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LPV Decoupling for Multivariable Control System Design

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Abstract— **The paper explores methods for decoupled linear parameter varying (LPV) control. The proposed approach seeks to benefit the multi-variable control of multi-input multi-output (MIMO) systems with variable operating conditions, variable parameters or nonlinear behavior. The method can improve the performance and reduce the variability of such MIMO systems with significant coupling in the system dynamics. We design MIMO decoupled feedback LPV controllers to address coupling effects. In particular, the method uses a parameter-dependent static inversion or SVD decomposition of the system to minimize the effects of the off-diagonal terms in the MIMO system transfer function matrix. The parameter-dependent decoupling matrices are selected along with the appropriate LPV controller design to guarantee the closed-loop performance specifications. A new parameter-dependent interaction measure is also introduced based on SVD decomposition and static inversion and is examined for the adaptive control design purposes to address the variability and coupling of the multivariable systems.**

I. INTRODUCTION

An idealized requirement in MIMO control system design is that of decoupling. Decoupling can be performed in several ways from static, where decoupling is only demanded for constant reference signals, to full dynamic decoupling, where decoupling is achieved over all frequencies. Since full dynamic decoupling is a stringent demand, it is more common to seek dynamic decoupling over a desired frequency bandwidth. If a plant is dynamically decoupled, then changes in the set-point of one process variable leads to a response in only that process variable and all others remain unchanged [8]. In the literature, different decoupling approaches have been proposed for MIMO LTI systems and successfully applied in many engineering applications. There has been a particular interest to the problem of dynamic decoupling for stable minimum phase systems, dynamic decoupling for stable non-minimum phase systems and dynamic decoupling for open loop unstable systems (see [8], [11] and references therein). As expected, full dynamic decoupling is a strong requirement that can rarely be achieved. This brings other relaxed forms of decoupling as more feasible and practical. For example, static decoupling is quite prevalent in practical applications. The question then becomes, over what bandwidth decoupling will be valid. Different decoupling strategies come at a cost. It turns out that the additional cost of decoupling is a function of open-loop poles and zeros in the right half plane [8]. Thus, if one is interested in restricting the decoupling to some bandwidth then by paying attention to those open-loop poles and zeros that fall within this bandwidth, the cost of decoupling over that region can be assessed.

Gain-scheduling control is a popular design technique often used by practicing control engineers when the controlled plant is highly nonlinear. In gain-scheduling, linear control techniques are applied for control design over a partition of the operating envelope. A necessary number of operating points are usually selected by the designer to cover the entire operating region, and a local controller is designed at each one of these points. In between the operating points, for which the linear controllers were designed, the parameters of the controllers are interpolated to cover the entire operating envelope. The main drawback of traditional gain-scheduling control is that the design does not guarantee stability of the closed-loop system. This is because the global feedback law obtained from gain-scheduling is a nonlinear controller and the guarantees of the linear synthesis methods do not hold. In contrast to the classical gain-scheduling techniques, the recently developed robust gain-scheduling methods (see [9] and references therein) in the linear parameter varying (LPV) framework provide systematic ways to design controllers scheduled based upon the operating point of the system. An LPV system is a system whose describing state-space matrices depend on a time-varying parameter vector. LPV-based design methods do not require the interpolation of the local controllers. In fact, they provide a family of linear controllers with guarantee of stability and performance for the entire operating range.

In many multivariable industrial plants and processes, single-loop PID controller design is often implemented by controlling individual feedback loops, where decoupling is employed to decouple the interaction between input and output variables. The use of standard PID design procedures and the ease of fine tuning are the main advantages of this approach [5]. Nevertheless, as mentioned earlier, this control design procedure for MIMO systems involves two stages: first the need to decouple the different SISO subsystems, and then the need to individually control them so that stability and satisfactory performance is provided. Traditional decoupling, however, does not address the challenge of system nonlinearities and changes in operating conditions. This is often a reason why the standard (static or dynamic) decoupling fails to provide satisfactory decoupling for systems in which the operating conditions constantly change. The objective of this paper is to propose an adaptive way of updating the decoupling transformation matrices by employing a set of *parameter-dependent bases* generated off-line and used

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to update the decoupling transformation matrices in real-time. The assumption here is that the operating points are measurable in real-time and slowly varying.

II. Problem Statement

Consider a linear parameter varying (LPV) plant $G(s,\rho)$ represented by the following state-space realization

$$\dot{x} = A(\rho)x + B(\rho)u \quad (1)$$
$$y = C(\rho)x + D(\rho)u \quad (2)$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^{n_u}$ is the control input and $y \in \mathbf{R}^{n_y}$ includes the measured outputs. In the above formulation, the time-varying parameter vector $\rho = [\rho_1, \ldots, \rho_s]^T$ is assumed to be unknown *a priori* but can be measured or estimated in real-time. In this paper, we are particularly interested in the frequency-domain control design problem for multivariable systems where the coupling effects due to the non-diagonal elements in the system transfer matrix $G(s,\rho)$ are significant. The decoupling problem for LTI systems has been well studied in the past couple of decades. Our objective is to propose a methodology for designing decoupling controllers to guarantee the design specifications in the presence of operating condition variability and system nonlinearities. The design approach is simple yet applicable to many applications in which an invariant decoupling procedure fails to provide satisfactory closed-loop performance due to nonlinearities and/or parameter variability. In this paper, we only consider systems with square transfer function matrices. However, the ideas discussed in the paper can be readily extended to nonsquare systems.

III. LPV Decoupled Control of MIMO Systems

A. Analytical Framework for LPV Decoupled Control

A multivariable system often exhibits coupling interactions between its inputs and outputs. These interactions cause inherent problems in the multivariable control design. Decoupling control is based on appropriate cancellation of these cross-coupling loop interactions and allows improved control and subsequent ease of tuning and calibration. Similar to most decoupling techniques, the LPV decoupling methodology for parameter-dependent multivariable control design proposed in this paper uses a two-step procedure as follows: (i) we first design a parameter-dependent *compensator* to decouple the interactions between the off-diagonal terms of the LPV open-loop system whose transfer function matrix is represented by $G(s,\rho)$, and (ii) we design a *diagonal controller* using standard methods applicable to SISO systems. The approaches we consider for decoupled MIMO control design are based on designing a pre-compensator to counteract the cross-coupling interactions in $G(s,\rho)$ or pre- and post-compensators by employing an SVD decomposition to diagonalize $G(s,\rho)$. In the next sections, we describe in detail how the method is applied for each decoupling approach.

The proposed LPV decoupling control design method provides significant advantages in terms of dealing with the coupling issue in MIMO systems subject to *nonlinear behavior* and *operating condition variability*. In particular, the proposed methods result in:

1) a systematic methodology for on-line updating of the decoupling matrices to guarantee system decoupling in the presence of parameter variability.
2) a combined LPV control design that uses the decoupled system to guarantee improved closed-loop performance.

We are also interested in developing conditions that guarantee appropriate selection of the parameter-dependent decoupling matrices and dynamic controllers for closed-loop stability and performance based on the computation of parameter-dependent Lyapunov functions.

B. MIMO LPV Decoupling Based on Inversion

LPV decoupling can be simply achieved using a pre-compensator $W(s,\rho)$ that results in a new shaped plant in the following form:

$$G_s(s,\rho) = G(s,\rho)W(s,\rho) \quad (3)$$

which is expected to have a desired structure, e.g. diagonal or approximately diagonal and, hence, easier to control compared to the original plant $G(s,\rho)$. Once such a parameter-dependent pre-compensator is found, then a diagonal LPV controller $K_s(s,\rho)$ can be designed for the shaped plant $G_s(s,\rho)$. The overall controller will then be

$$K(s,\rho) = W(s,\rho)K_s(s,\rho). \quad (4)$$

Decoupling is achieved if $G_s(s,\rho)$ defined in (3) is diagonal at a selected frequency range. There are two approaches for designing such a pre-compensator. One is the *dynamic decoupling*, where (assuming that $G_s(s,\rho)$ is invertible) $W(s,\rho) = G^{-1}(s,\rho)$ is selected to diagonalize $G_s(s,\rho)$ at all frequencies and all admissible time-varying parameters. Realizing such a compensator could be difficult due to the possible problems involved in taking the inverse of the plant transfer function [11], [8]. A second design approach is the *static decoupling* at a certain frequency w_0, where one looks for a parameter-dependent $W(\rho)$ to make $G_s(jw_0,\rho)$ as close to a diagonal matrix as possible. This may be obtained by selecting a constant pre-compensator $W(\rho) = G_0^{-1}(\rho)$ where G_0 is a real approximation of $G(jw_0)$. Steady-state decoupling by setting $w_0 = 0$ is one common way of achieving the static decoupling. The bandwidth frequency is also another good selection for w_0 since the effect on performance of the reduced coupling is normally maximum at this frequency [11]. For both dynamic and static decoupling, if the open-loop system's transfer matrix is non-square, then the pseudo-inverse can be utilized provided that the matrix is full-rank for all admissible time-varying parameters ρ. Due to its ease of implementation, we focus our attention to the static decoupling throughout this paper.

C. MIMO LPV Decoupling Based on SVD Decomposition

The basic idea is to shape the open-loop system as $G_s(s,\rho) = W_2(s,\rho)G(s,\rho)W_1(s,\rho)$, where W_1 and W_2 are appropriate parameter-dependent decoupling matrices, such that $G_s(s,\rho)$ becomes diagonal. Taking into account the fact that we are interested in the static decoupling problem, the compensators are parameter-dependent static matrices. Then, the overall controller will be

$$K(s,\rho) = W_1(\rho)K_s(s,\rho)W_2(\rho) \tag{5}$$

where $K_s(s,\rho)$ is the MIMO diagonal controller designed for the decoupled plant $G_s(s,\rho)$. The SVD-controller is a special case of a pre- and post-compensator design as explained above. Here

$$W_1(\rho) = V_0(\rho) \quad \text{and} \quad W_2(\rho) = U_0^T(\rho)$$

where $U_0(\rho)$ and $V_0(\rho)$ are obtained from a *parametric* singular value decomposition $G_0(\rho) = U_0(\rho)\Sigma_0(\rho)V_0^T(\rho)$, where $G_0(\rho)$ is a real approximation of $G(j\omega_0,\rho)$ at a given frequency ω_0. SVD decoupling may be performed around $\omega_0 = 0$ (steady-state decoupling) or $\omega_0 = \omega_b$ (bandwidth frequency decoupling), in which cases the matrices $U_0(\rho)$ and $V_0(\rho)$ are generated. Hence, the proposed decoupling approach is similar to a parameter-dependent SVD decomposition except that the decomposition is performed off-line and a basis is produced to be used for updating the decoupling matrices in real-time.

IV. Discussion

Decoupling is susceptible to erroneous or incomplete cross-coupling cancellations due to modeling errors. Hence, appropriate robustness of the design should be guaranteed. Partial decoupling can be employed if the effect of some of the interacting loops is deemed negligible. In the first part of this section, we present some results on the post-analysis of the closed-loop system to ensure that the decoupled LPV design will guarantee closed-loop stability and desired performance even when the decoupling is not perfectly achieved.

Interaction analysis of MIMO systems is crucial for control design and decentralized control problems. There are different measures used to quantify the interaction between the inputs and outputs which include the relative gain array (RGA) introduced by Bristol [6], the steady-state interaction indices developed by Chang and Davison [7] and a recently proposed method by Astrom *et. al* [3]. In the second part of this section, we extend the idea developed in [3], [4] and introduce a new interaction measure based on SVD decomposition and static inversion and show how it works for LPV decoupled control design.

A. Stability and Performance Degradation Analysis

In the previous section, we discussed the problem of LPV decoupled controller design for nonlinear systems in an LPV form. According to [10], [12], the decoupled controller has to meet two criteria: nominal stability and robust performance. Nominal stability is achieved if the decoupled controller, designed for the diagonal elements of the decoupled system, provides stability for the closed-loop system. Robust performance is achieved if the performance requirements are satisfied for all plants within the uncertainty set. It is noted that both nominal stability and robust performance will result in bounds on the sensitivity and complementary sensitivity functions of all the independent SISO control loops in the decoupled controller design, which can be readily used in an $\mathcal{H}^\infty$ loop shaping design of the SISO controllers.

Following the notation presented in the previous section, we have

$$G_s(s,\rho) = W_2(\rho)G(s,\rho)W_1(\rho)$$

Let us define $D_s(s,\rho) = diag\{G_{s_{ii}}(s,\rho)\}$ where $G_{s_{ii}}$ are the diagonal elements of the compensated plant G_s. In the case of a perfect decoupling, $G_s(s,\rho) = D_s(s,\rho)$. However, in reality, $D_s(s,\rho)$ is just an approximation of $G_s(s,\rho)$. Finally, $D_s(s,\rho)$ may be further manipulated to generate another diagonal matrix $D_d(s,\rho)$ using static or dynamic scaling.

The control design method discussed earlier is a decentralized one consisting of q independent SISO controllers designed based on $D_d(s,\rho)$. Let us assume that such a controller is represented by $K_s(s) = diag\{K_1(s),\ldots,K_q(s)\}$ designed for the compensated parameter-dependent system $D_d(s,\rho)$. It is noted that the controller itself could be parameter-dependent as well. The final controller then becomes (5). The sensitivity and complementary sensitivity based on $D_s(s,\rho)$ and the independent SISO controllers are defined as

$$\begin{aligned} S_D(\rho) &= (I + D_s(\rho)K_s)^{-1} \\ T_D(\rho) &= (I + D_s(\rho)K_s)^{-1}D_s(\rho)K_s. \end{aligned} \tag{6}$$

Considering the fact that $D_s(s,\rho)$ and $K_s(s)$ are both diagonal, $S_D(\rho)$ will become diagonal as well, and hence the following relations hold

$$\|S_D(\rho)\|_\infty = \max_{\rho} \max_{i=1,\ldots,q} \{\|(1 + G_{s_{ii}}K_i)^{-1}\|_\infty\} \tag{7}$$

$$\|T_D(\rho)\|_\infty = \max_{\rho} \max_{i=1,\ldots,q} \{\|(1 + G_{s_{ii}}K_i)^{-1}G_{s_{ii}}K_i\|_\infty\} \tag{8}$$

Applying the final controller represented by (5) to the system leads to the following sensitivity and complementary sensitivity functions

$$\begin{aligned} S(\rho) &= (I + G_s(\rho)K_s)^{-1} \\ T(\rho) &= (I + G_s(\rho)K_s)^{-1}G_s(\rho)K_s. \end{aligned} \tag{9}$$

Nominal stability is achieved if $S(\rho)$ defined above is stable. Due to the imperfect decoupling, the nominal stability may not be guaranteed. A simple way to check the stability issue due to the non-ideal decoupling is to treat the difference between $G_s(\rho)$ and $D_s(\rho)$ as a multiplicative output uncertainty by generalizing the formulation in [12] as

$$E(s,\rho) = (G_s(s,\rho) - D_s(s,\rho))D_s^{-1}(s,\rho).$$

A posteriori analysis can then be performed to determine the stability and performance degradation caused by an imperfect decoupling. This can be accomplished by employing, for instance, parameter-dependent Lyapunov functions used for the closed-loop stability and performance satisfaction in an LPV framework. The results presented in [2] can then be employed to do this post analysis. It should be, however, noted that due to the parameter-dependence of all the transfer matrices in the above formulations, checking the nominal stability and robust performance needs to be done by appropriate gridding the parameter space to achieve a finite-dimensional optimization problem and examine the appropriate conditions.

B. Interaction Measure for Decoupling LPV Systems

Consider an LPV MIMO control problem, where the goal is to design a parameter-dependent controller $K(s,\rho)$ for an LPV system represented by $G(s,\rho)$. The designed controller is assumed to be a static pre- and post-compensator combined with a decentralized parameter-varying PI controller with set-point weighting as configured in [1]. The control law can be represented by

$$U(s) = W_1(\rho)\left(\bar{K}(s,\rho)R(s) - K(s,\rho)W_2(\rho)Y(s)\right) \quad (10)$$

where W_1 and W_2 are the pre- and post-compensators, U is the control input, Y is the system measurement, and R is the reference input. The PI controller $\bar{K}$ is selected different from K to allow for set-point weighting [1]. Due to the decentralized nature of the design, both of these matrices are parameter-dependent diagonal for a system with the same number of inputs and outputs. We consider the following structure for the controllers

$$\begin{aligned} K_j &= k_{P_{(j)}} + \frac{k_{I_{(j)}}}{s} \\ \bar{K}_j &= \frac{k_{I_{(j)}}}{s} \end{aligned}$$

where the controller gains are parameter-dependent. In the literature it has been shown that excluding the proportional term in $\bar{K}_j$ is essential to achieve an improved performance in decentralized PID control for LTI systems [1], [3], [4].

Considering the SVD decomposition as in the static decoupling method at steady-state provides $W_1 = V_0$ and $W_2 = U_0^T$ where $G_0(\rho) = U_0(\rho)\Sigma_0(\rho)V_0^T(\rho)$ is the SVD decomposition of the LPV system transfer function evaluated at $s = 0$. Defining the decoupled system transfer function as $Q(s,\rho) = U_0^T(\rho)G(s,\rho)V_0(\rho)$, considering the compensated plant output $Y_t = U_0^T(\rho)Y$ and using (10) results in

$$Y_t = U_0^T G(s) V_0 \left(\bar{C}R(s) - CY_t\right)$$

or

$$\begin{aligned} Y_t(s) &= H(s,\rho)R(s) \\ H(s,\rho) &= (I + Q(s,\rho)C(s,\rho))^{-1}Q(s,\rho)\bar{C}(s,\rho) \end{aligned}$$

For simplicity of the formulations, we focus our attention to the case of two-input/two-output LPV systems. Defining $Q(s,\rho) = [q_{ij}]_{i,j\in\{1,2\}}$ leads to

$$H(s,\rho) = \begin{bmatrix} 1+q_{11}c_1 & q_{12}c_2 \\ q_{21}c_1 & 1+q_{22}c_2 \end{bmatrix}^{-1} \begin{bmatrix} q_{11}\bar{c}_1 & q_{12}\bar{c}_2 \\ q_{21}\bar{c}_1 & q_{22}\bar{c}_2 \end{bmatrix}$$

The definition of $Q(s,\rho)$ results in $|\det(Q(s,\rho))| = |\det(G(s,\rho))|$ considering the fact that the matrices U_0 and V_0 are unitary for all admissible LPV parameters. Following the discussion presented in [4], the elements of $H(s,\rho)$ may be simplified further. The closed-loop bandwidth w_b is limited by the right half-plane zeros of the open-loop system $G(s,\rho)$. Assuming that there exists a single zero at $z > 0$, approximately distributed between the two loops, then the bandwidth must be less than $\frac{z}{2}$ rad/sec [1]. Assuming that the zeros of the frozen plant $G(s)$ are not within the bandwidth of the closed-loop system and that $|\det(G(s,\rho))| \gg 0$ for all $w < w_b$, it is deduced that $|q_{11}(s,\rho)q_{22}(s,\rho)| \gg |q_{12}(s,\rho)q_{21}(s,\rho)|$. Using the latter inequality, the matrix H can then be approximated by

$$H(s,\rho) \approx \begin{bmatrix} \frac{q_{11}\bar{c}_1}{1+q_{11}c_1} & \frac{q_{12}\bar{c}_2}{(1+q_{11}c_1)(1+q_{22}c_2)} \\ \frac{q_{21}\bar{c}_1}{(1+q_{11}c_1)(1+q_{22}c_2)} & \frac{q_{22}\bar{c}_2}{1+q_{22}c_2} \end{bmatrix}$$

The off-diagonal elements in H will indicate the amount of interaction in the MIMO design. Due to the integral action in the controllers, the interaction is small at low frequencies. It is easy to observe that

$$H_{12} = q_{12}\bar{c}_2 S_1 S_2, \qquad H_{21} = q_{21}\bar{c}_1 S_1 S_2 \quad (11)$$

where S_1 and S_2 are the sensitivity functions associated with the first and second loops ignoring the loop interactions. An upper bound on the magnitude of the off-diagonal terms can be calculated as

$$|H_{12}(jw)| \le |q_{12}\bar{c}_2| M_{s1} M_{s2}\ , \quad |H_{21}(jw)| \le |q_{21}\bar{c}_1| M_{s1} M_{s2}$$

where M_{s1} and M_{s2} are the peaks associated with S_1 and S_2, respectively. It is noted that at steady-state, the following approximation is valid, where (12) is obtained by expanding $G(s,\rho)$ using the MacLaurin series in s around $s = 0$.

$$\begin{aligned} Q(s,\rho) &= U_0^T(\rho)G(s,\rho)V_0(\rho) \\ &\approx U_0^T \left(G(s=0,\rho) + s\frac{dG(s,\rho)}{ds}(s=0) \right) V_0 \\ &= \Sigma_0 + s\Sigma_1 \end{aligned} \quad (12)$$

where

$$\begin{aligned} \Sigma_0 &= U_0^T G(s=0,\rho)V_0 = diag(\sigma_0(\rho), \sigma_1(\rho)) \\ \Sigma_1 &= U_0^T \frac{dG(s,\rho)}{ds}(0)V_0 = [\sigma_{ij}]_{i,j\in\{1,2\}}. \end{aligned}$$

Then

$$Q(s,\rho) = \begin{bmatrix} \sigma_0(\rho) & s\sigma_{12}(\rho) \\ s\sigma_{21}(\rho) & \sigma_1(\rho) \end{bmatrix} \quad (13)$$

and this results in the following upper bounds on the off-diagonal terms at low frequencies.

$$\begin{aligned} |H_{12}(jw)| &\le h_{12} = |\sigma_{12}k_{I_{(2)}}| M_{s1} M_{s2} \\ |H_{21}(jw)| &\le h_{21} = |\sigma_{21}k_{I_{(1)}}| M_{s1} M_{s2} \end{aligned} \quad (14)$$

Also, it is noted that at low frequencies, $H_{11}(\rho) \approx 1$, $H_{22}(\rho) \approx 1$ since, e.g.

$$H_{11}(\rho) = \frac{q_{11}\bar{c}_1}{1 + q_{11}c_1} \approx \frac{\sigma_0 k_{I_{(1)}}/s}{1 + \sigma_0 k_{I_{(1)}}/s} = \frac{\sigma_0 k_{I_{(1)}}}{s + \sigma_0 k_{I_{(1)}}} \approx 1$$

It should be noted that the upper bounds presented in (14) will become conservative if the bandwidths associated with the two loops are significantly different.

The results presented in this subsection apply to the SVD decomposition-based LPV decoupling. Taking into account the static inverse-based LPV decoupling along with the pre- and post-compensators as $W_1(\rho) = G^{-1}(s = 0, \rho) \triangleq \mathcal{D}_0$ and $W_2 = I$ in (10) and following the same lines as in the SVD-based decoupling case results in a set of similar results to (14) except that the diagonal elements of matrix Q in (13) are now equal to one.

V. NUMERICAL EXAMPLE

In this section, we apply the design procedures presented in the previous section to decouple and control multivariable systems. The presented example, which is a parameter-varying 2×2 transfer function, is given to validate the LPV decoupling procedure along with adaptive PI control design method based on the interaction measure presented in this paper.

For this example, we follow the theoretical results presented in the previous section to design a parameter-dependent decoupled PI controller. Consider the multivariable LPV system represented by the following 2×2 transfer function matrix.

$$G(s, \rho) = \begin{bmatrix} \frac{\rho^2}{s+\rho} & \frac{\rho}{s+1} \\ \frac{\rho}{s+\rho} & \frac{2}{s+1} \end{bmatrix} \quad (15)$$

A difficulty to control this system is that the open-loop system's bandwidth heavily depends on the parameter ρ, and standard controller design methods such as QFT will result in conservative designs. We will, however, show that by taking advantage of the measurement of the LPV parameter, this can be alleviated. It is readily verified that the system has no RHP zero, and that after static LPV decoupling the decoupled system transfer function becomes

$$Q(s, \rho) = \begin{bmatrix} \frac{(2\rho-1)s+\rho}{(s+1)(s+\rho)} & \frac{\rho(1-\rho)s}{(s+1)(s+\rho)} \\ \frac{2(\rho-1)s}{\rho(s+1)(s+\rho)} & \frac{(2-\rho)s+\rho}{(s+1)(s+\rho)} \end{bmatrix}.$$

The interaction values can be calculated to be

$$\begin{aligned} \sigma_{12} &= 1 - \rho \\ \sigma_{21} &= \frac{2(\rho - 1)}{\rho^2}. \end{aligned}$$

It is noted that for $\rho = 1$, the decoupling becomes perfect in the sense that there is no interaction between the off-diagonal terms, in which case, the SISO control design is a trivial task. Throughout the rest of the example under study, we focus on the case where $\rho \neq 1$. Requiring the couplings h_{12} and h_{21} to be 0.2 and the maximum sensitivities M_{s1} and M_{s2} to be 0.45, the integral gains are determined from

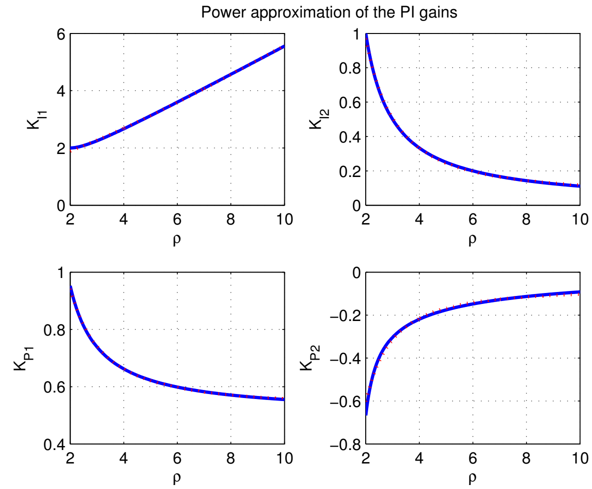

Fig. 1. Power approximation ($K_{P/I} = a\rho^b + c$) of the PI control gains: solid lines and dotted lines represent the original data and the power fit outputs, respectively.

$$\begin{aligned} k_{I_{(1)}} &= \frac{\rho^2}{2|\rho - 1|} \\ k_{I_{(2)}} &= \frac{1}{|\rho - 1|} \end{aligned}$$

To determine the proportional gains in the PI controllers for the two loops, we use a direct pole placement [1]. It is noted that the position of the poles will be dependent on the LPV parameter ρ and it is ensured that the poles are all placed in the LHP for any ρ. Following a parameter-dependent procedure for the pole-placement, we obtain that

$$\begin{aligned} k_{p_{(1)}} &= \frac{\rho}{2\rho^2 - 1} + \frac{\rho^2}{2|\rho - 1|(\rho + 1)} \\ k_{p_{(2)}} &= \frac{\rho}{2 - \rho^2} + \frac{1}{|\rho - 1|(\rho + 1)} \end{aligned}$$

In order to obtain simpler expressions for the PI gains, we solve a nonlinear least-square problem to fit power approximations in the form of the $y = ax^b + c$ to the PI gains. Figure 1 illustrates the comparison between the PI control gains and their approximations using a two-term power equation.

Next, we show the time-domain simulation results to compare the closed-loop performance of two scenarios using: (i) LPV decoupling where the PI control gains are not parameter-dependent, and (ii) LPV decoupling, where in addition to the decoupling matrices, the PI control gains in both loops are also scheduled based on the measurement of the LPV parameter ρ. Note that the latter one uses the fitting results discussed earlier in this section.

Figure 2 represents the outputs y_1 and y_2 of the closed-loop system resulted from the interconnection of the open-loop LPV system $G(s, \rho)$, the LPV decoupling compensator and the PI controller. As observed, the closed-loop system

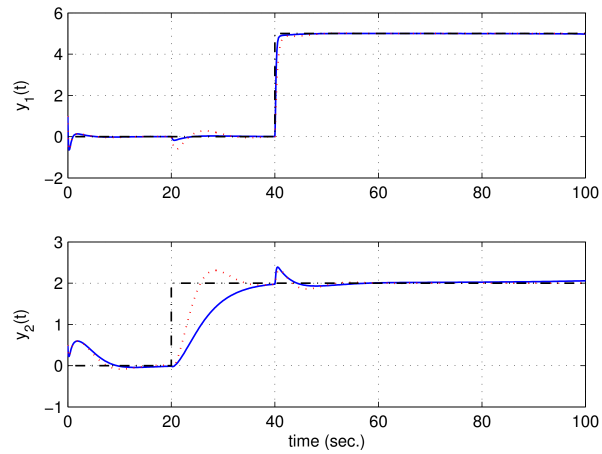

Fig. 2. Output signals of the closed-loop system using adaptive controllers (solid line) and fixed gain PI controllers (dotted line)

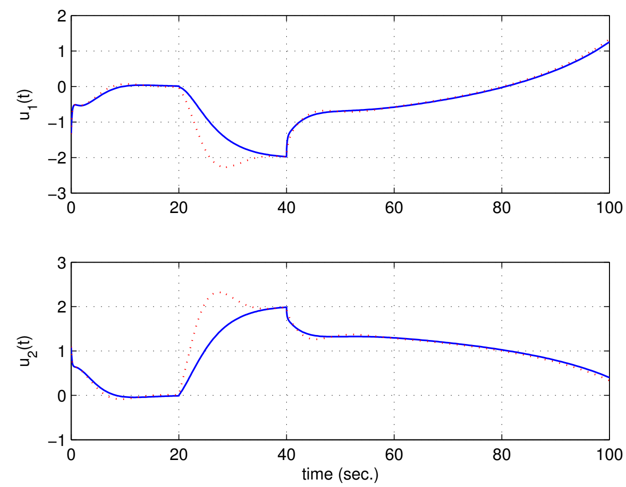

Fig. 3. PI controller output signals using adaptive controllers (solid line) and fixed gain controllers (dotted line)

using the PI controllers whose gains are adapted by the LPV parameter ρ outperforms the one with fixed control gains. Shown in Figure 3 is the comparison between the control signals u_1 and u_2 of the two control design schemes. Calculations show that the average control input u_1 and u_2 for the adaptive control strategy is 0.87 and 1.44, respectively, and for the fixed control gain is 1.33 and 1.91, respectively. The results presented verify that the adaptive PI control scheme demonstrates improved tracking performance, and at the same time, less control effort compared to the fixed gain controllers. The time-domain simulation performed in this section uses the LPV parameter profile shown in Figure 4.

VI. Conclusion

The paper presents an effective decoupling method for addressing nonlinearities and changes in operating conditions in a multivariable system. The proposed control design method for such systems uses a two-step procedure: first,

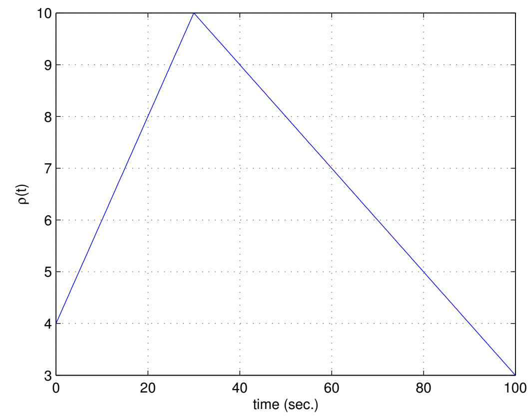

Fig. 4. Profile of the LPV parameter $\rho(t)$

we use parameter-dependent transformations, adapted in real-time, to reduce the coupling between the undesired set of inputs and outputs, and then for the decoupled system, we design SISO controllers (fixed or with adaptive gains). The methods extend the standard decoupling design results in the literature to the LPV case. Systematic post-design LPV system stability and performance analysis can be conducted for the closed-loop system to validate the designs. The paper also presents a new interaction measure that, in conjunction with the proposed LPV decoupling technique, can be used for the adaptive controller design. The computational example presented in the paper validates the benefits of the proposed LPV decoupling methodology for nonlinear and parameter-varying systems compared to standard decoupling methods.

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