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GPC-Based Remote Control for Hydraulic Position Control Systems in A Networked Environment

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Abstract— **This paper is concerned with the design of the remote hydraulic position control system (HPCS) using the modified generalized predictive control (M-GPC) method. Both sensor-to-controller (S-C) and controller-to-actuator (C-A) network-induced delays are modeled as Markov chains. The M-GPC uses the available output and previously predicted control information at the controller node to compute the future control sequences. Different from the conventional generalized predictive control (GPC) in which only the first element in control sequences is used, the M-GPC employs the whole control sequences to compensate for the network-induced delays in both S-C and C-A links. The closed-loop system is further formulated as a special jump linear system. The sufficient and necessary conditions for the stochastic stability are derived. The experimental tests for an HPCS are given to verify the effectiveness of the proposed method.**

I. INTRODUCTION

Hydraulic position control systems (HPCSs) are very important for the industrial application of control systems, e.g., aircraft flight control, remote robot position control, due to their characteristics of fast response, accurate positioning, and so on. Hence, the control of hydraulic systems has garnered significant research attention. There have been a large number of contributions on control synthesis methods applied to HPCSs, e.g., sliding mode control, neural network control, adaptive control, and model predictive control (MPC) [1]–[3]. However, most of the works are concerned with the local control of hydraulic systems while very few papers have paid attention to the networked control system (NCS) design for hydraulic plants. The use of networks as the medium to exchange information between the remote controller and hydraulic plant has several attractive advantages such as reduced equipment wiring, low installation cost, ease of maintenance, and enhanced mobility. There are, nevertheless, several problems such as network-induced time delays and packet dropouts, which can greatly degrade the closed-loop performance of HPCSs or even deteriorate the stability. In the networked environment, the local control schemes do not give satisfactory performance. For this reason, the predictive control method which can compensate for the time delays and packet dropouts is used in the networked control environment. In [4], the authors present an approach for analyzing the performance of dropout compensation strategies in the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ senses for NCSs with data losses under the framework of Markovian jump linear systems (MJLSs), which is tested on an experimental hydraulic servo system. In [5], the authors extend the generalized predictive control (GPC) to compensate for the delays, propose an adaptive predictive control method, and further apply the control scheme to a dual-axis hydraulic position system; but, the stability is not addressed.

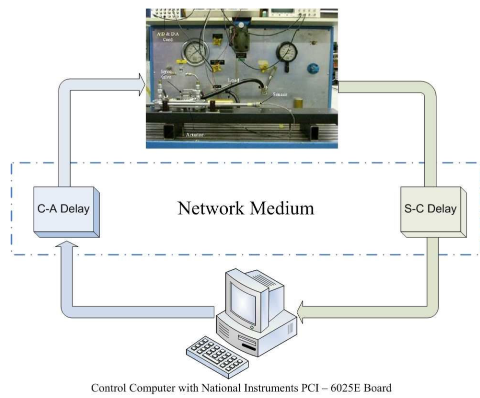

Fig. 1. Networked hydraulic position control system.

In a networked HPCS illustrated in Fig. 1, there are two types of time delays: Sensor-to-controller (S-C) and controller-to-actuator (C-A) delays. The S-C delays exist in the S-C link and can be obtained by using the time-stamping technique. Hence, the S-C delay information is available at the controller node. The other type of time delay is the C-A delay existing in the C-A links. Different from the S-C delay, the current C-A delay cannot be known at the controller node when the control actions are generated by the controller. In fact, the control signals that have been sent out and transmitted to the actuator node always suffer from the unknown C-A delays. Therefore, it is more challenging yet demanding to compensate for the C-A delays to improve the control system performance. In this paper, the S-C and C-A network-induced delays are modeled by homogenous Markov chains as in [6]–[8]. The Markov chain model of delays is widely used in the NCS design. The advantages lie in that it takes the dependency of the delays into account and the packet dropouts can be included naturally [6].

How to compensate for the C-A network-induced delay is a challenging issue. A natural idea is to employ a predicted signal (if available) to replace the delayed one whenever the C-A delay exists. MPC does have the prediction feature: At each time step, it not only generates the current control signal, but also a sequence of future control signals, under certain optimal settings. MPC has been one of the most

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popular advanced control methods in industry [9]. Recently, MPC strategies have been used in the NCS design [10]–[14]. In [11], Liu *et al.* propose a modified GPC method to compensate for the time delays and data packet dropouts. In [12], model-based estimation algorithms are developed to compensate for time delays and packet losses, but no stability analysis is given. Novel observer-based predictive controllers are proposed by incorporating only the C-A delays in [13], and both S-C and C-A delays in [14], respectively; the stability analysis is addressed via a switched system approach.

To the best of the authors' knowledge, the modified MPC (M-GPC) approach for NCSs has not been fully investigated, especially for NCSs with S-C and C-A delays modeled as Markov chains, which is the focus of this paper. On the other hand, application oriented research for networked HPCSs has received relatively less attention in the literature, which is another important motivation for this work.

The contributions of this paper are as follows.

1) Unlike [11], [13], [14] in which the S-C and C-A delays are assumed to be constant or random, both S-C and C-A delays are modeled as Markov chains in this study, and further the M-GPC is employed to compensate for the network-induced delays in both S-C and C-A links.
2) The closed-loop system is further formulated to be a special jump linear system, and the sufficient and necessary conditions to guarantee the stochastic stability are provided in terms of easily checked linear matrix inequalities (LMIs).
3) To move a step further towards practical applications, the developed M-GPC scheme is applied to an experimental HPCS.

The remainder of this paper is organized as follows. Section II describes the networked HPCS. In Section III, compensation schemes using the M-GPC algorithm for both S-C and C-A delays are given in detail. In Section IV, the sufficient and necessary conditions to guarantee the stochastic stability are presented. Section V details the controller design, experiments, and result analysis. Finally, the concluding remarks are addressed in Section VI.

II. Description of the Networked HPCS

The experimental HPCS shown in Fig. 2 is comprised of a double-rod cylinder, a proportional servo valve, and an inertia load force. The similar hydraulic system setup can be found in a wide variety of industrial processes. The objective of this paper is to design controllers to remotely control the HPCS to achieve the step tracking.

The identified continuous-time transfer function for the HPCS is [3]

$$G(s) = \frac{Y(s)}{U(s)} = \frac{422000}{s^3 + 141s^2 + 12100s}. \tag{1}$$

The output y is the cylinder position and the input u is the voltage. Its discrete-time model with the sampling time $t_s = 0.03$s can be obtained as

$$G(z^{-1}) = \frac{0.5946z^{-1} + 0.6178z^{-2} + 0.05628z^{-3}}{1 - 0.802z^{-1} - 0.1834z^{-2} - 0.01455z^{-3}}. \tag{2}$$

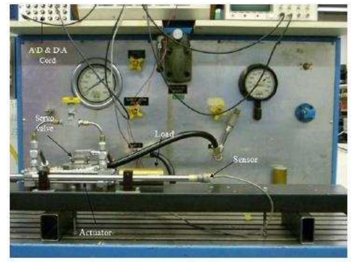

Fig. 2. Hydraulic position control system.

Now, consider a general single-input single-output discrete-time plant described as follows:

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k-1), \tag{3}$$

where $d \geq 0$ is the dead time of the system and

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_{n_a} z^{-n_a}, \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_{n_b} z^{-n_b}. \end{aligned}$$

From the discrete-time transfer function, we obtain $d = 0$.

Bounded random delays exist in the links from sensor to controller and controller to actuator as shown in Fig. 1. Here, $\bar{\tau} \geq \tau_k \geq 0$ represents the S-C delay and $\bar{d} \geq d_k \geq 0$ stands for the C-A delay. In this paper, τ_k and d_k are modeled as two homogeneous Markov chains that take values in $\mathcal{M} = \{0, 1, ..., \bar{\tau}\}$ and $\mathcal{N} = \{0, 1, ..., \bar{d}\}$, and their transition probability matrices are $\Lambda = [\lambda_{ij}]$ and $\Pi = [\pi_{rs}]$, respectively, meaning that τ_k and d_k jump from mode i to j and from mode r to s, respectively, with probabilities λ_{ij} and π_{rs}, which are defined by

$$\lambda_{ij} = \Pr(\tau_{k+1} = j | \tau_k = i), \quad \pi_{rs} = \Pr(d_{k+1} = s | d_k = r)$$

with the constraints $\lambda_{ij}, \pi_{rs} \geq 0$ and

$$\sum_{j=0}^{\tau} \lambda_{ij} = 1, \qquad \sum_{s=0}^{d} \pi_{rs} = 1,$$

for all $i, j \in \mathcal{M}$ and $r, s \in \mathcal{N}$.

III. Modified Generalized Predictive Control for NCSs

The conventional GPC algorithm consists of applying a control sequence that minimizes the following objective function [9]:

$$\begin{aligned} J(N_1, N_2, N_u) &= \sum_{j=N_1}^{N_2} \delta(j) \left[\hat{y}(k+j|k) - \omega(k+j)\right]^2 \\ &\quad + \sum_{j=1}^{N_u} \rho(j) \left[\Delta u(k+j-1)\right]^2, \end{aligned} \tag{4}$$

where $\hat{y}(k+j|k)$ is j step ahead prediction of the system output based on data up to k; N_1 and N_2 are the minimum and maximum prediction horizons, respectively; N_u is the control horizon; $\delta(j)$ and $\rho(j)$ are weighting sequences and

$\omega(k+j)$ is the future reference trajectory; $\Delta = 1 - z^{-1}$. The control constraint here is

$$\Delta u(k+j-1) = 0, \qquad j > N_u. \tag{5}$$

The future control predictions can be obtained based on the past output $y(k)$ up to time k and the past control signal $u(k)$ up to time $k-1$. For more details of calculating the predictive control signals using the conventional GPC, the readers are refereed to [9].

In an NCS shown in Fig. 1, due to the network-induced delays, the output signal $y(k)$ and control signal $u(k)$ at the plant node may not be received by the controller node immediately. Meanwhile, the current control signal might not reach the plant node in time. Hence, the conventional GPC cannot be directly applied to NCSs, which will be shown soon in the design examples. To compensate for the network-induced delays in a network environment, we need to modify the GPC scheme. During the network transmission, a sequence of signals can be packed up and transmitted together. To start with, some assumptions on the data transmission are made in the following:

A1: A sequence of output signals with length of n_a: $\begin{bmatrix} y(k)^{\mathbb{T}} & y(k-1)^{\mathbb{T}} & \cdots & y(k-n_a+1)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}$ are packed and sent to the controller node together.

A2: A sequence of predictive control signals with length of N_u: $\begin{bmatrix} u(k|k)^{\mathbb{T}} & u(k+1|k)^{\mathbb{T}} & \cdots & u(k+N_u-1|k)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}$ are packed and sent to the plant node together.

In the following, the compensation schemes for the S-C and C-A network-induced delays will be presented in detail.

A. Compensation for S-C delays

Considering the time delays in the S-C link, at current time k, the measurement information received by the controller is delayed by τ_k. Hence, the received output signal is $\begin{bmatrix} y(k-\tau_k)^{\mathbb{T}} & y(k-\tau_k-1)^{\mathbb{T}} & \cdots & y(k-\tau_k-n_a+1)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}$. Meanwhile, the control signal at the plant node up to time instance $k-1$ may not be available. To handle this, we can employ the previous predictive control signal at the controller node $\begin{bmatrix} u(k-1|k-1)^{\mathbb{T}} & u(k-2|k-2)^{\mathbb{T}} & \cdots \end{bmatrix}^{\mathbb{T}}$ instead. These two information vectors will be used to obtain the prediction of $y(t+j)$.

In order to minimize the cost function (4) and take the S-C delays τ_k into account, the prediction of $y(k+j)$ will be obtained by considering the following delay-dependent Diophantine equation:

$$\begin{aligned} 1 \;&=\; E_j(z^{-1})\tilde{A}(z^{-1}) + z^{-j-\tau_k}F_j(z^{-1}) \\ &\quad \text{with } \; \tilde{A}(z^{-1}) = \Delta A(z^{-1}). \end{aligned} \tag{6}$$

The polynomials E_j and F_j are uniquely defined with degrees $j+\tau_k-1$ and n_a, respectively. Then, the future prediction of $y(k+j)$ is

$$\hat{y}(k+j) = G_j(z^{-1})\Delta u(k+j-d-1) + F_j(z^{-1})y(k-\tau_k), \tag{7}$$

for $d+1 \le j \le N_2$, where

$$\begin{aligned} G_j(z^{-1}) &= E_j(z^{-1})B(z^{-1}) \\ &= g_0 + g_1 z^{-1} + \cdots + g_{j+\tau_k-1+n_b} z^{1-j-\tau_k-n_b}. \end{aligned}$$

Equation (7) can be rewritten as

$$\begin{aligned} \hat{y}(k+j) &= \bar{G}_j(z^{-1})\Delta u(k+j-d-1|k) \\ &\quad + z^{j-d-1}\left[G_j(z^{-1}) - \bar{G}_j(z^{-1})\right]\Delta u(k|k) \\ &\quad + F_j(z^{-1})y(k-\tau_k), \end{aligned} \tag{8}$$

where

$$\bar{G}_j(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{j-d-1} z^{d+1-j}.$$

Note that the last two terms in (8) only depend on the past data and the first term is related to future control actions. Further, we have

$$\mathbf{y}(k) = \Gamma\mathbf{u}(k) + G(z^{-1})\Delta u(k-1|k-1) + F(z^{-1})y(k-\tau_k), \tag{9}$$

where

$$\mathbf{y}(k) = \begin{bmatrix} \hat{y}(k+d+1|k) \\ \hat{y}(k+d+2|k) \\ \vdots \\ \hat{y}(k+N_2|k) \end{bmatrix}, \; \mathbf{u}(k) = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+N_u-1|k) \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_u-1} & \cdots & \cdots & g_0 \\ \vdots & \vdots & \cdots & \vdots \\ g_{N_2-1} & g_{N_2-2} & \cdots & g_{N_2-N_u} \end{bmatrix},$$

$$G(z^{-1}) = \begin{bmatrix} \left(G_{d+1}(z^{-1}) - \bar{G}_{d+1}(z^{-1})\right) z \\ \left(G_{d+2}(z^{-1}) - \bar{G}_{d+2}(z^{-1})\right) z^2 \\ \vdots \\ \left(G_{N_2}(z^{-1}) - \bar{G}_{N_2}(z^{-1})\right) z^{N_2-d} \end{bmatrix},$$

$$F(z^{-1}) = \begin{bmatrix} F_{d+1}(z^{-1}) \\ F_{d+2}(z^{-1}) \\ \vdots \\ F_{N_2}(z^{-1}) \end{bmatrix}.$$

The objective function (4) can be rewritten as

$$\begin{aligned} &J(N_1, N_2, N_u) \\ &= \left[\Gamma\mathbf{u}(k) + G(z^{-1})\Delta u(k-1|k-1) + F(z^{-1})y(k-\tau_k)\right. \\ &\quad \left.-\varpi(k)\right]^{\mathbb{T}} Q \times \left[\Gamma\mathbf{u}(k) + G(z^{-1})\Delta u(k-1|k-1)\right. \\ &\quad \left.+F(z^{-1})y(k-\tau_k) - \varpi(k)\right] + \mathbf{u}(k)^{\mathbb{T}} R\mathbf{u}(k), \end{aligned} \tag{10}$$

where

$$Q = \mathrm{diag}\left\{ \delta(N_1) \quad \delta(N_1+1) \quad \cdots \quad \delta(N_2) \right\};$$
$$R = \mathrm{diag}\left\{ \rho(1) \quad \rho(2) \quad \cdots \quad \rho(N_u) \right\};$$
$$\varpi(k) = \begin{bmatrix} \omega(k+N_1)^{\mathbb{T}} & \omega(k+N_1+1)^{\mathbb{T}} & \cdots & \omega(k+N_2)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}.$$

Here, $N_1 = d+1$ and $N_u \ge \bar{d}+1$.

By making the gradient of J to be zero, the optimal control increment signal can be obtained as

$$\begin{aligned} \mathbf{u}(k) &= (\Gamma^{\mathbb{T}}Q\Gamma + R)^{-1}\Gamma^{\mathbb{T}}Q\left[\varpi(k)\right. \\ &\quad \left.-G(z^{-1})\Delta u(k-1|k-1) - F(z^{-1})y(k-\tau_k)\right]. \end{aligned} \tag{11}$$

Further, the control signal can be determined by

$$\begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N_u-1|k) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u(k-1|k-1) + \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N_u} \end{bmatrix}$$
$$\times \left[\varpi(k) - G(z^{-1})\Delta u(k-1|k-1) - F(z^{-1})y(k-\tau_k)\right], \quad (12)$$

where

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{N_u} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \times \left[(\Gamma^{\mathbb{T}}Q\Gamma + R)^{-1}\Gamma^{\mathbb{T}}Q\right]. \quad (13)$$

Remark 1 The S-C time delays τ_k can be obtained by using the time-stamping technique. The effect of S-C time delays τ_k has been considered in (6) in the compensation scheme. It is worth noting that the matrices and vectors $G(z^{-1})$ and $F(z^{-1})$ are functions of τ_k, which will be used in the stability analysis. Also, $G(z^{-1})$, $F(z^{-1})$, and Γ can be calculated offline.

B. Compensation for C-A delays

Different from the conventional GPC in which only the current control signal $u(k|k)$ is used, the M-GPC uses the whole control signal sequences, which are packed and sent to the actuator/plant node together. Considering the time delay d_k from controller to actuator, the control signal received at the actuator/plant node at current time k is

$$\begin{bmatrix} u(k-d_k|k-d_k) \\ u(k-d_k+1|k-d_k) \\ \vdots \\ u(k|k-d_k) \\ \vdots \\ u(k-d_k+N_u-1|k-d_k) \end{bmatrix}. \quad (14)$$

We have set $N_u \geq \bar{d}+1$. Therefore, even though the control signal $u(k|k)$ may not be received at time k at the actuator/plant node, the previous prediction control signal for time k, $u(k|k-d_k)$ in the control package (14), is always already available at the actuator/plant node. Then, $u(k|k-d_k)$ will be chosen to be implemented on the plant.

Remark 2 A similar modified GPC method for NCSs was reported in [11], but the S-C time delay is assumed to be fixed. In our paper, the S-C delays are random and governed by Markov chains, which are more general in NCSs and also can include the fixed delays as special cases. However, the random S-C delays make the algorithm and the following stability analysis much more complex and challenging.

IV. Stability Analysis

Stability analysis is of great importance for the control system. In this section, we first formulate the closed-loop system as a special jump linear system. Then, the sufficient and necessary conditions for the stochastic stability are derived in the form of LMIs.

A. Closed-loop system

Without losing the generality, the reference input $\omega(k)$ is assumed to be zero. Let G_0 and F_0 be the coefficient matrices of polynomial vectors $G(z^{-1})\Delta$ and $F(z^{-1})$ in (12), respectively; n_g and n_f be the highest order of polynomials in vectors $G(z^{-1})\Delta$ and $F(z^{-1})$. Note that G_0 and F_0 vary with τ_k. Hence, we denote them as $G_0(\tau_k)$ and $F_0(\tau_k)$, respectively.

Then, (12) can be rewritten in the following form

$$\bar{U}(k) = G_1(\tau_k)\tilde{U}(k-1) + F_1(\tau_k)\tilde{Y}(k-\tau_k), \quad (15)$$

where

$$\begin{aligned} \bar{U}(k) &= \begin{bmatrix} u(k|k)^{\mathbb{T}} & u(k+1|k)^{\mathbb{T}} & \cdots & u(k+N_u-1|k)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}, \\ \tilde{U}(k-1) &= \begin{bmatrix} u(k-1|k-1)^{\mathbb{T}} & u(k-2|k-2)^{\mathbb{T}} & \cdots & u(k-n_g-1|k-n_g-1)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}, \\ \tilde{Y}(k-\tau_k) &= \begin{bmatrix} y(k-\tau_k)^{\mathbb{T}} & y(k-\tau_k-1)^{\mathbb{T}} & \cdots & y(k-\tau_k-n_f)^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}, \\ G_1(\tau_k) &= \left[\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^{\mathbb{T}} \; 0_{N_u\times n_g}\right] - HG_0(\tau_k), \\ F_1(\tau_k) &= -HF_0(\tau_k). \end{aligned}$$

Then, the control prediction sequence received at the actuator/plant node is

$$\begin{aligned} &\bar{U}(k-d_k) \\ =&G_1(\tau_{k-d_k})\tilde{U}(k-d_k-1)+F_1(\tau_{k-d_k})\tilde{Y}(k-d_k-\tau_{k-d_k}) \\ =&\begin{bmatrix} 0_{N_u\times d_k} & G_1(\tau_{k-d_k}) & 0_{N_u\times(\bar{d}-d_k)} \end{bmatrix}\hat{U}(k-1) \\ &+\left[0_{N_u\times(\tau_{k-d_k}+d_k)} \; F_1(\tau_{k-d_k}) \; 0_{N_u\times(\bar{d}+\bar{\tau}-d_k-\tau_{k-d_k})}\right]Y(k), \end{aligned}$$

where

$$\begin{aligned} \hat{U}(k) &= \begin{bmatrix} u(k|k)^{\mathbb{T}} & u(k-1|k-1)^{\mathbb{T}} & \cdots & u(k-n_g-\bar{d}|k-n_g-\bar{d})^{\mathbb{T}} \end{bmatrix}^{\mathbb{T}}, \\ Y(k) &= \left[y(k)^{\mathbb{T}} \; y(k-1)^{\mathbb{T}} \; \cdots \; y(k-\bar{\tau}-\bar{d}-n_f)^{\mathbb{T}}\right]^{\mathbb{T}}. \end{aligned}$$

Further, the control input of the plant is the $(1+d_k)$th element in vector $\bar{U}(k-d_k)$, which is

$$\begin{aligned} u(k) &= u(k|k-d_k) \\ &= \begin{bmatrix} 0_{1\times d_k} & 1 & 0_{1\times(N_u-d_k-1)} \end{bmatrix}\bar{U}(k-d_k) \\ &= c(\tau_k, d_k, \tau_{k-d_k})\hat{U}(k-1) + d(\tau_k, d_k, \tau_{k-d_k})Y(k), \end{aligned} \quad (16)$$

where

$$\begin{aligned} c(\tau_k, d_k, \tau_{k-d_k}) &= \begin{bmatrix} 0_{1\times d_k} & 1 & 0_{1\times(N_u-d_k-1)} \end{bmatrix} \\ &\quad\times\begin{bmatrix} 0_{N_u\times d_k} & G_1(\tau_{k-d_k}) & 0_{N_u\times(\bar{d}-d_k)} \end{bmatrix}, \\ d(\tau_k, d_k, \tau_{k-d_k}) &= \begin{bmatrix} 0_{1\times d_k} & 1 & 0_{1\times(N_u-d_k-1)} \end{bmatrix} \\ &\quad\times\left[0_{N_u\times(\tau_{k-d_k}+d_k)} \; F_1(\tau_{k-d_k}) \; 0_{N_u\times(\bar{d}+\bar{\tau}-d_k-\tau_{k-d_k})}\right]. \end{aligned}$$

Thus, based on (16), the control vector on the plant side can be expressed by

$$\begin{aligned} U(k) \;=\; & EU(k-1) + C(\tau_k, d_k, \tau_{k-d_k})\hat{U}(k-1) \\ & +D(\tau_k, d_k, \tau_{k-d_k})Y(k), \end{aligned} \quad (17)$$

where

$$U(k) = \left[\begin{array}{cccc} u(k)^{\mathbb{T}} & u(k-1)^{\mathbb{T}} & \cdots & u(k-n_b-d)^{\mathbb{T}} \end{array} \right]^{\mathbb{T}},$$

$$C(\tau_k, d_k, \tau_{k-d_k}) = \left[\begin{array}{c} c(\tau_k, d_k, \tau_{k-d_k}) \\ 0_{(n_b+d)\times(n_g+\bar{d}+1)} \end{array} \right],$$

$$D(\tau_k, d_k, \tau_{k-d_k}) = \left[\begin{array}{c} d(\tau_k, d_k, \tau_{k-d_k}) \\ 0_{(n_b+d)\times(\bar{\tau}+\bar{d}+n_f+1)} \end{array} \right],$$

$$E = \left[\begin{array}{cc} 0_{1\times(n_b+d)} & 0_{1\times 1} \\ I_{(n_b+d)\times(n_b+d)} & 0_{(n_b+d)\times 1} \end{array} \right].$$

It is clear from (3) that the output vector of the plant can be described by

$$Y(k) = A_1 Y(k-1) + B_1 U(k-1), \qquad (18)$$

where

$$A_1 = \left[\begin{array}{cc} \left[\begin{array}{cccc} -a_1 & -a_2 & \cdots & -a_{n_a} \end{array} \right] & 0_{1\times(\bar{\tau}+\bar{d}+n_f+1-n_a)} \\ I_{(\bar{\tau}+\bar{d}+n_f)\times(\bar{\tau}+\bar{d}+n_f)} & 0_{(\bar{\tau}+\bar{d}+n_f)\times 1} \end{array} \right],$$

$$B_1 = \left[\begin{array}{c} \left[\begin{array}{ccccc} 0_{1\times d} & b_0 & b_1 & \cdots & b_{n_b} \end{array} \right] \\ 0_{(\bar{\tau}+\bar{d}+n_f)\times(n_b+d+1)} \end{array} \right].$$

In addition, since $u(k|k)$ is the first row of $\bar{U}(k)$ in (15), it can be calculated by

$$\begin{aligned} u(k|k) \;&=\; \left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right]_{N_u\times 1} G_1(\tau_k)\tilde{U}(k-1) \\ &\quad + \left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right]_{N_u\times 1} F_1(\tau_k)\tilde{Y}(k-\tau_k). \qquad (19) \end{aligned}$$

Let

$$\begin{aligned} &\left[\begin{array}{cccc} \bar{g}_0(\tau_k) & \bar{g}_1(\tau_k) & \cdots & \bar{g}_{n_g}(\tau_k) \end{array} \right] \\ =\; &\left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right]_{N_u\times 1} G_1(\tau_k), \\ &\left[\begin{array}{cccc} \bar{f}_0(\tau_k) & \bar{f}_1(\tau_k) & \cdots & \bar{f}_{n_f}(\tau_k) \end{array} \right] \\ =\; &\left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right]_{N_u\times 1} F_1(\tau_k). \end{aligned}$$

Using (19), the vector $\hat{U}(k)$ can be constructed by

$$\hat{U}(k) = G_2(\tau_k)\hat{U}(k-1) + F_2(\tau_k)Y(k), \qquad (20)$$

where

$$G_2(\tau_k) = \left[\begin{array}{c} \left[\begin{array}{ccccc} \bar{g}_0(\tau_k) & \bar{g}_1(\tau_k) & \cdots & \bar{g}_{n_g}(\tau_k) & 0_{1\times\bar{d}} \end{array} \right] \\ \left[\begin{array}{cc} I_{(n_g+\bar{d})\times(n_g+\bar{d})} & 0_{(n_g+\bar{d})\times 1} \end{array} \right] \end{array} \right],$$

$$F_2(\tau_k) = \left[\begin{array}{c} \left[\begin{array}{cccccc} 0_{1\times\tau_k} & \bar{f}_0(\tau_k) & \bar{f}_1(\tau_k) & \cdots & \bar{f}_{n_f}(\tau_k) & 0_{1\times(\bar{d}+\bar{\tau}-\tau_k)} \end{array} \right] \\ \left[0_{(n_g+\bar{d})\times(\bar{\tau}+\bar{d}+n_f+1)} \right] \end{array} \right].$$

Further, combining (17), (18), and (20) yields the following closed-loop system

$$X(k) = A_c(\tau_k, d_k, \tau_{k-d_k})X(k-1), \qquad (21)$$

where

$$X(k) \;=\; \left[\begin{array}{c} Y(k) \\ U(k) \\ \hat{U}(k) \end{array} \right],$$

$$A_c(\tau_k, d_k, \tau_{k-d_k}) \;=\; \left[\begin{array}{ccc} A_1 & B_1 & 0 \\ D(\tau_k, d_k, \tau_{k-d_k})A_1 & E + D(\tau_k, d_k, \tau_{k-d_k})B_1 & C(\tau_k, d_k, \tau_{k-d_k}) \\ F_2(\tau_k)A_1 & F_2(\tau_k)B_1 & G_2(\tau_k) \end{array} \right].$$

Remark 3 The closed-loop system in (21) is a special jump linear system. Due to the random time delay in both S-C and C-A links, the state matrix A_c varies with $\tau_k, d_k, \tau_{k-d_k}$, which is different from and more complex than the closed-loop system in [11]. Meanwhile, the closed-loop system in (21) is not the standard Markovian jump linear system (MJLS) [15], because it depends not only on τ_k, d_k, but also τ_{k-d_k}. Hence, the existing results of stability analysis on MJLSs cannot be directly applied to this system.

B. Stochastic stability

Definition 1: [8] The system in (21) is stochastically stable if and only if for every finite $X_{-1} = X(-1)$, initial mode $\tau_{-d_0} = \tau(-d_0) \in \mathcal{M}$, and $d_0 = d(0) \in \mathcal{N}$, there exists a finite $W > 0$ such that the following holds:

$$\mathcal{E}\left\{ \sum_{k=0}^{\infty} \|X_k\|^2 |_{X_{-1},\ \tau_{-d_0},\ d_0} \right\} < X_{-1}{}^{\mathbb{T}} W X_{-1}. \qquad (22)$$

The sufficient and necessary conditions to guarantee the stochastic stability of system in (21) are shown in Theorem 1.

Theorem 1: The closed-loop system in (21) is stochastically stable if and only if there exists symmetric $P(i,r) > 0$ such that the following linear matrix inequality:

$$\begin{aligned} &\left(\sum_{j=1}^{\bar{d}} \sum_{s_1=0}^{\bar{\tau}} \sum_{s_2=0}^{\bar{\tau}} \pi_{rj} \Lambda_{is_2}^{1+r-j} \Lambda_{s_2 s_1}^{j-1} + \sum_{s_1=0}^{\bar{\tau}} \sum_{s_2=0}^{\bar{\tau}} \pi_{r0} \Lambda_{is_1}^{r} \Lambda_{s_1 s_2} \right) \\ &\quad \times A_c(s_1, r, i)^{\mathbb{T}} P(s_2, j) A_c(s_1, r, i) - P(i, r) < 0 \qquad (23) \end{aligned}$$

holds for all $i \in \mathcal{M}$ and $r \in \mathcal{N}$.

Proof: The proof is omitted here due to the space limitation. ■

The condition in (23) is sufficient and necessary, which is comprised of a set of LMIs. This feasibility problem can be efficiently verified using the LMI toolbox in Matlab.

V. Application for a Networked Hydraulic Position Control System

In this section, the hardware-in-the-loop (HIL) test is provided to verify the M-GPC algorithm. The design objective is to design a controller that can remotely control the HPCS over networks to achieve the step tracking.

First, we examine the local tracking control performance of GPC and apply the conventional GPC to the HPCS. The input $\omega(k)$ is $5u(t)$, where $u(t)$ is the step signal. The parameters of GPC are chosen as $N_1 = 1$, $N_2 = 12$, $N_u = 10$, $Q = I_{12\times 12}$, and $R = 50 \times I_{10\times 10}$. The result is shown in Fig. 3.

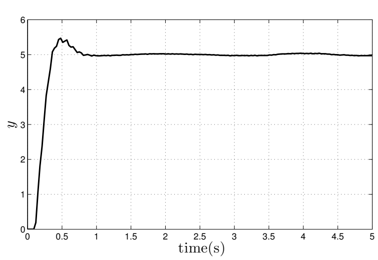

Fig. 3. Tracking performance of conventional GPC applied to local HPCS.

Next, we consider the networked HPCS shown in Fig. 1. Both S-C and C-A delays exist. The random delays involved are assumed to be $\tau_k \in \{0, 1, 2, 3\}$ and $d_k \in \{0, 1, 2, 3\}$, and their transition probability matrices are given by

$$\Lambda = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.5 & 0.2 \end{bmatrix}, \ \Pi = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.5 & 0.2 \end{bmatrix}.$$

The time delays τ_k and d_k are shown in Figs. 4 and 5, respectively. Directly apply the designed conventional GPC (as used in the first example) to the networked HPCS, the tracking performance is shown in Fig. 6. It is observed that the system becomes unstable due to the networked-induced delays. Obviously, the stability cannot be guaranteed, let alone the tracking performance. Therefore, the conventional GPC cannot be directly applied in a network environment.

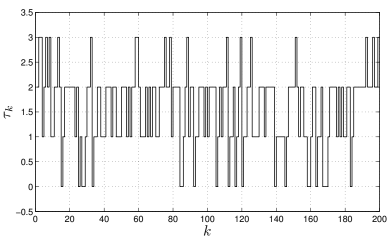

Fig. 4. S-C delays τ_k.

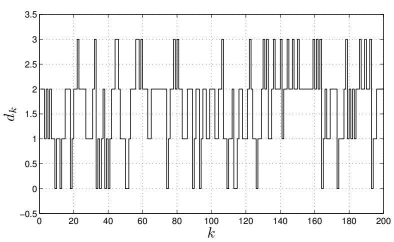

Fig. 5. C-A delays d_k.

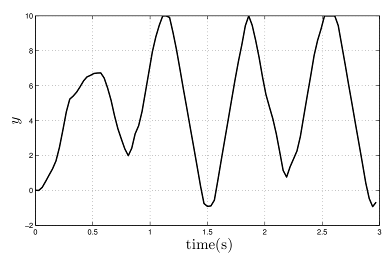

Fig. 6. Tracking performance of conventional GPC applied to networked HPCS.

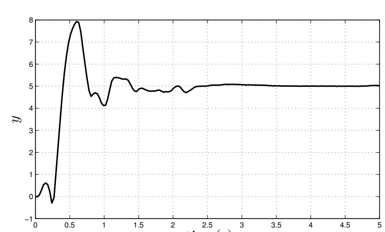

Fig. 7. Tracking performance of M-GPC applied to networked HPCS.

By applying the proposed M-GPC method, the results are shown in Fig. 7. It is observed that the system is stable. The output can track the desired input and the M-GPC guarantees the stability. Through the experimental tests, we can see that the proposed M-GPC is effective in compensating for both S-C and C-A delays.

VI. Conclusion

This paper investigates the predictive controller design problem for networked HPCSs. Both the S-C and C-A time delays are random and modeled by Markov chains. The M-GPC method is proposed to compensate for the time delays in both links. The closed-loop system is formulated to be a special jump linear system. The sufficient and necessary conditions for the stochastic stability are provided in the form of LMIs, which can be conveniently checked. The experimental results indicate that the proposed theoretical tools can be useful in real NCSs. Important issues such as robust G-MPC design against model uncertainties, disturbance attenuation, and tracking performance improvement need further study.

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