

# RRO Compensation of HDD Based on RPTC Method with Re-learning Scheme for Discrete Track Recording Media

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**Abstract**—This paper presents an improvement method of repetitive controller based on perfect tracking control (PTC) in order to reject variable repeatable runout (RRO) of hard disk drive. Authors' group proposed the repetitive PTC (RPTC) with switching mechanism. RPTC is constructed by using periodic signal generator (PSG) and PTC. The PSG works to make feedforward signal from the periodic disturbance and the PTC generates control input to cancel the periodic error in steady-state. However, we have not considered the difference of RRO between tracks. This paper proposes a re-learning scheme considering correlation of adjacent tracks. Finally, the advantages of RPTC using proposed scheme are demonstrated through simulations and experiments with HDD equipment with discrete track recording media (DTR).

## I. INTRODUCTION

Head-positioning controllers of hard disk drives (HDDs) are generally composed of two control modes; the track-seeking mode and the track-following mode. The feedforward performance is important in the track-seeking mode, and the disturbance rejection performance is important in the track-following mode. Digital two-degree-of-freedom controllers generally have two samplers for the reference signal  $r(t)$  and the output  $y(t)$ , and one holder on the input  $u(t)$  as shown in Fig.1. Therefore, there exist three time periods  $T_r$ ,  $T_y$ , and  $T_u$  which represent the period of  $r(t)$ ,  $y(t)$ , and  $u(t)$ , respectively. In the case of HDDs, the position error is detected by the discrete servo signals embedded in the disks. Therefore, the output sampling period  $T_y$  is decided by the number of these signals and the rotation frequency of the spindle motor. However, it is possible to set the control period  $T_u$  shorter than  $T_y$  because of the recent development of CPU. Thus, the controller can be regarded as the multirate control system which has the hardware restriction of  $T_u < T_y$ . Then, many multirate controllers have been proposed both for track-seeking and track-following modes [1][2][3].

Recently, magnetic printing media, discrete track recording media (DTR) and preformatted media with perpendicular anisotropy are studied for high track density as new generation media. These new generation media can record servo information in its manufacturing process without using the servo track writer. Because servo information is patterned on the disk, the repeatable runout (RRO) generally becomes bigger in steady-state. Consequently, it is important issue to reject not only low-order RRO but also high-order RRO.

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There have been a lot of studies of RRO compensation. Repetitive control is a widely used technique to reject periodic disturbances or to track a periodic reference signal in [4] and [5]. In [6], the disturbance is suppressed by peak-filters with phase compensator for the high frequency RRO. The application of iterative learning controller is reported in [7] and [8]. Adaptive feedforward cancellation scheme is utilized in [9] and [10].

Authors' group proposed repetitive controller based on perfect tracking control (PTC) with switching mechanism in order to reject high-order repeatable runout (RRO) with periodic signal generator (PSG) [11] and [12]. In the RPTC, multirate feedforward control is utilized to overcome the unstable zero problem of discrete-time plant. The PSG works to make the compensation signal from the periodic disturbance of the following track. To generate the compensation signal, the PSG needs to average position error signal (PES) at the each sector to reject non repeatable runout (NRRO). However, the RRO of one track is different from that of another track. When the track is changed, the compensation signal which is generated on one track would be degrade the following performance on the other tracks. However, it is not realistic to wait several rotations of disk to make the averaged signal at every track.

In this paper, we propose a re-learning scheme considering correlation of adjacent tracks. The correlation means that the periodic disturbance has the similarity in near adjacent tracks. After the compensation of RPTC in the current track, the PSG learns the residual position error signal to generate the new compensate signal for adjacent tracks.

## II. REPETITIVE PERFECT TRACKING CONTROL (RPTC)

### A. PTC [11]

In this paper, it is assumed that the control input can be changed  $N$  times during the sampling period of output signal  $T_y$ . For simplification, the input multiplicity  $N$  is set to be equal with the order of nominal plant  $n$  since  $N \geq n$  is the necessary condition of perfect tracking. But, by using the formulation of [11], this assumption can be relaxed to deal with more general system with  $N \neq n$ .

Consider the continuous-time  $n$ th-order plant described by

$$\dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x}(t) + \mathbf{b}_c u(t), \quad y(t) = \mathbf{c}_c \mathbf{x}(t). \quad (1)$$

The discrete-time state equation discretized by the shorter period  $T_u$  becomes

$$\mathbf{x}[k+1] = \mathbf{A}_s \mathbf{x}[k] + \mathbf{b}_s u[k], \quad (2)$$

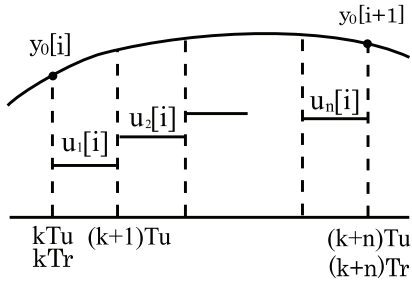


Fig. 1. Multirate hold.

where  $\mathbf{x}[k] = \mathbf{x}(kT_u)$  and

$$\mathbf{A}_s = e^{\mathbf{A}cT_u}, \mathbf{b}_s = \int_0^{T_u} e^{\mathbf{A}c\tau} \mathbf{b}_c d\tau. \quad (3)$$

By calculating the state transition from  $t=iT_y=kT_u$  to  $t=(i+1)T_y=(k+n)T_u$  in Fig.1, the discrete-time plant  $P[z]$  can be represented by

$$\mathbf{x}[i+1] = \mathbf{A}\mathbf{x}[i] + \mathbf{B}\mathbf{u}[i], \quad y[i] = \mathbf{c}\mathbf{x}[i]. \quad (4)$$

where  $\mathbf{x}[i] = \mathbf{x}(iT_y)$ ,  $z := e^{sT_y}$  and multirate input vector  $\mathbf{u}$  is defined in the lifting form as

$$\begin{aligned} \mathbf{u}[i] &:= [u_1[i], \dots, u_n[i]]^T, \\ &= [u(kT_u), \dots, u((k+n-1)T_u)]^T, \end{aligned} \quad (5)$$

and the coefficients are given by

$$\mathbf{A} = \mathbf{A}_s^n, \mathbf{B} = [\mathbf{A}_s^{n-1}\mathbf{b}_s, \mathbf{A}_s^{n-2}\mathbf{b}_s, \dots, \mathbf{A}_s\mathbf{b}_s, \mathbf{b}_s], \quad (6)$$

$$\mathbf{c} = \mathbf{c}_c.$$

From (4), the transfer function from  $\mathbf{x}[i+1] \in \mathbf{R}^n$  to the multirate input  $\mathbf{u}[i] \in \mathbf{R}^n$  can be derived as

$$\mathbf{u}[i] = \mathbf{B}^{-1}(\mathbf{I} - z^{-1}\mathbf{A})\mathbf{x}[i+1] \quad (7)$$

From the definition in (6), the nonsingularity of matrix  $\mathbf{B}$  is assured for a controllable plant. (7) is a stable inverse system.

$$\mathbf{u}_0[i] = \mathbf{B}^{-1}(\mathbf{I} - z^{-1}\mathbf{A})\mathbf{r}[i]. \quad (8)$$

Then, if the control input is calculated by (8) as shown in Fig.2, perfect tracking is guaranteed at sampling points for the nominal system because (8) is the exact inverse plant. Here,  $\mathbf{r}[i](= \mathbf{x}_d[i+1])$  is previewed desired trajectory of plant state. The nominal output can be calculated as

$$y_0[i] = \mathbf{c}\mathbf{x}_d[i] = z^{-1}\mathbf{c}\mathbf{r}[i]. \quad (9)$$

When the tracking error  $y[i] - y_0[i]$  is caused by disturbance or modeling error, it can be attenuated by the robust feedback controller  $C_2[z]$ .

### B. RPTC [12]

The RPTC is based on perfect tracking control with periodic signal generator (PSG). Because the PSG can be constructed by the series of memories  $z^{-1}$ , the computation cost is very low.

First, the PTC is designed using multirate feedforward control as minor-loop system to obtain the ideal command

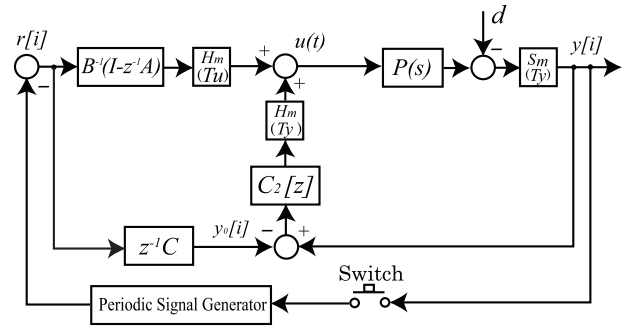


Fig. 2. Repetitive perfect tracking controller.

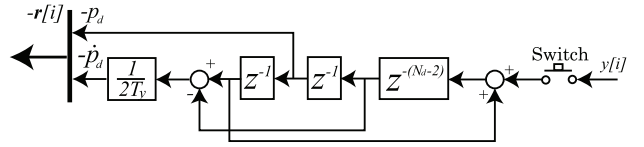


Fig. 3. Periodic signal generator for 2nd order system.

response. The measured output  $y[i]$  is assumed to have the output disturbance  $d[i]$  as

$$y[i] = p[i] - d[i] := \mathbf{c}\mathbf{x}[i] - d[i], \quad (10)$$

where  $p[i]$  is the plant output. In this section, the disturbance is assumed to be repetitive signal with period  $T_d$ . When the tracking error  $y[i] - y_0[i]$  is caused by unmodeled disturbance or modeling error, it can be attenuated by the robust feedback controller  $C_2[z]$  as shown in Fig.2. Second, the periodic signal generator is designed to generate desired trajectory  $\mathbf{r}[i]$ . Because perfect tracking ( $\mathbf{x}[i] = \mathbf{x}_d[i]$  or  $\mathbf{x}[i] = z^{-1}\mathbf{r}[i]$ ) is assured, the minor-loop nominal system is expressed as

$$y[i] = z^{-1}\mathbf{r}[i] - d_s[i], \quad \mathbf{r}[i] := \mathbf{c}\mathbf{r}[i], \quad (11)$$

where  $d_s[i] := (1 - P[z]C_2[z])^{-1}d[i]$  and  $P[z]$  is the single-rate plant with  $T_y$  if the minor-loop feedback controller  $C_2[z]$  is a single-rate system.

The PSG is can be designed as the outer-loop controller by

$$\mathbf{r}[i] = -\frac{z}{z^{N_d} - 1}y[i], \quad (12)$$

where the integer  $N_d$  is defined as  $T_d/T_y$ . From (11) and (12), the total closed-loop system is represented by

$$\mathbf{r}[i] = -\frac{z^{N_d} - 1}{z^{N_d}}d_s[i]. \quad (13)$$

Therefore, the repetitive disturbance which is modeled as  $d[i] = (z^{N_d} - 1)^{-1}$  is completely rejected at every sampling point in steady-state. In (11), there exists redundancy to decide  $\mathbf{r}[i]$  from the PSG output  $\mathbf{r}[i]$  since we have freedom to select the state variable  $\mathbf{x}$ . In order to make the multirate input smooth, it should be given as the derivative from  $\mathbf{x} = [p, \dot{p}, \ddot{p}, \dots]$ . Fig.3 shows one example of the 2nd order plant with  $\mathbf{x} = [p, \dot{p}]$ , in which the velocity command is generated by  $\dot{p}_d[i] = (p_d[i+1] - p_d[i-1])/2T_y$ .

In the steady-state, the switch turns on during one disturbance period  $T_d$  and the PSG stores the output. Here, the PTC generates control input  $u_0[i]$  to cancel the periodic steady error. Thus, the plant output  $p[i]$  perfectly tracks the periodic disturbance  $d[i]$  and the tracking error becomes zero at every sampling point ( $y[i] = 0$ ). Since the switch turns on just  $N_d$  sampling time, the  $N_d$  memories work as complete feedforward controller.

### C. Averaging runout considered adjacent tracks

The periodic disturbance in the HDDs appears as the synchronization with the spindle rotation speed. This is caused by the eccentric disk, the torque ripple of the spindle motor and each sector position noise.

The periodic disturbance of each track is different. Especially, the RRO of inner disk is completely different from that of outer disk. The PSG needs to memorize the periodic disturbances of each track. Therefore, when using the periodic disturbance in the previous track, the compensation signal would be different from the periodic disturbance of that track.

Here, this paper proposes re-learning scheme considering correlation of adjacent tracks with the weighting factor. First, it is assumed that PTC compensation is not applied. From (11) and  $r[l] = 0$ , position error signal  $y[l]$  is calculated as  $y[l] = -d_s[l]$ , where  $l$  means the sector number. It is assumed that  $y_0[l]$  is calculated with the average of  $y[l]$  at the each sector of the initial track, where the suffix of  $y_0[l]$  is incremented at every re-learning. It is assumed that the average times is  $N_0$  rotations. The initial compensation signal  $r_1[l]$  is calculated as  $r_1[l] = -zy_0[l] = zS[z]d_0[l]$ , and  $d_0[l]$  is the RRO of the initial track. Second, we consider that the head position move to another track. Then, the next position error signals  $y_1[l]$  is calculated as

$$\begin{aligned} y_1[l] &= z^{-1}r_1[l] - S[z](d_0[l] + \tilde{d}_1[l]), \\ &= z^{-1}zS[z]d_0[l] - S[z](d_0[l] + \tilde{d}_1[l]), \\ &= -S[z]\tilde{d}_1[l], \end{aligned} \quad (14)$$

where  $d_1[l] = d_0[l] + \tilde{d}_1[l]$  and  $\tilde{d}_1[l]$  is the variation of track runout from  $d_0[l]$ . Finally, we consider to generate the next compensation signal. The  $\tilde{d}_1[l]$  includes not only RRO signal but also NRRO signal which should be eliminated to make compensation signal. We consider the averaging times of several position error signal. Considering  $y_0[l]$  is calculated with the average of  $N_0$  times and  $y_1[l]$  is one period,  $r_2[l]$  is calculated as

$$\begin{aligned} r_2[l] &= zS[z] \left( \frac{N_0}{N_0+1}d_0[l] + \frac{1}{N_0+1}d_1[l] \right), \\ &= zS[z] \left( d_0[l] + \frac{1}{N_0+1}\tilde{d}_1[l] \right), \\ &= -z \left( y_0[l] + \frac{1}{N_0+1}y_1[l] \right). \end{aligned} \quad (15)$$

(15) means that the residual position error is added to  $r_1[l]$  with weight  $1/(N_0+1)$ . (15) is a simple method to re-learn

TABLE I  
SAMPLING TIME PERIOD.

$T_r[\mu s]$	$T_u[\mu s]$	$T_y[\mu s]$
79.4	39.7	79.4

TABLE II  
PARAMETERS OF RESONANCE MODES.

$r$	$f_r[\text{Hz}]$	$\zeta_r$	$A_r$
1	4640	0.03	-0.05
2	8000	0.03	-1.0

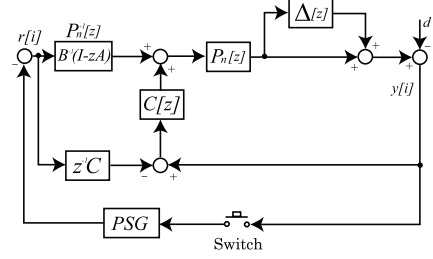


Fig. 4. Block diagram of modeling error.

the residual position error. By generalizing (15), the control law  $r_2[l]$  can be rewritten as

$$r_2[l] = r_1[l] - \alpha y_1[l], \quad (16)$$

where  $\alpha$  is a free parameter to give higher weight to new data. The weighting factor  $\alpha$  should be chosen as

$$\frac{1}{N_0+1} \leq \alpha \leq \frac{N_0}{N_0+1}. \quad (17)$$

Furthermore, by using Q filter [5], NRRO can be further reduced. When cut-off frequency of Q filter is set to be equal with the cut-off frequency of the feedback controller, NRRO is not influenced so much.

### D. Considering modeling error

Next, we consider the robustness of the re-learning scheme against the modeling error of plant model. Here, we assumed that there is not variation of track runout. Before starting the PTC compensation, the output  $y_0[l]$  is calculated as

$$\begin{aligned} y_0[l] &= -\frac{1}{1 - P_n[z](1 + \Delta[z])C[z]}d_0[l], \\ &= -\frac{1}{1 - P_n[z]C[z]} \frac{1}{1 + \frac{P_n[z]C[z]}{1 - P_n[z]C[z]}\Delta[z]}d_0[l], \\ &= -S[z] \frac{1}{1 + T[z]\Delta[z]}d_0[l], \end{aligned} \quad (18)$$

where  $\Delta[z]$  is the multiplicative modeling error. Therefore, the compensation signal is calculated as

$$r_1[l] = -zy_0[l] = (zS[z]/(1 + T[z]\Delta[z])d_0[l]), \quad (19)$$

where  $S[z] = 1/(1 - P_n[z]C[z])$  and  $T[z] = P_n[z]C[z]/(1 - P_n[z]C[z])$ . After feedforward compensation is started, the output signal  $y_1[l]$  and  $u_1[l]$  are described as

$$\begin{aligned} y_1[l] &= P_n(1 + \Delta[z])u_1[l] - d_0[l], \\ u_1[l] &= z^{-1}P_n^{-1}[z]r_1[l] + C[z](-z^{-1}r_1[l] + y_1[l]) \end{aligned} \quad (20)$$

In this analysis, PTC is approximately described as  $z^{-1}P_n^{-1}[z]$  in (20). Then, the output is calculated as

$$\{1 - P_n[z]C[z](1 + \Delta[z])\} y_1[l] =$$

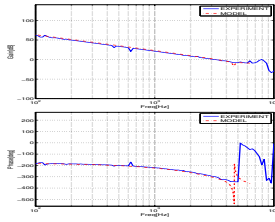


Fig. 5. Plant bode diagram.

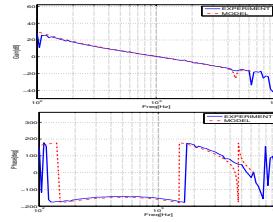


Fig. 6. Open loop bode diagram.

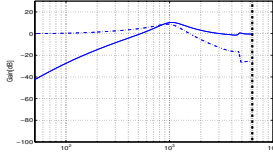


Fig. 7. Sensitivity functions  $S[z]$ ,  $T[z]$ .

$$(1 + \Delta[z])z^{-1}r_1[l] - P_n[z]C[z](1 + \Delta[z])z^{-1}r_1[l] - d_0[l]. \quad (21)$$

Then, we obtain

$$y_1[l] = z^{-1}r_1[l] + S[z]\frac{\Delta[z]}{1 + T[z]\Delta[z]}z^{-1}r_1[l] - S[z]\frac{1}{1 + T[z]\Delta[z]}d_0[l]. \quad (22)$$

Using (19), the output is rewritten as

$$y_1[l] = \left( S[z]\frac{1}{1 + T[z]\Delta[z]} \right) \Delta[z]d_0[l]. \quad (23)$$

(23) means that the residual output is described as  $y_1[l] = \Delta[z]y_0[l]$ . Consequently, in the case  $|\Delta[z]| \ll 1$ , the re-learning scheme works to cancel the residual error. On the other hand, in the case  $|\Delta[z]| \gg 1$  the re-learning scheme worsen the error.

### III. APPLICATIONS TO RRO REJECTION IN HDD

#### A. Control system design and simulations

The plant model is calculated from the experimental setup with DTR. The sampling period of this drive is  $T_y = 79.4\mu s$  and the control input is changed  $N = 2$  times during this period. In the design of controller, the nominal plant  $P_n(s)$  is modeled as double integrator system as

$$P_n(s) = \frac{kp}{ms^2}. \quad (24)$$

The high-order detail model  $P(s)$  that has rigid mode and  $l$ th resonance modes is expressed as

$$P(s) = \frac{kp}{m} \left( \frac{1}{s^2} + \sum_{r=1}^2 \frac{A_r}{s^2 + 2\zeta_r\omega_r s + \omega_r^2} \right). \quad (25)$$

The parameters of the resonance modes show in Table. II. The rotation frequency of spindle motor is 60 Hz and the number of sector can be  $N_d = 210$ . The minor loop feedback controller  $C_2[z]$  is designed by the PID controller with 850 kHz cross-over frequency. Fig.5 and Fig.6 show the bode diagrams of plant and the open loop, respectively. In order to deal with the resonance mode is considered, notch filter

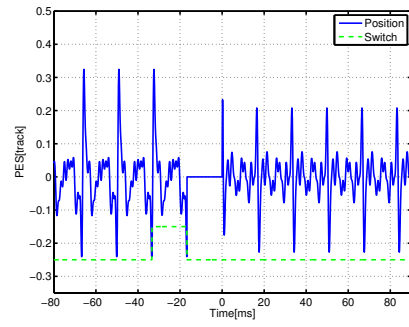


Fig. 8. Without re-learning method

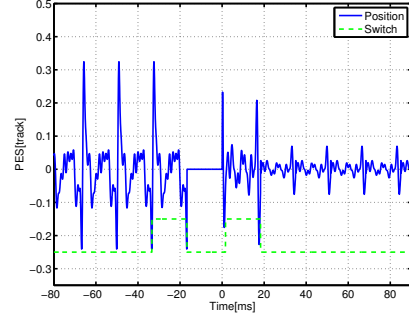


Fig. 9. With re-learning method

is used to suppress  $\omega_n = 2\pi 8000$  rad/sec resonance mode. The notch filter is given as

$$F(s) = \frac{s^2 + 2\zeta_2\omega_2 s + \omega_2^2}{s^2 + 2\zeta_d\omega_2 s + \omega_2^2}, \quad (26)$$

where  $\zeta_d = 0.3$ . The notch filter was discretized by prewarp tustin transformation, in which the sampling time is  $T_u = T_y/2$ .

Fig.7 shows the sensitivity  $S[z]$  (solid line) and complementary sensitivity  $T[z]$  (dash line) of the closed-loop. The injected disturbance signal is calculated from an approximate inverse of sensitivity function and position error signal (PES) obtained from experiments. The simulations use the only RRO signal which is extracted from experimental data by the averaging operation of the total PES.

The simulation results are shown from Fig.8 to Fig.10. The proposed re-learning scheme is simulated with the case of nominal model. In this simulation, we emulated the track or head change as the amplitude change of runout. Therefore, the track seeking is not included in these simulations. The averaging times  $N_0$  is 5 and the weighting factor  $\alpha$  is 0.4. Fig.8 and Fig.9 show the simulation results with the variation of disturbance. The RRO from the 1st to the 15th modes are injected and the amplitude from the 2nd to the 15th modes are changed at  $t = 0$ .

As shown in Fig.8, RPTC tracks the RRO with zero error after the switch turns on ( $t = -T_d$ ). When the disturbance changed ( $t = 0$ ), the residual error appeared. On the other hand, the proposed method in Fig.9, the residual error suppressed after the switch turns on again to regenerate the compensation signal. At the first switching action, the PSG learned the mainly periodic disturbance. When the

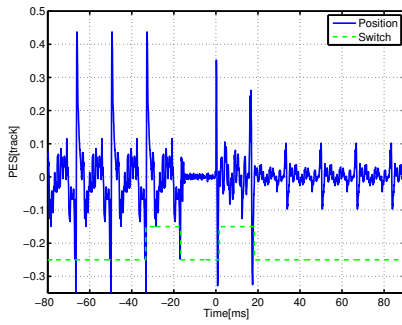


Fig. 10. Detail model with re-learning.

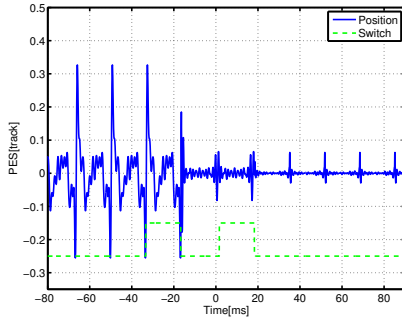


Fig. 11. With modeling error.

disturbance changed, the residual error appears in the same way as conventional method. When the switch turns on, PSG learned the residual error and weighted periodic disturbance. Then, after the 2nd switch turn off, PTC makes the better compensation signal, and the residual error is suppressed. Moreover, because RPTC maintains the compensation while the second switch is turning on, the following performance is not worsen. The simulation result of detail model  $P(s)$  is shown in Fig.10. The simulation condition is same with the nominal model. As shown in Fig.10, the residual error is suppressed well after the re-learning.

Next, the re-learning scheme for the plant with modeling error is simulated. The plant model with modeling error  $P_e(s)$  is chosen as

$$P_e(s) = 1.1 \frac{k_p}{m \cdot s^2} e^{-sT_d}. \quad (27)$$

The simulation result is shown in Fig.11. In this simulation, the variation of disturbance is not varied. After the first switch turns off, the residual error caused by modeling error is appeared. However, the residual error is suppressed by the re-learning scheme after the second switch turns off.

#### IV. EXPERIMENTS ON RPTC

In the experiments, the proposed RPTC with re-learning scheme is compared with the original RPTC without re-learning scheme. The average time at the initial track is set to  $N_0 = 5$  and the weighting factor  $\alpha$  is 0.4. Feedback controller  $C_2[z]$  is the same with the simulation. Then the cut-off frequency becomes 850 kHz. To attenuate 60 Hz disturbance, peak filter is used. Notch filter is used to suppress  $\omega_2 = 2\pi 8000$  rad/sec resonance mode. To remove the noise caused by resonance mode or to reduce NRRO

effect, Q filter is used on the output of PSG in Fig.2 [5]. The Q filter is given as

$$\mathbf{r}_f[k] = \left[ \frac{z + \gamma + z^{-1}}{\gamma + 2} \right]^{N_q} \mathbf{r}[k], \quad (28)$$

where  $\mathbf{r}[k]$  is the output of PSG,  $\mathbf{r}_f[k]$  is the output of Q filter. This filter is a low-pass filter with zero phase-delay, and the  $N_q$  sample ahead value is needed as (28). Here, we chose  $\gamma = 2$  and  $N_q = 10$ . Then, the cut-off frequency of Q filter is 700Hz.

The experimental results are shown from Fig.12 to Fig.14. We chose the initial track as 40,000 and the final track as 40,100, with every 10 tracks seeking. At the initial track, the first compensation signal is made from averaged position error signal at the each sector. The first compensation signal is used for the initial track and the second track (40,010 track). After the each 10 tracks seeking, the residual position error signal is re-learned with proposed method and the new compensation signal is used for the next track. Therefore, the compensation signal which is used at the final track is re-learned at the 40,090 track.

PES, RRO and NRRO signals are shown in Fig.12(a), Fig.13(a) and Fig.14(a), respectively. RRO and NRRO signals are displayed with intentional off-set to prevent overlapping each other. As shown in Fig.12(b), Fig.13(b) and Fig.14(b) which are the FFT of RRO signals, RRO under the cut-off frequency of Q filter is suppressed. On the other hand, the RRO in frequency band in which the sensitivity function has peak or over cut-off frequency of Q filter not suppressed. The FFT of NRRO signal is shown in Fig.12(c), Fig.13(c) and Fig.14(c). As shown in Fig.14(c), the NRRO of proposed method is almost same with the others. Table. III shows the  $\pm 3\sigma$  for each case. As shown in Table. III, the proposed method attenuate position error signal well. We can show that the proposed method can improve the track following performance.

#### V. CONCLUSION

In this paper, a novel re-learning scheme of repetitive controller based on PTC is proposed and the advantages are demonstrated through simulations and experimens using HDD equipment with Discreate Track Recording media. In the conventional RPTC without re-learning scheme, the runout change is attenuated by feedback controller. When the runout is different from the compensation signal, the conventional RPTC needs to learn the runout with long time averaging. Moreover, the feedforward compensation is stoped during learning the compensation signal. On the other hand, in the proposed RPTC with efficient re-learning scheme considering correlation of adjacent tracks, the RRO which has similarity with adjacent tracks can be attenuated with feedforward compensation. Additionally, the feedforward compensation is continued during re-learning the residual PES. Consequently, the proposed re-learning scheme can maintain the track following performance.



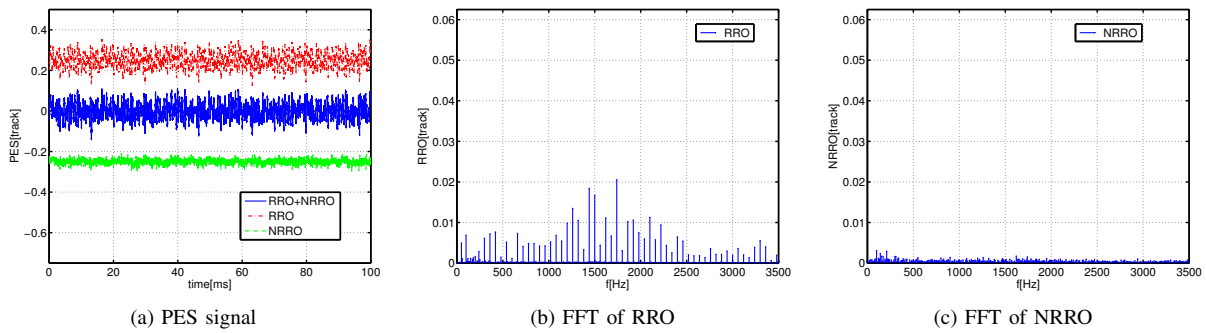


Fig. 12. Initial track (40000).

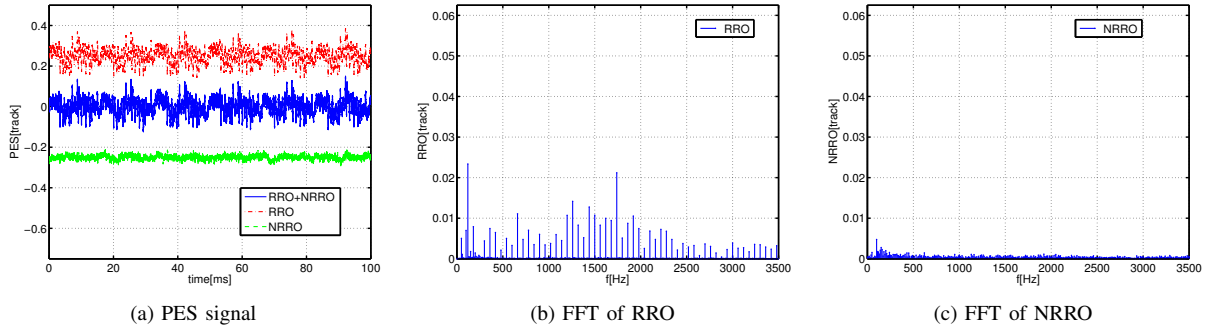


Fig. 13. Final track (40100) without re-learning scheme.

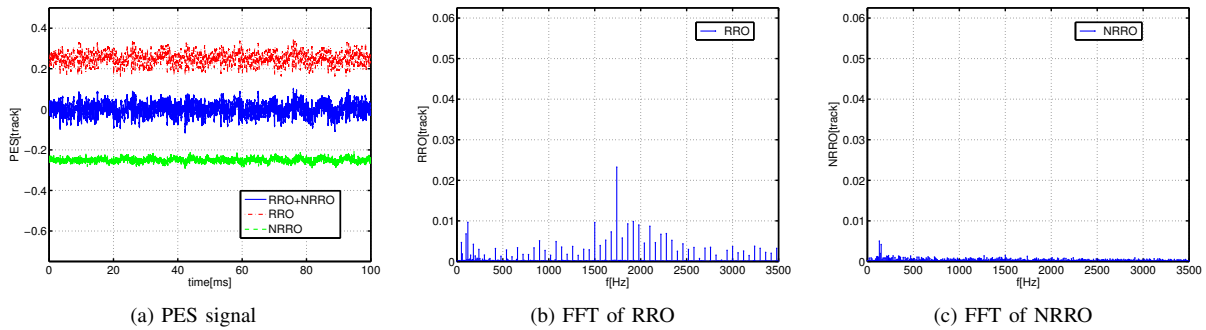


Fig. 14. Final track (40100) with re-learning scheme.

TABLE III  
 $\pm 3\sigma$  OF PES SIGNAL (TRACK) .

method	RPTC	without re-learning	with re-learning
track number	40000	40100	40100
$\pm 3\sigma$ [track]	0.249	0.253	0.197

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