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Mobile Sensor Networks for Learning Anisotropic Gaussian Processes

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Abstract— **This paper presents a novel class of self-organizing sensing agents that learn an anisotropic, spatio-temporal Gaussian process using noisy measurements and move in order to improve the quality of the estimated covariance function. This approach is based on a class of anisotropic covariance functions of Gaussian processes developed to model a broad range of anisotropic, spatio-temporal physical phenomena. The covariance function is assumed to be unknown a priori. Hence, it is estimated by the maximum likelihood (ML) estimator. The prediction of the field of interest is then obtained based on a non-parametric approach. An optimal navigation strategy is proposed to minimize the Cramér-Rao lower bound (CRLB) of the estimation error covariance matrix. Simulation results demonstrate the effectiveness of the proposed scheme.**

I. Introduction

In recent years, due to drastic global climate changes, it is necessary to monitor the changing ecosystems over vast regions in lands, oceans, and lakes. For instance, for certain environmental conditions, rapidly reproducing harmful algal blooms in the Great Lakes can produce cyanotoxins [1]. Wildland fires in dry seasons have been one of the devastating natural disasters destroying valuable natural resources [2]. Besides such natural disasters, there exist growing ubiquitous possibilities of the release of toxic chemicals and contaminants in the air, lakes, and public water systems. Hence, there are strong motivations to develop autonomous robotic systems that can perform a series of tasks such as estimation, prediction, monitoring, tracing and removal of a scalar field undergoing often complex transport phenomena[1] in a region of interest.

Significant enhancements have been made in the areas of mobile sensor networks and mobile sensing vehicles such as unmanned ground vehicles, autonomous underwater vehicles, and unmanned aerial vehicles. Emerging technologies have been reported on the coordination of mobile sensing agents [3], [4], [5], [6], [7], [8]. Mobile sensing agents form an ad-hoc wireless communication network in which each agent usually operates under a short communication range, with limited memory and computational power. Mobile sensing agents are often spatially distributed in an uncertain surveillance environment.

The mobility of the mobile agents can be designed in order to perform the optimal sampling of the field of interest. Recently distributed interpolation schemes for field estimation by mobile sensor networks are developed by [9]. Swarming agents with a gradient climbing strategy for tracking peaks of a field of interest using kernel regression were proposed by [8], [10]. In general, we design the mobility of sensing agents to find the most informative locations to make observations for a spatio-temporal phenomenon.

To find these locations that predict the phenomena best, one needs a model of the spatio-temporal phenomenon itself. In our approach, we focus on Gaussian processes to model fields undergoing transport phenomena. A Gaussian process (or Kriging in geostatistics) has been widely used as a nonlinear regression technique to estimate and predict geostatistical data [11], [12], [13], [14], [15]. A Gaussian process is a natural generalization of the Gaussian probability distribution. It generalizes the Gaussian distribution with a finite number of random variables to a Gaussian process with an infinite number of random variables in the surveillance region. Gaussian process modeling enables us to predict physical values, such as temperature and plume concentration, at any of spatial points with a predicted uncertainty level efficiently. For instance, near-optimal static sensor placements with a mutual information criterion in Gaussian processes were proposed by [16], [17]. Distributed Kriged Kalman filter for spatial estimation based on mobile sensor networks are developed by [18]. Asymptotic optimality of multicenter Voronoi configurations for random field estimation is reported by [19]. Multi-agent systems that are versatile for various tasks by exploiting predictive posterior statistics of Gaussian processes were developed by [20], [21].

The motivation of our work is as follows. Even though, there have been efforts to utilize Gaussian processes to model and predict the spatio-temporal field of interest, most of recent papers assume that Gaussian processes are isotropic implying that the covariance function only depends on the distance between locations. Many studies also assume that the corresponding covariance functions are known a priori for simplicity. However, this is not the case in general as pointed out in literature [22], [16], [17], in which they treat the non-stationary process by fusing a collection of isotropic spatial Gaussian processes associated with a set of local regions. Hence our motivation is to develop theoretically-sound algorithms for mobile sensor networks to learn the anisotropic covariance function of a spatio-temporal Gaussian process. Mobile sensing agents can then predict the Gaussian process based on the estimated covariance function in a non-parametric manner.

The contribution of this paper is to develop covariance function learning algorithms for the sensing agents to perform non-parametric prediction based on a properly adapted Gaussian process for a given spatio-temporal phenomenon. By introducing a generalized covariance function, we expand

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[1]Common examples are diffusion, convection, and advection.

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the class of Gaussian processes to include the anisotropic spatio-temporal phenomena. Maximum likelihood optimization is used to estimate hyperparameters for the associated covariance function. The proposed optimal navigation strategy for autonomous vehicles will maximize the determinant of the Fisher Information Matrix [23], improving the quality of the estimated covariance function.

This paper is organized as follows. In Section II, we briefly review the mobile sensing network model and the notation related to a graph. A non-parametric approach to predict a field of interest based on measurements is presented in Section III. Section IV introduces a covariance function learning algorithm for an anisotropic, spatio-temporal Gaussian process. An optimal navigation strategy is described in Section V. In section VI, we summarize our scheme by a protocol for mobile sensor networks. In section VII, simulation results illustrate the usefulness of our approach.

The standard notation will be used in the paper. Let $\mathbb{R}, \mathbb{R}_{\geq 0}, \mathbb{Z}$ denote, respectively, the set of real, non-negative real, and integer numbers. The positive semi-definiteness of a matrix A is denoted by $A \succeq 0$. $\mathbb{E}$ denotes the expectation operator.

II. Mobile Sensor Networks

First, we explain the mobile sensing network and sensor models used in this paper. Let N_s be the number of sensing agents distributed over the surveillance region $\mathcal{M} \in \mathbb{R}^2$. Assume that $\mathcal{M}$ is a compact set. The identity of each agent is indexed by $\mathcal{I} := \{1, 2, \cdots, N_s\}$. Let $q_i(t) \in \mathcal{M}$ be the location of the i-th sensing agent at time $t \in \mathbb{R}_{\geq 0}$. We assume that the measurement $y(q_i(t), t)$ of agent i is the sum of the scalar value of the Gaussian process $z(q_i(t), t)$ and sensor noise $w_i(t)$, at its position $q_i(t)$ and some measurement time t,

$$y(q_i(t), t) := z(q_i(t), t) + w_i(t).$$

The communication network of mobile agents can be represented by a graph with edges. Let $G(t) := (\mathcal{I}, \mathcal{E}(t))$ be an undirected communication graph such that an edge $(i, j) \in \mathcal{E}(t)$ if and only if agent i can communicate with agent $j \neq i$. We define the neighborhood of agent i at time t by $N(i, t) := \{j : (i, j) \in \mathcal{E}(t),\ i \in \mathcal{I}\}$. We also define the closed neighborhood of agent i at time t by the union of its index and its neighbors, i.e., $N[i, t] := \{i\} \cup N(i, t)$.

III. The Non-parametric Approach

With the spatially distributed sampling capability, agents need to estimate and predict the field of interest by fusing the collective samples from different space and time coordinates. We introduce recursively learning algorithms for each mobile sensing agent to update its own prediction of the field integrating the current noisy cooperative observations along with the previous prediction (based on past measurements) in an optimal fashion. We assume that a field undergoing a physical transport phenomenon can be modeled by a spatio-temporal Gaussian process, which can be used for non-parametric prediction.

Consider a spatio-temporal Gaussian process with a nonzero mean:

$$z(s, t) \sim \mathcal{GP}(\mu(s, t), \kappa(s, t; s^*, t^*)), \tag{1}$$

where $s, s^* \in \mathcal{M}$, $t, t^* \in \mathbb{R}_{\geq 0}$ and $\mu(s, t)$ denotes a mean at location s and time t. We then propose the following generalized covariance function $\kappa(s_i, t_i; s_j, t_j; \Psi)$ with a parameter vector $\Psi = [\psi_1\ \psi_2^x\ \psi_2^y\ \sigma_t]^T$:

$$\kappa(s_i, t_i; s_j, t_j; \Psi) = \psi_1^2 \exp\left(-\frac{1}{2} \sum_{l \in \{x, y\}} \frac{(s_i^l - s_j^l)^2}{(\psi_2^l)^2}\right) \exp\left(-\frac{1}{2} \frac{(t_i - t_j)^2}{\sigma_t^2}\right), \tag{2}$$

where s_i^l is the l-th entry of s_i. $\{\psi_2^x, \psi_2^y\}$ and σ_t are kernel bandwidths for space and time, respectively. (2) shows that points close in the measurement space and time space are strongly correlated and produce similar values. In reality, the larger temporal distance two measurements are taken with, the less correlated they become, which strongly supports our generalized covariance function in (2). This may also justify the truncation (or windowing) of the observed time series data to limit the size of covariance matrix for less computational cost.

In this paper, we only consider the case that the global coordinates are the same as the local model coordinates. In the case that the global coordinates are different from the local model coordinates, a similarity transformation can be used to address this issue. For instance, a rotational relationship between the model basis $\{\vec{e}_x, \vec{e}_y\}$ and the global basis $\{\vec{E}_x, \vec{E}_y\}$ is:

$$\begin{bmatrix} \vec{e}_x \\ \vec{e}_y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \vec{E}_x \\ \vec{E}_y \end{bmatrix},$$

where θ represents the angle of rotation. We then can use the following relationship to change the coordinates:

$$\begin{cases} x = X\cos\theta + Y\sin\theta \\ y = -X\sin\theta + Y\cos\theta \end{cases},$$

where x and y indicate the position in local coordinates and X and Y indicate their counterparts in global coordinates. (2) can then be rewritten in terms of global coordinates.

From (1), the collection of $z(s, t)$ is denoted by Z with a distribution

$$p(Z|\Psi) := \frac{\exp\left(-\frac{1}{2}(Z - \mu_Z)^T \Sigma_Z^{-1}(Z - \mu_Z)\right)}{(2\pi)^{\frac{n}{2}} |\Sigma_Z|^{\frac{1}{2}}}, \tag{3}$$

where n is the total number of data, $\mu_Z := \mathbb{E}(Z)$ is the mean of Z, $\Sigma_Z := \mathbb{E}\left((Z - \mu_Z)(Z - \mu_Z)^T\right)$ is the covariance matrix of Z obtained by $[\Sigma_Z]_{ij} = \kappa(s_i, t_i; s_j, t_j; \Psi)$, and $|\Sigma_Z|$ is the determinant of Σ_Z.

Up to time t_k, agent i has noisy collective data $\{y(s_j(t_m), t_m) \,|\, m \in \mathbb{Z}, j \in N[i, t_m], 1 \leq m \leq k\}$, where $N[i, t_m]$ denotes the closed neighborhood of agent i at time t_m. The measurements $y(s_j(t_m), t_m) = z(s_j(t_m), t_m) + w_j(t_m)$ are taken at different positions

$s_j(t_m) \in \mathcal{M}$ and different times $t_m \in \mathbb{R}_{\geq 0}$. The measurements are corrupted by the sensor and communication noises represented by Gaussian white noise $w_j \sim \mathcal{N}(0, \sigma_{w_j}^2)$. The column vectorized measurements collected by agent i is denoted by

$$Y := \text{col}\left(y(s_j(t_m), t_m) \,|\, m \in \mathbb{Z}, j \in N[i, t_m], 1 \leq m \leq k\right)$$

with a joint distribution

$$p(Y|\Psi) := \frac{\exp\left(-\frac{1}{2}(Y-\mu_Y)^T \Sigma_Y^{-1}(Y-\mu_Y)\right)}{(2\pi)^{\frac{n}{2}}|\Sigma_Y|^{\frac{1}{2}}},$$

where μ_Y, Σ_Y, and $|\Sigma_Y|$ are defined accordingly as in (3). Notice that the covariance matrix of Y is obtained by

$$\Sigma_Y := \Sigma_Z + \text{diag}(\cdots, \sigma_{w_j}^2, \cdots),$$

where j is the agent index associated with the corresponding measurement. If the covariance function is known a priori, the prediction of the random field $z(s,t)$ at location s and time t is then obtained by

$$z(s,t|t_k) := z(s,t) \,|\, Y \sim \mathcal{N}\left(\hat{z}(s,t|t_k), \sigma^2(s,t|t_k)\right), \quad (4)$$

where $\hat{z}(s,t|t_k) := \mathbb{E}\left(z(s,t|t_k)\right)$ is

$$\hat{z}(s,t|t_k) := \mu(s,t) + \Sigma_{zY}\Sigma_Y^{-1}(Y-\mu_Y),$$

and the prediction error variance is

$$\sigma^2(s,t|t_k) := \Sigma_z - \Sigma_{zY}\Sigma_Y^{-1}\Sigma_{Yz},$$

where Σ_z is the covariance of z, obtained by $\kappa(s,t;s,t;\Psi)$, $\Sigma_{zY} = \Sigma_{Yz}^T$ is the covariance matrix between z and Y, obtained by $[\Sigma_{zY}]_j = \kappa(s,t;s_j,t_j;\Psi)$. Each agent can then predict the field of interest at any location and time with the associated uncertainty in a non-parametric way. In the next section, we present a learning approach for unknown covariance function.

IV. Gaussian Process Learning

Without loss of generality, we use a zero mean Gaussian process $z(s,t) \sim \mathcal{GP}(0, \kappa(s,t;s^*,t^*))$, i.e., $\mu_Z = 0$ for modeling the field undergoing a physical transport phenomenon.

If the covariance function of a Gaussian process is not known a priori, mobile agents need to estimate parameters of the covariance function (Ψ) based on the observed samples. Using Bayes' rule, the posterior has the form of

$$p(\Psi|Y) = \frac{p(Y|\Psi)p(\Psi)}{p(Y)}.$$

The maximum likelihood (ML) estimate Ψ^{ML} of the hyperparameter vector is obtained by

$$\Psi^{ML} = \arg\max_{\Psi} p(Y|\Psi). \quad (5)$$

Maximizing the likelihood function is equivalent to maximizing the log likelihood function:

$$\ln p(Y|\Psi) = -\frac{1}{2}Y^T\Sigma_Y^{-1}Y - \frac{1}{2}\ln|\Sigma_Y| - \frac{n}{2}\ln 2\pi,$$

where n is the size of Y. A gradient-based algorithm is used to find a ML estimate of Ψ:

$$\Psi_{t+1} = \Psi_t + \epsilon \nabla_x \ln p(Y|x)|_{x=\Psi_t},$$

where ϵ is a small positive constant and $\nabla_x f(x)$ is the partial derivative of $f(x)$ with respect to x. The partial derivative of the log likelihood function with respect to a hyperparameter ψ_j is given by

$$\frac{\partial \ln p(Y|\Psi)}{\partial \psi_j} = \frac{1}{2}Y^T\Sigma_Y^{-1}\frac{\partial \Sigma_Y}{\partial \psi_j}\Sigma_Y^{-1}Y - \frac{1}{2}\text{tr}\left(\Sigma_Y^{-1}\frac{\partial \Sigma_Y}{\partial \psi_j}\right).$$

Let $\Psi^{(k)}$ be an estimate of Ψ from (5) based on measurements up to time t_k.

Alternatively, a simplex search method [24] can be used to find a ML estimate of Ψ. This is a direct search method that does not use numerical or analytic gradients.

After finding a ML estimate of Ψ, agents can proceed the prediction of the field of interest using (4).

V. Optimal Navigation Strategies

Agents should find new sampling positions to improve the quality of the estimated covariance function in the next iteration at time t_{k+1}. For instance, to precisely estimate the anisotropic phenomenon, i.e., processes with different covariances along x-axis and y-axis directions, sensing agents need to explore and sample measurements along different directions. In this section, we consider a centralized scheme for this purpose. Suppose that a leader agent (or a central station) knows the communication graph at the next iteration time t_{k+1} and also has access to all measurements collected by agents. Let Y_{k+1} and $Y_{\leq k}$ be the measurements at time t_{k+1} and the collective measurements up to time t_k, respectively, i.e.,

$$\begin{aligned} Y_{k+1} :=& \text{col}\left(y(s_i(t_{k+1}), t_{k+1}) \,|\, i \in \mathcal{I}\right), \\ Y_{\leq k} :=& \text{col}\left(y(s_i(t_m), t_m) \,|\, m \in \mathbb{Z}, i \in \mathcal{I}, 1 \leq m \leq k\right). \end{aligned}$$

To derive the optimal navigation strategy, we compute the log likelihood function of observations in $Y_{\leq k+1}$:

$$\begin{aligned} \mathcal{L}(Y_{\leq k+1}, \Psi) :=& \ln p(Y_{\leq k+1}|\Psi) \\ =& -\frac{1}{2}Y_{\leq k+1}^T\Sigma_{Y_{\leq k+1}}^{-1}Y_{\leq k+1} \\ & -\frac{1}{2}\ln|\Sigma_{Y_{\leq k+1}}| - \frac{n_{\leq k+1}}{2}\ln 2\pi, \end{aligned} \quad (6)$$

where $n_{\leq k+1}$ is the size of $Y_{\leq k+1}$.

Since the locations of observations in $Y_{\leq k}$ were already fixed, we represent the log likelihood function in terms of a vector of future sampling points $\tilde{s} := s(t_{k+1})$ at time t_{k+1} only and the hyperparameter vector Ψ:

$$\mathcal{L}(\tilde{s}, \Psi) := \ln p(Y_{\leq k+1}(\tilde{s})|\Psi).$$

Now consider the Fisher Information Matrix (FIM) that measures the information produced by measurements $Y_{\leq k+1}$ for estimating the hyperparameter vector at time t_{k+1}. The Cramér-Rao lower bound (CRLB) theorem states that the

inverse of the FIM is a lower bound of the estimation error covariance matrix [23], [25]:

$$\mathbb{E}\left((\Psi^{(k+1)}-\Psi)(\Psi^{(k+1)}-\Psi)^T\right) \succeq \text{FIM}^{-1},$$

where $\Psi^{(k+1)}$ represents the estimation of Ψ at time t_{k+1}. The FIM [23] is given by

$$\begin{aligned}[\text{FIM}(\tilde{s},\Psi)]_{ij} &= -\mathbb{E}\left(\frac{\partial^2\mathcal{L}(\tilde{s},\Psi)}{\partial\psi_i\partial\psi_j}\right)\\ &= \frac{1}{2}\text{tr}\left(\Sigma^{-1}_{Y_{\le k+1}}\frac{\partial\Sigma_{Y_{\le k+1}}}{\partial\psi_i}\Sigma^{-1}_{Y_{\le k+1}}\frac{\partial\Sigma_{Y_{\le k+1}}}{\partial\psi_j}\right),\end{aligned} \tag{7}$$

where the expectation is taken with respect to $p(Y_{\le k+1}|\Psi)$. Since the true value of Ψ is not available, we will evaluate the FIM in (7) at the currently available best estimate $\Psi^{(k)}$ [26].

We can expect that minimizing the CRLB results in a decrease of uncertainty in estimating Ψ. Using the D-optimality criterion [27], [28], the objective function J is given by

$$J(\tilde{s},\Psi^{(k)}) := \det(\text{FIM}(\tilde{s},\Psi^{(k)})). \tag{8}$$

A gradient ascent strategy can be obtained by for the objective function $J(\tilde{s})$ in (8).

Alternatively, a control law for the mobile sensor network can be formulated as follows:

$$q(t_{k+1}) = \arg\max_{\tilde{s}\in\prod_{i=1}^{N_s}Q_i} J(\tilde{s},\Psi^{(k)}), \tag{9}$$

where

$$Q_i = \prod_{j=1}^{n_d}[-\delta_j\ \ \delta_j] + q_i(t_k),$$

where $n_d = 2$ denotes the dimension of the surveillance region $\mathcal{M}$ and δ_j is the maximum step size for each agent to move in x and y directions.

However, optimization on $\ln p(Y_{\le k+1}|\Psi)$ in (6) and $J(\tilde{s})$ in (8) can be numerically costly due to the increasing size of $\Sigma_{Y_{\le k}}$ used in (6) and (8). A way to deal with this problem is to use a truncated date set

$$Y_{k-\delta\le,\le k} := \text{col}\,(y(s_i(t_m),t_m)\,|\,m\in\mathbb{Z}, i\in\mathcal{I}, \quad k-\delta\le m\le k)$$

instead of using $Y_{\le k}$.

On the other hand, this approach can be viewed as a strategy to deal with a slowly time varying parameter vector Ψ.

VI. A Protocol for Mobile Sensor Networks

The overall protocol for the sensor network is summarized as follows.

A. Prediction

A time t_k, the sensor network updates $\Psi^{(k)}$ using maximum likelihood optimization for a data set $Y_{\le k}$. Start this optimization with the initial point $\Psi^{(k-1)}$. For given $Y_{\le k}$ and $\Psi^{(k)}$, agent can compute prediction at any point and time using (4), i.e.,

$$p(z(s,t)|Y_{\le k},\Psi^{(k)}).$$

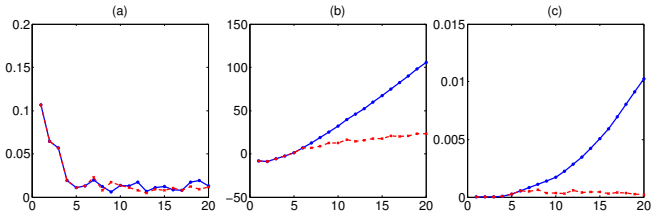

Fig. 1. The blue solid line represents simulation results without truncation; the red dotted line represents simulation results with truncation. (a) Normalized Root Mean Square (RMS) value of the spatially sampled prediction errors vs. iteration; (b) Maximum log likelihood value vs. iteration; (c) Cost function value vs. iteration.

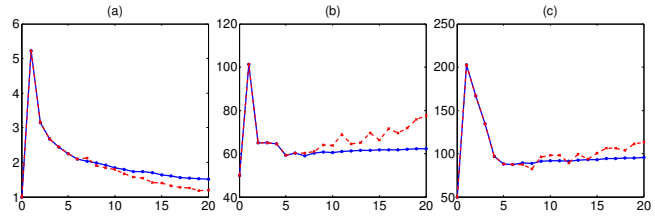

Fig. 2. The blue solid line represents simulation results without truncation; the red dotted line represents simulation results with truncation. (a) ψ_1 vs. iteration; (b) ψ_2^x vs. iteration; (c) ψ_2^y vs. iteration.

B. Sampling

Based on $\{\Psi^{(k)}, Y_{\le k}\}$, the sensor network computes the control (9) in order to maximize $J(\tilde{s},\Psi^{(k)})$. Update the positions of agents accordingly and collect measurements at time t_{k+1}. Repeat the prediction and sampling steps.

VII. Simulation Results

In this section, we apply our approach to a spatio-temporal field generated by physical phenomena (advection and diffusion). To generate the experimental data numerically, the advection-diffusion model developed in [29] was used. We used the simulated process in order to compare the predicted

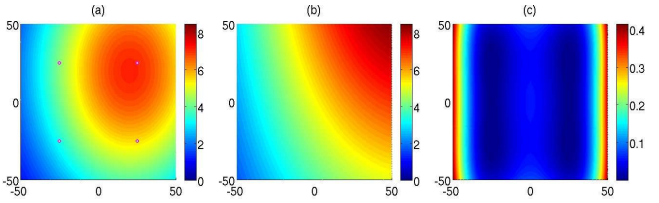

Fig. 3. Simulation results at step $k = 1$. (a) True field and agents' trajectories; (b) Predicted field; (c) Prediction error variance.

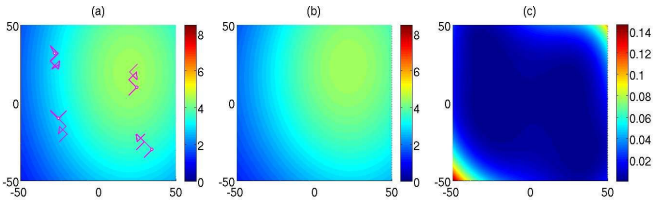

Fig. 4. Simulation results at step $k = 10$. (a) True field and agents' trajectories; (b) Predicted field; (c) Prediction error variance.

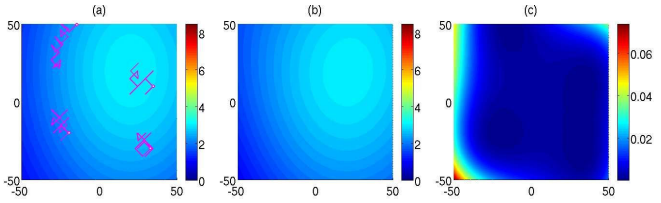

Fig. 5. Simulation results at step $k = 20$. (a) True field and trajectories of agents; (b) Predicted field; (c) Prediction error variance.

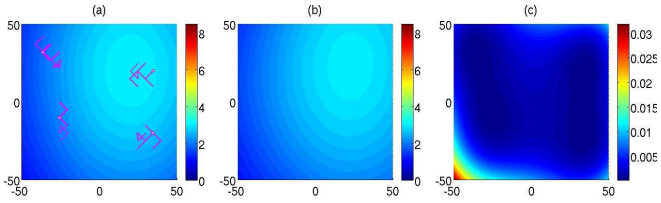

Fig. 6. Simulation results at step $k = 20$ using truncated data. (a) True field and trajectories of agents; (b) Predicted field; (c) Prediction error variance.

values with respect to the true values. For the model coordinates, a global Cartesian coordinate system was used. An instantaneous release of Qkg of gas occurs at a location (x_0, y_0, z_0). Assuming that all measurements are recorded at $z = 0$, and the release occurs at a ground level (i.e. $z_0 = 0$), the concentration C at an arbitrary location $(x, y, 0)$ and time t is described by the following analytical solution [30]:

$$C(x, y, 0, t) = \frac{Q}{4\pi^{\frac{3}{2}}(K_x K_y K_z)^{\frac{1}{2}}(\Delta t)^{\frac{3}{2}}} \times e^{-\frac{\Delta x^2}{4K_x \Delta t} - \frac{\Delta y^2}{4K_y \Delta t}} \tag{10}$$

where $\Delta x = x - x_0$, $\Delta y = y - y_0$ and $\Delta t = t$. The parameters used in the simulation study are shown in Table I. In particular, this process generates an anisotropic concentration field with parameters $K_x = 10m^2/s$ and $K_y = 20m^2/s$ as in Table I.

For simplicity, we have pre-selected the kernel bandwidth for time σ_t, which specifies the temporal part of the process. Moreover, we assume all agents have same level of measurement noises so that $\sigma_{w_i} = \sigma_w$.

As discussed in Section V, we consider a situation where at each time, measurements of agents are transmitted to a leader (or a central station) that uses our Gaussian learning algorithm and sends optimal control back to individual agents for next iteration to improve the quality of the estimated covariance function. In our simulation study, agents start sampling at $t_0 = 100s$ and take measurements at time t_k with a sampling time of $t_s = 4s$ as in Table I.

TABLE I

PARAMETERS USED IN SIMULATION.

Parameter	Notation	Unit	Value
Number of agents	N_s	-	4
Sampling time	t_s	s	4
Initial time	t_0	s	100
Gas release mass	Q	kg	10^6
Eddy diffusivity in x axis	K_x	m^2/s	10
Eddy diffusivity in y axis	K_y	m^2/s	20
Eddy diffusivity in z axis	K_z	m^2/s	0.2
Location of explosion	x_0	m	20
Location of explosion	y_0	m	20
Location of explosion	z_0	m	0
Time kernel bandwidth	σ_t	s	20
Sensor noise level	σ_w	g	0.01

After converging to a good estimate of Ψ, agents can switch to a decentralized configuration and collect samples for other goals such as peak tracking and prediction of the process [8], [21], [20].

Fig. 1 illustrates that (a) the Root Mean Square (RMS) value of spatially sampled normalized prediction errors (evaluated at 41×41 grid points) vs. iteration; (b) the maximum log likelihood value vs. iteration; and (c) the cost function at each iteration in blue solid lines. Since the concentration field diffuses, the normalized error by the maximum value of the true field at each iteration was used. As shown in Fig. 1-(a), the normalized RMS prediction error value decreases as iteration time increases. Fig. 1-(b) shows that the value of the maximum log likelihood function increases as iteration increases. Also our control cost function increases as iteration increases as shown in Fig. 1-(c).

Fig. 2 shows that (a) ψ_1, (b) ψ_2^x, and (c) ψ_2^y converge to some values that estimate the proposed covariance function in (2). As can be seen in Fig. 2-(b) and Fig. 2-(c), ψ_2^x, and ψ_2^y converge to different values, and, more importantly, $\psi_2^x < \psi_2^y$ validate that our approach can effectively cope with anisotropic field produced by (10) with $K_x < K_y$ as in Table I. All ML estimates of hyperparameters seem to converge quickly after about 5 iterations as depicted in Fig. 2.

Figs. 3, 4 and 5 show that (a) the true field along with trajectories of mobile agents, (b) the predicted field and (c) the prediction error variance at different times $\{t_k \,|\, k \in \{1, 10, 20\}\}$. Fairly good predictions are obtained after about 5 iterations. The error variance tends to decrease as time increases. Notice that the mobility of agents was not optimized for minimizing the prediction error variance as in [8], [21], [20].

Applying our scheme using only a finite number of truncated (or windowed) measurement data can significantly reduce the computational cost and still provide a reasonable quality of the estimate of Ψ. This way we can obtain a tradeoff between a precise estimation and computational efficiency. To make a fair comparison, we have used the same initial points and the same sensor noise sequences. The size of truncation was 20. This "truncated data scheme" is illustrated in red dotted lines in Figs. 1 and 2. Fig. 1 shows (a) the normalized RMS value of prediction errors vs. iteration; (b) the maximum log likelihood value vs. iteration; and (c) the cost function vs. iteration obtained from the truncated data scheme in red dotted lines.

The estimated hyperparameters from the truncated data scheme vs. iteration are depicted in red dotted lines in Fig. 2. This figure illustrates that ML estimates of ψ_1, ψ_2^x, and ψ_2^y only using the truncated data (in red dotted lines) are quite close to those using all 80 measurements (in solid blue lines).

Fig. 6 shows the prediction and corresponding error variance at iteration 20. By comparing Fig. 5 and Fig. 6, we can

see that the quality of prediction based on the truncated data is comparable to that of prediction based on all data.

VIII. Summary

In this paper, we presented a novel class of self-organizing sensing agents that learn an anisotropic, spatio-temporal Gaussian process using noisy measurements and move in order to improve the quality of the estimated covariance function. The maximum likelihood approach was used to estimate the hyperparameters in the unknown covariance function. The prediction of the field of interest was obtained based on a non-parametric approach. An optimal navigation strategy was proposed to minimize the Cramér-Rao lower bound of the estimation error covariance matrix. Simulation study indicated the effectiveness of the proposed scheme. In particular, our scheme was also implemented based on a finite number of truncated observations as compared to the one using all the measurements. The tradeoff between a precise estimation and computational efficiency will be studied in the future work.

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