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Decentralized Overlapping Tracking Control of a Formation of Autonomous Unmanned Vehicles

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Abstract— **In this paper a new methodology for decentralized overlapping tracking control of formations of autonomous unmanned vehicles based on the expansion/contracion paradigm is proposed. The methodology is based on a specific linear formation model. Decentralized controllers for the extracted subsystems are contracted to the original space after convenient modifications. It is proved that the overall closed loop system is stable if the formation graph has a directed spanning tree. An extension to the case of dynamic output feedback control law based on decentralized observers is also proposed. Experimental results give an illustration of the performance of the proposed controller when the local design is based on the LQ methodology.**

I. INTRODUCTION

During the last decade there has been an increasing interest for conducting research in analysis and control of formations of Autonomous Unmanned Vehicles (AUVs). This interest has been highly motivated by numerous applications such as distributed sensing and transportation. One of the main challenges in the analysis is that the problems considering multi-agent systems are still very much open and very difficult to solve, in general. Recently, important results in this area have been presented in various publications (*e.g.*, see [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12] and references reported therein).

In this paper we present a novel design methodology for decentralized overlapping tracking control law of planar formations based on the expansion/contraction paradigm and the inclusion principle (e.g. [15], [6]). In Section II, a specific formation state-space model is formulated on the basis of information structure constraints, using the initial results presented in [13], [6]; this approach enables treating formation as an interconnection of subsystems formally attached to all the vehicles. Section III deals with a general control design methodology for a formation to track given references of velocity and relative distances of the vehicles with respect to their neighbors, which allows local application of diverse controller design methodologies (LQ or LMI design). As the resulting overall feedback and feedforward matrix gains do not allow proper contraction to the original system space for implementation, a special attention is paid to the contractibility issue. It is shown that suitably modified feedback and feedforward gains can be constructed. Section IV is devoted to the stability issue. It is proved, starting from a digraph representation of the information flow, that asymptotic convergence of all the states to the the desired references can be achieved provided the underlying digraph has a spanning tree. This result, derived directly on the basis of the proposed formation state model and the expansion/contraction methodology, is in accordance with the recent results related to the second order consensus schemes [8], [9], [14]. In Section V a dynamic output controller with local observers is proposed in the case when the velocities of the neighboring vehicles are not known [15], [13], [16]. Section VI contains some simulation results obtained by using a specific design methodology based on decentralized LQ optimization, formulated as a generalization of the methodology presented in [13].

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II. FORMATION MODEL

Consider a set of N vehicles moving in a plane, in which the i-th vehicle is represented by the linear double integrator model

$$\dot{z}_i = A_v z_i + B_v u_i = \begin{bmatrix} 0_{2\times 2} & I_2 \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix} z_i + \begin{bmatrix} 0_{2\times 2} \\ I_2 \end{bmatrix} u_i, \quad (1)$$

($i = 1, \ldots, N$), where $z_i \in \mathcal{R}^4$ and $u_i \in \mathcal{R}^2$ are the state and the control input vectors, respectively ($0_{m\times n}$ denotes the $m \times n$ zero matrix, and I_n the $n \times n$ identity matrix). The state z_i and the input u_i are related to the physical state and input through standard transformations, *e.g.* [6]. We shall assume that the i-th vehicle is provided with the information about the set of neighboring vehicles, indices of which define the set of sensing indices $S_i = \{s_1^i, \ldots, s_{m_i}^i\}$; this information includes velocities and relative distances of the neighboring vehicles with respect to the i-th vehicle, the velocity of the vehicle itself, as well as the relative distance references and the velocity reference (which is supposed to be the same for all the vehicles). Decomposing z_i as $z_i = \begin{bmatrix} z_i'^T & z_i''^T \end{bmatrix}^T$, where $z_i' = \begin{bmatrix} z_{i,1}' & z_{i,2}' \end{bmatrix}^T = \begin{bmatrix} z_{i,1} & z_{i,2} \end{bmatrix}^T$ and $z_i'' = \begin{bmatrix} z_{i,1}'' & z_{i,2}'' \end{bmatrix}^T = \begin{bmatrix} z_{i,3} & z_{i,4} \end{bmatrix}^T$, we introduce the following change of variables

$$x_i' = \sum_{j\in S_i} \alpha_j^i z_j' - z_i', \qquad x_i'' = z_i'', \quad (2)$$

where $\alpha_j^i \geq 0$ and $\sum_{j\in S_i} \alpha_j^i = 1$; $x_i' = \begin{bmatrix} x_{i,1}' & x_{i,2}' \end{bmatrix}^T$ represents the distance between the i-th vehicle and a "centroid" of the set of vehicles selected by S_i, with *a priori* selected weights α_j^i (in the case of formation leaders when $S_i = \emptyset$, $x_i' = -z_i'$). Therefore, one obtains

$$\dot{x}_i' = \sum_{j\in S_i} \alpha_j^i z_j'' - z_i'' = \sum_{j\in S_i} \alpha_j^i x_j'' - x_i'', \qquad \dot{x}_i'' = u_i, \quad (3)$$

$i = 1,\ldots,N$, using the fact that $z_i'' = \dot{z}_i'$, so that $x_i'' = \left[x_{i,1}'' \; x_{i,2}'' \right]^T = z_i'' = \left[z_{i,1}'' \; z_{i,2}'' \right]^T = \dot{z}_i' = \left[\dot{z}_{i,1}' \; \dot{z}_{i,2}' \right]^T$ [6].

Defining the formation state and control input vectors x and u as concatenations of all the vehicle state and control input vectors $x_i = \left[x_i'^T \; x_i''^T \right]^T$ and u_i, $i = 1,\ldots,N$, we obtain the following formation state model

$$\mathbf{S}: \quad \dot{x} = Ax + Bu = [(G-I)\otimes A_v]x + [I \otimes B_v]u, \quad (4)$$

where $\otimes$ denotes the Kronecker's product. We shall assume that each vehicle has the information about the reference state trajectories $r_i = \left[r_i'^T \; r_i''^T \right]^T$, so that the control task to be considered is the task of tracking the desired references.

The above described set of N vehicles with their sensing indices and the corresponding weights can be considered as a directed weighted graph $\mathcal{G}$ in which each vertex represents a vehicle, and an arc with the weight α_j^i leads from vertex j to vertex i if $j \in S_i$. Consequently, the weighted adjacency matrix $G = [G_{ij}]$ is an $N \times N$ square matrix defined by $G_{ij} = \alpha_j^i$ for $j \in S_i$, and $G_{ij} = 0$ otherwise. We shall define the weighted Laplacian of the graph as $L = [L_{ij}]$, $L_{ij} = G_{ij}$, $i \neq j$, $L_{ii} = -\sum_j \alpha_j^i$ (*e.g.*, see [2]).

III. Decentralized Tracking Design by Expansion/Contraction

The structure of the formulated model (4) indicates that it is possible to consider the formation as an interconnection of N *overlapping subsystems*. Extending the reasoning successfully applied within the platooning problem (*e.g.*, [17], [13], [6]), we shall assign to the i-th vehicle in a formation a formally defined *subsystem* $\tilde{\mathbf{S}}_i$ with the state vector containing the vehicle state coordinates x_i' and x_i'', together with the second components x_j'' (velocity components) of the state vectors of all the vehicles sensed by the i-th vehicle, and the input vector $\tilde{u}_i$ containing the vehicle control vector u_i, together with the control vectors u_j associated with all the vehicles sensed by the i-the vehicle, *i.e.* $\tilde{x}_i = \left[x_{s_1^i}''^T \cdots x_{s_{m_i}^i}''^T \; x_i'^T \; x_i''^T \right]^T$ and $\tilde{u}_i = \left[u_{s_1^i}^T \cdots u_{s_{m_i}^i}^T \; u_i^T \right]^T$. Consequently, the subsystem models are:

$$\tilde{\mathbf{S}}_i: \quad \dot{\tilde{x}}_i = \tilde{A}_i \tilde{x}_i + \tilde{B}_i \tilde{u}_i, \quad (5)$$

where $\tilde{A}_i = \left[\begin{array}{c:c} 0_{2m_i\times 2m_i} & 0_{2m_i\times 4} \\ \hdashline A_\alpha^i & -A_v \end{array} \right]$, $A_\alpha^i = \left[\begin{array}{c} \alpha_{s_1^i} I_2 \vdots \cdots \vdots \alpha_{s_{m_i}^i} I_2 \\ 0_{2\times 2m_i} \end{array} \right]$ and $\tilde{B}_i = \left[\begin{array}{c:c} I_{2m_i\times 2m_i} & 0_{2m_i\times 2} \\ \hdashline 0_{4\times 2m_i} & B_v \end{array} \right]$.

We define the expansion $\tilde{\mathbf{S}}$ of $\mathbf{S}$ as a system whose state and input vectors are defined as concatenations of the subsystem state and input vectors, that is, $\tilde{x} = \left[\tilde{x}_1^T \cdots \tilde{x}_N^T \right]^T$ and $\tilde{u} = \left[\tilde{u}_1^T \cdots \tilde{u}_N^T \right]^T$. Consequently,

$$\tilde{\mathbf{S}}: \quad \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}, \quad (6)$$

where $\tilde{A} = \mathrm{diag}\{\tilde{A}_1,\ldots,\tilde{A}_N\}$ and $\tilde{B} = \mathrm{diag}\{\tilde{B}_1,\ldots,\tilde{B}_N\}$.

The expanded state and control vectors $\tilde{x}$ and $\tilde{u}$ can be represented as full rank linear transformations of the original state and control vectors x and u, *i.e.* $\tilde{x} = Vx$ and $\tilde{u} = Ru$, where $V^T = \left[V_1^T \vdots \cdots \vdots V_N^T \right]^T$ and $R^T = \left[R_1^T \vdots \cdots \vdots R_N^T \right]^T$, with $V_i = \left[\begin{array}{c} V_i' \\ \hdashline V_i'' \end{array} \right]$ and $R_i = \left[\begin{array}{c} R_i' \\ \hdashline R_i'' \end{array} \right]$, where V_i' is an $m_i \times 2N$ (2×2)-block matrix containing I_2 in j-th row at the column index $2s_j^i$, $j = 1,\ldots,m_i$ and zeros elsewhere, V_i'' a $2\times 2N$ (2×2)-block matrix containing I_2 at the $(2i-1)$-st place in the first tow and at the $2i$-th place in the second row, R_i' is an $m_i \times N$ block matrix containing I_2 in j-th row at the column index s_j^i, $j = 1,\ldots,m_i$ and zeros elsewhere and R_i'' a $1 \times N$ block matrix containing I_2 at the i-th place.

It is not difficult to verify on the basis of the structure of $\mathbf{S}$, $\tilde{\mathbf{S}}$, V and R, that $\mathbf{S}$ and $\tilde{\mathbf{S}}$ satisfy, in general, the following conditions:

$$\tilde{A}V = VA, \quad \tilde{B}R = VB. \quad (7)$$

According to the inclusion principle, the original model $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$ (see *e.g.* [18], [19], [16], [20], [15] for basic results related to the inclusion principle). Consequently, stability of $\tilde{\mathbf{S}}$ implies stability of $\mathbf{S}$.

Once $\tilde{\mathbf{S}}$ is defined and the subsystems $\tilde{\mathbf{S}}_i$ extracted, the next task is to design the local control laws for the subsystems. If $\tilde{r}_i(t)$ represents a given reference signal for the i-th subsystem (the desired state trajectory of $\tilde{\mathbf{S}}_i$), then we have to determine pairs of constant feedback and feedforward matrices $(\tilde{K}^i, \tilde{M}^i)$ in the local tracking control laws for (6)

$$\tilde{\mathbf{F}}_i: \quad \tilde{u}_i = \tilde{K}^i \tilde{x}_i + \tilde{M}^i \tilde{r}_i, \quad (8)$$

$i = 1,\ldots,N$. Notice that the references for x_i', denoted as r_i^d, $i = 1,\ldots,N$, are related to the set of references for individual inter-vehicle distances with respect to the sensed vehicles, denoted as $r_{i-s_j^i}^d$, simply by $r_i^d = \sum_{j=1}^{m_i} \alpha_{s_j^i}^i r_{i-s_j^i}^d$.

In the case when $S_i = \emptyset$ (formation leaders), $\tilde{u}_i = u_i$, and we have only the velocity feedback, so that $\tilde{K}^i = \left[0 \; K^{Li} \right]$ and $\tilde{M}^i = \left[0 \; M^{Li} \right]$, where K^{Li} and M^{Li} are 2×2 matrices.

When $S_i = \{s_1^i,\ldots,s_{m_i}^i\} \neq \emptyset$, we assume that the control signals are $u_j = \hat{K}_j^i x_j'' + \hat{M}_j r^v$ for all $j \in S_i$, where r^v is the velocity reference; the design of $\hat{K}_j^i$ and $\hat{M}_j^i$ can, in principle, be done as in the case of the vehicles with $S_i = \emptyset$. However, the control vector u_i is obtained using all the measurements available in $\tilde{\mathbf{S}}_i$, *i.e.*, $u_i = \bar{K}^i \tilde{x}_i + \bar{M}^i \tilde{r}_i$, where both $\bar{K}^i$ and $\bar{M}^i$ can be decomposed as $\bar{K}^i = \left[\bar{K}_1^i \vdots \cdots \vdots \bar{K}_{m_i}^i \vdots \bar{K}_{m_i+1}^i \vdots \bar{K}_{m_i+2}^i \right]$ and

$\bar{M}^i = \left[\bar{M}_1^i \vdots \cdots \vdots \bar{M}_{m_i}^i \vdots \bar{M}_{m_i+1}^i \vdots \bar{M}_{m_i+2}^i \right]$, having in mind the structure of $\tilde{x}_i$ (and $\tilde{r}_i$).

Therefore, the tracking control law for $\tilde{\mathbf{S}}_i$ given by (8) is characterized by matrices $\tilde{K}^i = \begin{bmatrix} \hat{K}^i & 0_{2m_i \times 4} \\ \hline \multicolumn{2}{c}{\bar{K}^i} \end{bmatrix}$ and $\tilde{M}^i = \begin{bmatrix} \hat{M}^i & 0_{2m_i \times 4} \\ \hline \multicolumn{2}{c}{\bar{M}^i} \end{bmatrix}$, where $\hat{K}^i = \mathrm{diag}\{\hat{K}^i_{s_1^i}, \ldots, \hat{K}^i_{s^i_{m_i}}\}$, and $\hat{M}^i = \mathrm{diag}\{\hat{M}^i_{s_1^i}, \ldots, \hat{M}^i_{s^i_{m_i}}\}$; the structure of $\tilde{K}^i$ and $\tilde{M}^i$ reflects the fact that the i-th vehicle senses the vehicles selected by S_i.

The overall control law $\tilde{\mathbf{F}}$ for the whole expanded system $\tilde{\mathbf{S}}$ is characterized by the pair $(\tilde{K}, \tilde{M})$, where $\tilde{K} = \mathrm{diag}\{\tilde{K}^1, \ldots, \tilde{K}^N\}$ and $\tilde{M} = \mathrm{diag}\{\tilde{M}^1, \ldots, \tilde{M}^N\}$, so that

$$\tilde{\mathbf{F}}: \quad \tilde{u} = \tilde{K}\tilde{x} + \tilde{M}\tilde{r}, \tag{9}$$

where $\tilde{r}$ is the desired trajectory of $\tilde{x}$.

The final step in the formation control design is the contraction of the obtained tracking controller for the expansion $\tilde{\mathbf{S}}$ to the controller for the original system $\mathbf{S}$, given by

$$\mathbf{F}: \quad u = Kx + Mr, \tag{10}$$

where r is the desired trajectory of x ($\tilde{r} = Vr$). The contractibility conditions given by

$$RK = \tilde{K}V, \quad RM = \tilde{M}V, \tag{11}$$

ensure that the closed-loop system $(\mathbf{S}, \mathbf{F})$ represents a restriction of the closed-loop system $(\tilde{\mathbf{S}}, \tilde{\mathbf{F}})$ (for more details on contractibility, see [21]). However, relations (11) do not have any solutions for K and M in the case when $\tilde{K}$ and $\tilde{M}$ are in the form of block diagonal matrices [16], [18], [22], [15].

One way to overcome this problem is to suitably modify both $\tilde{K}$ and $\tilde{M}$ in such a way as to achieve contractibility [16], [6]. We define $\tilde{K}_m$ (or $\tilde{M}_m$) by $\tilde{K}_m = \tilde{K}_{m1} + \tilde{K}_{m2}$, where $\tilde{K}_{m1} = RR^T\tilde{K}$, while $\tilde{K}_{m2}$ is constructed in such a way as to reduce the number of off-block-diagonal terms in $\tilde{K}_{m1}$, and to satisfy, at the same time, the restriction condition $\tilde{K}_{m1}V = 0$. More specifically, in order to construct the l-th block-row of $\tilde{K}_{m2}$ ($l = 1, \ldots, N + \sum_{i=1}^N m_i$), we first locate the part of the l-th block-row in $\tilde{K}_{m1}$ which belongs to some $\tilde{K}^i$, $i = 1, \ldots, N$ (diagonal blocks), and then identify the block-column index ν_l in the following way: a) when $S_i = \emptyset$, ν_l is the column index of K^{Li}; b) when $S_i \neq \emptyset$, this is the block-column index of either $\hat{K}^i_j$ ($j = 1, \ldots, m_i$) within the first m_i block-rows in $\tilde{K}^i$, or of $\bar{K}^i_{m_i+2}$ in the last block-row of $\tilde{K}_i$. Then, we identify the block-column of V having "I" at its ν_l-th block-row; the block-row indices of the remaining "I"'s in the same block-column compose a set V_l^{NZ}. Then, the nonzero terms in l-th block-row of $\tilde{K}_{m2}$ are taken to be the blocks from the l-th row of $\tilde{K}_{m1}$ at the block-column indices defined by V_l^{NZ} with the reversed sign, while the sum of these blocks is put at the column index ν_l. Therefore, the resulting contracted gains are

$$K = R^+\tilde{K}_m V, \quad M = R^+\tilde{M}_m V. \tag{12}$$

The vehicle control u_i in the case when $S_i \neq \emptyset$ is generated by

$$u_i = \left[\bar{K}_1^i \vdots \cdots \vdots \bar{K}_{m_i}^i \vdots \bar{K}_{m_i+1}^i \vdots \bar{K}_{m_i+2}^i + \sum_{k \in \bar{S}_i} \hat{K}_i^k \right] \tilde{x}_i + \left[\bar{M}_1^i \vdots \cdots \vdots \bar{M}_{m_i}^i \vdots \bar{M}_{m_i+1}^i \vdots \bar{M}_{m_i+2}^i + \sum_{k \in \bar{S}_i} \hat{M}_i^k \right] \tilde{r}_i. \tag{13}$$

IV. STABILITY

The resulting closed-loop system is represented by

$$\mathbf{S_{cl}}: \quad \dot{x} = A_{cl}x + B_{cl}r \tag{14}$$

where $A_{cl} = [(G - I) \otimes A_v + [(I \otimes B_v)R^+\tilde{K}_m V]$ and $B_{cl} = [(I \otimes B_v)R^+\tilde{M}_m V]$. Both matrices $K = R^+\tilde{K}_m V$ and $M = R^+\tilde{M}_m V$ are composed of $N \times N$ (4×4)-blocks, such that for $S_i \neq 0$ we have the block $\begin{bmatrix} 0 & 0 \\ \bar{K}^i_{m_i+1} & \bar{K}^i_{m_i+2} + \sum_{k \in \bar{S}_i} \hat{K}_i^k \end{bmatrix}$ at the corresponding block diagonal and the blocks $\begin{bmatrix} 0 & 0 \\ 0 & \hat{K}^i_j \end{bmatrix}$, $j = 1, \ldots, m_i$, at the block indices (i, s^i_j) determined by S_i; for $S_i = 0$ we have in the i-th block row only the diagonal block $\begin{bmatrix} 0 & 0 \\ 0 & K^{Li} \end{bmatrix}$, $i = 1, \ldots, N$. Therefore, the state matrix A_{cl} contains in the i-th block row for $S_i \neq 0$ the diagonal block $\begin{bmatrix} 0 & -I \\ \bar{K}^i_{m_i+1} & \bar{K}^i_{m_i+2} + \sum_{k \in \bar{S}_i} \hat{K}_i^k \end{bmatrix}$ and the blocks $\begin{bmatrix} 0 & \alpha^i_{s^i_j} I \\ 0 & \hat{K}^i_j \end{bmatrix}$, $j = 1, \ldots, m_i$, at the block indices (i, s^i_j), and $\begin{bmatrix} 0 & -I \\ 0 & K^{Li} \end{bmatrix}$ at the diagonal for $S_i = 0$, $i = 1, \ldots, N$. The indices of the nonzero (4×4)-blocks in A_{cl} are the same as the indices of the nonzero elements in the adjacency matrix G of the formation graph. Therefore, the matrix A_{cl} is cogredient (amenable by permutation transformations) to the following matrix

$$A^P_{cl} = \begin{bmatrix} 0 & (G - I) \otimes I_2 \\ \mathrm{diag}\{\bar{K}^1_{m_1+1}, \ldots, \bar{K}^N_{m_N+1}\} & K_{cl} \end{bmatrix}, \tag{15}$$

where K_{cl} contains (2×2)-blocks $\bar{K}^i_{m_i+2} + \sum_{k \in \bar{S}_i} \hat{K}^k_i$ at the block diagonal and $\hat{K}^i_j$, $j = 1, \ldots, m_i$, at the block indices (i, s^i_j), $i = 1, \ldots, N$. The eigenvalues of A_{cl} are the solutions of the equation $\det(\lambda I_{4N} - A_{cl}) = 0$, or, equivalently, of $\det(\lambda I_{4N} - A^P_{cl}) = 0$, which gives rise to

$$\det(\lambda^2 I_{2N} - \lambda K_{cl} - \\ -\mathrm{diag}\{\bar{K}^1_{m_1+1}, \ldots, \bar{K}^N_{m_N+1}\}((G - I) \otimes I_2)) = 0. \tag{16}$$

We shall assume that all the constituent (2×2)-blocks of K are diagonal with nonnegative entries, so that K (and, consequently, A_{cl}) can be decomposed into two components K^I and K^{II} (A^I_{cl} and A^{II}_{cl}) which correspond to the components $x'_{i,I}$, and $x'_{i,II}$ (or $x''_{i,I}$ and $x''_{i,II}$) of the two-dimensional distance and velocity vectors in $\mathbf{S}$ (in the sequel, it is understood that the assumptions and conclusions about K^I and A^I_{cl} hold analogously for K^{II} and A^{II}_{cl}). We shall analyze solutions of (16) under simplifying assumptions emphasizing structural properties of the formation.

Assumption A. Matrices K^I_{cl} and $(\mathrm{diag}\{\bar{K}^1_{m_1+1}, \ldots, \bar{K}^N_{m_N+1}\})^I(G-I)$ can be transformed into the triangular form by the same unitary matrix W (Schur transformation [23]).

Assumption B. If $\mu_1,\ldots,\mu_N$ and $\nu_1,\ldots,\nu_N$ are the eigenvalues of K^I_{cl} and $(\mathrm{diag}\{\bar{K}^1_{m_1+1},\ldots,\bar{K}^N_{m_N+1}\})^I\ (G-I)$, respectively, then there are such real numbers $\gamma_i > 0$ and $\varepsilon_i > 0$ that $\mu_i = \gamma_i\nu_i - \varepsilon_i$, $i = 1,\ldots,N$.

Theorem 1. Let Assumptions A and B be satisfied, and let the formation digraph $\mathcal{G}$ have a directed spanning tree. Then, for γ_i large enough matrix A^I_{cl} has one simple eigenvalue at 0, and all the remaining eigenvalues have negative real parts.

Proof: Applying W^T and W to (16), one obtains

$$\det(\lambda I_{2N} - A^P_{cl}) = \prod_{i=1}^{N}(\lambda^2 - (\gamma_i\nu_i - \varepsilon_i)\lambda - \nu_i) = 0, \quad (17)$$

wherefrom the eigenvalues of A^I_{cl} are

$$\lambda_{i\pm} = \frac{\gamma_i\nu_i - \varepsilon_i \pm \sqrt{(\gamma_i\nu_i - \varepsilon_i)^2 + 4\nu_i}}{2}, \quad (18)$$

$i = 1,\ldots,N$.

When $S_i \neq \emptyset$, $i = 1,\ldots,N$, we have $G - I = L$, where L is the weighted Laplacian of the formation digraph $\mathcal{G}$. If this digraph has a directed spanning tree, L has one simple zero eigenvalue and the other eigenvalues have negative real parts [24], so that for $\nu_1 = 0$, one obtains $\lambda'_{1+} = 0$ and $\lambda_{1-} = -\gamma_i$. For the remaining ν_i, $i = 2,\ldots,N$, a simple geometric reasoning based on [9], [8] shows that the corresponding $\lambda_{i\pm}$ have negative real parts for γ_i large enough. Remark only that the condition $\gamma_i > \sqrt{\frac{2}{\mathrm{Re}\{\nu_i\}}}$ which can be derived from the results in [9], [8] is overly conservative: it is possible to check the case of real ν_i, when, in fact, $\mathrm{Re}\{\lambda_{i\pm}\} < 0$ for all positive γ_i.

If there is one vehicle satisfying $S_i = \emptyset$, $G - I$ is nonsingular if the digraph has a spanning tree. However, in this case $\bar{K}^i_{m_1+1} = 0$ (see Section 3), and, therefore, matrix $(\mathrm{diag}\{\bar{K}^1_{m_1+1},\ldots,\bar{K}^N_{m_N+1}\})^I((G-I)))$ has one simple eigenvalue at the origin, *i.e.* for $\nu_1 = 0$, one obtains again $\lambda_{1+} = 0$ and $\lambda_{1-} = -\gamma_i$, etc. Thus the result. ■

We shall adopt further simplifying assumptions implying Assumptions A and B in order to make clear the main structural properties of the analyzed formation control law.

Assumption C. (a) $(\bar{K}^i_{m_i+1})^I = \kappa > 0$, (b) $(\bar{K}^i_j)^I = \rho > 0$, (c)$(\bar{K}^i_{m_i+2} + \sum_{k\in\bar{S}_i}\hat{K}^k_i)^I = -m_i\rho - \varepsilon$, $\varepsilon > 0$, $i = 1,\ldots,N$.

Theorem 2. Let Assumption C be satisfied and let the underlying graph $\mathcal{G}$ have a directed spanning tree. Then A^I_{cl} has a single eigenvalue at zero and all the remaining eigenvalues have negative real parts for $\rho\kappa^{-1}$ large enough.

Proof: The proof is entirely based on Theorem 1, with $\rho\kappa^{-1}$ playing the role of γ_i. ■

The main result of this section, connecting the results of Theorems 1 and 2 with the specific structure of the proposed formation model, is given in the following theorem.

Theorem 3. Let $\tilde{M} = -\tilde{K}$. Then, under the assumptions of Theorem 2, for $\rho\kappa^{-1}$ large enough:

(a) when $S_i \neq \emptyset$, $i = 1,\ldots,N$, $\lim_{t\to\infty}[x'_i(t) - \bar{r}^d_i] = 0$ and $\lim_{t\to\infty}[x''_i(t) - \bar{r}^v] = 0$, $i = 1,\ldots,N$, where $\bar{r}^d = \begin{bmatrix}\bar{r}^d_1 & \cdots & \bar{r}^d_N\end{bmatrix}$ satisfies $\bar{r}^d = L\bar{r}^z$ and $\bar{r}^z$ and $\bar{r}^v$ are arbitrary predefined constant $2N$-dimensional and 2-dimensional vectors, respectively;

(b) when $S_j = \emptyset$ for some $j \in \{1,\ldots,N\}$, $x_j(t) \to_{t\to\infty} \bar{r}^v t$, $\lim_{t\to\infty}[x'_i(t) - \bar{r}^d_i] = 0$, $i = 1,\ldots,N$, $i \neq j$, and $\lim_{t\to\infty}[x''_i(t) - \bar{r}^v] = 0$, $i = 1,\ldots,N$, where $\bar{r}^d_i$, $i = 1,\ldots,N$, $i \neq j$, and $\bar{r}^v$ are arbitrary predefined 2-dimensional vectors.

Proof: Assume first that $S_i \neq \emptyset$, $i = 1,\ldots,N$. Then, according to Theorem 2,

$$e^{(A^P_{cl})^I t} = P\begin{bmatrix}1 & 0\\ 0 & e^{Jt}\end{bmatrix}P^{-1}, \quad (19)$$

where $P = \begin{bmatrix}r_1 & \vdots & \cdots & \vdots & r_{2N}\end{bmatrix}$ and $P^{-1} = \begin{bmatrix}s_1^T\\ \cdots\\ s_{2N}^T\end{bmatrix}$, r_i representing the right eigenvectors (or generalized eigenvectors) and s_i the left eigenvectors (or generalized eigenvectors) of $(A^P_{cl})^I$ and where the $(2N-1)\times(2N-1)$ matrix J is Hurwitz. Without loss of generality, we choose $r_1^T = \begin{bmatrix}\mathbf{1}^T & \kappa\varepsilon^{-1}\mathbf{1}^T\end{bmatrix}$ and $s_1^T = \begin{bmatrix}p_1^T & 0\end{bmatrix}$, where $\mathbf{1}^T = [\,1\cdots 1\,]$ and p_1 is a nonnegative vector such that $p_1^T L = 0$ and $p_1^T\mathbf{1} = 0$ as a consequence of the fact that L has a simple zero eigenvalue; also, $s_1^T r_1 = 1$. Consequently, we obtain, having in mind that $\tilde{M} = -\tilde{K}$, that when $t\to\infty$

$$\begin{bmatrix}X_1^I(t)\\ X_2^I(t)\end{bmatrix} \to \begin{bmatrix}\mathbf{1}\\ \kappa\varepsilon^{-1}\mathbf{1}\end{bmatrix}\begin{bmatrix}p_1^T & 0\end{bmatrix}\begin{bmatrix}X_1^I(0)\\ X_2^I(0)\end{bmatrix}, \quad (20)$$

where $X_1(t)^{IT} = \begin{bmatrix}(x'_{1,I} - \bar{r}^d_{1,I})^T & \cdots & (x'_{N,I} - \bar{r}^d_{N,I})^T\end{bmatrix}$ and $X_2(t)^{IT} = \begin{bmatrix}(x''_{1,I} - \bar{r}^v_I)^T & \cdots & (x''_{N,I} - \bar{r}^v_I)^T\end{bmatrix}$ ($x'_{j,I}$ denotes the first component of x'_j, $x''_{j,I}$ the first component of x''_j, etc., $j = 1,\ldots N$). Obviously, $X_1^I(t) \to \mathbf{1}p_1^T X_1^I(0)$ and $X_2^I(t) \to \kappa\varepsilon^{-1}\mathbf{1}p_1^T X_1^I(0)$. However, according to the model definition in Section 2, we have the transformation $\begin{bmatrix}x'_{1,I}(t)\\ \vdots\\ x''_{1,I}(t)\\ \vdots\end{bmatrix} = \begin{bmatrix}L & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}z'_{1,I}(t)\\ \vdots\\ z''_{1,I}(t)\\ \vdots\end{bmatrix}$, so that, according to the assumption of the theorem that $\bar{r}^d = L\bar{r}^z$ for some $\bar{r}^z$, we obtain

$$\begin{bmatrix}X_1^I(t)\\ X_2^I(t)\end{bmatrix} = \begin{bmatrix}L & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}Z_1^I(t)\\ Z_2^I(t)\end{bmatrix}, \quad (21)$$

where $Z_1(t)^{IT} = \left[(z'_{1,I} - \bar{r}^z_{1,I})^T \cdots (z'_{N,I} - \bar{r}^z_{N,I})^T \right]$ and $Z_2(t)^{IT} = \left[(z''_{1,I} - \bar{r}^v_I)^T \cdots (z''_{N,I} - \bar{r}^v_I)^T \right]$. Introducing $\begin{bmatrix} X_1^I(0) \\ X_2^I(0) \end{bmatrix}$ back into (20), one obtains that $\lim_{t\to\infty} X_1^I(t) = \lim_{t\to\infty} X_2^I(t) = 0$ for any $\bar{r}^z$ and $\bar{r}^v$, having in mind that $p_1^T L = 0$.

Suppose now, without loss of generality, that $S_1 = \emptyset$. According to Theorem 2, $(A_{cl}^P)^I$ has a simple zero eigenvalue and $G - I$ is nonsingular. It is straightforward to deduce that now $r_1^T = [1\ 0 \cdots 0]$ and $s_1^T = \left[1\ 0 \cdots 0 \vdots (\varepsilon - \rho)^{-1}\ 0 \cdots 0 \right]$, so that $x_1(t) \to x'_{1,I}(0) + (\varepsilon - \rho)x''_{1,I}(0) + \bar{r}^v_I t$ when $t \to \infty$, and that the remaining components of the vector $\begin{bmatrix} X_1^I(t) \\ X_2^I(t) \end{bmatrix}$ tend to zero for any $\bar{r}_i^d$, $i = 2, \ldots, N$, and $\bar{r}^v$.

Hence the result. ■

Remark 1. The above analysis can be extended to the case when the velocity reference is available only to the leading vehicle. It is possible to use Theorem 3, and to conclude that the steady state error in both spacing and velocity tracking is nonzero in the general case when $S_i \neq \emptyset$ and proportional to the velocity reference. Then, the steady state spacing error is influenced directly by the controller parameter $\bar{K}^i_{m_i+2}$, as in the case of the so-called heading control strategy for vehicles on highways [17], having in mind possibilities of large transients. In this case, however, it is possible to recur to the introduction of the integral action aimed at reducing the steady state error. ■

V. Output Feedback with Decentralized Observers

Assume that the measurements available to the vehicles do not contain the velocities of the sensed vehicles, so that y_i, the measurement vector of the i-th vehicle, is composed of the distances with respect to the sensed vehicles and its own velocity, *i.e.* $y_i = \left[(z'_{s^i_{m_1}} - z'_i)^T \cdots (z'_{s^i_{m_i}} - z'_i)^T\ (z''_i)^T \right]^T$. If our task is to construct local state estimators, we shall attach to the vehicles specific subsystem models $\mathbf{\Xi_i}$ having the form

$$\mathbf{\Xi_i}: \quad \dot{\xi}_i = A_i^* \xi_i + B_i^* \tilde{u}_i \tag{22}$$

with the state vectors $\xi_i = \left[(z''_{s^i_1})^T \cdots (z''_{s^i_{m_i}})^T\ (z'_{s^i_1} - z'_i)^T \cdots (z'_{s^i_{m_i}} - z'_i)^T\ (z''_i)^T \right]^T$, where $A_i^* = \begin{bmatrix} 0_{2m_i \times 2N} \\ \bar{A}_i^* \\ 0_{2\times 2N} \end{bmatrix}$, in which $\bar{A}_i^*$ is a $m_i \times N$ (2×2)-block matrix in which all block rows contain $-I_2$ at the last column index and I_2 at the column index $s^i_{m_j}$, $j = 1, \ldots, m_i$, and $B_i^* = \begin{bmatrix} I_{2m_i} & 0_{2m_i\times 2} \\ 0_{2m_i\times 2m_i} & 0_{2m_i \times 2} \\ 0_{2\times 2m_i} & I_2 \end{bmatrix}$ ($\tilde{u}_i$ is defined as $\tilde{u}_i = \left[u^T_{s^i_1} \cdots u^T_{s^i_{m_i}}\ u_i^T \right]^T$). Subsystem models $\mathbf{\tilde{S}_i}$ used for control design in Sections 3 and 4 can be easily obtained from $\mathbf{\Xi_i}$ as aggregations (see [15], [18], [16]), *i.e.* $\tilde{x}_i = U\xi_i$ where U is a full rank $(2m_i + 4) \times (4m_i + 2)$ matrix of the form $U = \begin{bmatrix} I_{2m_i} & & & & \\ & \alpha^i_{s^i_1} I_2 & \cdots & \alpha^i_{s^i_{m_i}} I_2 & \\ & & & & I_2 \end{bmatrix}$, so that we have the aggregation conditions $UA_i^* = \tilde{A}_i U$ [.]. Notice that $\mathbf{\Xi_i}$ cannot be used for control design purposes, having in mind that it is uncontrollable from $\tilde{u}_i$. However, it can used as a basis for defining the following local observers of Luenberger type

$$\mathbf{E_i^*}: \quad \dot{\hat{\xi}}_i = A_i^* \hat{\xi}_i + B_i^* \tilde{u}_i + L^*[y_i - C^* \hat{\xi}_i], \tag{23}$$

where L^* is the estimator gain (*e.g.* Kalman gain) and $C^* = \left[0_{2(m_i+1)\times 2m_i} \vdots I_{2(m_i+1)} \right]$. Essentially, the main problem related to $\mathbf{E_i^*}$ is how to define the control vector $\tilde{u}_i$, since the real control inputs of the neighboring vehicles are generally unknown at the i-th vehicle. We shall adopt here approximations, motivated by the idea to generate $\tilde{u}_i$ by using the subsystem control law $\tilde{\mathbf{F}}_i$ in (8) in which $\tilde{x}_i$ is replaced by its estimate obtained by using $\mathbf{E_i^*}$ in such a way that $\hat{\tilde{x}}_i = U\hat{\xi}_i$, where $\hat{\xi}_i$ is generated by (23), so that $\tilde{u}_i = \tilde{u}_i^* = \left[u^{*T}_{s^i_1} \cdots u^{*T}_{s^i_{m_i}}\ u_i^{*T} \right]^T = \tilde{K}^i \hat{\tilde{x}}_i + \tilde{M}^i \tilde{r}_i$. According to the description of the structure of $\tilde{\mathbf{F}}_i$ given in Section 4, the control vector components $u^*_{s^i_1}, \ldots, u^*_{s^i_{m_i}}$ are generated by the local feedback designed for the leading vehicles as $u_j^* = \hat{K}^i_j \hat{z}''_j + \hat{M}_j r^v$, $j = s^i_1, \ldots, s^i_{m_i}$, where $\hat{z}''_j$ is a part of the state estimation vector $\hat{\xi}_i$. According to (13), the last component u_i^* in $\tilde{u}_i^*$ is defined by

$$u_i^* = \left[\bar{K}_1^i \vdots \cdots \vdots \bar{K}^i_{m_i} \vdots \bar{K}^i_{m_i+1} \vdots \bar{K}^i_{m_i+2} + \sum_{k\in \bar{S}_i} \hat{K}_i^k \right] U\hat{\xi}_i + \left[\bar{M}_1^i \vdots \cdots \vdots \bar{M}^i_{m_i} \vdots \bar{M}^i_{m_i+1} \vdots \bar{M}^i_{m_i+2} + \sum_{k\in \bar{S}_i} \hat{M}_i^k \right] \tilde{r}_i,$$

where $\hat{x}'_i$ is easily obtained from $\hat{x}_i^*$ according to the definition of the vector x_i as a function of the distances with respect to the sensed vehicles (this mapping is incorporated in the transformation U).

VI. Controller Realization and Experiments

The above exposed general methodology for formation tracking control design has been implemented by using the suboptimal hierarchical LQ strategy for local controller design and Kalman filters as local observers, based on the results presented in [25], [26], [17], [13]. A formation of five vehicles has been simulated, assuming that one vehicle plays the role of the formation leader. It has been assumed that the second vehicle observes the first, the third vehicle observes the first, the fourth observes the second and the third and the fifth vehicle observes the third. It is possible to demonstrate that for such formation graphs with no closed contours formation stability is ensured when the subsystems are locally stabilized. The proposed design methodology has been applied for the dynamic output feedback controller design, assuming that the measurements of the local velocity and the distances to the neighboring vehicles are available in the vehicles. The references of the distances (with respect to the centroid of the neighboring vehicles) and velocities have been composed in such a way as to obtain reconfiguration of the formation starting from the "V" form and ending with a line (platoon). Figures 1 and 2 represent the x-components of

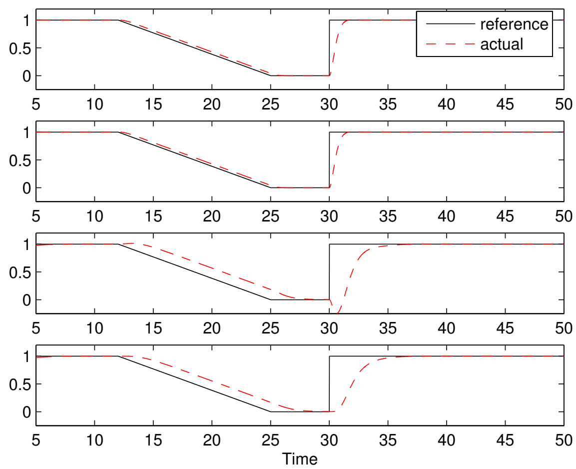

Fig. 1. Distance plots

the distances and velocities of four vehicles in the formation, excluding the leader. Obviously, tracking is very successful, even in the regime of fast changes of the references. It is very important to emphasize that the presented curves correspond to a specific choice of the weighting matrices in the quadratic criterion; different choices of these matrices provide different tracking properties.

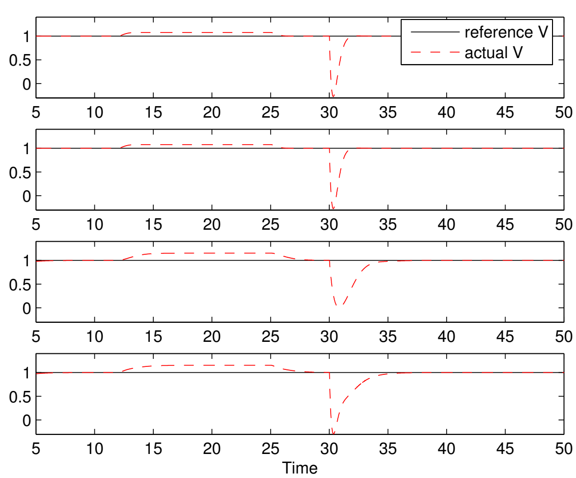

Fig. 2. Velocity plots

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