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Terminal Iterative Learning Control design with Singular Value Decomposition Decoupling for Thermoforming Ovens

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***Abstract*—Terminal Iterative Learning Control (TILC) is a cycle-to-cycle control approach that can be used on thermoforming oven. TILC automatically tune the heater temperature setpoints such that the temperature at the surface of the plastic sheet tracks a desired temperature profile. Industrial thermoforming ovens can have a large number of temperature sensor (inputs) and heaters (outputs) which makes the design of TILC difficult. This paper presents the design of a TILC using the singular value decomposition decoupling technique. With this tool, the TILC design is facilitated for industrial thermoforming oven.**

I. Introduction

THE thermoforming process consists of heating sheets of plastic before molding them [1, 2]. Each heater is locally controlled by PID loop, since its temperature is measured with an embedded thermocouple. However, the heater temperature setpoints of these PID loops are adjusted manually by trial and errors [1, 2]. This can result in plastic sheets being heated because they come out of the oven with a wrong temperature profile. Therefore, a financial lost is incurred from bad adjustment of heater temperature setpoints.

The use of Terminal Iterative Learning Control (TILC) can solve this problem by tuning automatically the heaters temperature setpoint [3, 4]. This cycle-to-cycle control was successfully used on an industrial thermoforming machine. To be able to use the TILC, the plastic sheet surface temperature profile needs to be measured. This can be done with infrared temperature sensors [3-5].

TILC was first introduced in [6, 7]. The application where TILC is used is mainly for rapid thermal processing chemical vapor deposition [6-9] and recently in the plastic sheet surface temperature control in thermoforming machine [3, 4]. TILC is derived from Iterative Learning Control (ILC). The main difference between the two approaches is the access to measurements during the cycle [8]. For ILC, there is measurement taken throughout the cycle [10], whereas TILC have access to only one measurement at the end of the cycle.

When the number of inputs (infrared temperature measurements) and the number of outputs (heater temperature setpoints) are high, it may become difficult to design the TILC, especially if it is a high order one [4].

A way to simplify the design of the TILC is to use singular value decomposition (SVD) decoupling [4, 11]. SVD seems rarely used for TILC, but it can facilitate the design of optimal reduced-order ILC [12], it can help to suppress residual vibration with ILC [13] and also, to achieve decoupling on ILC [11].

Section II presents the system to be controlled by TILC algorithm. Section III introduces the SVD decoupling as a tool for the design of TILC. Section IV presents a second-order TILC algorithm. Simulation results, using TILC designed using SVD decoupling and the thermoforming oven model developed in [3, 5, 14], are shown in Section V. Section VI concludes.

II. Problem Set-Up

The TILC algorithm is applied to a continuous-time linear time-invariant (LTI) system. This LTI system can be the linearized model of the reheat phase of a thermoforming machine [3, 5, 14, 15] and is represented by:

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where *t* is the time and the subscript *k* is the cycle number ($k \in \mathbb{N}$). Matrices *A*, *B*, and *C* are time-invariant. The state vector is $x_k(t) \in \mathbb{R}^n$, the (constant over one cycle) input vector is $u_k \in \mathbb{R}^m$ and the output vector is $y_k(t) \in \mathbb{R}^p$.

The control task is to update the control input u_k (heaters temperature setpoints) such that the terminal output $y_k(T)$ (sheet surface temperature at some locations) converges to a desired terminal value y_d at time *T*. From linear system theory, one can write the solution of (1) at $t = T$ as:

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$$x_k(T) = e^{AT} x_k(0) + \left(\int_0^T e^{A(T-\tau)} B d\tau \right) u_k . \quad (2)$$

From this terminal state, we calculate the corresponding terminal output as:

$$y_k(T) = Ce^{AT} x_k(0) + \left(C \int_0^T e^{A(T-\tau)} B d\tau \right) u_k . \quad (3)$$

Now, we change the notation to emphasize the fact that for the cycle-to-cycle control, the cycle k is equivalent to the time argument of a discrete time system [4]. Equation (3) is rewritten as:

$$y_T[k] = \Gamma x_0[k] + \Psi u[k], \quad (4)$$

where $y_T[k] = y_k(T)$, $x_0[k] = x_k(0)$. Matrix $\Gamma \in \mathbb{R}^{p\times n}$ is therefore defined by:

$$\Gamma := Ce^{AT} \quad (5)$$

and matrix $\Psi \in \mathbb{R}^{p\times m}$ by:

$$\Psi := C \int_0^T e^{A(T-\tau)} B d\tau . \quad (6)$$

We can apply discrete-time control on system (4) that will appear like cycle-to-cycle control to the system (1).

The z-transform of (4) is:

$$\hat{y}_T(z) = \Psi \hat{u}(z) + \Gamma \hat{x}_0(z) \quad (7)$$

where $\hat{u}(z)$, $\hat{y}_T(z)$ and $\hat{x}_0(z)$ are the z-transforms of $u[k]$, $y_T[k]$ and $x_0[k]$, respectively.

The following assumptions are made in this paper:

A1) The initial state is invariant throughout all cycles. Then $x_k(0) = x_0[k] = \chi_0$ is a constant vector for all cycle. This corresponds to the assumption that all plastic sheets are at the same initial temperature before being heated.

A2) The desired terminal output is constant for all cycles k: $y_d[k] = \gamma$. Since we want to thermoform plastic sheets to obtain a desired part in a repetitive way, the desired temperature must remains constant.

The system (7) can be controlled by a first order TILC defined by:

$$u[k+1] = u[k] + K\left(y_d[k] - y_T[k] \right). \quad (8)$$

The z-transform of (8) is:

$$\hat{u}(z) = \left(z-1\right)^{-1} K\left(\hat{y}_d(z) - \hat{y}_T(z) \right) \quad (9)$$

and lead to this closed-loop system:

$$\hat{y}_T(z) = \left\{ zI_p + \left(\Psi K - I_p \right) \right\}^{-1} \left\{ \Psi K \hat{y}_d(z) + \left(z-1\right) \Gamma \hat{x}_0(z) \right\} \quad (10)$$

or

$$\hat{u}(z) = \left\{ zI_m + \left(K\Psi - I_m \right) \right\}^{-1} \left\{ K\Psi \hat{y}_d(z) - \Gamma \hat{x}_0(z) \right\} . \quad (11)$$

From the first order TILC (9) we can define:

$$C(z) = K\left(z-1\right)^{-1} . \quad (12)$$

Lemma 1: The closed loop system (10) and (11) is internally stable if and only if the following matrix is invertible [16-18]:

$$\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} . \quad (13)$$

Proof: The proof can be found in [17, 18]. □

Theorem 1: Suppose that a system represented by the full-rank matrix Ψ is controlled with the TILC algorithm expressed by (12). Then, there is at least one controller gain K that makes the system internally stable.

Proof: From Lemma 1, the system is internally stable if (13) is invertible. Using the controller structure defined in (12), (13) becomes:

$$\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} = \begin{bmatrix} I_m & K\left(z-1\right)^{-1} \\ -\Psi & I_p \end{bmatrix} . \quad (14)$$

Using properties of matrix determinants, the determinant of (14) is:

$$p(z) = \det\left(\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} \right) = \det\left(I_p + \Psi K \left(z-1\right)^{-1} \right) . (15)$$

Or, we can also write:

$$\begin{aligned} p(z) &= \left(z-1\right)^{-p} \det\left(I_p \left(z-1\right) + \Psi K \right) \\ &= \left(z-1\right)^{-p} \det\left(zI_p + \left(\Psi K - I_p \right) \right) \end{aligned} . \quad (16)$$

The roots of the equation $p(z) = 0$ defined in (16) correspond to the poles of the closed-loop system. It is possible to find at least one TILC matrix gain K such that the system is internally stable. One of the possible choices is to select matrix K as the pseudoinverse of Ψ. □

Theorem 2: Suppose matrices Ψ and K are such that the closed-loop is stable. The system converges to any desired terminal vector γ_d if and only if the rank of the system matrix Ψ, the rank of the controller K and the rank of the product ΨK are all equal to p.

Proof: Using the final value theorem on (10) leads to:

$$\begin{aligned} y_T(\infty) &= \lim_{z\to 1} \left(z-1\right) z^{-1} \left\{ zI_p + \left(\Psi K - I_p \right) \right\}^{-1} \times \\ &\quad \left\{ \Psi K \gamma_d z \left(z-1\right)^{-1} + \Gamma \chi_0 z \right\} \\ &= \left\{ \Psi K \right\}^{-1} \left\{ \Psi K \right\} \gamma_d \end{aligned} \quad (17)$$

Suppose that the rank of the two matrices Ψ and K are both equal to p. If the rank of the product ΨK is also equal to p, then the last line of (17) involve the product of a full rank matrix with its inverse. Therefore $y_T(\infty) = \gamma_d$, and convergence to γ_d is obtained.

Conversely, if the rank of the product ΨK is lower than p, then the product $\left\{ \Psi K \right\}^{-1} \left\{ \Psi K \right\} \neq I_p$ and the convergence to γ_d will not happen. □

III. Singular Value Decomposition Decoupling

In some applications, like industrial thermoforming, it may become difficult to adjust the TILC matrix gain K such that the system will have poles located at some desired locations. That is the case when the system matrix Ψ has a big size.

How can the design be made easier? This can be achieved by using the SVD decoupling approach [19] to diagonalize the matrices of the system. Once all matrices are diagonal, the internal stability analysis becomes very easy to do.

The SVD of the system matrix Ψ is expressed by:

$$\Psi = W\Sigma V^T \tag{18}$$

where $W \in \mathbb{R}^{p\times p}$ and $V \in \mathbb{R}^{m\times m}$ are unitary (orthogonal) matrices, and $\Sigma \in \mathbb{R}^{p\times m}$ is a matrix filled with 0's, except on its diagonal which contains the singular values of Ψ in descending order.

Once the SVD of Ψ is included in the system equation (7) , we can write:

$$\hat{y}_T(z) = W\Sigma V^T \hat{u}(z) + \Gamma \hat{x}_0(z)\ . \tag{19}$$

Left-multiplying both sides of (19) by the transpose of W leads to:

$$W^T \hat{y}_T(z) = \Sigma V^T \hat{u}(z) + W^T \Gamma \hat{x}_0(z)\ . \tag{20}$$

Define the new variables:

$$\hat{\eta}_T(z) = W^T \hat{y}_T(z)\ , \tag{21}$$

$$\hat{\eta}_0(z) = W^T \Gamma \hat{x}_0(z)\ , \tag{22}$$

$$\hat{v}(z) = V^T \hat{u}(z)\ . \tag{23}$$

Then, we can rewrite (20) using these new variables so as to obtain the equivalent decoupled system:

$$\hat{\eta}_T(z) = \Sigma\, \hat{v}(z) + \hat{\eta}_0(z)\ . \tag{24}$$

The desired input of this equivalent system is:

$$\hat{\eta}_d(z) = W^T \hat{y}_d(z)\ . \tag{25}$$

For the equivalent system the first-order TILC algorithm becomes:

$$\hat{v}(z) = \left(z-1\right)^{-1} K_\Sigma \left(\hat{\eta}_d(z) - \hat{\eta}_T(z)\right). \tag{26}$$

where $K_\Sigma = V^T K W$. From the TILC algorithm in (26) we can define:

$$C_\Sigma(z) = K_\Sigma \left(z-1\right)^{-1}. \tag{27}$$

The gain $K_\Sigma \in \mathbb{R}^{m\times p}$ is a matrix filled with 0's, except for its main diagonal which contains the value of the controller gain. The controller (26) with the system (24) yields the closed-loop transfer function:

$$\hat{\eta}_T(z) = \left\{zI + \left(\Sigma K_\Sigma - I\right)\right\}^{-1} \left\{\Sigma K_\Sigma \hat{\eta}_d(z) + \left(z-1\right)\hat{\eta}_0(z)\right\} \tag{28}$$

Proposition 1: The closed-loop poles of the equivalent system are the same as those of the real system.

Proof: The matrix (13) is:

$$\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} = \begin{bmatrix} I_m & V K_\Sigma W^T \left(z-1\right)^{-1} \\ -W\Sigma V^T & I_p \end{bmatrix}. \tag{29}$$

Factoring out the two unitary matrices from (29) leads to:

$$\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} = \begin{bmatrix} V & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} I_m & K_\Sigma \left(z-1\right)^{-1} \\ -\Sigma & I_p \end{bmatrix} \begin{bmatrix} V^T & 0 \\ 0 & W^T \end{bmatrix} \tag{30}$$

The determinant of a unitary matrix is equal to 1. Hence, calculating the determinant of both side of (30), we obtain:

$$\det\left(\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix}\right) = \det\left(\begin{bmatrix} I_m & K_\Sigma \left(z-1\right)^{-1} \\ -\Sigma & I_p \end{bmatrix}\right). \tag{31}$$

This equality shows the direct equivalence between the poles of the real system and the poles of the equivalent system. □

Theorem 3: Consider a system represented by a full-rank matrix Σ and the TILC algorithm in (27). There is at least one matrix K_Σ that makes the system internally stable.

Proof: From the previous proposition, the system is internally stable if (29) is invertible. Modifying (30) with the equivalent system and the equivalent controller, we can write:

$$\begin{bmatrix} I_m & C_\Sigma(z) \\ -\Sigma & I_p \end{bmatrix} = \begin{bmatrix} I_m & K_\Sigma \left(z-1\right)^{-1} \\ -\Sigma & I_p \end{bmatrix}. \tag{32}$$

From (32), we obtain:

$$\begin{aligned} p(z) &= \det\left(\begin{bmatrix} I_m & K_\Sigma \left(z-1\right)^{-1} \\ -\Sigma & I_p \end{bmatrix}\right) \\ &= \det\left(I_p + \Sigma K_\Sigma \left(z-1\right)^{-1}\right) \end{aligned} \tag{33}$$

Since all matrices in (33) are diagonal:

$$\begin{aligned} p(z) &= \left(z-1\right)^{-p} \det\left(I_p \left(z-1\right) + \Sigma K_\Sigma\right) \\ &= \left(z-1\right)^{-p} \prod_{i=1}^{p} \left(z + \sigma_i k_{\Sigma_i} - 1\right) \end{aligned} \tag{34}$$

In (34), σ_i is the *i*-th singular value of Σ and k_{Σ_i} the *i*-th gain of the matrix K_Σ. Then, from (34), on can see that the range of gains that leads to a stable-closed loop system is $0 < k_{\Sigma_i} < 2/\sigma_i$ for all *i*. □

Thus the poles are easily obtained when we use the SVD decomposition. As a result, doing a TILC design with pole placement becomes straightforward. This is because the poles of the real system are exactly at the same locations of the poles of the equivalent system (Proposition 1).

Theorem 4: Suppose matrices Σ and K_Σ are such that the system is stable. The system output will converge to any desired terminal value γ_d if and only if the product ΣK_Σ is full rank.

Proof: Applying the final value theorem to (28), one can find:

$$\begin{aligned} \eta_T(\infty) &= \lim_{z\to 1} z^{-1}\left(z-1\right)\hat{\eta}_T(z) \\ &= \left\{\Sigma K_\Sigma\right\}^{-1} \left\{\Sigma K_\Sigma\right\} W^T \gamma_d \end{aligned} \tag{35}$$

If the rank of the matrices Σ and K_Σ are equal to *p*, product ΣK_Σ will be full rank and (35) reduces to $\eta_T(\infty) = W^T \gamma_d$.

Left-multiplying both sides by W gives:

$$y_T(\infty) = \gamma_d \tag{36}$$

showing the convergence of the equivalent system's output to the desired terminal value.

If at least one of the two matrices Σ or K_Σ have a rank less than *p*, then the product ΣK_Σ will not be full rank and convergence to γ_d is not obtained. □

Theorem 5: Suppose that the matrices Σ and K_Σ have a rank equal to *p*. The system output converges in only one step (deadbeat convergence) if and only if all gains on the diagonal of K_Σ are equal to: $k_{\Sigma_i} = 1/\sigma_i$.

Proof: From (34), one can see that all poles will be at $z = 0$ if all gains of K_Σ are selected as the inverse of the singular value:

$$k_{\Sigma_i} = \frac{1}{\sigma_i}. \tag{37}$$

This case corresponds to the deadbeat convergence of the equivalent TILC controller. The convergence to the desired output happens in only one cycle. □

Using (37) for all gains of K_Σ is equivalent to define $K_\Sigma = \Sigma^+$ where "+" corresponds to the pseudoinverse operation when Σ is not square and to the inverse operation if Σ is square.

Once the design is done on the equivalent decoupled system, the TILC gain matrix K of the real system is computed from K_Σ using:

$$K = VK_\Sigma W^T. \tag{38}$$

IV. SVD WITH A SECOND ORDER CONTROL

The TILC control can have a higher order than one. Consider for instance a second-order control defined by [4]:

$$\begin{aligned} u[k+1] &= L_1 u[k] + L_2 u[k-1] \\ &+ K_1\left(y_d[k] - y_T[k]\right) + K_2\left(y_d[k-1] - y_T[k-1]\right) \end{aligned} \tag{39}$$

The z-transform of this system is expressed by:

$$\hat{u}(z) = \left(z^2 I_m - zL_1 - L_2\right)^{-1}\left(zK_1 + K_2\right)\left(\hat{y}_d(z) - \hat{y}_T(z)\right). \tag{40}$$

In (39) and (40), $K_1 \in \mathbb{R}^{m\times p}$ is the matrix gain for the error of the recently finished cycle, $K_2 \in \mathbb{R}^{m\times p}$ is the matrix gain for the error of the second-to-last cycle, $L_1 \in \mathbb{R}^{m\times m}$ is the matrix gain for the input of the recently finished cycle and $L_2 \in \mathbb{R}^{m\times m}$ is the matrix gain for the input of the second-to-last cycle.

From (40), we can define:

$$C(z) = \left(z^2 I_m - zL_1 - L_2\right)^{-1}\left(zK_1 + K_2\right). \tag{41}$$

Theorem 6: Suppose the matrix Ψ is full rank. Then, there is at least one set of matrices K_1 and K_2 that stabilizes the closed-loop system.

Proof: As in Theorem 1, the system is internally stable if (13) is invertible. Using the controller algorithm defined in (41), (13) becomes:

$$\begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} = \begin{bmatrix} I_m & \left(z^2 I_m - zL_1 - L_2\right)^{-1}\left(zK_1 + K_2\right) \\ -\Psi & I_p \end{bmatrix} \tag{42}$$

Thus, the right hand side of (42) can be factored as the product:

$$\begin{aligned} \begin{bmatrix} I_m & C(z) \\ -\Psi & I_p \end{bmatrix} &= \begin{bmatrix} \left(z^2 I_m - zL_1 - L_2\right)^{-1} & 0 \\ 0 & I_p \end{bmatrix} \times \\ &\begin{bmatrix} \left(z^2 I_m - zL_1 - L_2\right) & \left(zK_1 + K_2\right) \\ -\Psi & I_p \end{bmatrix} \end{aligned} \tag{43}$$

The closed-loop poles are given by the zeros of the determinants of (43):

$$p(z) = \det\left(\begin{bmatrix} \left(z^2 I_m - zL_1 - L_2\right)^{-1} & 0 \\ 0 & I_p \end{bmatrix} \times \begin{bmatrix} \left(z^2 I_m - zL_1 - L_2\right) & \left(zK_1 + K_2\right) \\ -\Psi & I_p \end{bmatrix}\right) \tag{44}$$

We can rewrite (44) as:

$$\begin{aligned} p(z) &= \det\left(\left(z^2 I_m - zL_1 - L_2\right)^{-1}\right) \\ &\det\left(\left(z^2 I_m - zL_1 - L_2\right) + \left(zK_1 + K_2\right)\Psi\right) \end{aligned} \tag{45}$$

Suppose we choose $K_1 = L_1\Psi^+$ and $K_2 = L_2\Psi^+$. Then (45) simplify to:

$$p(z) = \det\left(\left(z^2 I_m - zL_1 - L_2\right)^{-1}\right)\det\left(z^2 I_m\right) \tag{46}$$

and then all finite poles are located at $z = 0$. Hence, there is at least one set of gains K_1 and K_2 that stabilize the closed-loop system. □

To design this second order TILC, we need to select the four gain matrices L_1, L_2, K_1 and K_2. For a large multi-input multi-output system this task seems very difficult. However, the SVD can help in the design of a second-order TILC.

From (40), (21), (22) and (23), a second order TILC controller of the equivalent decoupled system can be defined [4]:

$$\begin{aligned} \hat{v}(z) &= V^T\left(z^2 I_m - zL_1 - L_2\right)^{-1}\left(zK_1 + K_2\right)W \times \\ &\left(\hat{\eta}_d(z) - \hat{\eta}_T(z)\right) \end{aligned} \tag{47}$$

Since $VV^T = I_m$, we can rewrite (47) as:

$$\begin{aligned} \hat{v}(z) &= V^T\left(z^2 I_m - zL_1 - L_2\right)^{-1} VV^T\left(zK_1 + K_2\right)W \times \\ &\left(\hat{\eta}_d(z) - \hat{\eta}_T(z)\right) \end{aligned} \tag{48}$$

or:

$$\begin{aligned} \hat{v}(z) &= \left(z^2 I_m - zL_{\Sigma 1} - L_{\Sigma 2}\right)^{-1}\left(zK_{\Sigma 1} + K_{\Sigma 2}\right) \times \\ &\left(\hat{\eta}_d(z) - \hat{\eta}_T(z)\right) \end{aligned} \tag{49}$$

The two gain matrices $K_{\Sigma 1}$ and $K_{\Sigma 2}$ are chosen such that they have the same shape as the transpose of the equivalent system matrix Σ, with values only on the diagonal and zeros elsewhere. The matrices of gain $L_{\Sigma 1}$ and $L_{\Sigma 2}$ are the L matrices of the equivalent system. Since we want a decoupled system, the matrices $L_{\Sigma 1}$ and $L_{\Sigma 2}$ must also be diagonal.

The closed-loop equivalent system (24) with the control in (49) is:

$$\hat{\eta}_T(z) = \left\{ I + \Sigma\left(z^2 I_m - zL_{\Sigma 1} - L_{\Sigma 2}\right)^{-1}\left(zK_{\Sigma 1} + K_{\Sigma 2}\right)\right\}^{-1} \times \left\{ \Sigma\left(z^2 I_m - zL_{\Sigma 1} - L_{\Sigma 2}\right)^{-1}\left(zK_{\Sigma 1} + K_{\Sigma 2}\right)\hat{\eta}_d(z) + \hat{\eta}_0(z)\right\} \quad (50)$$

The internal stability of the equivalent system can be shown using Theorem 6, replacing Ψ by Σ, and adding Σ to the subscripts of all controller gain. Using the same approach as Theorem 6 the resulting characteristic polynomial is:

$$p'(z) = \prod_{j=1}^{p}\left(z^2 + z\left(k_{\Sigma 1_j}\sigma_j - l_{\Sigma 1_j}\right) + \left(k_{\Sigma 2_j}\sigma_j - l_{\Sigma 2_j}\right)\right). \quad (51)$$

There exists a combination of gains $k_{\Sigma 1_j}$, $k_{\Sigma 2_j}$, $l_{\Sigma 1_j}$, and $l_{\Sigma 2_j}$ (values on the main diagonal of the corresponding gain matrices), with the value of σ_i, such that the poles are strictly inside the unit circle for all *j*, from 1 to *p*.

With the two assumptions A1 and A2, (50) becomes:

$$\hat{\eta}_T(z) = \left\{ I + \Sigma\left(z^2 I_m - zL_{\Sigma 1} - L_{\Sigma 2}\right)^{-1}\left(zK_{\Sigma 1} + K_{\Sigma 2}\right)\right\}^{-1} \times \left\{ \begin{matrix} \Sigma\left(z^2 I_m - zL_{\Sigma 1} - L_{\Sigma 2}\right)^{-1}\left(zK_{\Sigma 1} + K_{\Sigma 2}\right)z(z-1)^{-1}W^T\gamma_d \\ +z(z-1)^{-1}W^T\Gamma\chi_0 \end{matrix} \right\} \quad (52)$$

Theorem 7: Suppose all matrices in (52) are such that the system is closed-loop stable. The system output will converge to any desired terminal value γ_d, if and only if $L_{\Sigma 1} + L_{\Sigma 2} = I_m$, $L_{\Sigma 1}$ does not have an eigenvalue equal to 2, the rank of Σ is equal to *p* and the sum of matrices $K_{\Sigma 1}$ and $K_{\Sigma 2}$ has also a rank equal to *p*.

Proof: The proof is too long for this paper and can be found in [4]. □

Once the second-order TILC design is completed, we can find back the gain matrices of the real system since:

$$L_j = VL_{\Sigma j}V^T, \; j = 1,2, \quad (53)$$

$$K_j = VK_{\Sigma j}W^T, \; j = 1,2. \quad (54)$$

V. Simulation Results

This design approach was used to design a TILC controller for a simulation on a model of thermoforming oven [5, 14]. The thermoforming model corresponds to an industrial AAA thermoforming machine pictured in Figure 1. We make simulation with different size of thermoforming oven model by grouping heater banks and using some of the infrared sensors.

For the design, we linearize the thermoforming model around its operating temperature. For the simulation, the nonlinear model is used with noise and initial plastic sheet temperature variation.

The first simulation is done with a thermoforming oven model with two heaters (top and bottom) and two infrared sensors (IR_{T1} and IR_{B1}). A first order TILC is designed using SVD with all closed loop poles located at 0.25. That pole location provides fast convergence and some robustness.

The resulting gain matrix is:

$$K = \begin{bmatrix} 1.97 & -0.70 \\ -0.70 & 1.97 \end{bmatrix}. \quad (55)$$

With this gain, the system error fall under 5°C at the second cycle, as shown in Figure 2. After the error remains under 3°C despite the noise (mean 1°C, standard deviation 1°C) present in the system.

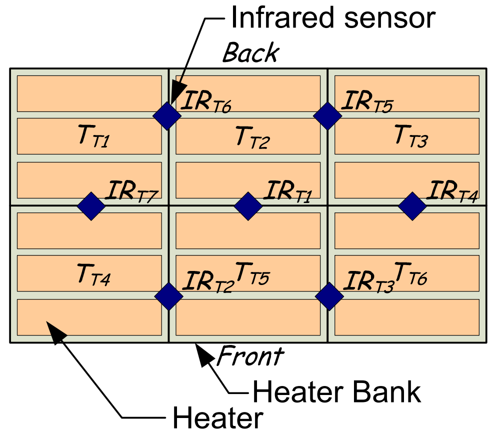

Figure 1: Location of heaters and infrared sensors (bottom heaters and sensors at the same location)

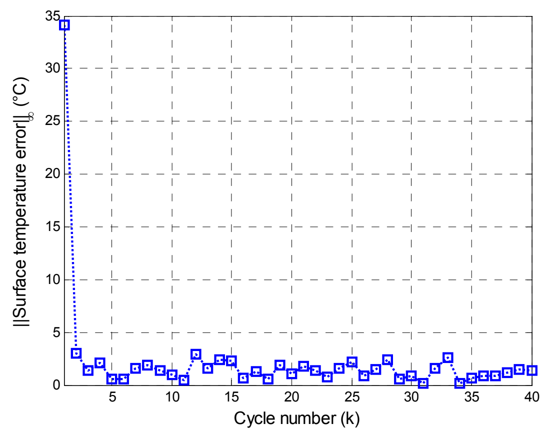

Figure 2: Error on sheet surface temperature (1st sim.)

A second simulation is done with an oven model having six heaters ($T_{T1\text{-}T4}$, $T_{T2\text{-}T5}$, $T_{T3\text{-}T6}$, $T_{B1\text{-}B4}$, $T_{B2\text{-}B5}$ and $T_{B3\text{-}B6}$) and six sensors (IR_{T1}, IR_{T2}, IR_{T5}, IR_{B1}, IR_{B2}, and IR_{B5}). The designed TILC controller is a second order one and is such that all closed loop poles are at 0.25. Figure 3 shows the convergence under 5°C in 3 cycles. After, the error remains under 4°C.

A third simulation is performed using again a second order TILC designed with the SVD approach with 5 poles at 0.125 and 5 poles at 0.25. The model used is based on a thermoforming oven with 10 heaters (All heater banks are independent except banks 2 and 5 which are grouped together) and 10 sensors (IR_{B1}, IR_{B2}, IR_{B3}, IR_{B5}, IR_{B6}, IR_{B1}, IR_{B2}, IR_{B3}, IR_{B5}, and IR_{B6}). The model used for the linearization is different from the model used from the simulation since the main parameters have been changed by 10% [4].

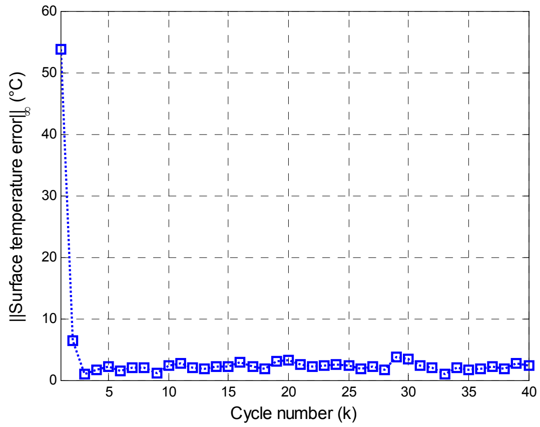

Figure 3: Error on sheet surface temperature (2nd sim.)

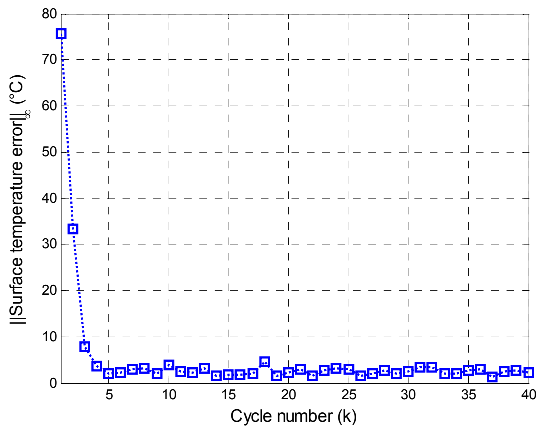

Figure 4: Error on the sheet surface temp. (3rd sim.)

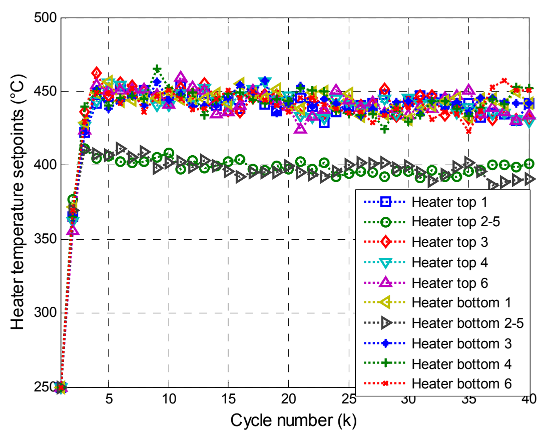

Figure 5: Heater temperature setpoints (3rd simulation)

Furthermore, the measurement is subjected to noise and there is a ±10°C slow sinusoidal variation of initial sheet temperature. Despite that, the surface temperature error falls below 5°C in 4 cycles. The sinusoidal variation of the initial sheet temperature appears on the heater temperature setpoints. Note that the TILC algorithm is able to automatically adjust the heater temperature setpoint despite the variation of sheet parameters and the slow variation of the initial sheet temperature.

VI. Conclusion

This paper has shown that one can easily design a TILC controller for a system that has a large number of inputs and outputs using SVD decoupling. Industrial thermoforming machine are very big machines and this SVD decoupling tool is needed. As future work, we need to analyze the effect of the condition number of the system matrix on the design because it can have an effect on the SVD decoupling design on big thermoforming ovens.

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