2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Active Noise Blocking: Non-Minimal Modeling, Robust Control, and Implementation

Tansel Yucelen, *Student Member, IEEE,* and Farzad Pourboghrat*, Senior Member, IEEE*

Abstract — **This paper presents a new modeling and robust control approach for active noise blocking (ANB). The proposed modeling technique is based on a new non-minimal state-space realization (NSSR) of continuous-time multiple-input multiple-output (MIMO) linear time-invariant (LTI) systems. The NSSR model generates a non-minimal set of states for a given system, using measured inputs and outputs, without differentation. From the NSSR model, using an H∞ model reduction technique, a reduced-order state space (RSS) model is derived with known states. A multi-model H∞ state-feedback (MHSF) control is then designed, in an LMI framework, for multiple RSS models of the system. This control design has an increased robustness against modeling uncertainty when different frequency responses of the system belong to a bounded convex set. Hardware experiments using a digital signal processor (DSP) have been carried out in order to verify the applicability and the performance of the proposed NSSR-based modeling and vibration control of a plate for active noise blocking (ANB) in a 3D acoustic enclosure.**

I. Introduction

Active noise control (ANC) with the aim of reducing the effects of unwanted audio signals has received a great deal of attention in the control community over the last decade. Current reaserch on low frequency audio noise reduction has mainly considered feedforward and feedback ANC methods [18], [19], [23], since the passive methods are ineffective. Feedforward ANC techniques, in general, cannot deal well with post design structural variations. On the other hand, feedback ANC schemes, developed by Olsen and May in 1953, are based on feedback control and can deal with model variations [20]. However, feedback ANC using microphones and speakers are effective only in small regions around the error microphones, known as the zones of quiet, while the noise may be increased outside these regions [15], [16]. Alternatively, successful active noise reduction can be achieved in large spaces, such as in aircraft/vehicle cabins, and coal mines, using active noise blocking (ANB) panels utilizing piezoelectric patches as sensors and actuators [19]. Here, a modeling and robust feedback control method is proposed for an aluminum panel with piezoelectric patches for active noise blocking (ANB) in a 3D acoustic enclosure.

That is, in this paper, a new non-minimal state space realization (NSSR) technique [22] is reported for modeling continuous-time multiple-input multiple-output (MIMO) linear time-invariant (LTI) systems. The states of this NSSR model can be found directly from input output measurements, without any differentiation. Moreover, a reduced-order state-space (RSS) approximation of this model can be found, also with known states, using an H∞ model reduction technique, [10], [13], [24]. The RSS model of the system is then controlled using an H∞ state feedback (HSF) control [1], [8], which can guarantee the closed-loop stability despite bounded disturbances. However, to improve robustness against model uncertainties, several estimated frequency response models of the system are simultaneously considered for H∞ control design, in an LMI framework.

Here, vibration control of a plate is considered for ANB application in a 3D acoustic enclosure. A 2-input 2-output NSSR model of this plate is found to be of 32^{nd} order, using continuous-time Kalman filter (KF) parameter estimation. Reducing the order of the NSSR model, a 4^{th} order RSS model is found, with known states, for HSF control of the ANB plate. Given various frequency responses of the system with RSS models, an MHSF control is designed for all these models, simultaneously, in LMI framework [2], [12], for ANB application with increased robustness.

This paper is organized as follows. Section 2 presents a modeling technique based on nonminimal state-space realization (NSSR), Kalman filter (KF) parameter estimation and H∞ model reduction. Section 3 presents robust control methodology in an LMI framework. Section 4 presents the experimental results. Finally, conclusions are summarized in Section 5.

II. Modeling

Given the mathematical structure of a linear time-invariant (LTI) model of a system with unknown parameters there are various identification techniques for estimating its parameters. The parameter estimation technique considered in this study is the Kalman filter (KF) method [4], [23]. The technique is applicable to both the LTI and the slowly time-varying systems and can be applied to systems that are corrupted by white noise.

A. Non-minimal state-space realization

Consider a controllable and observable MIMO system in state-space form, as

$$\dot{x} = Ax + Bu, \quad y = Cx \tag{2.1}$$

where $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $C \in \Re^{p\times n}$ are unknown constant matrices, and $x \in \Re^{n}$, $u \in \Re^{m}$, $y \in \Re^{p}$ are the state, input and output vectors, respectively. It is assumed that the state vector x is not measurable. An equivalent pn-th

T. Yucelen is with the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332 (e-mail: tansel@gatech.edu).

F. Pourboghrat is with the Department of Electrical and Computer Engineering, Southern Illinois University Carbondale, Carbondale, IL 62901-6603 USA (e-mail: pour@siu.edu).

978-1-4244-4524-0/09/$25.00 ©2009 AACC

order non-minimal observer-canonical state-space (NOSS) model of this system can be written as

$$\dot{\bar{x}} = \underbrace{\begin{bmatrix} 0 & I & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I \\ -a_0 I & -a_1 I & \cdots & -a_{n-1} I \end{bmatrix}}_{\bar{A}} \bar{x} + \underbrace{\begin{bmatrix} CB \\ CAB \\ \vdots \\ CA^{n-1}B \end{bmatrix}}_{\bar{B}} u \tag{2.2}$$
$$y = \underbrace{\begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix}}_{\bar{C}} \bar{x}$$

where a_i's are the coefficients of the characteristic polynomial of matrix A, and its state vector $\bar{x} \in \Re^{pn}$ is unknown. Using the above, we can write

$$\begin{aligned} a_0 y &= a_0 \bar{C}\bar{x} \\ a_1 \dot{y} &= a_1 \left(\bar{C}\bar{A}\bar{x} + \bar{C}\bar{B}u \right) \\ &\vdots \\ a_{n-1} y^{(n-1)} &= a_{n-1} \left(\bar{C}\bar{A}^{n-1}\bar{x} + \bar{C}\bar{A}^{n-2}\bar{B}u + \ldots + \bar{C}\bar{B}u^{(n-2)} \right) \\ y^{(n)} &= \bar{C}\bar{A}^{(n)}\bar{x} + \bar{C}\bar{A}^{n-1}\bar{B}u + \ldots + \bar{C}\bar{A}\bar{B}u^{(n-2)} + \bar{C}\bar{B}u^{(n-1)} \end{aligned} \tag{2.3}$$

Adding all the $n+1$ equations in the above, we get

$$y^{(n)} + \begin{bmatrix} a_0 I & a_1 I & \cdots & a_{n-1} I \end{bmatrix} Y = \begin{bmatrix} \bar{B}_0 & \bar{B}_1 & \ldots & \bar{B}_{n-1} \end{bmatrix} U \tag{2.4}$$

where, by Cayley-Hamilton theorem [3], we used the fact that $A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 I = 0$. Moreover,

$$\begin{aligned} \bar{B}_0 &= \bar{C}\left(a_1 I + a_2 \bar{A} + \ldots + a_{n-1}\bar{A}^{n-2} + \bar{A}^{n-1} \right)\bar{B} \\ \bar{B}_1 &= \bar{C}\left(a_2 I + a_3 \bar{A} + \ldots + a_{n-1}\bar{A}^{n-3} + \bar{A}^{n-2} \right)\bar{B} \\ &\vdots \\ \bar{B}_{n-1} &= \bar{C}\bar{B} \end{aligned} \tag{2.5}$$

$$Y = \begin{bmatrix} y^T & \dot{y}^T & \ldots & y^{(n-1)^T} \end{bmatrix}^T, \quad U = \begin{bmatrix} u^T & \dot{u}^T & \ldots & u^{(n-1)^T} \end{bmatrix}^T \tag{2.6}$$

Define $\zeta \in \Re^{(p+m)n}$ by augmenting *Y* and *U*, as

$$\zeta = \begin{bmatrix} Y^T & U^T \end{bmatrix}^T = \begin{bmatrix} y^T & \ldots & y^{(n-1)T} & u^T & \ldots & u^{(n-1)^T} \end{bmatrix}^T \tag{2.7}$$

Then, equation (2.4) in the above can be written as

$$y^{(n)} = \Theta\zeta \tag{2.8}$$

where $\Theta = \begin{bmatrix} -a_0 I & -a_1 I & \cdots & -a_{n-1} I & \bar{B}_0 & \bar{B}_1 & \cdots & \bar{B}_{n-1} \end{bmatrix} \in \Re^{p \times (p+m)n}$.

Using the above, a new (m+p)n-th order non-minimal state-space realization (NSSR) model of the original system can be written as

$$\dot{\zeta} = \underbrace{\begin{bmatrix} 0 & I & 0 & \ldots & & & & \ldots & 0 \\ \vdots & & \ddots & & & & & & \vdots \\ 0 & \ldots & 0 & I & 0 & \ldots & & \ldots & 0 \\ -a_0 I & \ldots & \ldots & -a_{n-1} I & \bar{B}_0 & \ldots & & \ldots & \bar{B}_{n-1} \\ 0 & \ldots & & \ldots & 0 & I & 0 & & 0 \\ \vdots & & & & & & \ddots & & \vdots \\ 0 & \ldots & & & & & \ldots & 0 & I \\ 0 & \ldots & & & & & & \ldots & 0 \end{bmatrix}}_{A_\zeta} \zeta + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ \\ \\ \\ \vdots \\ 0 \\ I \end{bmatrix}}_{B_\zeta} u^{(n)} \tag{2.9}$$
$$y = \underbrace{\begin{bmatrix} I & 0 & \ldots & & \ldots & 0 \end{bmatrix}}_{C_\zeta} \zeta$$

where the state vector $\zeta \in \Re^{(m+p)n}$ is defined in (2.7) and is known. However, it is not practical to differentiate measured input-output signals. To avoid differentiation and to eliminate $u^{(n)}$, using the filter $1/\Lambda(s)$, where $\Lambda(s)=(s+\lambda)^n$ is an arbitrary monic Hurwitz polynomial of degree *n*, we filter the system's inputs and outputs as

$$\zeta_f = \begin{bmatrix} Y_f^T & U_f^T \end{bmatrix}^T = \begin{bmatrix} y_f^T & \ldots & y_f^{(n-1)^T} & u_f^T & \ldots & u_f^{(n-1)^T} \end{bmatrix}^T \tag{2.10}$$

where subscript *f* denotes filtered version of the signal. Also, let us define $\bar{\lambda} = \begin{bmatrix} \lambda_0 & \ldots & \lambda_{n-1} \end{bmatrix}^T$. Then we have

$$\begin{aligned} u_f^{(n)} &= \frac{s^n}{(s+\lambda)^n} u = \frac{s^n - (s+\lambda)^n + (s+\lambda)^n}{(s+\lambda)^n} u \\ &= \left(s^n - (s+\lambda)^n \right) u_f + u = P_{n-1}(s)\, u_f + u \end{aligned} \tag{2.11}$$

where

$$P_{n-1}(s)\, u_f = -\left(\lambda_{n-1}s^{n-1} + \ldots + \lambda_1 s + \lambda_0 \right) u_f = -\bar{\lambda}^T U_f \tag{2.12}$$

Similarly, one can write

$$y_f^{(n)} = P_{n-1}(s)\, y_f + y = -\bar{\lambda}^T Y_f + y \tag{2.13}$$

Then, filtering equation (2.8), it can be written as

$$y_f^{(n)} = \Theta\zeta_f \tag{2.14}$$

Combining the above two equations, we get

$$y = y_f^{(n)} + \bar{\lambda}^T Y_f = \Theta\zeta_f + \begin{bmatrix} \bar{\lambda}^T & 0 \end{bmatrix}\zeta_f = \underbrace{\left(\Theta + \begin{bmatrix} \bar{\lambda}^T & 0 \end{bmatrix} \right)}_{\bar{C}_\zeta} \zeta_f \tag{2.15}$$

Now, filtering the state-apace equation (2.9), and using equations (2.11) and (2.15), a non-minimal state-space realization (NSSR) of the original system can be written as

$$\begin{aligned} \dot{\zeta}_f &= \bar{A}_\zeta \zeta_f + \bar{B}_\zeta u \\ y &= \bar{C}_\zeta \zeta_f \end{aligned} \tag{2.16}$$

where $\zeta_f \in \Re^{(m+p)n}$, $\bar{A}_\zeta = A_\zeta(\Theta) - \begin{bmatrix} 0, B_\zeta \bar{\lambda}^T \end{bmatrix}$, $\bar{B}_\zeta = B_\zeta$, and $\bar{C}_\zeta = \Theta + \begin{bmatrix} \bar{\lambda}^T, 0 \end{bmatrix}$ are constant parameter matrices such that $\bar{B}_\zeta$ is known, but $\bar{C}_\zeta$ and $\bar{A}_\zeta$ are in terms of the unknown parameter matrix Θ, defined in (2.5). However, unlike the original model (2.1), in this NSSR model of the system, the state vector ζ_f is completely known. Furthermore, the unknown parameter matrix Θ, and hence the matrices $\bar{A}_\zeta$ and $\bar{C}_\zeta$, can be identified from equation (2.11), using any appropriate parameter estimation technique.

B. Kalman Filter Based Modeling

This section considers the problem of estimating the unknown parameter matrix Θ, which appears in the NSSR model (2.16) of a MIMO system. To do that, note that equation (2.14) can be rewritten, as [6]

$$y_f^{(n)}(t) = \Phi_f^T(t)\,\theta \tag{2.17}$$

where the regression matrix $\Phi_f^T(t) \in \Re^{p \times pn(p+m)}$ and the parameter vector $\theta \in \Re^{pn(p+m)}$ are given as

$$\Phi_f^T(t) = \begin{bmatrix} \zeta_f^T(t) & & \\ & \ddots & \\ & & \zeta_f^T(t) \end{bmatrix} \tag{2.18}$$

$$\theta = \mathrm{vec}\left(\Theta^T \right) \tag{2.19}$$

where 'vec' operator concatenates the columns of a matrix into a vector. Noting that the unknown parameter vector θ is constant, an associated parameter dynamics (APD) can be written as

$$\begin{aligned}&\dot{\theta} = \omega \\ &y_f^{(n)}(t) = \Phi_f^T(t)\theta \ + v\end{aligned} \tag{2.20}$$

where the parameter vector θ is its state vector, $\Phi_f^T(t)$ is its time-varying output matrix, and ω and v are zero-mean random disturbances. Now, the parameter estimation of the NSSR model of the system can be converted to state estimation problem for the above APD system. Clearly, a Kalman filter (KF) technique can be used for the state estimation of the above APD system, which equivalently estimates the unknown parameters of the NSSR model of the MIMO system.

Defining $\hat{\theta}(t)$ as the estimate of the unknown state (parameter) vector θ, the estimate of the associated output $y_f^{(n)}(t)$ will be given as

$$\hat{y}_f^{(n)}(t) = \Phi_f^{\ T}(t)\hat{\theta}(t) \tag{2.21}$$

Denoting the corresponding state (parameter) estimation error as $\tilde{\theta} = \theta - \hat{\theta}$, the corresponding output estimation error (equation error) will be given as

$$e_n = y_f^{(n)}(t) - \hat{y}_f^{(n)}(t) = \Phi_f^{\ T}(t)\tilde{\theta}(t) \tag{2.22}$$

The objective of the state estimation is to minimize the cost function

$$V(\tilde{\theta}) = \lim_{t\to\infty} \mathrm{E}\left\{ \tfrac{1}{t}\int_{t_0}^{t} e_n(\tau)^T e_n(\tau) d\tau \right\} = \lim_{t\to\infty} \mathrm{E}\left\{ e_n(t)^T e_n(t) \right\} \tag{2.23}$$

A Kalman filter (KF) technique, with forgetting factor for exponential discounting of the old data, for state (parameter) estimation is given as, [22], [23],

$$\dot{\hat{\theta}}(t) = P(t)\Phi_f(t)\overline{\overline{R}}^{-1}(t)\varepsilon(t) \tag{2.24}$$

$$\dot{P}(t) = -\lambda^{-1}P(t)\Phi_f(t)\overline{\overline{R}}^{-1}(t)\Phi^T{}_f(t)P(t) + Q \tag{2.25}$$

with $P(0) = I$ and $\overline{\overline{R}}(t) = \lambda R + \Phi^T{}_f(t)P(t)\Phi_f(t)$, where $\lambda>0$ is the forgetting factor. A small λ may lead to non-robust estimation, while higher values for λ improve the convergence. Moreover, for successful implementation of the KF estimation algorithm the input and output signals should be filtered by band-pass filters before the estimation process, so as to remove both low and high frequency components of these signals outside the frequency range of interest [14].

It should be noted that the proposed NSSR model (2.16), with estimated parameters, transforms the problem of output feedback control for the MIMO system (2.1) with unknown states, to a state-feedback control with known state ζ_f. However, for practical control design, it is important that the equivalent model of the system is of low order.

C. H∞ Model Reduction

In this section, using an LMI-based H∞ model reduction, [12], the NSSR model (2.16) with known states is approximated by a reduced-order state-space (RSS) model, also with known states. Consider the $\bar{n} = n(m+p)$-th order NSSR model (2.16), which can be written, as

$$\bar{H}_\zeta(s) := \left[\begin{array}{c|c} \bar{A}_\zeta & \bar{B}_\zeta \\ \hline \bar{C}_\zeta & 0 \end{array}\right] \tag{2.26}$$

where $\bar{A}_\zeta \in \Re^{\bar{n}\times\bar{n}}$, $\bar{B}_\zeta \in \Re^{\bar{n}\times m}$, and $\bar{C}_\zeta \in \Re^{p\times\bar{n}}$. It is assumed that the minimal order model of system (2.1) is controllable, observable, and asymptotically stable. Denote the RSS model of the system as

$$H_r(s) = \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right] \tag{2.27}$$

with matrices $A_r \in \Re^{q\times q}$, $B_r \in \Re^{q\times m}$, $C_r \in \Re^{p\times q}$, and $D_r \in \Re^{p\times m}$ such that $\bar{n} \geq q \geq 1$. The above RSS model is found by solving the minimum norm problem,

$$\min_{H_r(s)} \|E(s)\|_\infty^2 < \sigma \tag{2.28}$$

where

$$E(s) := H_\zeta(s) - H_r(s) = \left[\begin{array}{cc|c} \bar{A}_\zeta & 0 & B_\zeta \\ 0 & A_r & B_r \\ \hline \bar{C}_\zeta & -C_r & -D_r \end{array}\right] \tag{2.29}$$

The above minimization problem can be solved, if there exist matrices $X > 0$ and $Z > 0$, and a scalar $\sigma > 0$, so that

$$\begin{bmatrix} \bar{A}_\zeta^{\ T}X + X\bar{A}_\zeta & C_\zeta^{\ T} \\ \bullet & -I \end{bmatrix} < 0, \ \begin{bmatrix} \bar{A}_\zeta^{\ T}Z + Z\bar{A}_\zeta & ZB_\zeta \\ \bullet & -\sigma I \end{bmatrix} < 0 \tag{2.30}$$

and that $X = Z + V\Sigma V^T$ where $\Sigma > 0$ and $V \in \Re^{\bar{n}xq}$ is to be chosen. The above minimization is equivalent to solving a convex optimization problem, as

$$\min_{\sigma,X,Z,\Sigma>0} \{\sigma : X = Z + V\Sigma V^T \text{ and } (2.30)\} \tag{2.31}$$

Then, the matrices of the RSS model can be found as, [12]

$$\begin{aligned} A_r &= \Sigma^{-1}\psi^{-1} \\ B_r &= \Sigma^{-1}\psi^{-1}V^T\{A^TZ + XA\}^{-1}XB_\zeta \\ C_r &= \bar{C}_\zeta\{A^TZ + XA\}^{-1}V\psi^{-1} \\ D_r &= -\bar{C}_\zeta\{A^TZ + XA\}^{-1}(I - V\psi^{-1}V^T\{A^TZ + XA\}^{-1})XB_\zeta \end{aligned} \tag{2.32}$$

where $\psi = V^T(A^TZ + XA)^{-1}V$. To choose V, according to [12], note that the equality appearing in (2.31) implies that $X > G_o > 0$, where $G_o \in \Re^{\bar{n}\times\bar{n}}$ is the observability grammian matrix of $H_\zeta(s)$. This, together with Schur complement applied to the second matrix inequality in (2.30), yields $\sigma G_c^{-1} > Z > G_o - V\Sigma V^T$ where $G_c \in \Re^{\bar{n}\times\bar{n}}$ is the controllability grammian of $H_\zeta(s)$. Now let $G_o = \bar{G}_o\bar{G}_o^{\ T}$ be the Cholesky factorization of the observability grammian G_o, and let $\Lambda = \bar{G}_o^{\ T}G_c\bar{G}_o > 0$, where Λ is a diagonal matrix with diagonal elements in descending order,

$$\Lambda = \begin{bmatrix} \Lambda_V & \\ & \Lambda_{\bar{V}} \end{bmatrix} \tag{2.33}$$

As in [12], V can be selected as those columns of $\bar{G}_o$, associated with the q greatest elements of Λ, which is by partitioning $\bar{G}_o = \begin{bmatrix} V & \bar{V} \end{bmatrix} \in \Re^{\bar{n}x\bar{n}}$. Then, Σ can be obtained from $X - Z = V\Sigma V^T$. Note that elements of $\bar{G}_o$ are also associated with the Hankel singular values (HSV) of $H_\zeta(s)$.

Therefore, discarding the columns of $\bar{G}_o$, associated with the $\bar{n}-q$ smallest elements of Λ, eliminates the less dominant components of $H_\zeta(s)$ and preserves the q dominant components of $H_\zeta(s)$. Moreover, the state vector x_r of the RSS model is found from the known state vector ζ_f, by discarding those elements of ζ_f associated with the $\bar{n}-q$ smallest Hankel singular values (HSV) of $H_\zeta(s)$. In other words, x_r consists of only those elements of ζ_f associated with the q largest HSVs of $H_\zeta(s)$, which are not eliminated.

III. Robust Control

In this section, a new LMI-based H∞ optimal control technique is presented for active noise blocking (ANB).

A. H∞ State-Feedback Control

The H∞ state feedback (HSF) control approach [8] is the state feedback version of the H∞ output feedback (HOF) control [22] – [24]. The method can be applied to the RSS model (2.27), or equivalently (3.1), in order to satisfy the inequality (3.2). That is,

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r u + B_w w \\ z &= C_z x_r + D_z u \end{aligned} \tag{3.1}$$

$$\|T_{zw}(s)\|_\infty < \gamma \tag{3.2}$$

where $T_{zw}(s)$ is the transfer function from the bounded disturbance w to performance output z, γ is the desired performance bound, A_r and B_r are the matrices of the RSS model, and x_r is the state vector, which is known. Also, for the ANB system, $B_w = B_r$, $D_z D_z^T = I$, and C_z is arbitrary. The solution is found by minimizing the cost function

$$J(x_r, u, w) = \int_0^\infty (z^T z - \gamma^2 w^T w) dt \tag{3.3}$$

where one needs to find a symmetric positive-definite matrix P from the LMI (3.4) to obtain the control (3.5) [2], [21],

$$\begin{bmatrix} A_r^T P + P A_r + C_z^T C_z & P B_r & P B_w \\ \bullet & I & 0 \\ \bullet & \bullet & -\gamma^{-2} I \end{bmatrix} < 0 \tag{3.4}$$

$$u = -K x_r,\ K = B_r P \tag{3.5}$$

where $\bullet$ denotes the corresponding parts of the symmetric matrix. This is equivalent to linear dynamic game [11], as

$$U(x_r(0)) = \min_u \max_{w,\, x_r(0)} J(x_r, u, w) < \infty \tag{3.6}$$

where, U is the upper value function. When the stabilizing control (3.5) exists, the inequality (3.2) will be satisfied.

B. Increased Robustness Against Uncertainty

Given the structure (3.1) of the RSS model of a system, one may find various independent estimates of its parameters, which could slightly differ from each other. Figure 3.1 shows that various frequency responses of a system belong to a convex set with an upper and lower bound frequency responses (vertices). Here, an LMI-based HSF control is introduced to guarantee robust performance for various system models that belong to such a convex set.

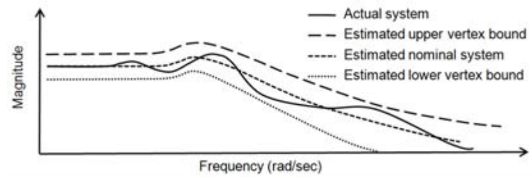

Figure 3.1. Estimated nominal frequency response of a system with upper and lower bound vertices

Consider N frequency response-based models of the system, including the upper and lower bound models (vertices), as

$$\begin{aligned} \dot{x}_{r,i} &= A_{r,i} x_{r,i} + B_{r,i} u + B_{w,i} w \\ z &= C_z x_{r,i} + D_z u \end{aligned} \tag{3.7}$$

where $i = 1, 2, \cdots, N$. Now, for HSF control design, one needs to find P and K for all N models, simultaneously. For this purpose, consider the Schur complement of (3.4), as

$$\begin{aligned} A_{r,i}^T P + P A_{r,i} - P B_{r,i} K - K^T B_{r,i}^T P + P B_{r,i} B_{r,i}^T P \\ + \gamma^{-2} P B_{w,i} B_{w,i}^T P + C_z^T C_z < 0 \end{aligned} \tag{3.8}$$

Let $\bar{P} = P^{-1}$, multiply both sides of the above equation by $\bar{P}$, and define $\bar{K} = K\bar{P}$. Then we get

$$\begin{aligned} \bar{P} A_{r,i}^T + A_{r,i} \bar{P} - B_{r,i} \bar{K} - \bar{K}^T B_{r,i}^T + B_{r,i} B_{r,i}^T \\ + \gamma^{-2} B_{w,i} B_{w,i}^T + \bar{P} C_z^T C_z \bar{P} < 0 \end{aligned} \tag{3.9}$$

Equivalently, using Schur complement [2], [7], we get

$$\begin{bmatrix} \bar{P} A_{r,i}^T + A_{r,i} \bar{P} - B_{r,i} \bar{K} - \bar{K}^T B_{r,i}^T + B_{r,i} B_{r,i}^T + \gamma^{-2} B_{w,i} B_{w,i}^T & \bar{P} C_z^T \\ \bullet & -I \end{bmatrix} < 0 \tag{3.10}$$

which is linear in $\bar{P}$ and $\bar{K}$ and can be solved to find $\bar{P}$ and $\bar{K}$. The closed-loop system $\dot{x}_r = (A_{r,i} - B_{r,i} K) x_r + B_{w,i} w$ will be quadratically stable if and only if (3.10) is feasible for $\bar{P} = \bar{P}^T > 0$, $\gamma > 0$, and $\bar{K}$, for all $i = 1, 2, \cdots, N$. Then, the MHSF control gain is given as

$$K = \bar{K}\, \bar{P}^{-1} \tag{3.11}$$

The above stabilizing MHFS control, guarantees that $\|T_{zw,i}(s)\|_\infty < \gamma$ for all system models (see [2]).

IV. Experiments

The proposed modeling and control strategy was implemented for vibration control of an ANB panel, using dSPACE ds1104 DSP processor, with Matlab/Simulink support. The DSP sampling rate was chosen to be 10kHz, and the proposed approach was realized in real-time. The experimental apparatus is a 120-cm long, 60-cm wide, 60-cm high rectangular Plexiglas 3D acoustic enclosure with an ANB panel assembled in the middle. The panel is made of foam and aluminum sheets. The foam is used to reduce mid and high frequency noise propogation. The aluminum sheet, with embedded piezoelectric patches, is used to reduce low frequency noise propogation, using the proposed control strategy. The experimental setup in Figure 4.1 shows the locations of the piezoelectric patches on the aluminum plate.

Figure 4.1. Experimental ANB Setup

A. Modeling and Model Reduction

Here, a 2-input 2-output arrangement of the ANB panel was considered. Kalman filter (KF) parameter estimation was applied to find the NSSR model of the system. The sampling time was t_s=0.0001sec and the forgetting factor was $\lambda = 0.01$. A first order Butterworth band-pass filter with low and high cut-off frequencies of w_l=0.05 rad/sec and w_h=1000 rad/sec was used for filtering the system's 2-input 2-output signals. The covariance matrices were $Q=10^{-3}I$, $R=1.250I$, and the initial parameter estimates were zero. A noise signal was added to the input to enhance the system excitation for KF process. The estimated NOSS model of the ANB panel was of 16th order and its NSSR model was of 32nd order.

Experimental estimates of the system's nominal model, using both the KF and an off-line frequency response technique, are shown in Figure 4.2. Both the KF and the off-line frequency response techniques produced equivalent transfer matrices. Applying the H$_\infty$ model reduction to the NSSR model, a 4th order RSS model of the ANB system was obtained with known states. The reduction error bound was $\sigma = 1.75$, and $\min_{H_r(s)\in\Upsilon} \left\| H_\zeta(s) - H_r(s) \right\|_\infty^2$ became 1.57, which satisfies (2.28) and shows the applicability of the H$_\infty$ technique for order reduction of the NSSR model in practice.

Figure 4.3 shows various frequency responses of the system that were collected at different times, independently. From this figure, the actual ANB panel model varies between two extreme upper and lower bound frequency responses. The equivalent RSS models of these frequency responses were used in the proposed LMI-MHSF control design, in order to simultaneously stabilize all these models.

B. Robust H∞ Control

Experimental implementation of the proposed LMI-MHSF control was considered for multiple RSS models of the system, using DSP. Other controls were also applied to the nominal model for comarison. Figure 4.4 shows the closed-loop frequency responses of the 16th order nominal NOSS model of the system with an LMI-based HOF control, and the 32nd order NSSR model of the system with LMI-based HSF control. From this figure, the LMI-HSF control, designed for the 32nd order NSSR model of the system, achieved better vibration reduction. This shows the advantage of knowing the system's states in a real-time implementation.

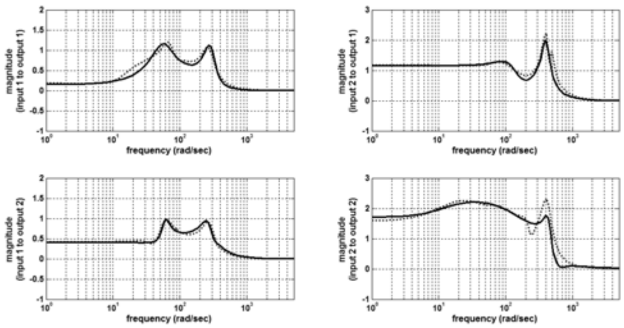

Figure 4.2. Frequency response of estimated nominal system model using Kalman filter (solid line), off-line sine response (dotted line)

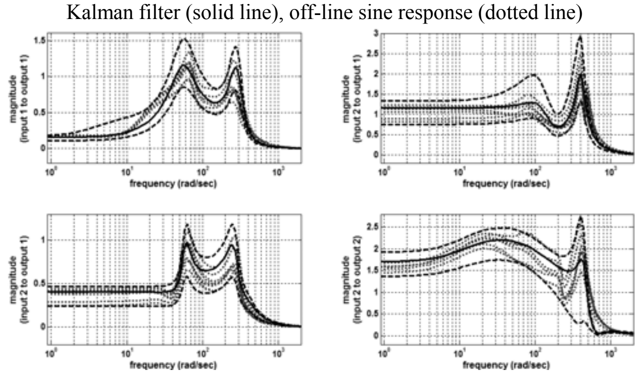

Figure 4.3. Frequency response of KF-based estimate of nominal system model (solid line), various independent models (dotted lines), lower bound and upper bound models (dashed lines)

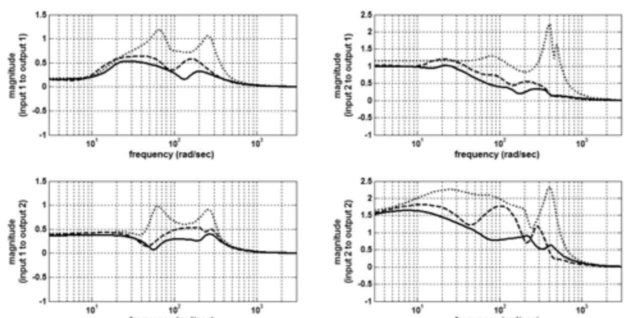

Figure 4.4. Frequency response of open-loop (dotted line), closed-loop with 32nd order NSSR model and LMI-HOF control (dashed line), closed-loop with 32nd order NSSR model with LMI-HSF control (solid line)

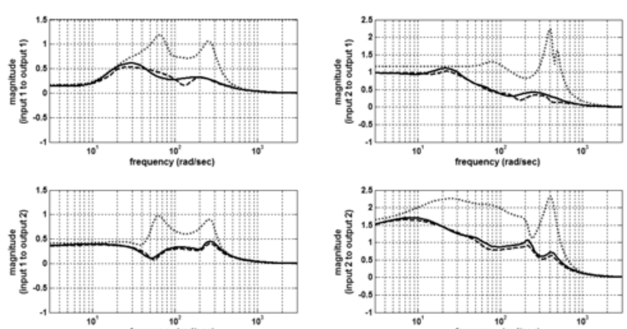

Figure 4.5. Frequency response of open-loop (dotted line), closed-loop with 32nd order NSSR model and LMI-HSF control (dashed line), closed-loop with 4th order nominal RSS model and LMI-HSF control (solid line)

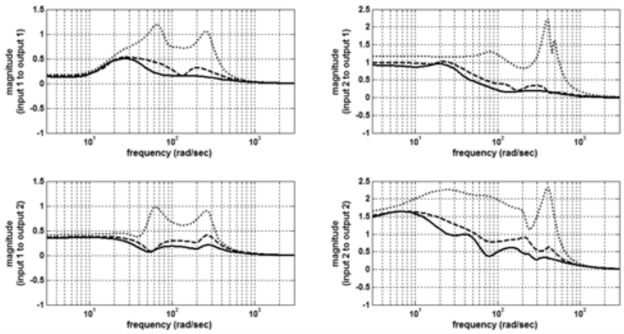

Figure 4.6. Frequency response for open-loop (dotted line), closed-loop with 32nd order NSSR model and LMI-HSF control (dashed line), closed-loop with multiple 4th order RSS models (nominal, lower and upper) and LMI-MHSF control (solid line)

The frequency responses of the closed-loop system with LMI-based H_∞ state feedback (HSF) control, designed for the 32nd order NSSR model, and for the 4th order RSS model of the system, are shown in Figure 4.5. Clearly, both 4th order RSS and 32nd order NSSR models give similar results. However, the HSF control for the 4th order RSS model is economically more appropriate for implementation.

In Figure 4.6, the closed-loop frequency responses of the system are shown for LMI-based HSF control designed for the nominal 32nd order NSSR model, and for the proposed LMI-based MHSF control designed for multiple 4th order RSS models of the system (nominal, upper and lower bound models). Clearly, the proposed LMI-based MHSF control resulted in a better noise reduction over a reasonable bandwidth. This proves the efficacy of the proposed robust control law, in real-time applications.

V. Conclusion

A new modeling and robust control technique was developed for active noise blocking (ANB). The modeling technique is based on a novel non-minimal state-space realization (NSSR) of continuous-time MIMO LTI systems, Kalman filter parameter estimation, and H_∞ model reduction. The algorithm produces an equivalent reduced-order state space (RSS) model of the system, with known states using input-output (I/O) measurements. The control design uses an LMI-based H_∞ approach to generate a multiple-model H_∞ state feedback (MHSF) control. The LMI-MHSF control simultaneously stabilizes multiple models of the system in a convex set. Laboratory experiments using HSF control for NSSR model and MHSF control for multiple RSS models of the system gave similar results. However, the proposed LMI-MHSF control achieved better noise reduction over a reasonable bandwidth. Therefore, the presented theory and the implementation results were compatible.

VI. Acknowledgement

Partial support of this research by the Illinois Department of Commerce and Economic Opportunity (DCEO) through Illinois Clean Coal Institute (ICCI) is gratefully acknowledged.

References

[1] M. R. Bai, H. H. Lin, "Comparison of active noise control structures in the presence of acoustical feedback by using the H_∞ synthesis technique", Journal of Sound and Vibration, Vol.206, pp.453-471, 1997.

[2] S. Boyd, L. Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM Studies in Applied Mathematics, 1994.

[3] M. Braun, Differential equations and their applications: An introduction to applied mathematics, 4th ed., Springer-Verlag, 1993.

[4] L. Cao, H. M. Schwartz, "Exponential converge of the Kalman filter based parameter estimation algorithm", Int'l J. of Adaptive Control and Signal Processing, Vol. 17, pp.763-783, 2003.

[5] L. Cao, H. M. Schwartz, "Analysis of the Kalman filter based estimation algorithm: An orthogonal decomposition approach", Automatica, Vo. 40, pp.5-19, 2004.

[6] L. Cirka, M. Fikar, "Identification tool for Simulink", Department of Process Control, FCFT SUT tech. report KAMF9803, 1998.

[7] S. Da Silva, V. L. Junior, M. J. Brennan, "Design of a control system using linear matrix inequalities for the active vibration control of a plate", J. of Intelligent Material Systems & Structures, Vol. 17, 2006.

[8] J. C. Doyle, K. Glover, P. P. Khargonekar, B. A. Francis, "State-Space Solutions to Standard H_2 and H_∞ control problems", IEEE Trans. On Automatic Control, Vol.38, No.8, pp.831-847, 1989.

[9] G. A. Dumont, M. Huzmezan, "Concepts, methods and techniques in adaptive control", IEEE Proc. of American Control Conference, pp.1137-1150, 2002.

[10] Y. Ebihara, T. Hagiwara, "On H_∞ Model Reduction using LMIs", IEEE Trans. on Automatic Control, Vol.49, No.7, pp.1187-1191, 2004.

[11] K. Ezal, P. V. Kokotovic, A. R. Teel, T. Basar, "Disturbance attenuating output-feedback control of nonlinear systems with local optimality", Automatica, Vol. 37, pp.805-817, 2001.

[12] J. C. Geromel, R. G. Egas, F. R. R. Kawaoka, "H_∞ Model Reduction with Application to Flexible Systems", IEEE Trans. on Automatic Control, Vol.50, No.3, pp.402-406, 2005.

[13] K. M. Grigoriadis, "Optimal H_∞ Model Reduction via Linear Matrix Inequalities: Continuous- and Discrete-Time Cases", Systems & Control Letters, Vol.26, pp.321-333, 1995.

[14] T. Hagglund, K. J. Astrom, "Supervision of adaptive control algorithms", Automatica, Vol. 36, pp.1171-1180, 2000.

[15] J. Hong, J. C. Akers, R. Venugopal, M .N. Lee, A. G. Sparks, P. D. Washabaugh, D.S. Bernstein, "Modeling, identification, and feedback control of noise in an acoustic duct", IEEE Trans. Contr. Syst. Technol., Vol.4, pp.283-291, 1996.

[16] A. J. Hull, C. L. Radcliffe, S. C. Southward, "Global active noise control of a one-dimensional acoustic duct using a feedback controller", Proc. ASME Winter Annual Meeting, Vol.91, WA-DSC-10, 1991.

[17] P. Ioannou, B. Fidan, Adaptive control tutorial, Society for Industrial and Applied Mathematics, SIAM Books, 2006.

[18] S. M. Kuo, D. R. Morgan, "Active Noise Control: A Tutorial Review", IEEE Proceedings, Vol.87, No.6, 1999.

[19] J. K. Lee, J. Kim, C. J. Rhee, C. H. Jo, S. B. Choi, "Noise Reduction of Passive and Active Hybrid Panels", Smart Materials and Structures, Vol. 11, pp.940-946, 2002.

[20] H. F. Olsen, E. G.May, "Electronic sound absorber", J. Acoustic Society of America, Vol.25, pp.1130-1136, 1953.

[21] C. Scherer, P. Gahinet, M. Chilali, "Multiobjective output-feedback control via LMI optimization", IEEE Trans. on Automatic Control. Vol.42, No.7, 1996.

[22] T. Yucelen, F. Pourboghrat, "Adaptive H_∞ Optimal Control Strategy Based on Nonminimal State Space Realization", ASME Int'l Mechanical Engineering Congress and Exposition, IMECE'07, 2007.

[23] T. Yucelen, F. Pourboghrat, "Kalman Filter Based Modeling and Constrained H_∞ Optimal Control for Active Noise Cancellation", IEEE Proc. of Control and Decision Conference, 2007.

[24] K. Zhou, J. C. Doyle, Essentials of Robust Control, Upper Saddle River, NJ: Prentice Hall, 1998.