maximal value of $2b\pi q$. From Equation (**??**), we have $\mathcal{L}_{max} = 2b\pi n_{max}$. In Equation (**??**), we need to determine a suitable value for δ_1. This can be found using Theorem **??** and n_{max}. We have θ_{max} representing the maximal slope angle for a signal, which in our case is $\tan^{-1}(\max_q \max_s \beta_q'(s)) = \tan^{-1}(2b\pi n_{max})$.*

$$\delta_1 > \frac{v_1}{K_p\cos(\tan^{-1}(2b\pi n_{max}))}$$

*Hence, from Equation (**??**), and the result (**??**), we have*

$$\begin{aligned}\mathcal{D}((x,\beta_p(x)),(x,\beta_q(x))) &= 1 \geq \mathcal{L}_{max}\delta_1 \\ \implies K_p &\geq \frac{2b\pi n_{max}v_1}{\cos(\tan^{-1}(2b\pi n_{max}))}\end{aligned}$$

We thus have determined a feasible and observable signal set $\mathcal{B}$, and the appropriate control law. □

More work needs to be done towards developing a codebook of messages that can be sent and understanding the effect of sensor observation noise on the reliability of observation. We are currently pursuing these notions as part of our current research.

D. Simulations

Figure **??** shows a simulation run of the signaling problem with R_1 moving along a straight line trajectory. The corresponding parameters are indicated in the plots. For these plots, $\beta(s)$ is chosen to be a Fourier cosine series. The question of a useful basis for the set of functions $\beta(s)$ is important. A Fourier basis like the one used can capture a wide range of signals. From our experience in the lab with approximations to Variable Speed Dubins Vehicles indicates that setting constant velocity profiles for each wheel of the Variable Speed Dubins Vehicle, for a duration of time, yields better odometry than an acceleration profile for the wheels. This in turn means that it is preferable for the vehicles to track trajectories represented by a sequence of constant curvature differentiable curves (essentially a sequence of circular arcs and line segments.) Developing an appropriate basis for such curves is one more direction of future research.

E. A Protocol for Signaling

For such signaling to work, clearly R_1 needs to know apriori that R_2 intends to signal a planar curve to it. There are various ways in which this can be done. One way is to simply send wireless data that sets up the trajectory of R_1 as well as the details regarding the starting of the transmission, the duration of the transmission and the termination of the transmission. But it is possible to complete this entire sequence purely using motion. The following is one such purely motion based protocol that can achieve this task:

1) **Initiation** In this phase, R_2, from an arbitrary location in the plane, approaches R_1 and attempts to maintain a constant distance and bearing from R_1. R_1 observes this motion of R_2 for a certain threshold of time, and *assumes* that R_2 wishes to signal information to R_1 and prepares for the reception in the next phase.
2) **Synchronization** R_1 has recognized that R_2 wishes to signal information to it. R_1 cooperates by choosing to

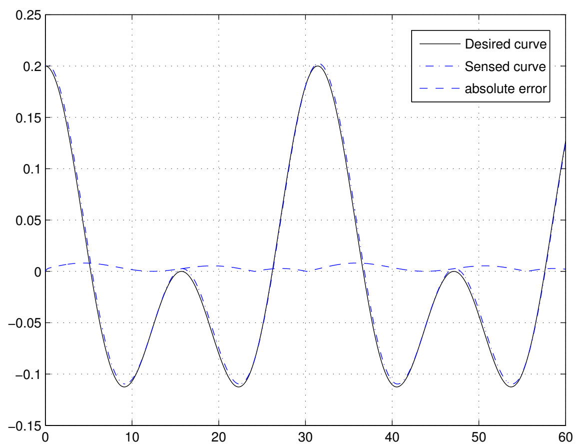

Fig. 6. Simulation with $v_1 = 0.1m/s, \omega_1 = 0.0$, $K_p = 1.0$, $\beta(s) = 0.1\cos(0.2s) + 0.1\cos(0.4s)$. The plot shows the desired curve, as parameterized by the path of R_1 (along the positive x-axis) and the actual sensed curve by R_1. It also shows the absolute error between the desired and actual readings.

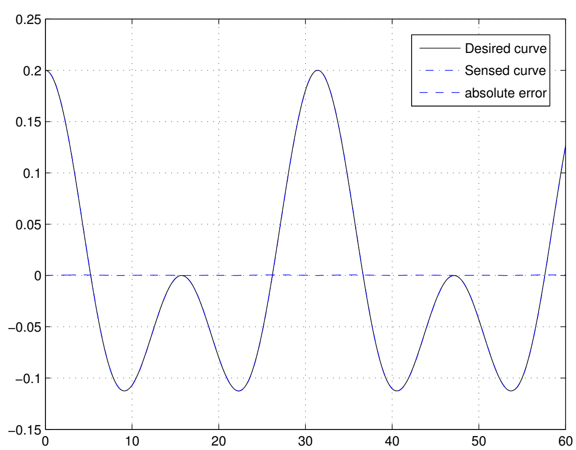

Fig. 7. Simulation with $v_1 = 0.1m/s, \omega_1 = 0.0$, $K_p = 10.0$, $\beta(s) = 0.1\cos(0.2s) + 0.1\cos(0.4s)$. The plot shows the desired curve, as parameterized by the path of R_1 (along the positive x-axis) and the actual sensed curve by R_1. It also shows the absolute error between the desired and actual readings. There exists a small error all the way through; this is better seen in Figure **??**.

travel in a trajectory that is "easy" to signal to (for instance, the straight line trajectory as discussed in the previous section.)

3) **Signaling** R_2 describes the planar curve using the control law described in the previous section(or a similar control law.)
4) **Termination** Once R_2 has completed signaling the curve, it continues to move parallel to R_1 to signal the completion of the transmission. This motion continues for a predetermined amount of time, beyond which the transmission is deemed complete.

Such a protocol becomes necessary for the actual implementation of this means for signaling.

IV. Conclusions

We have presented a novel method of signaling information between mobile autonomous agents. There are many open questions in continuing this line of research. Control laws based on the local information need to be developed for such signaling to be implementable on robots in a distributed and localized fashion. A formal protocol like the one described in Section **??** needs to be put in place in order for agents to successfully perform this signaling. We have presented a method for developing a codebook for motion-based signaling. Work needs to be done towards accounting for noisy observations by the sensors. The maximum achievable data rate using motion-based signaling is an open question. We note from our laboratory experience with mobile robots that closely approximate Variable Speed Dubins Vehicles that the most accurate odometry seems to be achieved when the trajectory to be tracked is comprised of a sequence of constant curvature segments rather than a trajectory with continuously varying curvature. This implies that signaling a curve composed of piecewise constant curvature segments may yield better signal overlay. One might thus consider a representation of the signal using a basis different from the Fourier basis. Finally, the question of using this mode of signaling for Unmanned Aerial Vehicles is still open.

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