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A Model-Based Predictive Supervisory Controller for Multi-Evaporator HVAC Systems

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Abstract—**Multi-evaporator vapor compression cooling systems are representative of the complex, distributed nature of modern HVAC systems. Earlier research efforts focused on the development of a decentralized control architecture for individual evaporators that exploits the constraint-handling capabilities of model predictive control while regulating the pressure and cooling setpoints. This paper presents a global controller that generates the setpoints for the local controllers; this controller balances the goals of cooling zone temperature tracking with optimal energy consumption. To accommodate the inherent limitations of the system, a Model Predictive Control (MPC) based approach is used. The improved efficiency and the effects of the tuning parameters are demonstrated upon an experimental system.**

I. Introduction

Vapor compression cooling (VCC) cycles are the primary means of mechanical cooling today; they can be found in settings ranging from household air conditioners to supermarket cases. Multiple evaporator systems are a variant of the basic VCC system that allow different amounts of cooling at different temperatures to be delivered to different regions in the same overall system, such as apartment units or large office buildings, with fewer losses from transporting chilled water. These multi-evaporator systems present significant control challenges due to cross-coupling of the dynamics of the evaporators; the application of advanced control strategies to these systems has the potential for increases in energy efficiency and significant worldwide cost savings.

Research has been performed exploring the relationship between system setpoints and energy optimization [1]. This is especially true of multiple evaporator, variable refrigerant flow systems, where the correct combination of control inputs can have significant impact on energy consumption for a given amount of cooling performed ([2],[3]).

An earlier paper by the authors presented a method of controlling a multiple evaporator system using a decentralized approach [4]. In this approach, the cooling capacity of each evaporator is regulated by its own multiple-input, multiple output (MIMO) MPC controller, which tracks a cooling setpoint while keeping superheat inside a narrow band. Additionally, the compressor and discharge valves use single-input, single-output (SISO) PI controllers to regulate system pressures. The separation of time scales of the various dynamics, as well as the variation in gains between the inputs and outputs, allows this configuration. The decentralized approach has the benefit of decoupling the system dynamics, and is modular; thus, the control system can be extended to an arbitrary number of evaporators without requiring individual controller redesign. Fig. 1 is a block diagram of the control architecture developed. The novelty of this paper lies in the development of a Global Controller for this decentralized architecture that will seek the optimal pressure and cooling setpoints for each evaporator, balancing energy consumption with setpoint tracking of a cooling zone temperature, while respecting the system's operational constraints. This combination of requirements suggests an MPC approach.

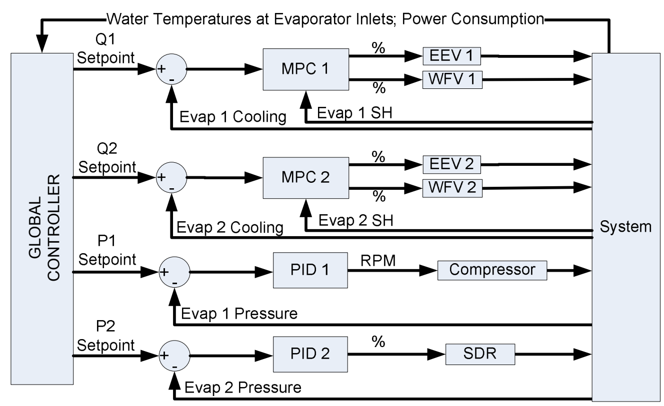

Fig. 1. Local Control Architecture

The term model predictive control (MPC) refers to a suite of control strategies that use an explicit model of the physical system to derive the set of controller actions that minimize a cost function, subject to a set of constraints. At each sampling instant, the controller selects the control actions that minimize the user-defined cost function over a prediction horizon. This process is repeated at each sampling instant, as the prediction horizon recedes. Since MPC can explicitly account for constraints, system performance can be improved over the long term [5]. MPC also has the advantage that additional constraints can be defined by the user to keep the system operating in a safe range, e.g., keeping evaporator superheat above a desired minimum. MPC has been adapted to HVAC systems as a system governor ([6], [7]), and used to control cooling of a single evaporator system [8] as well as a multiple evaporator system [9].

The next part of the paper is a review of vapor compression cycles, and an overview of the experimental apparatus used. This is followed by derivation of the global

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controller. Finally, experimental results displaying the efficacy of the control approach are discussed.

II. Background on Vapor Compression Cycles

A. *Generic Multi-Evap System*

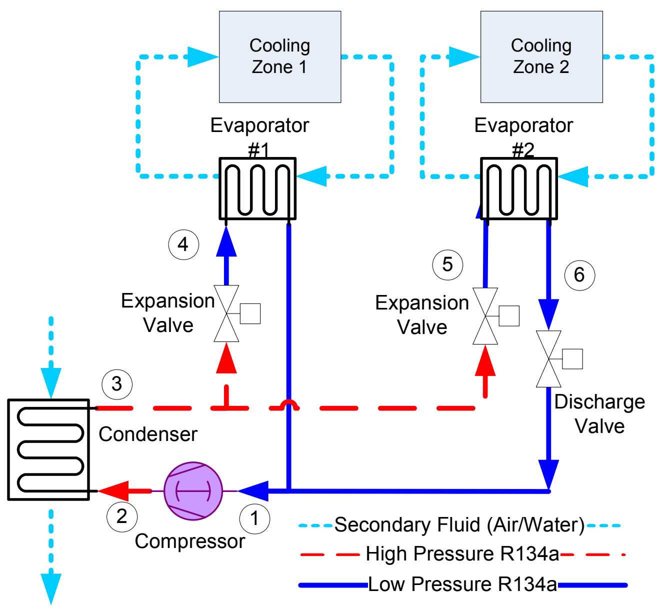

Fig. 2. General Multi-Evaporator System with designated states.

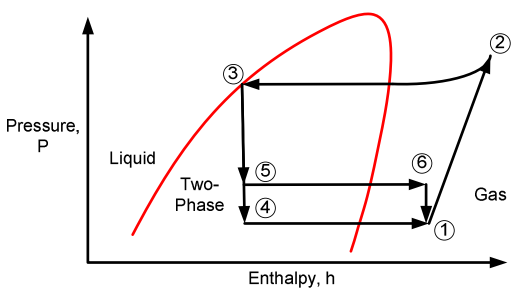

Fig. 3. Pressure-enthalpy (P-h) diagram

A two-evaporator vapor compression cycle is shown in Fig. 2; the Pressure-enthalpy (P-h) diagram of the system shown in Fig. 3 is adapted from the single evaporator system detailed in [10]. The first stage of the thermodynamic cycle is at the inlet of the variable speed compressor, where refrigerant is in a low pressure, gaseous state (denoted as state 1 in Fig. 2). The compressor adds energy to the fluid by compressing it to a high pressure, high temperature gas (state 2). This gas passes into the condenser, where heat energy is rejected from the refrigerant to the secondary fluid (water). This causes the refrigerant to condense to a high pressure liquid. A receiver at the end of the condenser ensures that the refrigerant becomes a saturated liquid (state 3). This saturated liquid is fed into a set of expansion valves, which meter the refrigerant flowing into the evaporators. The refrigerant is now a two-phase fluid (states 4 and 5). This two phase fluid absorbs heat from the water entering the evaporators, chilling the water and causing the refrigerant to evaporate. This low pressure gas exits the evaporators and returns to the compressor. The discharge valve on the secondary evaporators creates a pressure differential between evaporators, thus allowing them to provide cooling at different saturation temperatures.

B. *Experimental System*

For the research detailed in this paper, a two-evaporator water chiller test apparatus is used. Electronic expansion valves (EEVs) and an electronic discharge valve (SDR) are controlled by the user. Additionally, a variable speed compressor is used. Water flow valves (WFVs) regulate the flow of the secondary fluid through the evaporators; this chilled water is returned to the cooling zone. This arrangement simulates the operation of an air conditioner or refrigeration system, where air is cooled and returned to the chamber whose air temperature is being regulated. The refrigerant used is R134a, which is an HCFC widely used in automotive and industrial systems.

In order to measure system properties, including regulated variables, transducers are placed at salient points of the thermodynamic cycle. Pressure transducers are placed at the outlet of each evaporator and the condenser to measure the saturation pressures of the refrigerant. Thermocouples are immersed in the refrigerant flow at the inlet and outlet of each evaporator and the condenser. These temperature and pressure measurements allow computation of fluid properties such as enthalpy and density at the relevant points of the cycle. In order to measure mass flow of refrigerant through each evaporator, the calculated fluid densities are used in conjunction with turbine-type volumetric flow meters placed at the inlet of each EEV. These measurements allow on-line computation of cooling and superheat of each evaporator.

III. Global Controller

The role of the global controller is to regulate the cooling zone temperatures while seeking to maximize the energy efficiency of the system as a whole. In order to balance the competing goals of energy efficiency optimization and temperature regulation, weights specified by the user will govern the controller decisions. For example, if a very large weight is placed on the water temperature error relative to the weight placed on energy consumption of the plant, the controller will make decisions that will bring the temperature to its setpoint as quickly as possible, regardless of energy consumption. Similarly, if energy consumption is specified as very "expensive," the controller will minimize energy consumption and the zone temperatures will reach their setpoints slowly, or have a non-zero steady state error. In addition, this global controller must be easily expanded so that a similar approach could be used for systems with more than two evaporators. This continues the modular, networked approach pursued in the development of the local level controllers. Finally, the global controller must take into account the local level constraints of the various components; for example, the maximum amount of cooling that one evaporator is capable of delivering. The combination of tunable control weights and constraint handling again points toward adaptation of an MPC-based control algorithm for the global law.

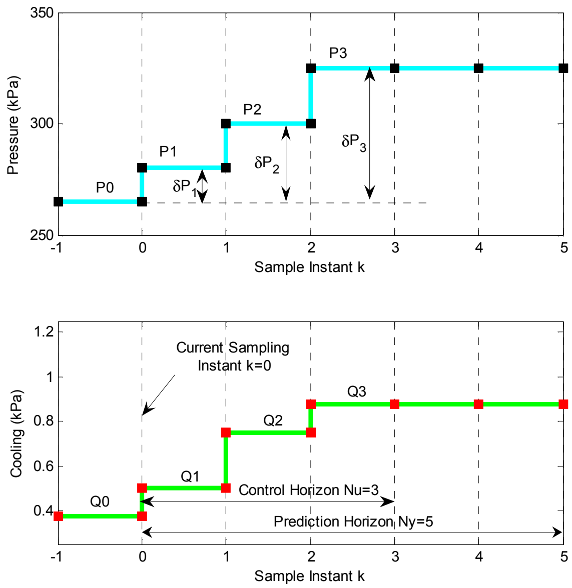

Fig. 3 Global controller conventions

TABLE I
GLOBAL CONTROLLER TERMS

Term	Description
Nu	control horizon
$^{n}\delta Q_i$	change in cooling for the nth evaporator at the ith future sampling instant
$^{m}\delta P_j$	change in pressure for the mth evaporator at the jth future sampling instant
H	positive definite matrix defined by system parameters
f	column vector defined by system parameters
A	constraint matrix
b	constraint vector

The heart of the global controller is a quadratic programming (QP) problem that seeks to minimize a constrained cost function, similar to the generalized predictive control (GPC) algorithm [11]. The vector space over which the minimization occurs is the set of all feasible changes in the local controllers' cooling and pressure setpoints over a control horizon. Due to the large time scale separation between the fast refrigerant dynamics and the slow water temperature dynamics, the global controller can ignore the transients occurring at the local controller level, treating the changes in evaporator cooling and pressure as instantaneous. Similarly, the local controllers will treat the cooling zone temperatures as constant. A lumped-capacitance model of the cooling zones is explicitly included in the QP problem. Hence, the solution of the QP problem will be the cooling and pressure setpoints for the evaporators that will balance the efficiency with the cooling zone temperature regulation, according to the weights specified by the user. The cost function is shown in (1) and (2); Fig. 3 and Table I describe the terms contained therein. The vector x is defined as the changes in the control inputs.

$$\min_x J = \frac{1}{2}x^T H x + f^T x \qquad Ax \le b \tag{1}$$

$$x \equiv \Delta U = [{}^1\Delta Q|{}^2\Delta Q|{}^1\Delta P|{}^2\Delta P\,]^T \tag{2}$$
$${}^{\mathrm{n}}\Delta Q = [{}^{\mathrm{n}}\delta Q_1 \ldots {}^{\mathrm{n}}\delta Q_{\mathrm{Nu}}]^{\mathrm{T}}$$
$${}^{\mathrm{m}}\Delta P = [{}^{\mathrm{m}}\delta P_1 \ldots {}^{\mathrm{m}}\delta P_{\mathrm{Nu}}]^{\mathrm{T}}$$

A. Energy Efficiency Function

The first step in devising this global control law is developing a function that expresses energy efficiency as a function of the operating conditions of the evaporators; if this function alone were used to determine the setpoints of the local controllers, it would select the pressure (P) and cooling (Q) setpoints for each evaporator that would minimize this function.

A common way of expressing the efficiency of VCC cycles is the Coefficient of Performance (COP), normally defined as the ratio of cooling performed by the system to the total work input required by the system [10]. Since the COP increases with increasing efficiency, and the global control law being developed seeks to minimize a cost function, an Inverse Coefficient of Performance (ICOP), denoted Φ, will be used:

$$\Phi({}^1Q, {}^2Q, {}^1P, {}^2P) = \frac{W_{COMP} + {}^1W_{FAN} + {}^2W_{FAN}}{{}^1Q + {}^2Q} \tag{3}$$

A relationship for compressor power consumption (W_{COMP}) as a function of evaporator pressures and cooling rates was developed experimentally. On a single evaporator, the EEV and compressor were slowly "walked" through their operating ranges—simulating steady state conditions—while cooling and power consumption were measured, and the inlet temperature was fixed. This allowed a curve fit function to be generated for power consumption as a function of evaporator pressure and cooling.

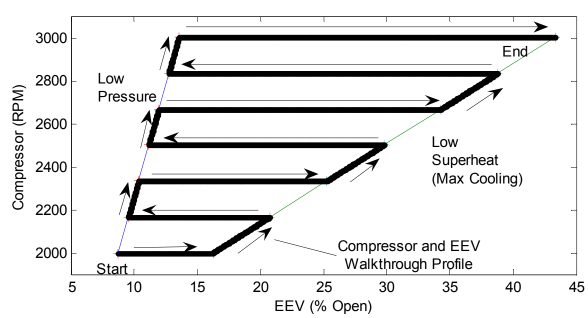

Fig. 4. Compressor/EEV Walkthrough. This allowed an expression for compressor power as a function of cooling and pressure to be developed.

In the experimental apparatus used, no energy is consumed to move the water through the evaporators, since all of the heat exchangers are gravity fed. However, this is not typical of most VCC systems, which generally use fans or pumps to create secondary fluid flow. Therefore, an energy consumption term (expressed as fan work, W_{FAN}) for the water flow was developed in order to emulate an actual

VCC system. The WFV and compressor for one evaporator were walked through a profile similar to that seen in Fig. 4, while the EEV was used with a PID loop to keep superheat low and constant at 6°C. By fixing superheat and the inlet water temperature, the required secondary fluid flow is a function of the evaporator cooling and pressure. A representative fan power curve is used, allowing the fan power to be expressed as an experimentally developed function of evaporator pressure and cooling, assuming a fixed, low superheat. Since cooling and pressure are measured directly, the ICOP for the system at any moment can be calculated using the above experimentally developed function. A Taylor series expansion puts (3) into a form that can be used in the cost function:

$$\Phi \approx \Phi_0 + \frac{\partial \Phi}{\partial({}^1Q)}\delta({}^1Q) + \frac{\partial \Phi}{\partial({}^2Q)}\delta({}^2Q) + \frac{\partial \Phi}{\partial({}^1\mathrm{P})}\delta({}^1\mathrm{P}) + \frac{\partial \Phi}{\partial({}^2\mathrm{P})}\delta({}^2\mathrm{P}) \quad (4)$$

Assuming that the ICOP function gradient will not change over the control horizon allows restatement of (4) as a linear function in ΔU:

$$\Phi \approx \Phi_0 + \left[\frac{\partial \Phi}{\partial({}^1Q)} w_1^T \middle| \frac{\partial \Phi}{\partial({}^2Q)} w_1^T \middle| \frac{\partial \Phi}{\partial({}^1P)} w_1^T \middle| \frac{\partial \Phi}{\partial({}^2P)} w_1^T\right] \Delta U \quad (5)$$

Where: $w_1 = [1 \quad \cdots \quad 1]^T{}_{N_u \times 1}$

B. Setpoint Error Terms

If the developed ICOP function were the only consideration for the control law, the system would simply operate at the conditions that minimize energy consumption, which implies not running the system at all. However, VCC systems have a primary role of regulating the temperature of a specified cooling zone (CZ), e.g., a room or chamber. Therefore, the error—defined in this case as the difference between the desired and actual temperatures of the individual cooling zones—must also play a role in the development of the control law. The error definition for the i[th] evaporator is:

$$^ie_0 = {}^iT_s - {}^iT_0 \quad (6)$$

In this equation, T_s is the water temperature setpoint, T_0 is the temperature at the current sampling instant, and e_0 is the error at the current sampling instant, assuming a lumped capacitance model of the zone. At each future instant, the future errors will change linearly with respect to the future levels of cooling:

$$\begin{cases} {}^ie_0 = {}^iT_s - {}^iT_0 \\ {}^ie_1 = {}^iT_s - {}^iT_0 + \frac{k}{mc}\,{}^iQ_1 \\ \cdots \\ {}^ie_{N_y} = {}^iT_s - {}^iT_0 + \frac{k}{mc}\left(\left(\sum_{j=1}^{Nu}{}^iQ_j\right) + \left(N_y - N_u\right){}^iQ_{N_u}\right) \end{cases} \quad (7)$$

In these equations, m is the mass of the water in the CZ, k is the sample time, and c is the specific heat of water. As before, N_u is the control horizon and N_y is the output prediction horizon. This set of equations can be expressed in vector form as:

$$^ie = {}^ie_0 v_1 + M\,{}^iQ \quad (8)$$

Where:

$$^ie = \left[{}^ie_1 \quad \cdots \quad {}^ie_{N_y}\right]^T$$

$$v_1 = [1 \quad \cdots \quad 1]^T{}_{N_y \times 1} \qquad {}^iQ = \left[{}^iQ_1 \quad \dots \quad {}^iQ_{N_u}\right]^T$$

$$M = \frac{k}{mc}\begin{bmatrix} 1 & 0 & \cdots & \dots & 0 \\ 1 & 1 & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & \dots & 1 & 2 \\ 1 & 1 & \vdots & 1 & \vdots \\ 1 & 1 & \cdots & 1 & 1 + Ny - Nu \end{bmatrix}$$

(Now define a positive tuning weight, λ_e, which assigns a weight to the predicted CZ temperature error. Multiplying this scalar by the identity matrix results in a symmetric, nonsingular, positive definite weighting matrix R:

$$R = \lambda_e I_{N_y \times N_y} \quad (9)$$

Define the terms of the $^i\boldsymbol{Q}$ vector as the variations from the initial condition $^iQ_0$:

$$\begin{cases} \begin{bmatrix} {}^iQ_1 \\ \vdots \\ {}^iQ_{N_u} \end{bmatrix}_{N_u \times 1} = {}^iQ_0 \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{N_u \times 1} + \begin{bmatrix} {}^i\delta Q_1 \\ \vdots \\ {}^i\delta Q_{N_u} \end{bmatrix} \\ {}^iQ = {}^iQ_0 w_1 + {}^i\Delta Q \end{cases} \quad (10)$$

Squaring the error vector $^i\boldsymbol{e}$, scaling it with the weighting matrix R, and substituting (8) and (10) yields a quadratic term (11). Thus we now have a quadratic function in ΔQ that is equal to the weighted sum of squares of the water temperature errors over the prediction horizon of the controller; each evaporator will have one of these equations.

$$^ie^T R\,{}^ie = {}^i\varepsilon + {}^if_e^T({}^i\Delta Q) + \tfrac{1}{2}\,{}^i\Delta Q^T({}^iH_e){}^i\Delta Q \quad (11)$$

Where:

$$^i\varepsilon = {}^ie_0^2 v_1^T R v_1 + 2\,{}^ie_0\,{}^iQ_0 v_1^T R M w_1 + {}^iQ_0^2 w_1^T M^T R M w_1$$

$$^if_e^T = 2\,{}^ie_0 v_1^T R M + 2\,{}^iQ_0 w_1^T M^T R M$$

$$^iH_e = 2M^T R M$$

C. Assembly of Cost Function

In the earlier sections, we derived terms for the Inverse Coefficient of Performance and the water temperature errors that were either linear or quadratic in terms of the changes in control inputs, where the changes are measured from the command profile at the sampling instant of the global controller. Now these terms must be assembled into a cost function that will be used by the global controller. As with any controller, a smooth control profile is desirable; that is, some damping on the controller is necessary so that the cooling and pressure setpoints do not oscillate wildly. This is achieved by adding slew rate weights λ_Q and λ_P to the quadratic matrix; these are assigned to changes in cooling and pressure, respectively, and chosen in an ad hoc manner. Note that λ_P must be greater than zero, so that the non-singularity of the quadratic matrix $\boldsymbol{H}$ is retained. Additionally, define a weight λ_Φ that penalizes energy consumption; the ICOP gradient terms are multiplied by this weight.

Equation (12) is the quadratic cost function that the global controller will minimize; it balances the cooling demand with energy efficiency according to the weights assigned to each by the user, and can be tuned to achieve desired performance characteristics.

$$J = \frac{1}{2}\,\Delta U^T \mathrm{H} \Delta U + \mathrm{f}\Delta U \tag{12}$$

Where:

$$\mathrm{H} = \begin{bmatrix} {}^1H_e + \lambda_Q I & 0 & 0 & 0 \\ 0 & {}^2H_e + \lambda_Q I & 0 & 0 \\ 0 & 0 & \lambda_P I & 0 \\ 0 & 0 & 0 & \lambda_P I \end{bmatrix}$$

$$\mathrm{f} = \Big[{}^1f_e^T + \lambda_\phi \frac{\partial \Phi}{\partial({}^1\delta Q)} w_1^T,\quad {}^2f_e^T + \lambda_\phi \frac{\partial \Phi}{\partial({}^2\delta Q)} w_1^T,\quad \frac{\partial \Phi}{\partial({}^1\delta P)} w_1^T,\quad \frac{\partial \Phi}{\partial({}^2\delta P)} w_1^T\Big]$$

IV. Experimental Results

A. Test 1: Baseline Test

For the first test shown, the entire control system was run to bring the water temperature down to a desired setpoint as shown. During this test, a small amount of warm water was continuously mixed into the evaporator water; due to the system construction, the same amount of mixed evaporator water overflows into the condenser water tank, allowing the mass of water in the CZ to be kept constant. This has the effect of adding a disturbance to the plant; namely, heating at an unknown rate is added to the water in addition to the cooling performed by the system. For this test, the parameters are found in Table II. The system constraints imposed are due to the feasible operational ranges of the system, and are found in Table III. Fig. 5 displays some of the functionality of the control architecture. At the beginning of the experimental run, the zone 1 error is approximately 3 °C; as the water is chilled, the error decreases, and the global controller begins to allow cooling to decrease, as the dominance of the error term in the cost function decreases. At approximately 1200 seconds, the setpoint is decreased, increasing the error significantly; this time, the controller responds by increasing cooling to the maximum value allowed. Again, as the error decreases, the cooling setpoint decreases, resulting in an asymptotic approach to final temperature by the cooling zone temperature. Cooling zone 2 shows a different example of the controller behavior during the same run. As the setpoint is increased at approximately 1100 seconds, the controller reduces cooling to its minimum of 0.6 kW. Due to the disturbance added, which is estimated to be approximately 0.6 kW, the cooling zone temperature remains almost constant throughout the run. This displays the effect of the weight attached to the ICOP term in the cost function; the steady state error of approximately 1 °C is acceptable to the controller.

TABLE II
TEST 1 PARAMETERS

Term	Description	Value
Ts	sample time	150 s
Nu	control horizon	3
Ny	prediction horizon	5
λ_e	temperature error weight	1.0
λ_Φ	ICOP weight	2000
λ_Q	cooling slew weight	4.0
λ_P	pressure slew weight	4.0

TABLE III
SYSTEM CONSTRAINTS

Variable	Value
evaporator maximum cooling	1 kW
evaporator minimum cooling	0.6 kW
evaporator maximum pressure	350 kPa
evaporator minimum pressure	200 kPa
maximum pressure slew rate	40 kPa/sample
max. evap pressure differential, P2-P1	100 kPa
min. evap pressure differential, P2-P1	0 kPa

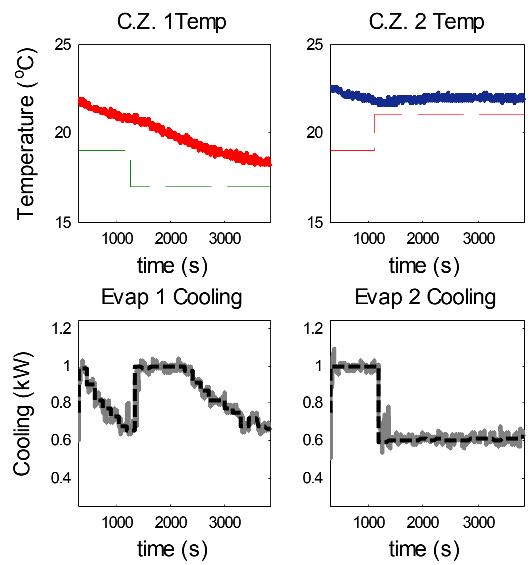

Fig. 5. Test 1 cooling zone temperatures and cooling. The top two graphs show the temperature of the two zones with their respective setpoints (as specified to the global controller). The bottom two graphs show the evaporator cooling for the respective zones (solid lines), along with the cooling setpoints generated by the global controller (dashed lines). The cooling setpoints are changed every 150 seconds; this illustrates the separation of time scales between the dynamics of the refrigerant cycle and the water temperature.

B. Test 2: Reduced Efficiency Weight

As noted, the global controller in Test 1 allowed a steady state error of 1 °C in the cooling zone temperature. For the second test, the weights of the global MPC controller were adjusted; namely, the weight associated with power consumption was decreased from 2000 to 750. In physical terms, this means that the controller will tolerate less steady state error before allowing the evaporator cooling setpoints to come to their minimum values. The temperature spikes at approximately 5000 and 10000 seconds are the result of a warm water disturbance being added, simulating an additional load being placed upon a cooling system. Fig. 6 shows the results of this test.

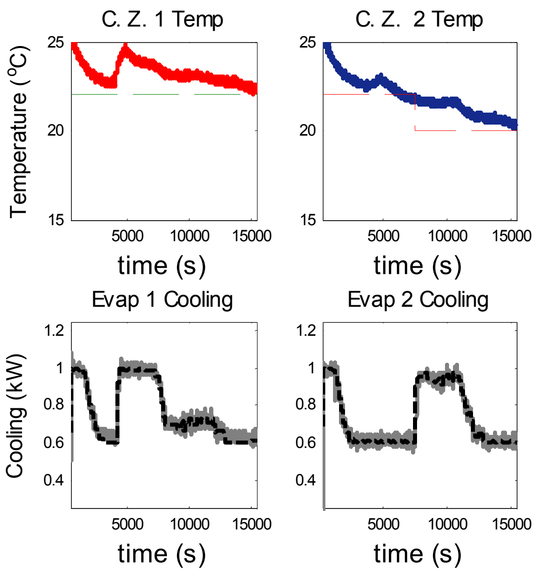

Fig. 6. Test 2 cooling zone temperatures and cooling. Note the presence of a disturbance at approximately 5000 and 10000 seconds.

C. Test 3: Energy Efficiency Seeking

The purpose of this test is to provide some verification that the controller will move the system from a randomly chosen set of operating points to a more energy efficient one. At the start of this test, the system was allowed to come to a steady state where the evaporator cooling and the disturbance were balanced, thereby maintaining a constant water temperature. This condition represents a scenario where zone temperatures are properly regulated, but the system efficiency could be improved. Then the global controller was activated at approximately 1200 seconds, which brought the pressure and cooling setpoints to a more efficient operating condition. Fig. 7 shows the 9.5% increase in COP as a result of the controller's actions.

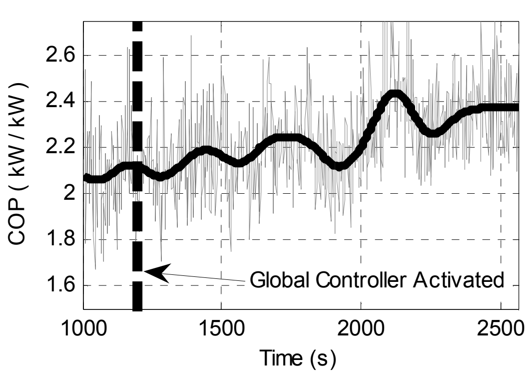

Fig. 7. Test 3 Coefficient of performance

V. Future Work

This MPC-based approach is capable of many extensions based upon the specifics of the application. For example, the individual evaporators' weights can be varied to allow prioritization of cooling demands. Additionally, the controller can ignore cooling requirements that exceed capacity, and allocate the available cooling among the evaporators; this is an important idea known as demand shedding. The tunable weights and system constraints can be time-varying; this allows demand limiting based upon projected power loads or variable electricity pricing.

Future research will seek a first-principles based derivation of the ICOP function, constructed from manufacturer supplied information about the heat exchangers, fans, compressors, etc. These models will also account explicitly for the superheat and cooling zone temperatures—which were assumed to be constant for the function developed here—and will eliminate the need for extensive experimental tests. The end result will be a global controller that accounts for variations in the secondary fluid temperatures, which will allow for better decisions with respect to energy consumption of the system.

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