# Model-Based Control of Multi-Unit Systems under Partial Shutdown Conditions

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Abstract—A systematic control strategy is proposed for the optimal operation of multi-unit systems undergoing a partial shutdown. The strategy entails manipulating the degrees-offreedom available during and after a shutdown such that production is restored in a cost-optimal fashion while meeting all safety and operational constraints. In this work, we investigate the coordination of buffer tanks, production rates and recycle streams during a partial shutdown. The problem is cast as a nonlinear dynamic optimization problem. Disturbances are handled using a nonlinear predictive controller that is explicitly aware of shutdown events.

# I. INTRODUCTION

In a typical chemical plant, raw materials are transported through a number of processing units to be fashioned into some end-product. A shutdown in an intermediate unit constitutes an disruption in the processing chain, which can adversely affect the plant's ability to continue operating. The severity of a disruption is typically a function of the configuration of the processing units and the extent of remedial actions taken.

In many plants, process units are shut down from time to time either for maintenance or due to equipment failure. From an operations perspective, unit shutdowns can be categorized into *critical* and *partial* shutdowns. Critical shutdowns are those that lead to the shutdown of the entire plant, and partial shutdowns are those that do not.

In the case of critical shutdowns, the entire plant is forced to go offline. Under such circumstances the usefulness of any control policy is limited. Under partial shutdown scenarios however, it is frequently possible for an operator to pursue certain courses of action that will permit the unaffected units to continue operating to some degree. Possible courses of action include reconfiguring the process pathways, re-routing material streams, slowing down production, making use of buffer capacities and so on. The end-product during the shutdown period may go off-specification, but in some cases, they can be recycled and reprocessed.

In this work, we propose an optimal control scheme that will compute control trajectories for equipment in unaffected parts of the plant, during (and after) a shutdown, with the view of restoring the system to its pre-shutdown state in a cost-optimal way.

#### II. BACKGROUND

We will primarily be examining the optimal use of buffer capacities and the manipulation of production rates and recycles during the shutdown period. The essential goal of our control scheme is to arrive at a set of optimal control inputs for effecting the process transitions during the shutdown and restoration periods.

The main function of buffer capacities is to decouple various segments of a plant. When a process unit downstream of a buffer tank fails, the tank can hold the material for a period of time until the process unit is brought back online. Likewise, when a process unit upstream of a buffer tank fails, the material held in the buffer tank can slowly be discharged to the downstream units in order that downstream processing may continue.

Having judiciously placed buffer capacities in a plant also helps to mitigate the propagation of process variations along a production line. These intermediate storage units are not only able to dampen the effects of short term fluctuations, they are also able to deal with larger processing disturbances such as total unit shutdowns if they can be coordinated correctly.

#### III. MODELING

To motivate this study, we will consider unit shutdowns in the fiber line of Kraft paper plant. The diagram in Fig. 1 shows a simplified schematic of the Kraft plant fiber line topology. The main departments in this section of the plant are the digestion, knotting & washing, and delignification departments. These departments are separated by buffer tanks, whose levels may be manipulated in order to keep the plant operational when any unit in any given department is shut down.

Wood chips and liquor are charged into the digester unit, which cooks the chips into pulp. The digested pulp is held in a buffer tank (Buffer 1), and is sent downstream to the knotting and washing units for further processing. The output of the washing units is conveyed into another buffer tank (Buffer 3). The contents of this tank are recycled to the washing units and to the first buffer tank (Buffer 1). The diluted pulp exiting the washing units is transported to the screening units and then held in a buffer tank (Buffer 2), and subsequently pumped to the delignification units.

In our model, the buffer tanks are dynamic units represented by the differential-algebraic equations (DAEs) below.

$$h = \frac{M}{\rho_{avg} \cdot A} \tag{1}$$

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Fig. 1. System topology of the fiber line of a Kraft paper plant.

 $\frac{d}{dt}M = F_{in} - F_{out} \tag{2}$ 

$$M \cdot \frac{d}{dt} x_{out}^P = F_{in} \cdot (x_{in}^P - x_{out}^P)$$
(3)

$$M \cdot \frac{d}{dt} x_{out}^{DS} = F_{in} \cdot (x_{in}^{DS} - x_{out}^{DS})$$
(4)

$$x_{out}^P + x_{out}^W + x_{out}^{DS} = 1 (5)$$

where h = tank level (m), M = mass holdup (tons),  $A = \text{area (m}^2)$  of the base of tank (the tank is assumed to be perfectly cylindrical),  $\rho_{avg} = \text{average density of the material in the tank (tons/m<sup>3</sup>)}$ ,  $F_i = \text{flowrates (tons/hr) of material in stream } i \in \{in, out\}, x_i^j = \text{mass fraction of material } j \in \{P, W, DS\}$  in stream  $i \in \{in, out\}$ . Three components are considered: pulp (P), water (W) and dissolved solids (DS).

The dynamics of the other process units are assumed to be sufficiently fast relative to the control interval so as to be represented by a set of nonlinear algebraic equations.

#### **IV. PROBLEM FORMULATION**

The problem of determining optimal control inputs for operating the plant under shutdown conditions is cast within a dynamic optimization framework.

The formulation is as follows:

**Objective Functional** 

$$\max_{\mathbf{u}(t)} \Phi_{economics} \tag{6}$$

subject to

**Economics-based Objective Function** 

$$\Phi_{economics} = \sum_{m \in K} \left[ C_m \int_0^{t_f} F_m \, dt \right] \tag{7}$$

See Table I for values.

**Model Equations and Constraints** 

$$\mathbf{f}(\mathbf{\dot{x}}(t), \mathbf{x}(t), \mathbf{z}(t), \mathbf{u}(t), t) = \mathbf{0}$$
(8)

$$\begin{aligned}
\mathbf{g}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{u}(t), t) &= \mathbf{0} \quad (9) \\
\mathbf{h}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{u}(t), t) &\leq \mathbf{0} \quad (10)
\end{aligned}$$

for 
$$t \in [0, t_f]$$

#### Variable bounds

 $\mathbf{x}_{\mathbf{L}} \le \mathbf{x}(t) \le \mathbf{x}_{\mathbf{U}} \tag{11}$ 

$$\mathbf{z}_{\mathbf{L}} \le \mathbf{z}(t) \le \mathbf{z}_{\mathbf{U}} \tag{12}$$

$$\mathbf{u}_{\mathbf{L}} \le \mathbf{u}(t) \le \mathbf{u}_{\mathbf{U}} \tag{13}$$

for 
$$t \in [0, t_f]$$

**Initial conditions** 

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{14}$$

#### **Restoration constraints**

$$\mathbf{x}_0 - \epsilon_x \le \mathbf{x}(t) \le \mathbf{x}_0 + \epsilon_x \quad \text{for } t_{res} < t \le t_f \quad (15)$$

$$\mathbf{u}_0 - \epsilon_u \le \mathbf{u}(t) \le \mathbf{u}_0 + \epsilon_u \qquad \text{for } t_{res} < t \le t_f \qquad (16)$$

# Shutdown constraints

$$F_{in,unit}(t) = f_{shutdown}$$
 for  $t_{start} \le t \le t_{end}$  (17)

where

 $t_f = \text{final time}$ 

 $t_{res}$  = time at the end of restoration period

 $t_{start}$  = time at which shutdown commences

 $t_{end}$  = time at which shutdown ends

K = set of materials consumed/produced (see Table I) = {pulp, chips, chemicals, steam, liquor}

- $\mathbf{x}(t)$  = differential state vector
- $\mathbf{z}(t)$  = algebraic state vector
- $\mathbf{u}(t) = \text{control input vector}$

$$\epsilon_x, \epsilon_u =$$
tolerance

 $F_{in,unit}(t)$  = mass flow into a process unit, where *unit* denotes a specific process unit that is shut down

 $f_{shutdown}$  = shutdown flowrate threshold value, below which a unit is deemed to have shut down. Usually set to 0 or some small number.

 $\Phi_{economics}$  = economic objective function

m = materials produced or consumed

 $C_m$  = price of material m

 $F_m$  = flowrate of material m

Fig. 2 shows the different phases of the shutdown process. A plant will initially be operating at a certain steady-state point. The shutdown phase begins at  $t_{start}$ , when all input

Materials, m	<b>Prices</b> , $C_m$ (\$/ton)
Pulp	725
Chips	-25
Chemicals	-100
Steam	-7.31
Energy from Black Liquor	0.348

TABLE I

PRICES OF MATERIALS [1] (NEGATIVE PRICES INDICATE CONSUMED MATERIALS)

and output flows to the process unit being taken offline are forced be 0. The shutdown phase proceeds until  $t_{end}$ . Between  $t_{start}$  and  $t_{end}$ , we assume measures are taken to restore or repair the unit. At time  $t_{end}$ , the unit is deemed to be ready for for operation, and the restoration phase begins. In the restoration phase, control actions are prescribed to the plant to return it to its original steady-state operating point. The restoration phase terminates at time  $t_{res}$ , a juncture at which the plant has been successfully restored to normal steady-state operation.

# A. Optimization

Unlike most predictive control formulations, the objective here is not a weighted minimization of a setpoint tracking error and manipulated input variation; instead, the goal is to find a set of trajectories that maximizes an economic objective function such that the losses due to a shutdown might be minimized.

To solve the DAE system using an optimization (specifically, a nonlinear programming [NLP]) framework, the differential-algebraic equations are discretized in the time coordinate using a Backward Euler approximation and piecewise constant inputs (zero-order hold). There are also other options such as orthogonal collocation on finite elements (which corresponds to an implicit Runge-Kutta method), should a more precise integration procedure be desired. The resulting set of equations is posed as constraints in the optimization problem. In our case, the resulting sparsestructured NLP is modeled using a specialized in-house modeling system [2] and solved by means of a large-scale interior point optimizer, IPOPT [3].

#### B. Nonunique Trajectories and Two-Tiered Optimization

In some cases, the input trajectories calculated may be nonunique; that is, there exists more than one set of input trajectories that, when implemented, give the same objective value. Some of these trajectories may exhibit a high-



Fig. 2. Shutdown timeline

frequency chatter-like behavior, while others prescribe large changes in an input variable over a short period of time. These types of trajectories, although optimal in the economic sense, are undesirable, giving rise to actuator/valve wear.

To address this problem, a type of hierarchical optimization algorithm is applied. In our case, we perform a "twotiered optimization". In the first calculation, we solve the dynamic optimization problem that maximizes an economic objective function, and record the optimal value of the objective. This value represents an upper-bound on economics achievable by the system.

Tier 1: Obtain trajectory for optimal economics

$$J_{economics}^* = \max_{\mathbf{u}} \Phi_{economics} \tag{18}$$

where  $\mathbf{u}$  = discretized input vector.

In the second calculation, the objective function is replaced with another objective that minimizes control effort, subject to a minimum constraint on the economic performance.

Tier 2: Minimize control effort

$$\min_{\mathbf{u}} ||\Delta \mathbf{u}||_2 \tag{19}$$

s.t. 
$$\Phi_{economics} \ge (1 - \xi) \cdot J^*_{economics}$$
 (20)

where  $\xi$  = percentage of original economics the customer is willing to trade off to obtain a smoother trajectory (commonly set to 1%).

This method returns input trajectories that require a minimum control effort to achieve a specified level of achievable economic performance.

# C. Restoration constraints

A unit shutdown is a transient event which ought not to permanently shift the original operating point of the process. As such, we require states and inputs to return to their original pre-shutdown values through restoration constraints. This ensures that the result obtained from the optimization is meaningful; if the states and inputs were not restored, the optimizer would pursue avenues for optimizing the objective by coercing the system to a new operating point, which violates the mandate of a control system that is designed to handle only the transient event. It is also complicates the quantification of the economics of the system as inventory deviation costs will then have to be accounted for.

Therefore, state and input restoration constraints (15, 16) are imposed for the purpose of bringing the system to its preshutdown steady-state at the end of the restoration period.

From the viewpoint of reliability, it is imperative that the buffer tank levels be restored to their original levels after the shutdown/restoration transition period so that the system is at a state where it is prepared to accommodate subsequent shutdowns should they occur. The restoration of constraints also has a bearing on the optimality of the problem. Without these restoration constraints, the optimizer will prescribe control actions to empty out the tanks to obtain as much product as possible out of the last unit (due to the pulp production throughput's influence on the objective function). Allison [4] demonstrated that the optimal solution for such



Fig. 3. Shutdown Controller Configuration

problems over a fixed horizon is to drive the storage vessels empty. This is clearly undesirable as it deprives the plant of the ability to anticipate any further shutdowns.

# D. Assumptions

It is assumed that the system is ready to resume operation at the end of the shutdown phase (i.e. at  $t_{end}$ , see Fig. 2). The startup process is not modeled explicitly.

Another assumption made is that the shutdown of a unit is perfectly modeled by turning off the inlet/outlet flows to that unit, and that the relevant shutdown procedure for a unit is followed. This shutdown procedure itself is not modeled; we assume that a manual or an automated procedure for start-ups and shutdowns is in place, as in [5].

#### V. MODEL-BASED SHUTDOWN CONTROLLER

Operating a plant undergoing a partial shutdown requires a suitable feedback control scheme. In the previous section, the procedure for generating operating policies using dynamic optimization was described. However, during the period of transience, unforeseen process disturbances and model mismatch can invalidate these policies; therefore some kind of feedback mechanism is imperative for the control scheme to be practicable. Toward this end, we propose a model-based shutdown controller (based on ideas from Model Predictive Control [MPC]) as a means of implementing abnormal-situation control on the plant.

We envisage a scenario where the operator switches the plant mode from "normal operation" to "abnormal operation" when a shutdown occurs, in which the proposed modelbased shutdown controller activates and takes over from the standard control system. As soon as the system is restored to its original state, the shutdown controller then transfers the control back to the standard control system.

The goal of our study is to demonstrate this framework in the role of a controller for the fiber line of the Kraft paper mill under a shutdown scenario.

# A. Controller Setup

Referring to Fig. 3, the feedback mechanism works as follows:

 The predictive controller performs an open-loop dynamic optimization based on the current states of the system. A set of control trajectories is obtained.

- 2) The first step of the calculated trajectories is implemented on the plant. To carry out the control actions, the predictive controller either sends setpoints to lower-level PID controllers in local control loops, or sends control signals directly to the actuators in the plant itself.
- 3) Plant measurements are taken and fed into the state estimator, which typically takes the form of an Extended Kalman Filter (EKF). The estimate of the states is then conveyed back to the predictive controller, where it is used as initial values for the next iteration.
- 4) This process is repeated until the system is restored to its nominal steady state operation.

It is important to note that the predictive controller here is distinct from a conventional MPC setup in that its objective is not to track given setpoints, but to implement manipulated variable adjustments to optimize an economic objective.

Shutdowns are finite in duration. Therefore, in lieu of the ordinary receding prediction and control horizons found in conventional MPC, shrinking prediction and control horizons are used. (The length of our prediction and control horizons decrease as the the controller advances toward the end of the time horizon.)

In the shutdown problem, the prediction horizon length needs to correspond to the duration of the shutdown and restoration combined. If a shorter length is chosen, suboptimal control may result from the controller not having an adequate picture of the full transient process. Also, in our predictive controller, the control horizon equals the prediction horizon.

#### B. Explicitly Known Events

Explicitly known discrete events (such as time and duration of a planned shutdown and restoration) are embedded directly into the prediction model. These events can be specified either through operator invention or automatically prescribed by a fault monitoring module. This form of process event anticipation is distinct from traditional feedforward control in which disturbances are detected solely through plant measurements.

From a computation point of view, the temporal entry and exit points of this type of transient event are usually specified relative to the left-hand boundary of the prediction window. Because the shrinking-horizon causes this reference boundary to move as it advances to the end of the time horizon, the entry/exit points of the transient event must be shifted accordingly as time progresses.

Let us suppose a transient event  $T(\mathbf{x}, \mathbf{z}, \mathbf{u}, k)$  occurs in the period  $[k_{start}, k_{end}]$ , and the left-hand boundary of the prediction horizon is the current time, k. In the predictive control optimization problem at time k, the event constraint is expressed as follows:

# **Event Constraint**

$$T(\mathbf{x}, \mathbf{z}, \mathbf{u}, k) = 0 \quad \text{for } k \in [k_{lhs}, k_{rhs}], \qquad (21)$$

where

$$k_{lhs} = \begin{cases} k_{start} - k, \text{ if } k < k_{start} \\ k, \text{ if } k_{start} \le k \le k_{end} \end{cases}$$
$$k_{rhs} = k_{end} - k, \quad \text{if } k \le k_{end}$$

As k moves past the endpoint of the event (*i.e.*  $k > k_{end}$ ), (21) is dropped from the optimization problem.

#### VI. CASE STUDIES

A. Optimal Operating Policy for a Shutdown in the Delignification Department (Open-loop)

In this case study, the delignification department in the Kraft plant (Fig. 1) was shut down for 6 hours. Fig. 4 shows the plant operating at steady-state from t = 0-7 hrs. A shutdown occurs at t = 8 hr, the effect of which can be seen in the  $F_{out2}$  (the flowrate from the buffer tank directly upstream of the department) trajectory, as it shuts off between t = 8-14 hrs. Dynamic optimization was used to calculate the open-loop control policies that work to restore the system to its original steady-state point, using the economic objective function in (6). The two-tiered optimization method was applied.

Note that  $M_1, M_2, M_3$  are differential state variables (**x**),  $F_{digest}, F_{out1}, F_{out2}, F_{out3,1}, F_{out3,2}$  are input variables (**u**) and  $F_{in1}, F_{in2}, F_{in3}$  are algebraic state variables (**z**).

During the shutdown, t = 0-7 hrs

Throughout the shutdown, all flows into the delignification unit are cut off, hence we observe an increase in the level of the buffer tank directly upstream of it, as reflected by  $M_2$ making a steady climb in Fig. 4.

During the restoration, t = 8-56 hrs

In order to make up for lost production due to the shutdown, the contents in Buffer 1 are rapidly discharged for processing downstream, as witnessed by a descent in  $M_1$ . We also see rise in the  $M_3$  on account of the increased processing rate. However, the recycle flowrate  $F_{out3,1}$  is kept low in order to not dilute the contents of Buffer 1.

In the later part of the restoration, we see a burst in the recycle stream  $F_{out3,1}$ , which serves to bring  $M_1$  back to its original steady-state level (as enforced by endpoint constraints). The contents of Buffers 2 and 3 are discharged in order to bring  $M_2$  and  $M_3$  back to steady-state levels.

We note that throughout the shutdown and restoration process, the units that are not offline are able to continue operating.

# B. Disturbance Rejection using the Shutdown Controller (Closed-loop)

In this case study, the predictive controller was applied to the system to demonstrate the use of feedback to counter disturbances.

During the shutdown, t = 8-14 hrs

A shutdown in the knotting unit occurs between t = 8-14 hrs (duration 6 hrs). This shutdown can be observed in the  $F_{out1}$  trajectory (Fig. 5), where the flowrate into the knotter drops to zero during this time. The nominal trajectories are represented by the dashed lines.



Fig. 4. Optimal Operating Policy for Shutdown in Delignification Department (t = 8-14 hrs)

Material accumulates in Buffer 1, as seen by a steady increase in  $M_1$ . The digester feed  $F_{digest}$  flowrate and the blowtank recycle flowrate  $F_{out3,2}$  are throttled down in order to prevent Buffer 1 from violating its capacity constraints. Further downstream, the contents of Buffer 2 are depleted in a gradual fashion (see  $M_2$  trajectory) to provide the downstream units with material to continue operating.

During the restoration, t = 15-60 hrs

To restore the system to its original steady state, the contents of Buffer 1  $(M_1)$  are discharged to the knotter, and inventory in Buffer 2  $(M_2)$  is built back up.

Disturbance rejection

A disturbance was introduced between t = 22-37 hrs, where the composition of the  $F_{digest}$  stream was stepped from  $(x^P, x^W, x^{DS}) = (0.43, 0.53, 0.04) \rightarrow (0.38, 0.58, 0.04)$ . This represents a transient drop in the concentration of pulp (P) and an increase in the amount of water (W) in the digester feed stream.

To motivate the need for feedback, the open-loop optimal control trajectories (assuming no disturbance) were implemented on the plant without feedback. In this particular case, we found that the resulting operation became infeasible in the presence of the specified disturbance.

Then, the predictive controller was applied in a closed-



Fig. 5. Disturbance Rejection with Shutdown Controller for Shutdown in Knotting Unit (t = 8-14 hrs). (dashed lines = nominal trajectories, solid lines = actual trajectories)

loop simulation. Upon detection of the disturbance, the controller calculates a set of trajectories that compensates for the effect of the drop in pulp concentration. Fig. 5 shows the actual MPC trajectories superimposed on dotted lines representing trajectories that would have been obtained had the disturbance not occurred.

The predictive controller countered the drop in pulp concentration by increasing the residence time of pulp in the system. Production is scaled down during the disturbance period, as seen by a drop in  $F_{digest}$ . The liquor-carrying recycle stream flowrate in  $(F_{out3,1})$  is also brought down in order to prevent further dilution of the pulp.

This motivates the need for feedback for the implementation of control policies, in light of system uncertainties.

# VII. CONCLUSIONS AND FUTURE WORK

#### A. Conclusions

In this work, we presented the use of dynamic optimization for calculating optimal control inputs for plants undergoing partial shutdowns. Buffer capacities were used to decouple plant departments in order to circumscribe the effects of the shutdown, and also to permit units that are not offline to continue operating, albeit at reduced levels. This was accomplished through the cost-optimal adjustment of production rates, recycles and buffer levels. A two-tiered optimization method was used to address the problem of nonunique trajectories. A case study involving a partial shutdown in the delignification department of a Kraft pulp plant was presented.

Disturbances during and after the shutdown can cause the trajectories to deviate from the optimal control policy. A predictive control framework encapsulating a dynamic optimizer was developed for countering these effects by means of feedback. The proposed predictive controller incorporates a nonlinear first-principles model of the plant and uses an economic objective function. Because a partial shutdown is a transient event with a finite endpoint, controller calculations are performed on a shrinking-horizon basis. This framework also allows events to be embedded into the prediction model, thus enabling the controller to anticipate explicitly-known events. A case study was presented to illustrate the use of this scheme to address a feed disturbance.

#### B. Future Work

Induced shutdowns are shutdowns that occur following a shutdown in a particular unit. In some cases, one can avoid induced shutdowns by imposing a shutdown-cost penalty function in the objective that accurately captures the cost of the shutdowns. Discontinuous formulations of these penalty functions will be explored.

Specifications enforced under nominal conditions may cease to be optimal or feasible during partial shutdowns, necessitating the relaxation or release of a subset of the specifications during the transient period. An optimizationbased strategy is being explored to determine this subset and to obtain a dynamic operating window that will enable the system to effect an efficient restoration.

#### VIII. ACKNOWLEDGMENTS

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